

**Principles of
Computerized
Tomographic
Imaging**

OTHER IEEE PRESS BOOKS

Radar Applications, *Edited by M. I. Skolnik*
Robust Control, *Edited by P. Dorato*
Writing Reports to Get Results: Guidelines for the Computer Age, *By R. S. Blicq*
Multi-Microprocessors, *Edited by A. Gupta*
Advanced Microprocessors, II, *Edited by A. Gupta*
Adaptive Signal Processing, *Edited by L. H. Sibul*
System Design for Human Interaction, *Edited by A. P. Sage*
Advances in Local Area Networks, *Edited by K. Kümmerle, J. O. Limb, and F. A. Tobagi*
Computers and Manufacturing Productivity, *Edited by R. K. Jurgen*
Being the Boss, *By L. K. Lineback*
Effective Meetings for Busy People, *By W. T. Carnes*
VLSI Signal Processing, II, *Edited by S. Y. Kung, R. E. Owen, and J. G. Nash*
Modern Acoustical Imaging, *Edited by H. Lee and G. Wade*
Low-Temperature Electronics, *Edited by R. K. Kirschman*
Undersea Lightwave Communications, *Edited by P. K. Runge and P. R. Trischitta*
Multidimensional Digital Signal Processing, *Edited by the IEEE Multidimensional Signal Processing Committee*
Adaptive Methods for Control System Design, *Edited by M. M. Gupta*
Residue Number System Arithmetic, *Edited by M. A. Soderstrand, W. K. Jenkins, G. A. Jullien, and F. J. Taylor*
Singular Perturbations in Systems and Control, *Edited by P. V. Kokotovic and H. K. Khalil*
Space Science and Applications, *Edited by J. H. McElroy*
Medical Applications of Microwave Imaging, *Edited by L. Larsen and J. H. Jacobi*
Modern Spectrum Analysis, II, *Edited by S. B. Kesler*
The Calculus Tutoring Book, *By C. Ash and R. Ash*
Imaging Technology, *Edited by H. Lee and G. Wade*
Kalman Filtering: Theory and Application, *Edited by H. W. Sorenson*
Biological Effects of Electromagnetic Radiation, *Edited by J. M. Osepchuk*
Engineering Contributions to Biophysical Electrocardiography, *Edited by T. C. Pilkington and R. Plonsey*

Principles of Computerized Tomographic Imaging

Avinash C. Kak

*School of Electrical Engineering
Purdue University*

Malcolm Slaney

Schlumberger Palo Alto Research

Electronic Copy (c) 1999 by A. C. Kak and Malcolm Slaney
Copies can be made for personal use only.



The Institute of Electrical and Electronics Engineers, Inc., New York

IEEE PRESS
1987 Editorial Board
R. F. Cotellessa, *Editor in Chief*
J. K. Aggarwal, *Editor, Selected Reprint Series*
Glen Wade, *Editor, Special Issue Series*

James Aylor	J. F. Hayes	A. C. Schell
F. S. Barnes	W. K. Jenkins	L. G. Shaw
J. E. Brittain	A. E. Joel, Jr.	M. I. Skolnik
B. D. Carrol	Shlomo Karni	P. W. Smith
Aileen Cavanagh	R. W. Lucky	M. A. Soderstrand
D. G. Childers	R. G. Meyer	M. E. Van Valkenburg
H. W. Colborn	Seinosuke Narita	John Zaborsky
	J. D. Ryder	

W. R. Crone, *Managing Editor*
Hans P. Leander, *Technical Editor*
Laura J. Kelly, *Administrative Assistant*

Randi E. Scholnick and David G. Boulanger, *Associate Editors*

This book is set in Times Roman.

Copyright © 1988 by
THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.
345 East 47th Street, New York, NY 10017-2394
All rights reserved.

PRINTED IN THE UNITED STATES OF AMERICA

IEEE Order Number: PC02071

Library of Congress Cataloging-in-Publication Data

Kak, Avinash C.

Principles of computerized tomographic imaging.

“Published under the sponsorship of the IEEE Engineering in Medicine and Biology Society.”

Includes bibliographies and index.

1. Tomography. I. Slaney, Malcolm. II. IEEE Engineering in Medicine and Biology Society. III. Title.

RC78.7.T6K35 1987 616.07'572 87-22645

ISBN 0-87942-198-3

Contents

<i>Preface</i>	ix
1 Introduction	1
References	3
2 Signal Processing Fundamentals	5
2.1 One-Dimensional Signal Processing	5
Continuous and Discrete One-Dimensional Functions • Linear Operations • Fourier Representation • Discrete Fourier Transform (DFT) • Finite Fourier Transform • Just How Much Data Is Needed? • Interpretation of the FFT Output • How to Increase the Display Resolution in the Frequency Domain • How to Deal with Data Defined for Negative Time • How to Increase Frequency Domain Display Resolution of Signals Defined for Negative Time • Data Truncation Effects	
2.2 Image Processing	28
Point Sources and Delta Functions • Linear Shift Invariant Operations • Fourier Analysis • Properties of Fourier Transforms • The Two-Dimensional Finite Fourier Transform • Numerical Implementation of the Two-Dimensional FFT	
2.3 References	47
3 Algorithms for Reconstruction with Nondiffracting Sources	49
3.1 Line Integrals and Projections	49
3.2 The Fourier Slice Theorem	56
3.3 Reconstruction Algorithms for Parallel Projections	60
The Idea • Theory • Computer Implementation of the Algorithm	
3.4 Reconstruction from Fan Projections	75
Equiangular Rays • Equally Spaced Collinear Detectors • A Re-sorting Algorithm	
3.5 Fan Beam Reconstruction from a Limited Number of Views	93
3.6 Three-Dimensional Reconstructions	99
Three-Dimensional Projections • Three-Dimensional Filtered Backprojection	

3.7	Bibliographic Notes	107	
3.8	References	110	
4	<i>Measurement of Projection Data—The Nondiffracting Case</i>		113
4.1	X-Ray Tomography	114	
	Monochromatic X-Ray Projections • Measurement of Projection Data with Polychromatic Sources • Polychromaticity Artifacts in X-Ray CT • Scatter • Different Methods for Scanning • Applications		
4.2	Emission Computed Tomography	134	
	Single Photon Emission Tomography • Attenuation Compensation for Single Photon Emission CT • Positron Emission Tomography • Attenuation Compensation for Positron Tomography		
4.3	Ultrasonic Computed Tomography	147	
	Fundamental Considerations • Ultrasonic Refractive Index Tomography • Ultrasonic Attenuation Tomography • Applications		
4.4	Magnetic Resonance Imaging	158	
4.5	Bibliographic Notes	168	
4.6	References	169	
5	<i>Aliasing Artifacts and Noise in CT Images</i>		177
5.1	Aliasing Artifacts	177	
	What Does Aliasing Look Like? • Sampling in a Real System		
5.2	Noise in Reconstructed Images	190	
	The Continuous Case • The Discrete Case		
5.3	Bibliographic Notes	200	
5.4	References	200	
6	<i>Tomographic Imaging with Diffracting Sources</i>		203
6.1	Diffracted Projections	204	
	Homogeneous Wave Equation • Inhomogeneous Wave Equation		
6.2	Approximations to the Wave Equation	211	
	The First Born Approximation • The First Rytov Approximation		
6.3	The Fourier Diffraction Theorem	218	
	Decomposing the Green's Function • Fourier Transform Approach • Short Wavelength Limit of the Fourier Diffraction Theorem • The Data Collection Process		
6.4	Interpolation and a Filtered Backpropagation Algorithm for Diffracting Sources	234	
	Frequency Domain Interpolation • Backpropagation Algorithms		

6.5	Limitations	247	
	Mathematical Limitations • Evaluation of the Born Approximation • Evaluation of the Rytov Approximation • Comparison of the Born and Rytov Approximations		
6.6	Evaluation of Reconstruction Algorithms	252	
6.7	Experimental Limitations	261	
	Evanescent Waves • Sampling the Received Wave • The Effects of a Finite Receiver Length • Evaluation of the Experimental Effects • Optimization • Limited Views		
6.8	Bibliographic Notes	268	
6.9	References	270	
7	<i>Algebraic Reconstruction Algorithms</i>		275
7.1	Image and Projection Representation	276	
7.2	ART (Algebraic Reconstruction Techniques)	283	
7.3	SIRT (Simultaneous Iterative Reconstructive Technique)	284	
7.4	SART (Simultaneous Algebraic Reconstruction Technique)	285	
	Modeling the Forward Projection Process • Implementation of the Reconstruction Algorithm		
7.5	Bibliographic Notes	292	
7.6	References	295	
8	<i>Reflection Tomography</i>		297
8.1	Introduction	297	
8.2	B-Scan Imaging	298	
8.3	Reflection Tomography	303	
	Plane Wave Reflection Transducers • Reflection Tomography vs. Diffraction Tomography • Reflection Tomography Limits		
8.4	Reflection Tomography with Point Transmitter/Receivers	313	
	Reconstruction Algorithms • Experimental Results		
8.5	Bibliographic Notes	321	
8.6	References	321	
	<i>Index</i>		323
	<i>About the Authors</i>		329

Preface

The purpose of this book is to provide a tutorial overview on the subject of computerized tomographic imaging. We expect the book to be useful for practicing engineers and scientists for gaining an understanding of what can and cannot be done with tomographic imaging. Toward this end, we have tried to strike a balance among purely algorithmic issues, topics dealing with how to generate data for reconstruction in different domains, and artifacts inherent to different data collection strategies.

Our hope is that the style of presentation used will also make the book useful for a beginning graduate course on the subject. The desired prerequisites for taking such a course will depend upon the aims of the instructor. If the instructor wishes to teach the course primarily at a theoretical level, with not much emphasis on computer implementations of the reconstruction algorithms, the book is mostly self-contained for graduate students in engineering, the sciences, and mathematics. On the other hand, if the instructor wishes to impart proficiency in the implementations, it would be desirable for the students to have had some prior experience with writing computer programs for digital signal or image processing. The introductory material we have included in Chapter 2 should help the reader review the relevant practical details in digital signal and image processing. There are no homework problems in the book, the reason being that in our own lecturing on the subject, we have tended to emphasize the implementation aspects and, therefore, the homework has consisted of writing computer programs for reconstruction algorithms.

The lists of references by no means constitute a complete bibliography on the subject. Basically, we have included those references that we have found useful in our own research over the years. Whenever possible, we have referenced books and review articles to provide the reader with entry points for more exhaustive literature citations. Except in isolated cases, we have not made any attempts to establish historical priorities. No value judgments should be implied by our including or excluding a particular work.

Many of our friends and colleagues deserve much credit for helping bring this book to fruition. This book draws heavily from research done at Purdue by our past and present colleagues and collaborators: Carl Crawford, Mani Azimi, David Nahamoo, Anders Andersen, S. X. Pan, Kris Dines, and Barry Roberts. A number of people, Carl Crawford, Rich Kulawiec, Gary S. Peterson, and the anonymous reviewers, helped us proofread the manuscript;

we are grateful for the errors they caught and we acknowledge that any errors that remain are our own fault. We are also grateful to Carl Crawford and Kevin King at GE Medical Systems Division, Greg Kirk at Resonex, Dennis Parker at the University of Utah, and Kris Dines of XDATA, for sharing their knowledge with us about many newly emerging aspects of medical imaging.

Our editor, Randi Scholnick, at the IEEE PRESS was most patient with us; her critical eye did much to improve the quality of this work.

Sharon Katz, technical illustrator for the School of Electrical Engineering at Purdue University, was absolutely wonderful. She produced most of the illustrations in this book and always did it with the utmost professionalism and a smile. Also, Pat Kerkhoff (Purdue), and Tammy Duarte, Amy Atkinson, and Robin Wallace (SPAR) provided excellent secretarial support, even in the face of deadlines and garbled instructions.

Finally, one of the authors (M.S.) would like to acknowledge the support of his friend Kris Meade during the long time it took to finish this project.

AVINASH C. KAK
MALCOLM SLANEY