

**IMAGING WITH  
DIFFRACTION TOMOGRAPHY**

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**by**

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**This work is dedicated to  
my family  
and friends.**

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## PREFACE

We have not succeeded in solving all of your problems. The answers we have found only serve to raise a whole new set of questions. In some ways we are as confused as ever, but we feel that we are confused on a higher level and about more important things.

-- Gary S. Peterson

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## ABSTRACT

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This work reviews the theory and limits of first order diffraction tomography and studies iterative techniques that can be used to improve the quality of tomographic imaging with diffracting sources. Conventional (straight-ray) tomographic algorithms are not valid when used with acoustic or microwave energy. Thus more sophisticated algorithms are needed.

First order diffraction tomography uses a linearized version of the wave equation and gives an especially simple reconstruction algorithm. This work reviews first order approximations to the scattered field and studies the quality of the reconstructions when the assumptions behind these approximations are violated. It will be shown that the Born approximation is valid when the phase change across the object is less than  $\pi$  and the Rytov approximation is valid when the refractive index changes by less than two or three percent.

Better reconstructions will be based on higher order approximations to the scattered field. This work describes two fixed point algorithms (the Born and the Rytov approximations) and an algebraic approach to more accurately calculate the scattered fields. The limits of each of these approaches is discussed and simulated results are shown.

Finally a review of higher order inversion techniques is presented. Each of these techniques is reviewed and some of their limitations are discussed.