

**Coupling of integral acoustics methods with LES for jet noise prediction**

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**ABSTRACT**

This study is focused on developing a Computational Aeroacoustics (CAA) methodology that couples the near field unsteady flow field data computed by a 3-D Large Eddy Simulation (LES) code with various integral acoustic formulations for the far field noise prediction of turbulent jets. The LES code employs state-of-the-art numerical schemes and a localized version of the dynamic Smagorinsky subgrid-scale (SGS) model. The code also has the capability to turn off the SGS model and treat the spatial filter that is needed for numerical stability as an implicit SGS model. Noise computations performed for a Mach 0.9, Reynolds number 400,000 jet using various integral acoustic results are presented and the results are compared against each other as well with those from experiments at similar flow conditions. Our results show that the surface integral acoustics methods (Kirchhoff and Ffowcs Williams - Hawkings) give similar results to the volume integral method (Lighthill’s acoustic analogy) at a much lower cost. To the best of our knowledge, Lighthill’s acoustic analogy is applied to a Reynolds number 400,000 jet at Mach 0.9 for the first time in this study. The distribution of Lighthill sources that radiate noise in the direction of various observer locations is evaluated. A source decomposition shows significant cancellations among the individual components of the Lighthill source.

**NOMENCLATURE**

**Roman Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$C_I$</td>
<td>Compressibility correction constant in the subgrid-scale model</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$D_j$</td>
<td>Jet nozzle diameter</td>
</tr>
</tbody>
</table>
Coupling of integral acoustics methods with LES for jet noise prediction

\( f \) Arbitrary variable; frequency
\( \bar{f} \) Large scale component of variable \( f \)
\( f_{sg} \) Subgrid-scale component of variable \( f \)

\( G(\bar{x}, \bar{x}', \Delta) \) Filter function
\( M_r \) Reference Mach number
\( p \) Pressure
\( Pr_t \) Turbulent Prandtl number
\( Re_D \) Reynolds number based on jet diameter
\( r_o \) Jet nozzle radius
\( r \) Radial direction in cylindrical coordinates
\( St \) Strouhal number
\( t \) Time
\( T_{ij} \) Lighthill stress tensor
\( T_{ij}^m \) Mean component of \( T_{ij} \)
\( T_{ij}^l \) Component of \( T_{ij} \) linear in velocity fluctuations
\( T_{ij}^n \) Component of \( T_{ij} \) nonlinear in velocity fluctuations
\( T_{ij}^s \) Entropy component of \( T_{ij} \)
\( (u, v, w) \) Velocity vector in Cartesian coordinates
\( U_o \) Jet centerline velocity at nozzle exit
\( u_i \) Alternate notation for \((u, v, w)\)
\( V \) Integration volume
\( (x, y, z) \) Cartesian coordinates (\( x \) is the streamwise direction.)
\( x, \text{ or } x \) Alternate notation for \((x, y, z)\)

Greek Symbols
\( \alpha_f \) Filtering parameter of the tri-diagonal filter
\( \Delta \) Local grid spacing or eddy viscosity length scale
\( \delta_{ij} \) Kronecker delta
\( \nabla \) Divergence operator
\( \gamma \) Ratio of the specific heats of air
\( \rho \) Density
\( \tau \) Retarded time
\( \theta \) Angle from downstream jet axis
\( \varphi \) Azimuthal angle on the observer circle

Other Symbols
\( (\cdot) \) Mean quantity
\( (\cdot)' \) Spatially filtered quantity
\( (\cdot)' \) Perturbation from mean value; acoustic variable
\( (\cdot)_o \) Ambient flow value
\( (\cdot)_o \) Flow value at jet centerline on the nozzle exit
\( \langle \cdot \rangle \) Time averaging operator
1. INTRODUCTION
Jet noise remains one of the most complicated and difficult problems in aeroacoustics because the details of the noise generation mechanisms caused by the complex turbulence in a jet are still not well understood. Thus, there is a need for more research that will lead to improved jet noise prediction methodologies and further understanding of the jet noise generation mechanisms. Such advances will eventually aid in the design process of aircraft engines with low jet noise emissions.

With the recent improvements in the processing speed of computers, the application of Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) to jet noise prediction methodologies is becoming more feasible. Although there have been numerous experimental studies of jet noise to date, the experiments provide only a limited amount of information and the ultimate understanding of jet noise generation mechanisms will most likely be possible through numerical simulations, since the computations can literally provide any type of information needed for the analysis of jet noise generation mechanisms.

In a DNS, all the relevant scales of turbulence are directly resolved and no turbulence modeling is used. One of the first DNS of a turbulent jet was performed for a Reynolds number 2,000 jet at Mach 1.92 by Freund et al.\textsuperscript{1} The computed overall sound pressure levels were compared with experimental data and found to be in good agreement with jets at similar convective Mach numbers. Freund\textsuperscript{2} also simulated a Reynolds number 3,600, Mach 0.9 turbulent jet, matching the parameters of the experimental jet studied by Stromberg et al.\textsuperscript{3} Excellent agreement with the experimental data was obtained for both the mean flow field and the radiated sound. Such results clearly show the attractiveness of DNS to the jet noise problem. However, due to the wide range of length and time scales present in turbulent flows requiring fine meshes, DNS is still restricted to low-Reynolds-number flows and relatively simple geometries. DNS of high-Reynolds-number jet flows of practical interest would necessitate tremendous resolution requirements that are far beyond the capability of even the fastest supercomputers available today.
Therefore, turbulence still has to be modeled in some way to perform simulations for problems of practical interest. LES, with lower computational cost, is an attractive alternative to DNS. In an LES, the large scales are directly resolved and the effect of the small scales or the subgrid-scales on the large scales are modeled. The large scales are generally much more energetic than the small ones and are directly affected by the boundary conditions. The small scales are usually much weaker and they tend to have more or less a universal character. Hence, it makes sense to directly simulate the more energetic large scales and model the effect of the small scales. LES methods are capable of simulating flows at higher Reynolds numbers and many successful LES computations for different types of flows have been performed to date. Since noise generation is an inherently unsteady process, LES will probably be the most powerful computational tool to be used in jet noise research in the foreseeable future since it is the only way, other than DNS, to obtain time-accurate unsteady data computationally.

Although the application of Reynolds Averaged Navier Stokes (RANS) methods to jet noise prediction is also a subject of ongoing research, RANS methods heavily rely on turbulence models to model all the relevant scales of turbulence. Moreover, such methods try to predict the noise using the mean flow properties provided by a RANS solver. Since noise generation is a multi-scale problem that involves a wide range of length and time scales, it appears the success of RANS-based prediction methods will remain limited to specific cases for which semi-empirical noise models exist, unless very good turbulence and aeroacoustic models capable of accurately modeling a wide range of turbulence scales are developed. One specific case in which RANS-based methods have been particularly successful is the axisymmetric subsonic jet noise. However, LES seems to be a more appropriate, albeit more computationally expensive, choice for future studies of more complicated problems such as the noise of jets issuing from nozzle configurations with tabs, chevrons, or a lobed mixer. With the rapid advances taking place in supercomputing technology, such challenging simulations are likely to be performed in a decade or so from the time of this writing. The present LES studies of free jet flows can be considered as a first step before the more advanced LES of jets issuing from complicated nozzle or mixer geometries.

In noise calculations, one can separate the computation into two parts. In the first part, the nonlinear generation of sound is computed using techniques such as DNS or LES. In the second part, the sound propagation to the far field is calculated. There are several methodologies for describing the sound propagation once the source has been identified, including using the traditional acoustic analogy or Linearized Euler Equations (LEE). However, the most promising methods appear to be Kirchhoff’s method and the porous Ffowcs Williams - Hawking method. The non-porous version of the Ffowcs Williams - Hawking formulation assumes a non-porous integration surface coincident with a solid body surface, whereas the porous version allows surface integrals to be performed on a permeable integration surface that is not necessarily coincident with a solid body surface. Di Francescantonio demonstrated the use of the Ffowcs Williams - Hawking approach on a fictitious surface that does not correspond to a physical body. Both Kirchhoff’s and the Ffows Williams - Hawkins methods are hybrid and consist of the numerical calculation of the nonlinear near field
with the far field solutions computed from an integral formulation with all quantities evaluated on a control surface. The control surface is assumed to enclose all the nonlinear flow effects and noise sources. These approaches seem to have an advantage over the traditional acoustic analogy approach, which requires volume integrals. Pilon and Lyrantzius\cite{Pilon1991} as well as Brentner and Farassat\cite{Brentner1992} showed that the Ffowcs Williams – Hawkings formulation is equivalent to the Kirchhoff formulation when the integration surface is located in the linear wave propagation region. If the integration surface is located in the nonlinear flow region, some type of refraction correction\cite{Stevens1995,Tracy1996} is needed.

There have been numerous studies on the application of LES with or without integral acoustics methods to jet noise prediction. References\cite{Kawahara1993,Stevens1995,Tracy1996,Kawahara1997,Choi1998,DeBonis1999a,DeBonis1999b,DeBonis1999c,DeBonis1999d,DeBonis1999e,DeBonis1999f,DeBonis1999g,DeBonis1999h} provide the details of some of those studies. The recent jet noise computations of Bogey and Bailly\cite{Bogey2000,Bogey2001,Bogey2002} are perhaps the most successful LES calculations performed for reasonably high Reynolds number jets at the time of this writing. In this study, we will repeat a test case studied by Bogey and Bailly\cite{Bogey2000,Bogey2001} and make comparisons with their results as part of the validation of our CAA methodology. Bogey and Bailly\cite{Bogey2000,Bogey2001,Bogey2002} compute the noise in the immediate surroundings of the jet using direct LES data only, whereas in this work we employ integral acoustics methods to compute the noise both in the near and far field. Use of integral acoustics methods allows us to compute the noise at any observer point outside the nonlinear flow region. We use computational grids that are comparable in size to those used by Bogey and Bailly\cite{Bogey2000,Bogey2001,Bogey2002}. However, we pack most of the grid points inside the nonlinear jet flow and do relatively rapid grid stretching outside. Then, placing a control surface quite close to the jet flow allows us to obtain higher cutoff frequencies in the noise calculations.

In general, the LES results in the literature to date are encouraging and show the potential promise of LES applied to jet noise prediction. Except for the studies of Choi et al.,\cite{Choi1998} DeBonis and Scott\cite{DeBonis1999a} and DeBonis\cite{DeBonis1999b} which are not well-resolved LES calculations, and the recent studies of Bogey and Bailly\cite{Bogey2000,Bogey2001,Bogey2002} the highest Reynolds numbers reached in the LES simulations so far are still below those of practical interest. The cutoff frequency of the noise spectra in the LES computations is dictated by the grid resolution, hence only a portion of the noise spectra was computed in the LES computations to date. Well-resolved LES calculations of jets at higher Reynolds numbers close enough to practical values of interest would be very helpful for evaluating the suitability of LES to such problems, as well as for analyzing the broadband noise spectrum, and possibly looking into the mechanisms of jet noise generation at such Reynolds numbers.

2. OBJECTIVES OF THE PRESENT STUDY

With this motivation behind the current study given, we now present the two main objectives of this research:

1. Development and validation of a versatile 3-D LES code for turbulent jet simulations.
2. Accurate prediction of the far field noise.

The compressible LES code and the integral acoustics codes developed in this study form the core of a CAA methodology for jet noise prediction. Using these tools,
we attempt well-resolved LES computations for jet flows with Reynolds numbers as high as 400,000. As will be shown in the results section, the cutoff Strouhal number in the subsequent noise computations is as high as 4.0. Such a cutoff frequency appears to be greater than the cutoff frequencies captured in all other jet noise LES computations for similar Reynolds numbers available in the literature to date.

3. GOVERNING EQUATIONS FOR LES

Large Eddy Simulation (LES) can be thought of as a compromise between Direct Numerical Simulation (DNS) and solving the Reynolds Averaged Navier-Stokes (RANS) equations. In a DNS, all the relevant scales of turbulence have to be directly computed, whereas in a RANS calculation, all the relevant scales of turbulence need to be modeled. In an LES, the flowfield is decomposed into a large-scale or resolved-scale component ($f$) and a small-scale or subgrid-scale component ($f_{sg}$).

\[ f = \tilde{f} + f_{sg}. \]  

The large-scale component is obtained by filtering the entire domain using a grid filter function, $G$, as follows

\[ \tilde{f}(\vec{x}) = \int_V G(\vec{x}, \vec{x}', \Delta) f(\vec{x}') \, d\vec{x}'. \]  

The filtering operation removes the small-scale or subgrid-scale turbulence from the Navier-Stokes equations. The resulting governing equations are then solved directly for the large scale turbulent motions, while the effect of subgrid-scales is computed using a subgrid scale model, such as the classical Smagorinsky model\cite{Ger} or the more sophisticated dynamic Smagorinsky model proposed by Germano et al.\cite{Ger} and extended to compressible flows by Moin et al.\cite{Moin}

For problems involving complex geometries, the form of the governing equations in generalized curvilinear coordinates should be used. Extension of the equations from Cartesian coordinates to generalized curvilinear coordinates can be found in Rizzetta et al.\cite{Rizzetta} and in Uzun.\cite{Uzun} The Favre-filtered unsteady, compressible, non-dimensionalized Navier-Stokes equations formulated in generalized curvilinear coordinates are solved in this study using the numerical methods described in the next section. Formulation of the current single-block code in generalized curvilinear coordinates makes the code suitable for extension into a multi-block version which can be used for LES calculations in complicated geometries.

4. NUMERICAL METHODS

We first transform a given non-uniformly spaced curvilinear computational grid in physical space to a uniform grid in computational space and solve the discretized governing equations on the uniform grid. To compute the spatial derivatives at interior grid points away from the boundaries, we employ the non-dissipative sixth-order
compact scheme developed by Lele.\textsuperscript{51} For points on the boundaries, we apply a third-order one-sided compact scheme, and for the points next to the boundaries, we use a fourth-order central compact formulation. It is known from previous experience, e.g. the work of Koutsavdis,\textsuperscript{52} that numerical instabilities usually arise if high-order compact boundary and near-boundary closures are coupled with high-order compact interior schemes. Because of this issue, we employ low-order compact boundary and near-boundary closures (third-order on the boundary, and fourth-order on the point next to the boundary) to ensure stability during the computations. We make use of flow field information sufficiently away from the domain boundaries while computing the jet noise in this work and, hence, the use of low-order near-boundary compact schemes is not expected to have a significant effect on the noise prediction results.

The viscous stresses appearing in the governing equations are obtained by computing the first derivatives of the velocity. For computational efficiency, the terms in the inviscid and viscous fluxes are first added together and then the total flux is differentiated to compute the right hand side of the governing equations. As a consequence of such an implementation, the second-derivatives in the viscous terms are essentially evaluated by applying the first-derivative operator twice. Although this approach is not as accurate as when a compact scheme is used to directly compute the second-derivatives in the viscous terms, it is much cheaper to implement in curvilinear coordinates. The difference between the two approaches mainly occurs in the small scales, which are filtered as noted below. Also, only the viscous stress terms are adversely affected from such an implementation. However, as Lighthill pointed out in his pioneering work,\textsuperscript{10} the contribution of the viscous stresses to the quadrupole source term in his acoustic analogy is unimportant and, hence, the aforementioned loss of accuracy in the viscous stress terms is not expected to have an effect on the sound field of the jet.

Spatial filtering can be used as a means of suppressing unwanted numerical instabilities that can arise from the boundary conditions, unresolved scales and mesh non-uniformities.\textsuperscript{53} In our study, we employ a sixth-order tri-diagonal filter used by Visbal and Gaitonde.\textsuperscript{54}

For the localized dynamic subgrid-scale model, a fifteen-point explicit filter developed by Bogey and Bailly\textsuperscript{55} is used as the test-filter. The ratio of the test-filter width to the grid spacing is taken as 2. Details of the localized dynamic Smagorinsky model can be found in Uzun.\textsuperscript{50} The code also has the capability to turn off the localized dynamic Smagorinsky SGS model and treat the spatial filter as an implicit SGS model. The LES code which dynamically computes the Smagorinsky constant requires approximately 50\% more computing time relative to the LES code which treats the spatial filter as an implicit SGS model. Dynamic evaluation of the compressibility correction coefficient and the turbulent Prandtl number require test filtering of some additional quantities. To keep the additional cost of the dynamic model at an acceptable level, the compressibility correction coefficient, \( C_f \) and the turbulent Prandtl number, \( Pr_t \) are not computed dynamically in this study and are set to constant values instead. The turbulent Prandtl number is typically set to the same value as the laminar Prandtl number (usually 0.7), whereas the compressibility correction coefficient can be safely
set to zero for the jet Mach number considered in this study. A fixed Prandtl number may not be appropriate for a heated jet, but we do not consider heated jets in this work.

The standard fourth-order explicit Runge-Kutta scheme is used for time advancement. We apply Tam and Dong’s 3-D radiation and outflow boundary conditions\(^\text{56}\) on the boundaries of the computational domain as illustrated in figure 1. The original two-dimensional boundary conditions of Tam and Dong\(^\text{57}\) were extended to 3-D by Bogey and Bailly.\(^\text{56}\) We additionally use the sponge zone method\(^\text{58}\) in which grid stretching and artificial damping are applied to dissipate the vortices present in the flowfield before they hit the outflow boundary. This way, unwanted reflections from the outflow boundary are suppressed.

Since the actual nozzle geometry is not included in the present calculations, randomized perturbations in the form of a vortex ring are added to the velocity profile at a short distance downstream of the inflow boundary in order to excite the 3-D instabilities in the jet and cause the potential core of the jet to break up at a reasonable distance downstream of the inflow boundary. This forcing procedure has been adapted from Bogey \textit{et al.}\(^\text{59}\). More detailed information about the numerical methods employed in the LES code can be found in Uzun.\(^\text{50}\)

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**Figure 1.** Schematic of the boundary conditions. Vorticity magnitude contours are shown within the jet while divergence of velocity contours are shown in the outer part of the computational domain.
Far field noise computations are performed by coupling the time accurate near field data provided by the LES code with surface integral acoustics methods, such as Kirchhoff’s method and the Flowcs Williams-Hawkings method. A description of these methods as well as their relative merits can be found in the review paper by Lyrintzis. Lighthill’s acoustic analogy, which is discussed next, is also used in the noise computations.

5. Lighthill’s Acoustic Analogy

Lighthill’s equation can be written as

\[
\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho'}{\partial x_i \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j},
\]

where the Lighthill stress tensor, \( T_{ij} \) is given as

\[
T_{ij} = \rho u_i u_j + (p - c_\infty^2 \rho) \delta_{ij},
\]

with the viscous stress term neglected. In Lighthill’s equation, all effects other than propagation in a homogeneous stationary medium are lumped into the source term on the right hand side. It should be stated here that the right hand side of the above equation is by no means a true or unique representation of the acoustic sources in the turbulent flow. The double divergence of \( T_{ij} \) only serves as a nominal acoustic source built with information from the turbulent flow.

In Lighthill’s acoustic analogy, the sound generated by a turbulent flow is equivalent to what the quadrupole distribution \( T_{ij} \) would emit if placed in a uniform acoustic medium at rest. In other words, in this analogy the quadrupole source distribution replaces the actual fluid flow and moreover, the sources may move, but the fluid in which they are embedded may not. The sources are embedded in a medium at rest that has constant properties \( \rho_\infty, p_\infty \) and \( c_\infty \), which are the same as those in the ambient fluid external to the flow.

Lighthill shows that the following equation can be used to approximate the pressure fluctuations at points far enough from the flow region, i.e. at a distance that is large compared to \( (2\pi)^{-1} \) times a typical acoustic wavelength,

\[
p - p_\infty = (\rho - \rho_\infty)c_\infty^2 \int \frac{\left(\frac{x_i - y_i}{|x - y|^2}\right)\left(\frac{x_j - y_j}{|x - y|^2}\right)\frac{\partial^2}{\partial t^2} T_{ij}(y, t - \frac{|x - y|}{c_\infty})}{4\pi|x - y|^2} dy.
\]

For example, \( (2\pi)^{-1} \) times the wavelength of the sound wave whose frequency corresponds to Strouhal number 0.1 is about three jet radii. For higher frequencies, this
“typical distance” will be smaller. The above time derivative formulation of Lighthill’s volume integral is used later in this paper for computing the far field noise at a distance of 60 jet radii, which is substantially greater than the “typical distances” associated with the wavelengths of various frequencies. The volume integral is performed over the entire turbulent flow region, which is the source. The time derivative formulation of Lighthill’s volume integral seems to be less computationally intensive than the double divergence formulation, and hence, it is our preferred formulation.

Following Freund, we can split the Lighthill stress tensor, \( T_{ij} \), into a mean component, \( T_{ij}^m \), a component that is linear in velocity fluctuations, \( T_{ij}^l \), a component that is quadratic in velocity fluctuations, \( T_{ij}^q \) and the so-called entropy component, \( T_{ij}^s \), as follows

\[
T_{ij} = T_{ij}^m + T_{ij}^l + T_{ij}^q + T_{ij}^s, 
\]

where

\[
T_{ij}^m = \rho \bar{u}_i \bar{u}_j + (\bar{\rho} - c_w^2 \bar{\rho}) \delta_{ij}, 
\]

\[
T_{ij}^l = \rho \bar{u}_i \bar{u}_j + \rho u'_i u'_j, 
\]

\[
T_{ij}^q = \rho u'_i u'_j, 
\]

\[
T_{ij}^s = (p' - c_w^2 \rho') \delta_{ij}. 
\]

By definition, the mean component \( T_{ij}^m \) does not make noise. In the above equations, density in the \( \rho u_i u_j \) terms has not been decomposed into a mean and a fluctuating part. Freund shows that the noise from \( T_{ij} \) is nearly the same as that from

\[
T_{ij}^\bar{\rho} = \bar{\rho} u_i u_j + (p' - c_w^2 \rho') \delta_{ij}. 
\]

Hence, the effect of the density fluctuations in the \( \rho u_i u_j \) terms is not considered in this study. The noise from \( T_{ij}^l \) is called the shear noise since this component consists of turbulent fluctuations interacting with the sheared mean flow. On the other hand, the noise from \( T_{ij}^q \) is called the self noise since this component consists of turbulent fluctuations interacting with themselves, whereas the noise from \( T_{ij}^s \) is the so-called entropy noise. Lilley shows that the \( T_{ij}^q \) term contains both isentropic and non-isentropic components. This source decomposition allows evaluation of the noise from the different components of the Lighthill source term and is useful for studying the correlation among the noise of the different components. As will be shown later in the results section, the components are highly correlated and hence, such a source decomposition may not be that useful after all. Freund provides references to previous works in which the same source decomposition has been coupled with \( k - \varepsilon \) turbulence models for jet noise prediction.
6. PREDICTION OF THE NOISE RADIATED BY A MACH 0.9 ROUND JET AT A REYNOLDS NUMBER OF 400,000

The initial simulations performed for relatively low Reynolds number jets using the constant-coefficient Smagorinsky model can be found in Uzun et al., whereas the results of a Reynolds number 100,000 jet LES performed using the localized dynamic SGS model are given in Uzun et al.

An LES was performed without any explicit SGS model for a turbulent isothermal round jet at a Mach number of 0.9 and Reynolds number of 400,000. The tri-diagonal spatial filter was treated as an implicit SGS model. The filtering parameter was set to $\alpha_t = 0.47$. The jet centerline temperature was chosen to be the same as the ambient temperature and set to $286K$. A fully curvilinear grid consisting of approximately 16 million grid points was used in the simulation. The grid had 390 points along the streamwise $x$ direction and 200 points along both the $y$ and $z$ directions. Figure 2 shows a cross-section of the grid on the $x-y$ plane. Initially, the grid points are clustered around the shear layer of the jet in order to accurately resolve the shear layer. After the potential core of the jet breaks up (at around $x = 13r_o$), the grid is stretched in the $y$ and $z$ directions in order to redistribute the grid points and achieve a more uniform distribution. This test case corresponds to one of the test cases studied by Bogey and Bailly. The physical portion of the domain in this simulation extends to 35 jet radii in the streamwise direction and $\pm 15$ jet radii in the transverse directions.

Figure 2. The $x-y$ cross-section of the grid. (Every 4th grid node is shown for clarity.)
resolution in this simulation is estimated to be about 400 times the local Kolmogorov length scale. The following mean streamwise velocity profile is initially specified on the inflow boundary

\[ \bar{u}(r) = \frac{U_o}{2} \left[ 1 + \tanh \left( 10 \left( 1 - \frac{r}{r_o} \right) \right) \right]. \quad (12) \]

There are about 14 grid points in the initial jet shear layer. The following Crocco-Buseman relation for an isothermal jet is initially specified for the density profile on the inflow boundary\(^\text{39}\)

\[ \bar{\rho}(r) = \rho \left( 1 + \frac{\gamma - 1}{2} M_r^2 \frac{\bar{u}(r)}{U_o} \left( 1 - \frac{\bar{u}(r)}{U_o} \right) \right)^{-1}, \quad (13) \]

where \( M_r = 0.9 \). Application of Tam and Dong’s radiation boundary conditions on the inflow boundary allows outgoing acoustic waves to exit the domain through this boundary without reflection back into the domain. Small perturbations which are observed on the mean velocity and density profiles on the inflow boundary as the outgoing acoustic waves cross this boundary are expected and automatically determined by the boundary conditions.

In a recent study, Bogey and Bailly\(^\text{40}\) took a close look at the effects of the inflow conditions on the jet flow and noise. They used a total of 16 azimuthal modes in their vortex ring forcing. They found that reducing the amplitude of the initial disturbances results in an increase in radiated noise. Also, a thinner initial jet shear layer thickness was observed to cause increased noise levels in the sideline direction and reduced noise levels in the downstream direction. The most important changes were obtained when the first 4 azimuthal modes of forcing were removed. In this case, the noise levels were noticeably reduced. They concluded that further improvements are still needed to reduce the sideline noise levels which are overestimated with respect to experimental data. The reader is referred to Bogey and Bailly\(^\text{40}\) for more detailed information. Based on their findings, we decided to use the inflow forcing in which the first 4 modes of the 16 total azimuthal modes are excluded. This forcing was found by Bogey and Bailly\(^\text{40}\) to match the available experimental data best. The forcing parameter \( \alpha \) is set to 0.007.

A more detailed study of the inflow forcing can be found in Lew \( et \ al.\text{67} \) In addition, in a similar study, Kim and Choi\(^\text{68}\) recently investigated the effect of the initial momentum thickness on the acoustic source and the far field sound from an incompressible round jet. They found that the acoustic source distribution as well as the characteristics of the far field sound propagation depend on the initial momentum thickness. With increasing initial momentum thickness, the shear layer was found to become unstable at a further downstream location. Moreover, they observed that the dominant acoustic sources are located in the shear layer when the initial momentum thickness is small, but they are located downstream of the potential core in the case of a thick initial momentum thickness.
The computations were performed on an IBM-SP3 machine using 200 processors in parallel. A total of about 5.5 days (132 hours) of computing time was needed. The initial transients exited the domain over the first 10,000 time steps. The time history of the unsteady flow data inside the jet was saved at every 10 time steps over a period of 40,000 time steps. These data are used in the volume integrals later in this paper where the far field noise of the jet is computed using Lighthill’s acoustic analogy. The sampling period corresponds to a time scale in which an ambient sound wave travels about 23 times the domain length in the streamwise direction. The Strouhal number resolution of this sampling is about 0.00278.

The LES code used in this study was previously validated for a Reynolds number 100,000 jet test case. The mean flow parameters as well as Reynolds stresses and mean streamwise velocity profiles in the far downstream region predicted by LES in that test case compared very well with existing experimental measurements. The aerodynamic results for the present Reynolds number 400,000 jet test case can be found in Uzun. The one-dimensional spectra of the streamwise velocity fluctuations at various locations inside the jet exhibited Kolmogorov’s –5/3 decay rate before the grid cutoff wavenumber, meaning that a portion of the inertial range of turbulence is resolved in the simulation. The reader is referred to Uzun for more details on the aerodynamic results.

6.1. Aeroacoustic calculations using surface integral acoustics methods

As part of the aeroacoustic analysis, we investigated the sensitivity of far field noise predictions to the position of the control surface on which aeroacoustic data are collected. For this purpose, we put 3 control surfaces around our jet as illustrated in figure 3. This figure plots the divergence of velocity (i.e. dilatation) contours on a plane that cuts the jet in half. A way from the turbulent mixing region (i.e. in the linear wave propagation region), the divergence of velocity is directly related to the time derivative of pressure as follows

\[ \nabla \cdot u = -\frac{1}{\rho_\infty c_\infty^2} \frac{\partial p}{\partial r}. \]  

The divergence of velocity contours show the acoustic waves in the linear wave propagation region nicely. By examining these contours, one can visually identify the nonlinear sound generation region inside the jet flow as well as the linear acoustic wave propagation outside the jet (see Figure 3). In other words, the divergence of velocity contours give qualitative information that can be used to distinguish the sound generation region from the acoustic propagation region, which is generally linear but might be slightly nonlinear near the jet. The control surfaces start 1 jet radius downstream of the inflow boundary and extend to 35 jet radii along the streamwise direction. Hence the total streamwise length of the control surfaces is 34\(r_0\). Control surfaces 1, 2 and 3 are initially at a distance of 3.9, 5.9 and 8.1 jet radii from the jet centerline, respectively. They open up to a distance of 9.1, 10.8 and 12.2 jet radii from the jet centerline, respectively, at the far downstream location of 35\(r_0\). In a study where
Freund computed the noise of a low Reynolds number jet, he quoted that a minimum radial location of 8 radii is necessary for the control surface. However, in that study, Freund used a cylindrical grid and hence, his control surface was a cylindrical shell with a constant radius. In such a scenario, the minimum radial location of 8 radii is then necessary in order to keep the jet flow from hitting the control surface in the downstream region since the jet keeps opening up with streamwise distance after the potential core breaks up. One can certainly place the control surface at a distance that is closer than 8 jet radii in the initial portion of the jet flow and then adjust the control surface distance properly after the jet potential core breaks up. Use of curvilinear grids allows such flexibility. Going back to the details of the aeroacoustic calculations, we gathered flow field data on the control surfaces at every 5 time steps over a period of 25,000 time steps during our LES run. The total acoustic sampling period corresponds to a time scale in which an ambient sound wave travels about 14 times the domain length in the streamwise direction. The Strouhal number resolution of this sampling is about 0.00444. Both the FWH and Kirchhoff’s methods were employed in the far field noise predictions. It should be noted here that the time history gathered on the control surface is longer than that used to compute the acoustic pressure signals at the observer locations because of the need to account for the retarded time.

For simplicity, we decided to use open control surfaces in the first part of this study. Flow data needed by Kirchhoff’s and the FWH methods were gathered on the lateral
surfaces surrounding the jet only. The far field acoustic pressure signals were calculated at 36 equally spaced azimuthal points on a full circle at a given \( \theta \) location on a far field arc. See figures 4 and 5 for the location of the observer points. The radius of the arc is equal to 60 jet radii and its center is chosen as the centerline of the jet nozzle exit as shown in figure 4. \( \theta \) values on the arc range from 25\(^\circ\) to 90\(^\circ\) with an increment of 5\(^\circ\). Figure 5 shows the observer circle for a given \( \theta \) location. \( \phi \) is the azimuthal angle and the 36 observer points on this circle are distributed uniformly with 10\(^\circ\) intervals.

The run time of the Kirchhoff code is almost the same as that of the FWH code. For control surface 1, the computation of the 4096-point time history of acoustic pressure at a given far field location with each method took about 6 minutes of computing time using 136 processors in parallel on an IBM-SP3. Hence, for control surface 1, both methods needed a total of 50 hours to compute the acoustic pressure history at a total

![Figure 4. Schematic showing the center of the arc and how the angle \( \theta \) is measured from the jet axis. Contours shown are for the same variables as those in figure 1.](image-url)
of 504 far field points. Calculations on control surface 2 and 3 using 136 processors in parallel required 59 and 65 hours total, respectively.

Figure 6 shows the acoustic pressure spectra at the 60° observation location on the far field arc. The time signals were broken into 4 1024-point signals which were then used in the Fast Fourier Transforms (FFTs) to get the acoustic pressure spectra. This was done in order to smooth the spectra a bit. We then averaged the acoustic pressure spectra over the equally spaced 36 azimuthal points to obtain the final averaged spectrum at the given observer location. It should be noted here that a 4096-point FFT gives finer frequency resolution but not a higher resolved frequency, which is set by the grid resolution (as discussed earlier in the text). However, use of 1024-point FFTs allows 4 times as many samples to average the spectra over. (Note that there is a total
of 144 (= 36 × 4) 1024-point FFTs for each observer point on the arc.) Hence, the resulting averaged spectrum at a given arc location becomes much smoother when 1024-points FFTs are used. The averaged spectra are then integrated to compute the OASPL. The OASPL computed from a relatively smooth spectrum is expected to contain less error than that computed from a “less smooth” or “more noisy” spectrum. It should be also mentioned here that since the final averaged spectra are still a bit “noisy”, the spectra that are shown are polynomial fits to the actual computed spectra. The polynomial fits are useful for avoiding ambiguity while identifying the peak of the spectra. Figure 7 plots a typical “noisy” acoustic pressure spectrum and the polynomial fit to it. The maximum deviation of the noisy spectrum from the polynomial fit is less than 2 dB/St. Although not shown here for the sake of brevity, for a given control surface, the FWH and Kirchhoff’s methods were found to give almost identical results. This means that all of the control surfaces chosen in this study were indeed sufficiently far away from the jet (i.e. in the linear acoustic field) such that they did not encounter any nonlinear sound generating regions. Hence, the Kirchhoff’s method prediction was just as good as the FWH method prediction for all control surfaces.

Figure 6. The Ffowcs Williams - Hawkings method prediction for the acoustic pressure spectra at $R = 60r_o$, $\theta = 60^\circ$ location on the far field arc.
As will be evident shortly, at least 6 points per wavelength are needed to accurately resolve an acoustic wave using the 6th-order compact difference scheme implemented in the LES code. Therefore, we see that the maximum frequency resolved with our grid spacing around the control surfaces corresponds to a Strouhal number of approximately 3.0, 2.0 and 1.5 for control surface 1, 2 and 3, respectively. It should be noted that the grid spacing on the control surfaces is not uniform, however, we use the coarsest grid spacing on every control surface to compute the maximum Strouhal number resolved by each control surface. The Nyquist frequency based on the temporal sampling interval corresponds to a Strouhal number of about 1.11. However, we choose the maximum frequency that is based on spatial resolution as our cutoff frequency. Since grid stretching was employed in the computational grid used in this simulation, the grid spacing gets coarser towards the outer domain boundaries. This means that grid spacing around control surface 1 is finer than that around control surface 2 and similarly the grid spacing around control surface 2 is finer than that around control surface 3. Hence, the maximum frequency that can be captured by a control surface decreases as one puts the

Figure 7. A typical “noisy” acoustic pressure spectrum and the polynomial fit to it.
control surface further away from the jet because of the grid stretching. The spectra comparison given in figure 6 shows solid vertical lines corresponding to the cutoff frequency for every control surface. Until Strouhal number 1.5, the spectra predicted by the three control surfaces are seen to be very similar. Then, the spectrum predicted by control surface 3 starts to drop sharply due to the fact that the grid spacing around control surface 3 is too coarse to sufficiently resolve higher frequencies. Similarly, the spectrum predicted by control surface 2 is very similar to that predicted by control surface 1 until Strouhal number 2.0 and then we observe a sharp drop in the spectrum predicted by control surface 2 for the higher frequencies. Thus, it appears that 6 grid points per wavelength are adequate to accurately resolve an acoustic wave. Based on the data sampling rate (recall that the data on the control surfaces were saved at every 5 time steps), the number of temporal points per period in these highest resolved frequencies are 8, 12, and 16, respectively. We also employ the 6th-order compact difference scheme to compute the time derivatives of various quantities in the surface integral acoustics methods. Since at least 6 points per wavelength are needed to spatially resolve an acoustic wave using the 6th-order compact difference scheme, it makes sense to assume that at least 6 points per period are needed to temporally resolve an acoustic wave if one employs the 6th-order compact difference scheme to compute the time derivatives. Hence, we believe the temporal resolution of our data sampling rate in this simulation is adequate. From the findings in this study, we see that control surface 1 is the optimal surface to choose among the 3 control surfaces. Control surface 1 does not go through any flow nonlinearities even though it has been placed quite close to the jet flow, while at the same time it has a higher frequency resolution than the other two control surfaces since the grid spacing around it is finer than that around the other two surfaces.

It should be also mentioned at this point that although Bogey and Bailly\textsuperscript{39,40} used a computational grid that is comparable in size to our grid, their cutoff Strouhal number was 2.0 which is lower than our cutoff Strouhal number of 3.0 obtained with control surface 1. In their LES calculations, Bogey and Bailly\textsuperscript{39,40} use direct LES data in the immediate surroundings of the jet to evaluate the near field jet noise only. They do not employ integral acoustics methods to estimate the far field noise. Since they directly compute the noise in the near field only, their computational grid is more mildly stretched compared to ours. However, since we are employing integral acoustics methods in our methodology to compute the far field noise, we choose to pack most of the grid points inside the jet flow and do relatively rapid grid stretching outside. Then, placing a control surface such as control surface 1 quite close to the jet flow allows us to obtain higher cutoff frequencies in the subsequent noise calculations.

Figure 8 compares the overall sound pressure level (OASPL) prediction of the FWH method with the experimental data as well as with the SAE ARP 876C\textsuperscript{69} database prediction and the previous Reynolds number 100,000 jet LES result.\textsuperscript{65,66} Although not shown here, the Kirchhoff results are almost identical to those of the FWH method. The acoustic pressure spectra were integrated over the resolved frequency range to determine the OASPL values. It should also be noted here that since the control surface has a relatively short streamwise length of $34r_{ao}$, the acoustic waves travelling at the shallow...
angles, i.e. $\theta < 40^\circ$, cannot be accurately captured. Hence, the predicted OASPL values show a sharp drop-off at the shallow angles. Freund et al.\textsuperscript{70} indicate that for an open control surface, if the straight line between the source and the observer goes through part of the surface, then the result is acceptable; however, if the line goes through the open part, then the result is erroneous. In the previous Reynolds number 100,000 jet noise calculations,\textsuperscript{65,66} an open surface that extended 59$r_o$ in the streamwise direction was used. Hence, the OASPL predictions at the shallow angles show better accuracy for the Reynolds number 100,000 jet. Although the numerical predictions are a few $dB$ louder than the experiments, the overall agreement is encouraging. One reason for the overprediction of the numerical results relative to experimental measurements is believed to be the inflow forcing employed in the simulations. Several researchers have recently demonstrated the effects of inflow forcing on jet noise.\textsuperscript{38,40,67}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{The Ffowcs Williams - Hawkings method prediction for the overall sound pressure levels along the far field arc and comparison with other data.}
\end{figure}
Next, we compare our acoustic pressure spectra at 2 observation points with those of Bogey and Bailly.40 Figures 9 and 10 compare the results at the observation points \( x = 29r_o, r = 12r_o \), and \( x = 11r_o, r = 15r_o \), respectively. Our spectra at these points were computed by coupling the data on control surface 1 with the FWH method, while those of Bogey and Bailly40 are based on data directly provided by their LES. As can be seen from figure 9, the two spectra are quite similar at the observation point \( x = 29r_o, r = 12r_o \). At this observation point, the cutoff frequency of Bogey and Bailly40 corresponds to Strouhal number of 2.0, while our cutoff frequency is at Strouhal number 3.0, since that is the maximum frequency we can accurately resolve using the data gathered on control surface 1. These cutoff frequencies are also shown in the figure. It is seen that the spectrum of Bogey and Bailly is a few dB louder than ours and their spectrum starts to roll off compared to ours beyond \( St = 2 \). Figure 10 shows the spectra comparison at the observation point at

![Figure 9](image-url)

**Figure 9.** The Ffowcs Williams - Hawkings method prediction for the acoustic pressure spectrum at \( x = 29r_o, r = 12r_o \) and comparison with the spectrum of Bogey and Bailly.40
This time, Bogey and Bailly’s cutoff frequency correspond to Strouhal number of about 1.1 due to their coarsened grid spacing at the given observation point. Our cutoff frequency at this location is still 3.0. The figure shows that Bogey and Bailly’s spectrum is similar to ours until Strouhal number 1.1, then their spectrum exhibits a sharp drop-off, while ours continues until Strouhal number 3.0. Again, it is observed that their spectrum is a few dB louder than ours. At this stage, one might wonder as to why the far field noise predictions of Bogey and Bailly are not identical to ours since exactly the same inflow forcing was used and all other flow parameters were kept the same in both simulations. One possible reason is the slightly different acoustic source location due to the slightly different jet potential core lengths. The potential core of the jet simulated by Bogey and Bailly has a length of 12$r_o$, whereas our jet’s potential core length is about 13$r_o$. Another reason could be the
differences in the computational grids. Our grid was more densely packed in the jet flow region and rapidly stretched outside the flow, whereas the grid of Bogey and Bailly was less dense than ours in the jet flow region and more mildly stretched outside the jet flow. The slight difference in the jet potential core lengths could also be attributed to the grid differences.

The aeroacoustics results presented so far were obtained using an open control surface. Due to the relatively short streamwise control surface length, the acoustic pressure signals at observation angles less than 40° on the far field arc were not accurately captured. To see the effects of closing the control surface at the outflow, a new study was conducted. A new control surface that starts one jet radius downstream of the inflow boundary and extends to 31r_0 in the downstream direction was generated. This control surface is the same as the open control surface 1 used in the aeroacoustics studies presented earlier in this section, but with an added base surface at x = 31r_0. The current control surface is truncated at x = 31r_0 whereas the previous open control surface extended up to x = 35r_0. Since the physical portion of the domain ends at around x = 35r_0, placing the outflow surface a few radii away from the end of the physical domain ensures that the flow data gathered on the outflow surface are not affected by the presence of the sponge zone. The data gathered on the new control surface were coupled with the FWH method to compute the far field noise. No refraction corrections were included when the control surface was closed on the outflow surface. The far field acoustic pressure signals were calculated at 36 equally spaced azimuthal points on a full circle at a given θ location on the far field arc. θ values on the arc range from 5° to 120° with an increment of 5°. The FWH method required about 86 hours of computing time on 136 POWER3 processors of an IBM-SP3 to compute the 4096-point time histories at a total of 864 far field points. Figure 11 plots the OASPL predictions on the far field arc which were obtained using the data gathered on the closed control surface and makes comparisons with other data. The OASPL values at all observation angles are shifted up by some amount when the control surface is closed at the outflow. The effect of the closed control surface appears to be minimal for the range of observation angles where 50° < θ < 65°. The increase of the OASPL at the shallow observation angles is expected; however, we clearly see a spurious effect for the range where θ > 80°. The OASPL profile in this range is very slowly decreasing with the observation angle. A similar observation was recently made by Rahier et al. who conducted a study of surface integral acoustic methods and looked at the sensitivity of far field noise results to the placement of closed control surfaces in the nonlinear flow field. The spurious effect observed here is attributed to the fact that we have placed a control surface inside the nonlinear flow region ignoring the noise due to the quadrupole sources outside the control surface. Rahier et al. also gave the same reason for the similar spurious effects they have observed. As the MGB method shows that the sources beyond 32r_0 in a jet are not significant, we believe that an effective line of dipole sources is created as the quadrupoles exit the downstream surface. These dipole sources can also be partially responsible for the spurious effects observed here. Although this is a speculative argument at this point, a quadrupole convecting out of the domain would act like an effective dipole when only part of it is inside the domain. Hence, the effective line of
Moving the outflow surface further downstream will reduce the strength of the line of dipoles appearing on the outflow surface. This is supported by the observation of Rahier et al.\textsuperscript{71} which says that the spurious effects weaken as the outflow FWH control surface is moved further downstream. Finally, we look at the acoustic pressure spectra predictions at the $\theta = 30^\circ$ and $\theta = 60^\circ$ locations on the far field arc. Figure 12 shows the two spectra at the $\theta = 30^\circ$ observation point which were computed using the open and closed control surfaces. It is clear that at the $\theta = 30^\circ$ location, the spectral energy is shifted up at all frequencies when the control surface is closed on the outflow surface. The spectra at $\theta = 60^\circ$ computed using the open and closed control surfaces are shown in figure 13. The differences in the two spectra are
minor at this observation point, hence the outflow surface does not have much influence on the noise spectrum at the \( \theta = 60^\circ \) observation location.

6.2 Computation of the far field noise using Lighthill’s acoustic analogy

We also computed the far field noise of the Reynolds number 400,000 jet using Lighthill’s acoustic analogy. As mentioned previously, data from the simulation which was performed without any explicit SGS model were used for this purpose. The spatial filter was treated as the effective SGS model in the LES. Far field noise computations performed using Lighthill’s acoustic analogy are compared with the FWH method results as well as with some experimental noise spectra in this section. Due to space limitations, only a short summary of the noise computations using Lighthill’s acoustic analogy is presented in this section. More details can be found in Uzun.\(^{50}\)
The 5 primitive flow variables were saved in nearly 7.5 million cell volumes inside the jet flow at every 10 time steps over a period of 40,000 time steps during the LES run. The sampling period corresponds to a time scale in which an ambient sound wave travels about 23 times the domain length in the streamwise direction. The flow data were saved in double precision format and so the entire flow field database consists of almost 1.2 Terabytes (TB) of data. Figures 14 through 16 depict the distribution of the second time derivative of $T_{nn}$ (where $n$ is the direction of the observer) that radiates noise in the direction of the observers located at 30°, 60°, and 90° on an arc of radius 60$r_o$ from the jet nozzle. Moving animations can be found at http://roger.ecn.purdue.edu/~lyrintzi/. Note that

$$T_{nn} = \frac{1}{4\pi c_\infty} \frac{(x_i - y_i)(x_j - y_j)}{|x - y|^3} T_i(y, t - \frac{|x - y|}{c_\infty}) \Delta V,$$

Figure 13. Acoustic pressure spectra at $R = 60r_o$, $\theta = 60^\circ$ location on the far field arc. (Obtained using the open and closed control surfaces of streamwise length 30$r_o$)
where \( x, y \) are the observer and source location, respectively, and \( \Delta V \) is the local cell volume. The solid dark lines in these figures correspond to the boundaries of the volume in which time accurate LES data were saved. The volume starts at the inflow boundary and extends to 32 jet radii in the streamwise direction. The initial width and height of the volume are 10\( r_o \) at the inflow boundary. At \( x = 32r_o \), the width and height of the volume are 20\( r_o \). As can be seen from the figures, the lateral boundaries are sufficiently far away from the sources that radiate noise. After a careful analysis of the spatial extent of the sources that radiate noise and based on the grid resolution in the region where the sources are located, the cutoff frequency in the subsequent noise calculations was found to be located at around Strouhal number 4. Based on the data sampling rate, there are about 4 temporal points per period in this highest resolved frequency. An 8th-order accurate explicit scheme is employed for computing the time derivatives while computing the noise and 4 points per period is sufficient for this numerical scheme. The second time derivative in Lighthill’s volume integral is evaluated by means of two successive applications of the first time derivative operator. It should also be noted here that the cutoff frequency of Strouhal number 4 in Lighthill’s volume integral calculations is higher than the Strouhal number 3 cutoff frequency of the previous aeroacoustics computations that employed surface integral acoustics methods. This is because the grid spacing is finest inside the jet and gets coarser outside the jet flow.

Figure 14. Instantaneous distribution of the second time derivative of \( T_{nn} \) that radiates noise in the direction of the observer at \( R = 60r_o, \theta = 30^\circ \) on the far field arc.
Hence, the maximum frequency which can be accurately captured by the control surfaces placed around the jet flow is lower than that which can be captured in the volume integrals.

The patterning of the second time derivative of $T_{nn}$ contours shown in figures 14 through 16, of course, depends upon the observer location. As the observer location moves from $30^\circ$ to $90^\circ$ along the far field arc, the corresponding Lighthill source structures appear to align in the direction of the observer location and become steeper relative to the horizontal. The reason for this patterning is unknown at this point. Although not shown here, we also looked at the Lighthill source distributions for observer points located at $\theta = -30^\circ, -60^\circ, -90^\circ$, i.e. at observer locations on a far field arc that is “below” the jet. As expected, the corresponding Lighthill source structure distributions were found to align in the direction of these observer locations as well. Also, the Lighthill source structures appear to become more “fine-grained” as we move from $30^\circ$ to $90^\circ$. This is consistent with the fact that it is the large scale turbulence that is mostly responsible for the sound radiation at the smaller observation angles, and the sound radiation from the finer turbulence scales becomes more dominant with increasing observation angle.

Using the 5 primitive variables and the mean flow data, one can easily compute $T_{ij}$ and all of its components. To compute the noise of $T_{ij}$ or one of its components, the following

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**Figure 15.** Instantaneous distribution of the second time derivative of $T_{nn}$ that radiates noise in the direction of the observer at $R = 60r_O, \theta = 60^\circ$ on the far field arc.
procedure was used. A retarded time based on the distance between the observer location and the source point inside the jet was computed first. Then, a 24-point time stencil was constructed around the retarded time point such that 12 points fell before and 12 points fell after the retarded time point. These 24 points correspond to the time history points at which LES flow data were saved. Using the 8th-order accurate, 9-point explicit finite difference scheme developed by Bogey and Bailly, the second time derivative of the source term was computed at the 4 points to the left and 4 points to the right of the retarded time point. It should be noted here that the above explicit finite difference scheme was originally derived to compute the first derivative of a given variable, hence we applied it twice successively to get the second time derivative. Furthermore, to reduce the number of quantities whose second time derivative need to be computed, we first take the product of $T_{ij}$ (or one of its components) with $(x_i - y_i)$ and $(x_j - y_j)$ (see the time derivative formulation of the volume integral in equation 5) and compute the time derivative of the resulting quantity instead. This operation reduces the number of quantities to be time differentiated from 24 to 4. The source time derivatives at the 8 time history points were then used in a cubic spline interpolation subroutine of the NAG software library to get the second time derivative of the source term at the retarded time point. This value was then used in the integrand appearing in the volume integral. The turbulent flow region is divided into many cell volumes and the volume

Figure 16. Instantaneous distribution of the second time derivative of $T_{nn}$ that radiates noise in the direction of the observer at $R = 60r_o$, $\theta = 90^\circ$ on the far field arc.
integrals are performed in a discrete sense by adding up the contributions from all cell volumes. Lighthill’s volume integral was computed using 1160 processors in parallel on the *Lemieux* cluster at the Pittsburgh Supercomputing Center. Computation of the 3072-point time history of the noise from $T_{ij}$, $T_{ij}^{\prime}$, $T_{ij}^{\prime\prime}$, and $T_{ij}^{\prime\prime\prime}$ took about 8 minutes of computing time per observer point.

Overall sound pressure levels and acoustic pressure spectra were computed along a far field arc of radius $60r_o$ from the jet nozzle. There are 14 observer points on the far field arc. Even though acoustic pressure spectra were averaged over 36 equally spaced azimuthal points on a full circle at a given observer location in the previous section, we determined through numerical experimentation that averaging the spectra over 8 equally spaced azimuthal points gives almost the same averaged spectrum as that obtained by averaging the spectra over 36 equally spaced azimuthal points. Hence, in order to save computing time, the acoustic pressure signals were computed at only 8 equally spaced azimuthal points on a full circle at a given $\theta$ location on the arc. The 3072-point time signal at a given observer location was then broken up into 3 1024-point signals and the 1024-point signals were used in the spectral analysis. Hence, there were a total of 24 1024-point signals available for a given observer point on the arc. The 24 acoustic pressure spectra were used to obtain the averaged spectrum which was integrated later to compute the overall sound pressure level at the given $\theta$ location on the arc. For 112 far field points, the total run time needed was about 15 hours on 1160 processors.

We first looked at the effect of the integration domain size on the far field noise estimations. For this purpose, Lighthill’s volume integral was carried out for 3 different streamwise domain lengths. The domain lengths considered were $24r_o$, $28r_o$, and $32r_o$, respectively. Figure 17 shows the Lighthill’s volume integral predictions for the 3 different streamwise domain lengths and makes comparisons with experimental data as well as with the open and closed control surface predictions of the FWH method from the previous section. From this figure, it is clear that as the integration domain size is increased from $24r_o$ to $32r_o$, the observation angles smaller than $60^\circ$ see as much as 2 $\text{dB}$ decrease in the overall sound pressure levels. The OASPL changes for observation angles located in the range $60^\circ < \theta < 80^\circ$ are less than 1 $\text{dB}$, whereas the changes in OASPL for observation angles greater than $90^\circ$ are on the order of 2 to 3 $\text{dB}$. This observation implies that there are significant noise cancellations taking place as one includes a longer streamwise length in the volume integration and such cancellations cause a reduction in the overall sound pressure levels for certain observation angles in the far field. The OASPL curve that is obtained when the integration domain length is
32r_o has an almost flat portion in the range where 80° < θ < 120°. On the other hand, the experimental data show a continuous drop at those observation angles. The spurious effects observed here are quite similar to those observed in the previous section when the FWH method was applied on a closed control surface. It is also interesting to note that the MGB method\textsuperscript{72} shows that the sources beyond 32r_o in a jet are not significant. It can also be argued at this point that the sudden truncation of the domain in the current computations creates spurious dipole sources on the outflow surface as the quadrupole sources pass through the downstream surface. Such spurious dipole sources could be partially responsible for the behavior seen in the OASPL plot for the range where θ > 80°. Since all acoustic sources decay as we move downstream, a longer streamwise domain will improve the predictions for θ > 80°. In his study, Freund\textsuperscript{60} also observed substantial noise cancellations happening among the noise generated in different streamwise

**Figure 17.** Overall sound pressure levels of the noise from T_ij along the far field arc.
sections of a low Reynolds number ($Re_D = 3,600$) jet; however, he did not get the spurious effects we observe here. His streamwise domain size was $31r_o$. So it appears that although the acoustic sources decay substantially by $x = 31r_o$ for a low Reynolds number jet, they have a larger streamwise extent in the present high Reynolds number jet.

Figure 18 shows the OASPL values of the noise from $T_{ij}$ and its individual components $T_{ij}^l$, $T_{ij}^n$, and $T_{ij}^s$ when the streamwise integration domain extends up to $32r_o$. Even though the current jet is an isothermal jet ($T_o/T_w = 1$), the entropy noise from $T_{ij}^s$ is significant near the jet axis when the observation angle is small, but becomes insignificant at large angles. It is also observed from the figure that the shear noise from $T_{ij}^l$ and the self noise from $T_{ij}^n$ are louder than the total noise from $T_{ij}$ for observation angles $\theta < 40^\circ$, while the entropy noise from $T_{ij}^s$ is louder than the total noise for $\theta < 15^\circ$.

Figure 18. Overall sound pressure levels of the noise from $T_{ij}$ and its components along with far field arc. (Volume integrals are performed until $x = 32r_o$.)
The shear noise reaches its minimum OASPL value at around $\theta = 80^\circ$ and starts to increase for larger angles, whereas the self noise exhibits a continuous drop. The fact that the shear noise, self noise and entropy noise components are more intense than the total noise for some observation angles near the jet axis implies that the noise from the different components must be correlated, as suggested by Freund, so that significant cancellations happen among the noise generated by the individual components. To see the correlation between the noise components, we can define the following correlation coefficients:

$$
C_{ln} = \frac{\langle p^l p^n \rangle}{p_{rms}^l p_{rms}^n}, \quad C_{ls} = \frac{\langle p^l p^s \rangle}{p_{rms}^l p_{rms}^s}, \quad C_{ns} = \frac{\langle p^n p^s \rangle}{p_{rms}^n p_{rms}^s},
$$

(16)

where the superscripts $l$, $n$, and $s$ indicate the shear noise from $T_{ij}^l$, the self noise from $T_{ij}^n$, and the entropy noise from $T_{ij}^s$, respectively. The correlation coefficients $C_{ln}$, $C_{ls}$, and $C_{ns}$ are plotted in figure 19. For observation angles in the range $\theta < 40^\circ$, the shear noise from $T_{ij}^l$ cancels the self noise from $T_{ij}^n$ with a correlation coefficient on the order of $-0.6$. There is also some correlation between the self noise from $T_{ij}^n$ and the entropy noise from $T_{ij}^s$ at the small observation angles where the correlation coefficient has a value around $-0.3$. The shear noise from $T_{ij}^l$ and the entropy noise from $T_{ij}^s$ also cancel each other at very small angles near the jet axis with a correlation coefficient of almost $-0.3$. The correlation coefficient between these two noise components starts to rapidly move towards zero as the observation angle increases and reaches a positive value in between 0.1 and 0.2 at the $\theta = 40^\circ$ observation point. It then starts to approach the zero line for larger observation angles. All correlations reach a value close to zero at around the $\theta = 80^\circ$ and $\theta = 90^\circ$ locations.

Next, we look at the noise spectra of the individual components of $T_{ij}$ at the observation locations of 30° and 60° on the far field arc based on the integration domain length of 32$r_o$. Figures 20 and 21 plot the total noise from $T_{ij}$, shear noise from $T_{ij}^l$, self noise from $T_{ij}^n$, and entropy noise from $T_{ij}^s$ for these two observation angles, respectively. As can be seen in figure 20, at 30°, the peak of the shear noise spectrum coincides with that of the total noise at around Strouhal number 0.3. We see the high frequency part of both the shear noise and the self noise spectra is more energetic than that of the total noise. The entropy noise is relatively weak over most of the frequency range when compared with the shear noise and the self noise. Figure 19 shows that at the observation angle of 30°, the shear noise and the self noise cancel each other with a correlation coefficient of about $-0.6$, whereas the self noise and the entropy noise cancel each other with a correlation coefficient of about $-0.3$. Hence, the interaction of the shear noise, self noise and entropy noise at the 30° location results in a total noise spectrum that has reduced spectral energy levels in the high frequency range.

From figure 21, it is seen that the total noise spectrum is very similar to the self noise spectrum at the 60° location. The portion of the shear noise spectrum for Strouhal number greater than 2 has the same spectral energy levels as the self noise spectrum.
Except for the low frequency region, the entropy noise is very weak over the entire frequency range. From figure 19, we now see that at the observation angle of 60°, the shear noise and the self noise cancel each other with a correlation coefficient of about −0.4, whereas the self noise and the entropy noise cancel each other with a correlation coefficient of about −0.15. Such an interaction among the noise components at the given observation point causes the total noise spectrum to be essentially the same as the self noise spectrum. From our analysis so far, we see that the individual components of the Lighthill source term are highly correlated and, therefore, such a source decomposition may not be considered that useful.

It seems appropriate at this point to compare the current far field noise spectra predictions with the results previously obtained by using the FWH method and also with some experimental noise spectra recently obtained from the NASA Glenn Research Center. Experimental noise data from a Mach 0.85 cold jet are shown in the comparisons. The Mach number of this jet is close enough to that of our simulated jet. The estimated Reynolds numbers of the experimental jet is approximately 1.2 million,

Figure 19. Variation of the correlation coefficients along the far field arc.
while the ratio of the jet temperature to the ambient temperature is 0.88. The far field noise spectra of the experimental jet were obtained at 40 jet diameters away from the nozzle. The experimental data were made available to us after the numerical computations were completed. Hence, in order to facilitate the comparison with numerical results, the experimental noise spectra were shifted to 30 jet diameters away from the nozzle using the $1/R$ decay assumption of the acoustic waves. In this adjustment, the distances were measured relative to the acoustic source location which is the end of the jet potential core located at around 13 jet radii from the jet nozzle exit. On average, the SPL values of the experimental spectra were shifted upwards by about 2.5 dB/St. Moreover, since the Mach number of the experimental jet was not exactly 0.9, the experimental noise spectra were also adjusted for Mach 0.9 following the

Figure 20. Spectra of the noise from $T_{ij}$ and its components at the $R = 60r_o, \theta = 30^\circ$ location on the far field arc.
SAE ARP 876C guidelines. The adjustment depends upon the observer location; however, on average, the experimental spectra were shifted upwards by about 2 $\text{dB/St}$. No temperature adjustments were made, as SAE ARP 876C does not include $T_o/T_\infty < 1$. However, the effect of the temperature ratio difference is expected to be very small (about 0.2 $\text{dB}$).

The spectra obtained from the previous Reynolds number 100,000 jet simulation are also included in the comparisons. Figures 22 through 24 make comparisons at the observation locations of 30°, 60° and 90°, respectively. All spectra shown in these figures are curve fits to the actual data. In these figures, a logarithmic scale is used for the frequency axes so as to illustrate the spectra decay rates better. However, a linear scale was used for the frequency axes in the previous spectra plots for the sake of better

**Figure 21.** Spectra of the noise from $T_{ij}$ and its components at the $R = 60r_o$, $\theta = 60^\circ$ location on the far field arc.
The FWH method results for the Reynolds number 400,000 jet are shown both for the open and closed control surfaces. The reader is reminded here that the open control surface extends until $x = 31r_o$ and the outflow surface which closes the control surface is placed at the $x = 31r_o$ location, whereas the Lighthill volume integral was performed until the $x = 32r_o$ downstream location. The slight difference in the streamwise extent of the surface and volume integrals is not expected to cause a significant difference in the comparisons. The FWH method results for the Reynolds number 100,000 jet were obtained using an open surface that extended 59$r_o$ in the streamwise direction.

At the 30° location, we see that the closed surface FWH method prediction for the Reynolds number 400,000 jet is in fairly good agreement with Lighthill’s acoustic analogy until Strouhal number 2 or so. Then, we observe higher spectral energy levels in Lighthill’s acoustic analogy prediction for higher frequencies. The open surface FWH method prediction for the Reynolds number 400,000 jet, on the other hand, shows lower spectral energy levels at all frequencies. This is due to the fact that the relatively

\[ \text{SPL (dB/St)} \]

**Figure 22.** Acoustic pressure spectra comparisons at the $R = 60r_o$, $\theta = 30^\circ$ location on the far field arc.
short open control surface cannot effectively capture the acoustic waves travelling at the shallow angles. It is interesting to note that the agreement of the shape of the Reynolds number 100,000 jet noise spectrum with the experimental spectrum until Strouhal number 1 is better than that between the Reynolds number 400,000 jet noise spectra and the experiment. The reason for this is believed to be the fact that the larger domain in the Reynolds number 100,000 LES allows a better evaluation of the lower frequencies. For the Reynolds number 400,000 jet, the Lighthill prediction seems to be showing the best qualitative agreement with the experimental noise spectrum at this observation location. The peaks of all noise spectra in the figure are seen to be in the Strouhal number 0.25-0.3 range. However, the experimental spectrum exhibits a much stronger decay right after the peak. The decay rate of the spectrum obtained from Lighthill's acoustic analogy seems to be similar to the experimental spectrum decay rate in the frequency range where $1.5 < St < 3.0$. Then, the Lighthill spectrum decays with a faster rate for the higher frequencies.

**Figure 23.** Acoustic pressure spectra comparisons at the $R = 60r_o$, $\theta = 60^\circ$ location on the far field arc.
At the 60° location, for the Reynolds number 400,000 jet, the FWH method yields almost identical results for the open and the closed control surfaces. The Lighthill prediction is also in acceptable agreement with the FWH prediction, considering the fact that the two methods are based on completely different formulations. The comparison with the experimental noise spectrum at this observation location reveals that the experimental peak is located at a lower frequency than that of the numerical predictions. Furthermore, the numerical results for the Reynolds number 400,000 jet show a faster spectrum decay rate at the higher frequencies. The decay of the Reynolds number 100,000 jet spectrum after the peak takes place at a faster rate than that observed in the Reynolds number 400,000 jet spectra as well as in the experiment.

Finally, the comparison at the 90° location shows that the closed surface FWH prediction gives increased spectral energy levels relative to those given by the open surface FWH prediction. The Lighthill prediction is seen to be in between the two predictions given by the FWH method. It should be repeated here that from our previous analysis,
the spectra of the Reynolds number 400,000 jet for observation angles in the range $\theta > 80^\circ$ may be affected by a spurious line of dipoles appearing on the outflow surface as the quadrupole sources move out of the control volume. The numerical predictions at this observation location once again reveal a spectrum decay rate that is larger than that of the experimental noise spectra. The decay of the the Reynolds number 100,000 jet spectrum takes place at a faster rate than that of the Reynolds number 400,000 jet spectra. The experimental peak is again located at a lower frequency than that of the numerical predictions. The comparison of the numerical OASPL predictions with the OASPL values of the NASA Mach 0.85 cold jet and the SAE ARP 876C database prediction along the far field arc is plotted in figure 25. We see OASPL differences as high as 6 dB between the numerical predictions and the NASA experimental jet. The agreement between the numerical OASPL values and the SAE ARP 876C prediction seems to be better.

![Figure 25. Overall sound pressure levels along the far field arc.](image-url)
The differences observed between the shape of the numerical and experimental noise spectra might be due to various reasons. The difference in the Reynolds number (approximately 1.2 million in the experiment versus 400,000 in the LES) as well as the difference in the initial shear layer thickness (very thin in the experiment versus relatively thick in the LES due to grid resolution constraints) are two possible reasons. Another reason could be the mismatch of the inflow conditions in the numerical simulations with those in the actual experiment. The experiment was performed at a high enough Reynolds number so that the jet shear layers at the nozzle exit were fully turbulent. In the numerical simulations, since it was deemed computationally too expensive to include the nozzle geometry, laminar shear layers were fed into the domain and randomized velocity fluctuations in the form of a vortex ring were imposed into the jet shear layers. Moreover, it has been observed experimentally\textsuperscript{74,75} that high-frequency sources are located a small distance downstream of the jet nozzle and a significant portion of the noise spectrum originates from this near field region of the jet. Hence, the high-frequency noise generated in the near-nozzle jet shear layer within a few diameters downstream of the nozzle exit is missing in the current simulations. The absence of the noise generated just downstream of the nozzle could be responsible for the faster decay rates in the high frequency range of the spectra in the current computations. The present findings once again emphasize the importance of correctly modeling the inflow conditions in jet noise simulations.

It is also believed that the limited domain size in the simulations influences the low frequencies. A larger domain size allows a better resolution of the low frequencies. A better resolution of the low frequencies also implies a better prediction of the peak frequency. To re-iterate, the previous Reynolds number 100,000 jet LES\textsuperscript{65,66} was performed in a domain that extended 60 jet radii in the streamwise direction and \( \pm 20 \) jet radii along the transverse directions. The spectra comparisons plotted in figures 22 through 24 clearly show that the experimental spectra peak in the low frequency region and moreover, the noise spectra for the Reynolds number 100,000 jet peak at frequencies that are fairly close to the experimental peak frequencies.

On the other hand, the SGS model can also be important to the location of the peak frequency. The Reynolds number 100,000 jet simulation was performed with the dynamic Smagorinsky model (DSM) turned on, while the Reynolds number 400,000 jet simulation was performed with no explicit SGS model. To see the effects of an explicit SGS model on the far field noise of the Reynolds number 400,000 jet, a new simulation was performed by employing the dynamic Smagorinsky model (DSM) in the LES. All parameters in this simulation are kept the same as those used in the previous \( Re_D = 400,000 \) jet LES done without any explicit SGS model. Spatial filtering is also employed in this case so as to remove the very high frequency spurious oscillations not supported by the grid resolution. The tridiagonal spatial filter with the filtering parameter set to \( \alpha_f = 0.47 \) is used in the current simulation for that purpose. The turbulent Prandtl number is set to a constant value of 0.7, while the compressibility correction constant in the SGS model is set to 0. The acoustic pressure spectra obtained from the simulation done with the DSM are compared with those obtained from the previous LES with no SGS model as well as with the spectra of the NASA experimental...
jet at the observation angles of $\theta = 60^\circ$ and $\theta = 90^\circ$ on the far field arc in figures 26 and 27, respectively. The spectra obtained from the previous Reynolds number 100,000 jet simulation are again included in the comparisons. From the comparison at these observation angles, it is seen that the peak frequencies of the spectra from the Reynolds number 400,000 jet LES done with the DSM are in better agreement with the experimental peaks. The spectra comparisons also show that the spectral energy contained within the low frequencies is higher in the noise spectra computed from the simulation done with the DSM. This must be due to the presence of more energetic large turbulent scales in the simulation done with the DSM, since it is the large scales that are responsible for low frequency noise generation. On the other hand, the spectra computed from the simulation done with the DSM have significantly lower spectral energy at the higher frequencies. This is because the finer scales in the simulation done with the DSM have lower turbulent kinetic energy. More details of the Reynolds...
number 400,000 jet simulation done with the DSM and the effects of the SGS model on noise predictions can be found in Uzun. Bogey and Bailly also looked at the effects of SGS model on jet noise in another recent study.

In summary, from the results that we gathered so far, the peak frequency of the spectra seems to depend on the domain size, SGS model and jet Reynolds number. To see if the spectra peaks of the Reynolds number 400,000 jet would move even closer to the experimental peaks, a new simulation in a bigger domain may be performed in the future with the DSM model turned on.

Finally, we should also note here that the cut-off frequency of the experimental spectra for the NASA cold jet is located at around Strouhal number 19 which is higher than our cutoff frequency (Strouhal number 3 for calculations performed using surface integral method and Strouhal number 4 for calculations performed using Lighthill’s acoustic analogy) in the numerical simulations. Hence, it is of interest to see how much dB

**Figure 27.** Acoustic pressure spectra comparisons at the \( R = 60r_e, \theta = 90^\circ \) location on the far field arc.
difference would exist if one integrated the experimental spectra until Strouhal number 3 or 4 rather than over the whole resolved frequency range. We integrated the experimental spectra at the observation angles of 30°, 60°, 90° over three frequency ranges to determine this difference. The results are tabulated in table 1. As can be seen from the table, at 30°, the differences in the OASPL are less than 0.2 dB. At this angle, most of the spectral energy is contained within the low frequency portion of the spectrum and hence, integrating the spectrum until Strouhal number 3 or 4 gives almost the same result as that obtained by integrating the spectrum over the whole resolved frequency range. At the observation angles of 60° and 90°, high-frequency noise is more important and, therefore, integrating the spectrum until Strouhal number 3 or 4 gives an OASPL value that is a dB or so less than the value obtained by integrating the spectrum over the whole resolved frequency range.

## 7. CONCLUDING REMARKS

Using state-of-the-art numerical techniques, we have developed and tested a Computational Aeroacoustics (CAA) methodology for jet noise prediction. The CAA methodology has two main components. The first one is a 3-D Large Eddy Simulation (LES) code. The latest version of the LES code employs high-order accurate compact finite differencing as well as implicit spatial filtering schemes together with Tam and Dong’s boundary conditions on the LES domain boundaries. Explicit time integration is accomplished by means of the standard 4th-order, 4-stage Runge-Kutta method. The localized dynamic Smagorinsky subgrid-scale model is utilized to model the effect of the unresolved scales on the resolved scales. The code also has the capability to turn off the dynamic SGS model and perform simulations by treating the spatial filter as an implicit SGS model. The second component of the CAA methodology consists of integral acoustics methods. We have developed acoustics codes that employ Kirchhoff’s and Ffowcs Williams - Hawkings (FWH) methods as well as Lighthill’s acoustic analogy.

In this paper, we present results from an LES performed for a Reynolds number 400,000 jet. A much more detailed report of this research can be found in Uzun. The time accurate LES data were coupled with integral acoustics methods for far field noise calculations. Far field aeroacoustics results compared favorably with existing experimental measurements. The possible reasons for the discrepancies between

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Observation angle</th>
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<tbody>
<tr>
<td>0 &lt; Str ≤ 19</td>
<td>30°</td>
</tr>
<tr>
<td></td>
<td>60°</td>
</tr>
<tr>
<td></td>
<td>90°</td>
</tr>
<tr>
<td>0 &lt; Str ≤ 4</td>
<td>112.26 dB</td>
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<td></td>
<td>107.11 dB</td>
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<td>104.00 dB</td>
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<td>0 &lt; Str ≤ 3</td>
<td>112.12 dB</td>
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<td></td>
<td>105.92 dB</td>
</tr>
<tr>
<td></td>
<td>102.83 dB</td>
</tr>
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Table 1. Experimental OASPL of the NASA cold jet for three frequency ranges.
numerical predictions and experiments were discussed. In our Reynolds number 400,000 jet simulations, the highest noise frequency resolved in the surface integral acoustics calculations corresponds to Strouhal number 3, while the highest frequency resolved when Lighthill’s acoustic analogy was employed corresponds to Strouhal number 4. Both of these frequencies are larger than Bogey and Bailly’s cutoff frequency of Strouhal number 2 in their recent Reynolds number 400,000 jet LES. Moreover, our noise computations for the Reynolds number 400,000 jet have cutoff frequencies which are greater than the cutoff frequencies of all other jet noise LES results in the literature to date. Use of integral acoustics methods allows clustering of the majority of the grid points inside the jet flow where nonlinear noise generation takes place and rapid grid stretching outside the jet. Consequently, the maximum frequency resolved in noise computations using integral acoustics methods is higher compared to that captured in the simulations (with similar number of grid points) in which only the near field jet noise is computed using direct LES data.

Finally, to the best of our knowledge, computation of the Lighthill volume integral over the near field of a turbulent jet at a reasonably high Reynolds number has been carried out for the first time in this study. A database greater than 1 Terabyte (TB) in size was postprocessed using 1160 processors in parallel on a modern supercomputing platform for this purpose. The Lighthill stress tensor was decomposed into several components and the noise generated by the individual components was analyzed in detail. We found that significant cancellations occur among the noise generated by the individual components of the Lighthill stress tensor making the usefulness of such a decomposition questionable. Far field noise predictions using the FWH method on the closed control surface were found be comparable to those given by Lighthill’s volume integral. Moreover, Lighthill’s acoustic analogy was found to be about 40 times more computationally expensive than the FWH method. Hence, it is preferable to use the cheaper FWH method rather than the very expensive Lighthill volume integral if one’s sole purpose is to predict the far field noise. However, if a connection between the near field jet turbulence and the far field noise is sought, then an analysis of the Lighthill source term inside the jet may be useful. Some analyses that could be carried out are as follows. First, the reader should be reminded that Lighthill’s source term contains effects that have nothing to do with sound generation; however, using Fourier methods on DNS data for a low Reynolds number jet, Freund was able to isolate the portion of the Lighthill source that radiates noise to the far field. Fourier transform techniques similar to those used by Freund could be used to analyze the spectral makeup of the Lighthill stress tensor and to identify components capable of radiating to the far field. The wavenumber frequency makeup of the Lighthill stress tensor can be useful for gaining some insight into the physics of jet noise generation. Another alternative could be to examine Lilley’s source term, since the acoustic analogy based on Lilley’s equation makes an attempt to separate noise generation from propagation effects. Going back to the conclusions of this study, both FWH (applied on a closed control surface) and Lighthill’s methods show increased OASPL levels for observation angles greater than 80° on the far field arc. Such spurious effects are believed to be due to a spurious line of dipoles appearing on the outflow surface as quadrupoles exit the domain. This effect is
due to the relatively short domain size in the streamwise direction. A longer domain decreases the strength of the acoustic sources passing through the outflow surface and, therefore, reduces the magnitude of the spurious noise at high angles.

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