Coupling of Integral Acoustics Methods with LES for Jet Noise Prediction *

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This study is focused on developing a Computational Aeroacoustics (CAA) methodology that couples the near field unsteady flow field data computed by a 3-D Large Eddy Simulation (LES) code with various integral acoustic formulations for the far field noise prediction of turbulent jets. Noise computations performed for a Reynolds number 400,000 jet using various integral acoustic results are presented and the results are compared against each other as well with an experimental jet at similar flow conditions. Our results show that the surface integral acoustics methods (Kirchhoff and Ffowcs Williams - Hawkings) give similar results to the volume integral method (Lighthill's acoustic analogy) at a much lower cost. To our best knowledge, Lighthill's acoustic analogy was applied to a reasonably high Reynolds number jet for the first time in this study. A database greater than 1 Terabytes (TB) in size was post-processed using 1160 processors in parallel on a modern supercomputing platform for this purpose.

Introduction

Jet noise remains as one of the most complicated and difficult problems in aeroacoustics because the details of noise generation mechanisms by the complex turbulence phenomena in a jet are still not well understood. Thus, there is a need for more research that will lead to improved jet noise prediction methodologies and further understanding of the jet noise generation mechanisms, which will eventually aid in the design process of aircraft engines with low jet noise emissions.

With the recent improvements in the processing speed of computers, the application of Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) to jet noise prediction methodologies is becoming more feasible. Although there have been numerous experimental studies of jet noise to date, the experiments provide only a limited amount of information and the ultimate understanding of jet noise generation mechanisms will most likely be possible through numerical simulations since the computations can literally provide any type of information needed for the analysis of jet noise generation mechanisms. In a DNS, all the relevant scales of turbulence are directly resolved and no turbulence modelling is used. The first DNS of a turbulent jet was done for a Reynolds number 2,000, supersonic jet at Mach 1.92 by Freund et al.1 The computed overall sound pressure levels were compared with experimental data and found to be in good agreement with jets at similar convective Mach numbers. Freund2 also simulated a Reynolds number 3,600, Mach 0.9 turbulent jet using 25.6 million grid points, matching the parameters of the experimental jet studied by Stromberg et al.3 Excellent agreement with the experimental data was obtained for both the mean flow field and the radiated sound. Such results clearly show the attractiveness of DNS to the jet noise problem. However, due to the wide range of length and time scales present in turbulent flows, DNS is still restricted to low-Reynolds-number flows and relatively simple geometries. DNS of high-Reynolds-number jet flows of practical interest would necessitate tremendous resolution requirements that are far beyond the capability of even the fastest supercomputers available today.

Therefore, turbulence still has to be modelled in some way to do simulations for problems of practical interest. LES, with lower computational cost, is an attractive alternative to DNS. In an LES, the large scales are directly resolved and the effect of the small scales or the subgrid scales on the large scales are modelled. The large scales are generally much more energetic than the small ones and are directly affected by the boundary conditions. The small scales, however, are usually much weaker and they tend to have more or less a universal character. Hence, it makes sense to directly simulate the more energetic large scales and model the effect of the small scales. LES methods are capable of simulating flows at higher Reynolds numbers and many successful LES computations for different types of flows have been performed to date. Since noise generation is an inherently unsteady process, LES will probably be the most powerful computational tool to be used in jet noise research in the foreseeable future.

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future since it is the only way, other than DNS, to obtain time-accurate unsteady data. Although the application of Reynolds Averaged Navier Stokes (RANS) methods to jet noise prediction is also a subject of ongoing research,

RANS methods heavily rely on turbulence models to model all relevant scales of turbulence. Moreover, such methods try to predict the noise using the mean flow properties provided by a RANS solver. Since noise generation is a multi-scale problem that involves a wide range of length and time scales, it appears the success of RANS-based prediction methods will remain limited, unless very good turbulence models capable of accurately modelling a wide range of turbulence scales are developed and implemented into existing RANS solvers.

It is now widely accepted among the jet noise research community that the low frequency noise generated by the jet flow is associated with the large scale turbulent motions with length scales on the order of the jet diameter, whereas the high frequency noise is related to the finer scales of turbulence. Moreover, the large scales are known to be strongly affected by the jet nozzle geometry. So although LES seems to be very suitable for directly computing the low frequency jet noise and a portion of the higher frequencies, some ingenuity is still needed so as to estimate the noise from the unresolved higher frequencies generated by the very fine turbulent scales not resolved by the LES grid. Such an estimation necessitates subgrid-scale noise models that will somehow extrapolate the information contained in the scales well-resolved by the LES to predict the high frequency noise. Although there has been some preliminary efforts in this area,

t a satisfactory SGS noise model is yet to be developed.

At this point, it seems appropriate to give an overview of the application of LES to jet noise prediction. Although the overview given here is not a comprehensive list of all the jet noise LES computations done to date, it certainly includes a review of the state-of-the-art computations that are believed to be the most successful application of LES to jet noise prediction. One of the first attempts in using LES as a tool for jet noise prediction was carried out by Mankbadi et al.
They employed a high-order numerical scheme to perform LES of a supersonic jet flow to capture the time-dependent flow structure and applied Lighthill’s theory to calculate the far field noise. Lyriztis and Mankbadi used LES in combination with Kirchhoff’s method for jet noise prediction. LES has been used together with Kirchhoff’s method for the noise prediction of a Mach 2.0 jet also by Gamet and Estivalezes as well as for a Mach 1.2 jet with Mach 0.2 coflow by Choi et al. with encouraging results obtained in both studies. Zhao et al. did LES for a Mach 0.9, Reynolds number 3,600 jet obtaining mean flow results that compared well with Freund’s DNS and experimental data. Their overall sound pressure levels for this test case were in good agreement with experiments as well. They also studied the far field noise from a Mach 0.4, Reynolds number 5,000 jet. They compared Kirchhoff’s method results with the directly computed sound and observed good agreement. Morris et al. simulated high speed round jet flows using the Nonlinear Disturbance Equations (NLDE). In their NLDE method, the instantaneous quantities are decomposed into a time-independent mean component, a large-scale perturbation and a small-scale perturbation. The mean quantities are obtained using a traditional Reynolds Averaged Navier-Stokes (RANS) method. As in LES, they resolved the large-scale fluctuations directly and used a subgrid-scale model for the small-scale fluctuations in their unsteady calculations. They also analyzed the noise from a supersonic elliptic jet in another study. Chyczewski and Long conducted a supersonic rectangular jet flow simulation and also did far field noise predictions with a Kirchhoff method. Boersma and Lele did LES for a Mach 0.9 jet at Reynolds numbers of 3,600 and 36,000 without any noise predictions. Bogey et al. simulated a Reynolds number 65,000, Mach 0.9 jet using LES and obtained very good mean flow results, turbulent intensities as well as sound levels and directivity. Constantinescu and Lele did simulations for a Mach 0.9 jet at Reynolds numbers of 3,600 and 72,000. They directly calculated the near field noise using LES. Their mean flow parameters and turbulence statistics were in good agreement with experimental data and results from other simulations. The peak of the near field noise spectra was also captured accurately in their calculations. DeBonis and Scott simulated a Reynolds number 1.2 million, Mach 1.4 jet using 1.5 million grid points, but they did not do any noise predictions. Bodony and Lele did simulations for a Mach 0.9, Reynolds number 72,000 jet using 5.9 million grid points in an LES and experimented with the inlet conditions. They showed the important role of the inlet conditions in the far field noise of the jet. More recently, Bogey and Bailly’s LES for a Mach 0.9 round jet at Reynolds number, \( Re_D = 400,000 \) using 12.5 and 16.6 million grid points produced mean flow and sound field results that are in good agreement with the experimental measurements available in the literature. They also studied the effect of the various inflow conditions on the jet flow field as well as on the jet noise in another recent study. Their study once again revealed the importance of inflow conditions in jet noise LES. Moreover, they brought up the issue of the effects of the eddy viscosity based Smagorinsky SGS model on the jet noise in yet another recent study. They showed that the high-frequency portion of the noise spectra was significantly suppressed by the eddy viscosity. The recent jet noise computations of Bogey and Bailly are perhaps the most successful LES calculations done
for reasonably high Reynolds number jets at a test time of this writing. In this study, we will repeat a test case studied by Bogey and Bailly\cite{34,35} and make comparisons with their results as part of the validation of our CAA methodology.

In general, the LES results in the literature to date are encouraging and show the potential promise of LES application to jet noise prediction. Except for the studies of Choi et al.\cite{19} and DeBonis and Scott\cite{31} which are not well-resolved LES calculations, and the recent studies of Bogey and Bailly,\cite{33–35} the highest Reynolds numbers reached in the LES simulations so far are still below those of practical interest. The cutoff frequency of the noise spectra in the LES computations is dictated by the grid resolution, hence only a portion of the noise spectra was computed in the LES computations to date. Well-resolved LES calculations of jets at higher Reynolds numbers close enough to practical values of interest would be very helpful for evaluating the suitability of LES to such problems as well as for analyzing the broad-band noise spectrum and possibly looking into the mechanisms of jet noise generation at such Reynolds numbers.

**Objectives of the Present Study**

With this motivation behind the current study given, we now present the two main objectives of this research:

1. **Development and validation of a versatile 3-D LES code for turbulent jet simulations.** A high-order accurate 3-D compressible LES code utilizing a robust dynamic subgrid-scale (SGS) model has been developed to simulate high speed jet flows with high subsonic Mach numbers. Generalized curvilinear coordinates are used, so the code can be easily adapted for calculations in several applications with complicated geometries. Since the sound field is several orders of magnitude smaller than the aerodynamic field, we make use of high-order accurate non-dissipative compact schemes which satisfy the strict requirements of CAA. Implicit spatial filtering is employed to remove the high-frequency oscillations resulting from unresolved scales and mesh non-uniformities. Non-reflecting boundary conditions are imposed on the boundaries of the domain to let outgoing disturbances exit the domain without spurious reflection. A sponge zone that is attached downstream of the physical domain damps out the disturbances before they reach the outflow boundary. Initial simulations were performed with the constant-coefficient Smagorinsky SGS model. However, the results were found to be sensitive to the choice of the Smagorinsky constant.\cite{36,37} The latest version of the LES code has the localized dynamic SGS model implemented.

The code also has the capability to turn off the SGS model and treat the spatial filter as an implicit SGS model.

2. **Accurate prediction of the far field noise.** Even though sound is generated by a nonlinear process, the sound field itself is known to be linear and irrotational. This implies that instead of solving the full nonlinear flow equations out in the far field for sound propagation, one can use Lighthill’s acoustic analogy\cite{15} or surface integral acoustic methods such as Kirchhoff’s method\cite{17} and the Flowes Williams - Hawkings (FWH) method.\cite{38,39} In this study, we couple the near field data directly provided by LES with Kirchhoff’s and Flowes Williams - Hawkings (FWH) methods as well as with Lighthill’s acoustic analogy for computing the noise propagation to the far field.

The compressible LES code and the integral acoustics codes developed in this study form the core of a CAA methodology for jet noise prediction. Using these tools, we will attempt state-of-the-art well-resolved LES computations for jet flows with Reynolds numbers as high as 400,000. As will be shown in the results section, the cutoff non-dimensional frequency (Strouhal number) in the subsequent noise computations will be as high as 4.0. Such a cutoff frequency, to our best knowledge, is greater than the cutoff frequencies captured in all other jet noise LES computations for similar Reynolds numbers available in the literature to date. It is expected that the coupling of the present CAA methodology with a future SGS model for high-frequency noise from unresolved scales will be a powerful jet noise prediction tool.

**Governing Equations**

The governing equations for LES are obtained by applying a spatial filter to the Navier-Stokes equations in order to remove the small scales. The effect of the subgrid-scales is computed using the dynamic Smagorinsky subgrid-scale model proposed by Moin et al.\cite{40} for compressible flows. Since we are dealing with compressible jet flows, the Favre-filtered unsteady, compressible, non-dimensionalized Navier-Stokes equations formulated in curvilinear coordinates are solved in this study using the numerical methods described in the next section. More details about the code, including the governing equations can be found in Uzun.\cite{41}

**Numerical Methods**

We first transform a given non-uniformly spaced curvilinear computational grid in physical space to a uniform grid in computational space and solve the discretized governing equations on the uniform grid. To compute the spatial derivatives at interior grid
Spatial filtering can be used as a means of suppressing unwanted numerical instabilities that can arise from the boundary conditions, unresolved scales and mesh non-uniformities. In our study, we employ a sixth-order tri-diagonal filter used by Visbal and Gaitonde.

The standard fourth-order explicit Runge-Kutta scheme is used for time advancement. We apply Tam and Dong’s 3-D radiation and outflow boundary conditions on the boundaries of the computational domain as illustrated in figure 1. We additionally use the sponge zone method in which grid stretching and artificial damping are applied to dissipate the vortices present in the flowfield before they hit the outflow boundary. This way, unwanted reflections from the outflow boundary are suppressed. Since the actual nozzle geometry is not included in the present calculations, randomized perturbations in the form of a vortex ring are added to the velocity profile at a short distance downstream of the inflow boundary in order to excite the 3-D instabilities in the jet and cause the potential core of the jet to break up at a reasonable distance downstream of the inflow boundary. This forcing procedure has been adapted from Bogey et al.

For subgrid-scale modelling, both the standard constant-coefficient Smagorinsky as well as the dynamic Smagorinsky models have been implemented into the LES code. The main idea in the dynamic model is to compute the model coefficients as functions of space and time by making use of the information contained within the smallest resolved scales of motion. For the dynamic model, a fifteen-point explicit filter developed by Bogey and Bailly is used as the test-filter. The ratio of the test-filter width to the grid spacing is taken as 2. The usual practice in LES calculations is to average certain dynamically computed quantities over statistically homogeneous directions and then use these averaged quantities to compute the model coefficients. This is an ad hoc procedure that is used to remove the very sharp fluctuations in the dynamic model coefficients and to stabilize the model. Obviously, such an approach is not useful for turbulent flows for which there is no homogeneous direction. In our implementation, the dynamically computed model coefficients are locally averaged in space using a second-order three-point filter in order to avoid the sharp fluctuations in the model coefficient. No negative model coefficients are allowed. The upper limit for the model coefficients is set to 0.5. This procedure works reasonably well for the jet flows we are studying. The code also has the capability to turn off the localized dynamic Smagorinsky SGS model and treat the spatial filter as an implicit SGS model. The LES code which dynamically computes the Smagorinsky constant requires approximately 50% more computing time relative to the LES code which employs the constant-coefficient SGS model. Dynamic evaluation of the compressibility correction coefficient and the turbulent Prandtl number require test filtering of some additional quantities. To keep the additional cost of the dynamic model at an acceptable level, the compressibility correction coefficient, $C_f$ and the turbulent Prandtl number, $Pr_t$ are not computed dynamically in this study and are set to constant values instead.

**LES for a Mach 0.9 Round Jet at a Reynolds Number of 400,000**

The initial simulations performed for relatively low Reynolds number jets using the constant-coefficient Smagorinsky model can be found in Uzun et al., whereas the results of a Reynolds number 100,000 jet LES performed using the localized dynamic SGS model are given in Uzun et al.

In this section, we will present results from an LES that was performed without any explicit SGS model for a turbulent isothermal round jet at a Mach number of 0.9 and Reynolds number of 400,000. The tri-diagonal spatial filter was treated as an implicit SGS model. The filtering parameter was set to $\alpha_f = 0.47$. The jet centerline temperature was chosen to be the same as the ambient temperature and set to 286 K. A fully curvilinear grid consisting of approximately 16 million grid points was used in the simulation. The grid had 390 points along the streamwise $x$ direction and 200 points along both the $y$ and $z$ directions. This test case corresponds to one of the test cases studied by Bogey and Bailly. The physical portion of the domain in this simulation extended to 35 jet radii in the
where the far field noise of the jet is computed using Lighthill’s acoustic analogy. The sampling period corresponds to a time scale in which an ambient sound wave travels about 23 times the domain length in the streamwise direction.

The first task performed in this test case was to investigate the sensitivity of far field noise predictions to the position of the control surface on which aeroacoustic data are collected. For this purpose, we put 3 control surfaces around our jet as illustrated in figure 2. This figure plots the divergence of velocity contours on a plane that cuts the jet in half. By analyzing these contours, one can clearly identify the non-linear sound generation region inside the jet flow as well as the linear acoustic wave propagation outside the jet. The control surfaces start 1 jet radius downstream of the inflow boundary and extend to 35 jet radii along the streamwise direction. Hence the total streamwise length of the control surfaces is 34$\alpha_{r}$. Control surfaces 1, 2 and 3 are initially at a distance of 3.9, 5.9 and 8.1 jet radii from the jet centerline, respectively. They open up to a distance of 9.1, 10.8 and 12.2 jet radii from the jet centerline, respectively, at the far downstream location of 35$\alpha_{r}$. We gathered flow field data on the control surfaces at every 5 time steps during a period of 25,000 time steps during our LES run. The total acoustic sampling period corresponds to a time scale in which an ambient sound wave travels about 14 times the domain length in the streamwise direction. Both the FWH and Kirchhoff’s methods were employed in the far field noise predictions. Details of the FWH and Kirchhoff’s methods can be found in Lyrintzis.\(^{31}\) It should be noted that the control surfaces employed here are open surfaces. The main assumption in the surface integral acoustics methods is that the control surface must be placed outside the non-linear flow region. It is known from the study conducted by Brentner and Farassat\(^{52}\) that Kirchhoff’s method will yield inaccurate predictions if one puts a control surface in the non-linear region and does not include any further corrections to account for the non-linearities. On the other hand, the FWH method can produce some reasonable predictions depending on how strong the non-linearities are in the region where the control surface is placed. In fact, one can take the non-linearities into account accurately by including the quadrupole sources outside the control surface in the original FWH formulation. The disadvantage of including this term is that computationally expensive volume integrals are needed for its computation or sometimes (as it is the case here) this information

\[\bar{u}(r) = \frac{U_o}{2} \left[ 1 + \tanh \left( 10 \left( 1 - \frac{r}{\alpha_{r}} \right) \right) \right], \tag{1} \]

where $M_r = 0.9$.

In a recent study, Bogey and Bailly\(^{34}\) took a close look at the effects of the inflow conditions on the jet flow and noise. They found out that reducing the amplitude of the initial disturbances results in an increase in radiated noise. On the other hand, a thinner initial jet shear layer thickness was observed to cause increased noise levels in the sideline direction and reduced noise levels in the downstream direction. The most important changes were obtained when the first 4 azimuthal modes of forcing were removed. In this case, the noise levels were noticeably reduced. They concluded that further improvements are still needed to reduce the sideline noise levels which are overestimated with respect to experimental data. The reader is referred to Bogey and Bailly\(^{34}\) for more detailed information. Based on their findings, we decided to use the inflow forcing which was found by Bogey and Bailly\(^{34}\) to match the available experimental data best. There are 16 azimuthal modes total and the first 4 modes are not included in the forcing. The forcing parameter $\alpha$ is set to $0.007$. A more detailed study of the inflow forcing can be found in Lew et al.\(^{50}\)

Computations were done on an IBM-SP3 machine using 200 processors in parallel. A total of about 5.5 days (132 hours) of computing time was needed. The initial transients exited the domain over the first 10,000 time steps. Time history of the unsteady flow data inside the jet was saved at every 10 time steps over period of 40,000 time steps. These data are used in the volume integrals in the next section of this paper where the far field noise of the jet is computed using Lighthill’s acoustic analogy. The sampling period corresponds to a time scale in which an ambient sound wave travels about 23 times the domain length in the streamwise direction.
is beyond the end of the computational domain, and thus not available.

For simplicity, we decided to use open control surfaces in the first part of this study. Flow data needed by Kirchhoff’s and the FWH methods were gathered on the lateral surfaces surrounding the jet only. No data were gathered on the inflow and outflow surfaces. Towards the end of this section, we will also show far field noise predictions obtained by including the contribution of the data gathered on an outflow surface. No refraction corrections will be used when the outflow surface is included and only the FWH method will be employed in that case.

The far field acoustic pressure signals were calculated at 36 equally spaced azimuthal points on a full circle at a given $\theta$ location on a far field arc. The radius of the arc is equal to 60 jet radii and its center is chosen as the jet nozzle exit. $\theta$ values on the arc range from 25° to 90° with an increment of 5°. The run time of the Kirchhoff code is almost the same as that of the FWH code. For control surface 1, the computation of the 4096-point time history of acoustic pressure at a given far field location with both methods took about 6 minutes of computing time using 136 processors in parallel on an IBM-SP3. Hence, for control surface 1, both methods needed a total of 50 hours to compute the acoustic pressure history at a total of 504 far field points. Calculations on control surface 2 and 3 using 136 processors in parallel required 59 and 65 hours total, respectively.

Figures 3 and 4 plot the acoustic pressure spectra at the 60° observation location on the far field arc. The time signals were broken into 4 1024-point signals which were then used in the Fast Fourier Transforms to get the acoustic pressure spectra. We then averaged the acoustic pressure spectra over the equally spaced 36 azimuthal points to obtain the final averaged spectrum at the given observer location. It should be mentioned here that since the computed spectra are noisy, the spectra that are shown are polynomial fits to the actual computed spectra. As can be seen from figures 3 and 4, for a given control surface, the FWH and Kirchhoff’s methods give almost identical results. Although not shown here, comparisons at other locations along the arc yielded the same observations. This means that all of the control surfaces chosen in this study were indeed sufficiently far away from the jet (i.e. in the linear acoustic field) such that they did not encounter any non-linear sound generating regions. Also, the grid allows wave propagation without diffusion. Hence, the Kirchhoff’s method prediction was just as good as the FWH method prediction for all control surfaces.

Assuming at least 6 points per wavelength are needed to accurately resolve an acoustic wave using compact difference schemes, we see that the maximum frequency resolved with our grid spacing around the control surfaces corresponds to a Strouhal number of approximately 3.0, 2.0 and 1.5 for control surface 1, 2 and 3, respectively. The Nyquist frequency, which is the maximum frequency that can be resolved with the time increment of our data sampling rate, corresponds to a Strouhal number of about 11.11. However, we choose the maximum frequency that is based on spatial resolution as our cutoff frequency. The time increment used in the LES corresponds to a Strouhal number of about 55.56. As will be evident shortly, 6 points per wavelength are indeed sufficient to resolve an acoustic wave using compact difference schemes. When compared to the spectra predicted by control surface 1, the spectra predicted by control surface 2
will show a drop-off starting at around Strouhal number 2.0. Similarly, the spectra predicted by control surface 3 will show a drop-off at around Strouhal number 1.5. Based on the data sampling rate, the number of temporal points per period in these highest resolved frequencies are 8, 12, and 16, respectively. Hence, the temporal resolution is adequate. Since grid stretching was employed in the computational grid used in this simulation, the grid spacing gets coarser towards the outer domain boundaries. This means that grid spacing around control surface 1 is finer than that around control surface 2 and similarly the grid spacing around control surface 2 is finer than that around control surface 3. Hence, the maximum frequency that can be captured by a control surface decreases as one puts the control surface further away from the jet because of the grid stretching. The spectra comparison figures show solid vertical lines corresponding to the cutoff frequency for every control surface. Until Strouhal number 1.5, the spectra predicted by the three control surfaces are seen to be very similar. Then, the spectra predicted by control surface 3 start to drop sharply due to the fact that the grid spacing around control surface 3 is too coarse to sufficiently resolve higher frequencies. Similarly, the spectra predicted by control surface 2 are very similar to those predicted by control surface 1 until Strouhal number 2.0 and then we observe a sharp drop in the spectra predicted by control surface 2 for the higher frequencies. From the findings in this study, we see that control surface 1 is the optimal surface to choose among the 3 control surfaces. Control surface 1 does not go through any flow non-linearities even though it has been placed quite close to the jet flow and at the same time it has a higher frequency resolution than the other two control surfaces since the grid spacing around it is finer than that around the other two surfaces.

It should be also mentioned at this point that although Bogey and Bailly\textsuperscript{33,34} used a computational grid that is comparable in size to our grid, their cutoff Strouhal number was 2.0 which is lower than our cutoff Strouhal number of 3.0 obtained with control surface 1. In their LES calculations, Bogey and Bailly\textsuperscript{33,34} use direct LES data in the immediate surroundings of the jet to evaluate the near field jet noise only. They do not employ integral acoustics methods to estimate the far field noise. Since they directly compute the noise in the near field only, their computational grid is more mildly stretched compared to ours. However, since we are employing integral acoustics methods in our methodology to compute the far field noise, we choose to pack most of the grid points inside the jet flow and do rapid grid stretching outside. Then, placing a control surface such as control surface 1 quite close to the jet flow allows us to obtain higher cutoff frequencies in the subsequent noise calculations.

The acoustic pressure spectra predicted by the 3 control surfaces were integrated over their corresponding resolved frequency range to determine the overall sound pressure levels (OASPL). The overall sound pressure level predictions of the FWH and Kirchhoff’s methods using the data gathered on the 3 control surfaces are given in figures 5 and 6, respectively. Since control surface 3 has the lowest cutoff frequency, the OASPL values predicted by control surface 3 are slightly lower than those predicted by control surface 2. Similarly, the OASPL values predicted by control surface 2 are lower than those predicted by control surface 1 that has the highest cutoff frequency. It should also be noted here that since a relatively short domain
length of $34r_o$ was used in this study, the streamwise length of the control surfaces is not sufficient to capture the acoustic waves travelling at the shallow angles, i.e., $\theta < 40^\circ$. Hence, the predicted OASPL values show a sharp drop-off at the shallow angles. Figures 7 and 8 compare the OASPL predictions of the FWH and Kirchhoff’s methods with the experimental data as well as with the SAE ARP 876C database prediction and the previous Reynolds number 100,000 jet LES result. Although the numerical predictions are a few dB louder than the experiments, the overall agreement is encouraging. One reason for the overprediction of the numerical results relative to experimental measurements is believed to be the inflow forcing employed in the simulations.

Next, we will do comparisons of our acoustic pressure spectra at 2 observation points with those of Bogey and Bailly. Figures 9 and 10 do the comparisons at the observation points $x = 29r_o$, $r = 12r_o$, and $x = 11r_o$, $r = 15r_o$, respectively. Our spectra at these points were computed by coupling the data on control surface 1 with the FWH method, while those of Bogey and Bailly are based on data directly provided by their LES. As can be seen from figure 9, the two spectra are quite similar at the observation point...
Fig. 11 Axial profile of the root mean square of axial fluctuating velocity along \( r = r_o \) and comparison with the profile of Bogey and Bailly.\(^{34}\)

\[ (v_x')_{rms}/U_o \]

\[ x = 29r_o, \ r = 12r_o. \] At this observation point, the cutoff frequency of Bogey and Bailly\(^{34}\) corresponds to Strouhal number 2.0, while our cutoff frequency is at Strouhal number 3.0 since that is the maximum frequency we can accurately resolve using the data gathered on control surface 1. These cutoff frequencies are also shown in the figure. It is seen that the spectrum of Bogey and Bailly is a few dB louder than ours. Figure 10 shows the spectra comparison at the observation point at \( x = 11r_o, \ r = 15r_o. \) This time, Bogey and Bailly’s\(^{34}\) cutoff frequency correspond to Strouhal number of about 1.1 due to their coarsened grid spacing at the given observation point. Our cutoff frequency at this location is still 3.0. The figure shows that Bogey and Bailly’s\(^{34}\) spectrum is similar to ours until Strouhal number 1.1, then their spectrum exhibits a sharp drop-off while ours continues until Strouhal number 3.0. Again, it is observed that their spectrum is a few dB louder than ours. At this stage, one might wonder as to why the far field noise predictions of Bogey and Bailly\(^{34}\) are not identical to ours since exactly the same inflow forcing was used and all other flow parameters were kept the same in both simulations. The answer is to be found in the different numerical methods used in the two simulations. Bogey and Bailly\(^{34}\) employed high-order accurate explicit finite differencing and explicit spatial filtering schemes in their calculations, whereas we have used implicit compact differencing and implicit spatial filtering schemes in our simulation. Since different numerical techniques have different characteristics, the differences observed are not surprising.

The differences between the two simulations can be further examined by comparing some turbulence statistics. The streamwise profiles of the root mean square of the fluctuating axial and radial velocities along \( r = r_o \) are plotted in figures 11 and 12, respectively and comparisons are done with the corresponding profiles of Bogey and Bailly.\(^{34}\) Although our profiles and theirs are qualitatively similar, their profiles have higher peak levels than ours. Furthermore, their peaks are located slightly downstream of ours. Bogey and Bailly\(^{34}\) have observed that the sideline noise levels are directly linked to the radial velocity fluctuations in the shear layer. They showed that the lower the peak of the radial velocity fluctuations in the jet shear layer, the lower the sideline noise levels. This observation and the fact that the peak of our fluctuating radial velocity profile given in figure 12 is lower than that of Bogey and Bailly\(^{34}\) explain why our acoustic pressure spectrum at the \( x = 11r_o, \ r = 15r_o \) observation location is lower than theirs over the frequency range \( 0 < St < 1.1 \) in figure 10. The reader is once again reminded of the fact that the Bogey and Bailly’s\(^{34}\) spectrum has a cutoff frequency at \( St = 1.1 \) at this observation location. Figures 13 and 14 show the centerline variation of the root mean square of the fluctuating axial and radial velocities and compare them with those of Bogey and Bailly.\(^{34}\) Our centerline profiles peak at a location downstream of their peaks. Our peaks are also seen to be lower than those of Bogey and Bailly.\(^{34}\) Our peak values for \( (v_x')_{rms}/U_o \) and \( (v_r')_{rms}/U_o \) on the jet centerline are 0.117 and 0.100, respectively, while those of Bogey and Bailly\(^{34}\) are 0.120 and 0.106, respectively. Arakeri et al.\(^{55}\) obtained an experimental value of 0.12 for the peak \( (v_x')_{rms}/U_o \) on the centerline of a jet with initially transient shear layers. The experimental jet studied by Arakeri et al.\(^{55}\) was a Mach 0.9 jet with Reynolds number 500,000. Lau et al.\(^{56}\) measured a
peak value of 0.14 for \((v'_x)_{rms}/U_o\) on the centerline of a Mach 0.9, Reynolds number 1 million jet with initially turbulent shear layers. Lau et al.\(^5\) also reported a value of 0.11 for the peak \((v'_r)_{rms}/U_o\) on the jet centerline from the same experiment. The numerical predictions for the peak \((v'_x)_{rms}/U_o\) and \((v'_r)_{rms}/U_o\) on the jet centerline are seen to be in reasonable agreement with the experimental measurements. In their study, Bogey and Bailly\(^3\) have also noted that the downstream noise levels are related to the centerline turbulence intensities. They showed that the lower the peak of the centerline turbulence intensities, the lower the downstream noise levels. Since our jet’s centerline turbulence intensities are lower than those of Bogey and Bailly,\(^3\) we can now see why our spectrum at the \(x = 29r_o\), \(r = 12r_o\) location is lower than theirs.

The aeroacoustics results presented so far were obtained using an open control surface. Due to the relatively short streamwise control surface length, the acoustic pressure signals at observation angles less than 40° on the far field arc were not accurately captured. To see the effects of closing the control surface at the outflow, a new study was conducted. A new control surface that starts one jet radius downstream of the inflow boundary and extends to 31\(r_o\) in the downstream direction was generated. This control surface is the same as the open control surface 1 used in the aeroacoustics studies presented earlier in this section. The current control surface is truncated at \(x = 31r_o\) whereas the previous open control surface extended up to \(x = 35r_o\). The outflow surface of the new control surface was placed at \(x = 31r_o\). Since the physical portion of the domain ends at around \(x = 35r_o\), placing the outflow surface a few radii away from the end of the physical domain ensures that the flow data gathered on the outflow surface are not affected by the presence of the sponge zone. The data gathered on the new control surface were coupled with the FWH method to compute the far field noise. No refraction corrections were included when the control surface was closed on the outflow surface. The FWH method required about 59 hours of computing time on 136 POWER3 processors of an IBM-SP3 processor to compute the 4096-point time history at a total of 504 far field points. Figure 15 plots the OASPL predictions on the far field arc which were obtained using the data gathered on the closed control surface and makes comparisons with other data. The OASPL values at all observation angles are shifted up by some amount when the control surface is closed at 10° of 20°.

![Fig. 13](image1.png)  
**Fig. 13** Centerline profile of the root mean square of axial fluctuating velocity and comparison with the profile of Bogey and Bailly.\(^3\)

![Fig. 14](image2.png)  
**Fig. 14** Centerline profile of the root mean square of radial fluctuating velocity and comparison with the profile of Bogey and Bailly.\(^3\)

![Fig. 15](image3.png)  
**Fig. 15** Overall sound pressure levels along the far field arc obtained using the open and closed control surfaces of streamwise length 30\(r_o\).
the outflow. The effect of the closed control surface appears to be minimal for the range of observation angles where $50^\circ < \theta < 65^\circ$, though. The increase of the OASPL at the shallow observation angles is expected, however, we clearly see a spurious effect for the range where $\theta > 80^\circ$. The OASPL profile in this range is very slowly decreasing with the observation angle.

A similar observation was recently made by Rahier et al. who conducted a study of surface integral acoustic methods and looked at the sensitivity of far field noise results to the placement of closed control surfaces in the non-linear flow field. The spurious effect observed here is attributed to the fact that we have placed a control surface inside the non-linear flow region ignoring the noise due to the quadrupole sources outside the control surface. Rahier et al. also gave the same reason for the similar spurious effects they have observed. We also believe that an effective line of dipole sources is created as the quadrupoles exit the downstream surface. These dipole sources can also be partially responsible for the spurious effects observed here. It is believed that moving the outflow surface further downstream will reduce the strength of the line of dipoles appearing on the outflow surface. This is supported by the observation of Rahier et al. which says that the spurious effects weaken as the outflow FWH control surface is moved further downstream.

Finally, we look at the acoustic pressure spectra predictions at the $\theta = 30^\circ$ and $\theta = 60^\circ$ locations on the far field arc. Figure 16 shows the two spectra at the $\theta = 30^\circ$ observation point which were computed using the open and closed control surfaces. It is clear that at the $\theta = 30^\circ$ location, the spectral energy is shifted up at all frequencies when the control surface is closed on the outflow surface. The spectra at $\theta = 60^\circ$ computed using the open and closed control surfaces are shown in figure 17. The differences in the two spectra are minor at this observation point, hence the outflow surface does not have much influence on the noise spectrum at the $\theta = 60^\circ$ observation location.

**Computation of the Far Field Noise Using Lighthill’s Acoustic Analogy**

We also computed the far field noise of the Reynolds number 400,000 jet using Lighthill’s acoustic analogy. As mentioned in the previous section, data from the simulation which was performed without any explicit SGS model were used for this purpose. The spatial filter was treated as the effective SGS model in the LES. Far field noise computations done using Lighthill’s acoustic analogy will be compared with the FWH method results as well as with some experimental noise spectra in this section. Due to space limitations, only a short summary of the noise computations using Lighthill’s acoustic analogy will be presented in this section. More details can be found in Uzun.

![Figure 16: Acoustic pressure spectra at $R = 60r_o$, $\theta = 30^\circ$ location on the far field arc. (Obtained using the open and closed control surfaces of streamwise length 30$r_o$.)](image1)

![Figure 17: Acoustic pressure spectra at $R = 60r_o$, $\theta = 60^\circ$ location on the far field arc. (Obtained using the open and closed control surfaces of streamwise length 30$r_o$.)](image2)
above equation is by no means a true or unique representation of the acoustic sources in the turbulent flow. The double divergence of $T_{ij}$ only serves as a nominal acoustic source and what it provides is an exact connection between the near field turbulence and the far field noise.

In Lighthill’s acoustic analogy, the sound generated by a turbulent flow is equivalent to what the quadrupole distribution $T_{ij}$ per unit volume would emit if placed in a uniform acoustic medium at rest. In other words, in this analogy the quadrupole distribution replaces the actual fluid flow and moreover, the sources may move, but the fluid in which they are embedded may not. The sources are embedded in a medium at rest that has the constant properties $\rho_\infty$, $p_\infty$, and $c_\infty$, the same as those in the ambient fluid external to the flow.

Lighthill\textsuperscript{15} shows that an approximation to the pressure fluctuations at points far enough from the flow region is given by a volume integral whose integrand contains the second time derivative of $T_{ij}$ that is evaluated at retarded times. This volume integral is performed over the entire turbulent flow region that contains the acoustic sources. The time derivative formulation of Lighthill’s volume integral will be used in this section for computing the far field noise.

Following Freund,\textsuperscript{58} we can split the Lighthill stress tensor, $T_{ij}$, into a mean component, $T_{ij}^m$, a component that is linear in velocity fluctuations, $T_{ij}^l$, a component that is quadratic in velocity fluctuations, $T_{ij}^q$ and the so-called entropy component, $T_{ij}^s$, as follows

$$T_{ij} = T_{ij}^m + T_{ij}^l + T_{ij}^q + T_{ij}^s,$$

where

$$T_{ij}^m = \rho \bar{u}_i \bar{u}_j + (\bar{p} - c_\infty^2 \rho) \delta_{ij},$$

$$T_{ij}^l = \rho \bar{u}_i \bar{u}_j',$$

$$T_{ij}^q = \rho u_i' u_j',$$

$$T_{ij}^s = (p' - c_\infty^2 \rho') \delta_{ij}.$$  

By definition, the mean component $T_{ij}^m$ does not make noise. In the above equations, density in the $\rho u_i u_j$ terms has not been decomposed into a mean and fluctuating part. Freund\textsuperscript{58} shows that the noise from $T_{ij}$ is nearly the same as that from

$$T_{ij}^q = \bar{p} u_i u_j + (p' - c_\infty^2 \rho') \delta_{ij}.$$  

Hence, the effect of the density fluctuations in the $\rho u_i u_j$ terms is not considered in this study. The noise from $T_{ij}^l$ is called the shear noise since this component consists of turbulent fluctuations interacting with the sheared mean flow. On the other hand, the noise from $T_{ij}^q$ is called the self noise since this component consists of turbulent fluctuations interacting with themselves, whereas the noise from $T_{ij}^s$ is called the entropy noise since it is composed of the density and pressure fluctuations in the turbulent flow that differ from the isentropic relation.

The 5 primitive flow variables were saved in nearly 7.5 million cell volumes inside the jet flow at every 10 time steps over a period of 40,000 time steps during the LES run. The sampling period corresponds to a time scale in which an ambient sound wave travels about 23 times the domain length in the streamwise direction. The flow data were saved in double precision format and the entire flow field database consisted of almost 1.2 Terabytes (TB) of data. Figures 18 through 20 depict the distribution of the second time derivative of $T_{ij}$ (where $n$ is the direction of the observer) that radiates noise in the direction of the observers located at 30°, 60°, and 90° on an arc of radius 60$r_o$ from the jet nozzle. The solid dark lines in these figures correspond to the boundaries of the volume in which time accurate LES data were saved. The volume starts at the inflow boundary and extends to 32 jet radii in the streamwise direction. The initial width and height of the volume are 10$r_o$ at the inflow boundary. At $x = 32r_o$, the width and height of the volume are 20$r_o$. As can be seen from the figures, the lateral boundaries are sufficiently far away from the sources that radiate noise. After a careful analysis of the spatial extent of the sources that radiate noise and based on the grid resolution in the region where the sources are located, the cutoff frequency in the subsequent noise calculations was found to be located at around Strouhal number 4. Based on the data sampling rate, there are about 4 temporal points per period in this highest resolved frequency. An 8th-order accurate explicit scheme will be employed for computing the time derivatives while computing the noise and 4 points per period is sufficient for this numerical scheme.\textsuperscript{48} It should also be noted here that this cutoff frequency is higher than the Strouhal number 3 cutoff frequency of the previous aeroacoustics computations that employed surface integral acoustics methods. This is because the grid spacing is finest inside the jet and gets coarser outside the jet flow. Hence, the maximum frequency that can be accurately captured by the control surfaces placed around the jet flow is lower than that can be captured in the volume integrals. Lighthill’s volume integral was computed using 1160 processors in parallel on the Lemieux cluster at the Pittsburgh Supercomputing Center.

As we had done previously, we computed the overall sound pressure levels and acoustic pressure spectra along a far field arc of radius 60$r_o$ from the jet nozzle in this study. There are 14 observer points on the far field arc. Even though acoustic pressure spectra were averaged over 36 equally spaced azimuthal points on a full circle at a given observer location in the previous section, we determined through numerical experimentation that averaging the spectra over 8 equally spaced
azimuthal points gives almost the same averaged spectrum as that obtained by averaging the spectra over 36 equally spaced azimuthal points. Hence, in order to save computing time, the acoustic pressure signals were computed at only 8 equally spaced azimuthal points on a full circle at a given $\theta$ location on the arc. The 3072-point time signal at a given observer location was then broken up into 3 1024-point signals and the 1024-point signals were used in the spectral analysis. Hence, there were a total of 24 1024-point signals available for a given observer point on the arc. The 24 acoustic pressure spectra were used to obtain the averaged spectrum which was integrated later to compute the overall sound pressure level at the given $\theta$ location on the arc. For 112 far field points, the total run time needed was about 15 hours on 1160 processors.

The distribution of the Lighthill sources that radiate noise in the direction of the observers located at 30°, 60°, and 90° on the far field arc was previously shown in figures 18 through 20. As can be seen from these figures, although the sources near the outflow boundary at $x = 32r_o$ seem to be relatively weak for the observers located at 30° and 60° on the arc, the sources that radiate noise in the direction of the observer at 90° are somewhat stronger near the outflow boundary. Thus it seems that a longer streamwise domain length will help the convecting acoustics sources to decay sufficiently. The implications of truncating the domain at $x = 32r_o$ will be discussed in more detail shortly.

We first looked at the effect of the integration domain size on the far field noise estimations. For this purpose, Lighthill’s volume integral was carried out for 3 different streamwise domain lengths. The domain lengths considered were 24$r_o$, 28$r_o$, and 32$r_o$, respectively. Figure 21 shows the Lighthill’s volume integral predictions for the 3 different streamwise domain lengths and makes comparisons with experimental data as well as with the open and closed control surface predictions of the FWH method from the previous section. From this figure, it is clear that as the integration domain size is increased from 24$r_o$ to 32$r_o$, the overall sound pressure levels at all observation angles other than those in the range 60° < $\theta$ < 80°, decrease as much as 2 dB. The changes in OASPL for observation angles greater than 90°, on the other hand, are on the order of 2 to 3 dB. This observation implies that there are significant noise cancellations taking place as one includes a longer streamwise length in the volume integration and such cancellations cause a reduction in the overall sound pressure levels for certain observation angles in the far field.

The difference in the OASPL predictions when the integration domain size is increased from 28$r_o$ to 32$r_o$ is within 1 dB for the observation angles $\theta$ < 80°. The OASPL curve that is obtained when the integration domain length is 32$r_o$ has an almost flat portion in the range where 80° < $\theta$ < 120°. On the other
hand, the experimental data show a continuous drop at those observation angles. The spurious effects observed here are quite similar to those observed in the previous section when the FWH method was applied on a closed control surface. It is also interesting to note that the MGB method\(^5\) shows that the sources beyond \(32r_o\) in a jet are not significant. It can also be argued at this point that the sudden truncation of the domain in the current computations creates spurious dipole sources on the outflow surface as the quadrupole sources pass through downstream surface. Such spurious dipole sources could be partially responsible for the behavior seen in the OASPL plot for the range where \(\theta > 80^\circ\). Since all acoustic sources decay as we move downstream, a longer streamwise domain will improve the predictions for \(\theta > 80^\circ\). In his study, Freund\(^58\) also observed substantial noise cancellations happening among the noise generated in different streamwise sections of the jet, however he did not get the spurious effects we observe here. His streamwise domain size was \(31r_o\). So it appears that although the acoustic sources decay by \(x = 31r_o\) for a low Reynolds number jet, they have a larger streamwise extent in the present high Reynolds number jet.

Figure 22 shows the OASPL values of the noise from \(T_{ij}\) and its individual components \(T_{ij}^s\), \(T_{ij}^l\) and \(T_{ij}^n\) when the streamwise integration domain extends up to \(32r_o\). Even though the current jet is an isothermal jet (\(T_o/T_\infty = 1\)), the entropy noise from \(T_{ij}^n\) is significant near the jet axis where the observation angle is small, but becomes insignificant at large angles. It is also observed from the figure that the shear noise from \(T_{ij}^s\) and the self noise from \(T_{ij}^l\) are louder than the total noise from \(T_{ij}\) for observation angles \(\theta < 40^\circ\) while the entropy noise from \(T_{ij}^n\) is louder than the total noise for \(\theta < 15^\circ\). The shear noise reaches its minimum

\[ \text{Correlation coefficient} \]

\[ C_{ln} = \frac{\langle p^l p^n \rangle}{p^{ln}p^{ln}}, \quad C_{ls} = \frac{\langle p^l p^s \rangle}{p^{ls}p^{ls}}, \quad C_{ns} = \frac{\langle p^n p^s \rangle}{p^{ns}p^{ns}} \]  

(11)

Fig. 21  Overall sound pressure levels of the noise from \(T_{ij}\) along the far field arc.

Fig. 22  Overall sound pressure levels of the noise from \(T_{ij}\) and its components along the far field arc. (Volume integrals are performed until \(x = 32r_o\).)

Fig. 23  Variation of the correlation coefficients along the far field arc.

OASPL value at around \(\theta = 80^\circ\) and starts to increase for larger angles, whereas the self noise exhibits a continuous drop. The fact that the shear noise, self noise and entropy noise components are more intense that the total noise for some observation angles near the jet axis implies that the noise from the different components must be correlated as suggested by Freund\(^58\) so that significant cancellations are happening among the noise generated by the individual components. To see the correlation between the noise components, we can define the following correlation coefficients,\(^58\)

\[ C_{ln} = \frac{\langle p^l p^n \rangle}{p^{ln}p^{ln}}, \quad C_{ls} = \frac{\langle p^l p^s \rangle}{p^{ls}p^{ls}}, \quad C_{ns} = \frac{\langle p^n p^s \rangle}{p^{ns}p^{ns}} \]  

(11)
where the superscripts $l$, $n$, and $s$ indicate the shear noise from $T_{ij}^l$, the self noise from $T_{ij}^n$ and the entropy noise from $T_{ij}^s$, respectively. The correlation coefficients $C_{ln}$, $C_{ls}$ and $C_{ns}$ are plotted in figure 23. For observation angles in the range $\theta < 40^\circ$, the shear noise from $T_{ij}^l$ cancels the self noise from $T_{ij}^n$ with a correlation coefficient on the order of $-0.6$. There is also some correlation between the self noise from $T_{ij}^n$ and the entropy noise from $T_{ij}^s$ at the small observation angles where the correlation coefficient has a value around $-0.3$. The shear noise from $T_{ij}^l$ and the entropy noise from $T_{ij}^s$ also cancel each other at very small angles near the jet axis with a correlation coefficient of almost $-0.3$. The correlation coefficient between these two noise components starts to rapidly move towards zero as the observation angle increases and reaches a positive value in between 0.1 and 0.2 at the $\theta = 40^\circ$ observation point. It then starts to approach towards the zero line for larger observation angles. All correlations reach a value close to zero at around the $\theta = 80^\circ$ and $\theta = 90^\circ$ locations.

Figures 24 through 26 depict the acoustic pressure spectra at the observation angles of $30^\circ$, $60^\circ$, and $90^\circ$ on the far field arc. Comparisons are shown for 3 streamwise domain lengths over which Lighthill’s volume integral is carried out. As mentioned earlier, the cutoff noise frequency in the current computations is located around Strouhal number 4, hence the portion of the spectra for frequencies greater than Strouhal number 4 is cut off. The spectra at the $\theta = 60^\circ$ location show minor changes as the integration domain is increased from $24r_o$ to $32r_o$, whereas there are more significant changes at the other observation angles. At the $30^\circ$ and $90^\circ$ observation angles, the high frequency part of the spectra shifts down as the integration domain increases along the streamwise direction. Spurious dipole line strength is reduced as the outflow surface is moved further downstream.

Next, we look at the noise spectra of the individual components of $T_{ij}$ at the observation locations of $30^\circ$ and $60^\circ$ on the far field arc. Figures 27 and 28 plot the total noise from $T_{ij}$, shear noise from $T_{ij}^l$, self noise from $T_{ij}^n$ and entropy noise from $T_{ij}^s$ for these two observation angles, respectively. As can be seen in figure 27, at $30^\circ$, the peak of the shear noise spectrum coincides with that of the total noise at around Strouhal number 0.3. We see the high frequency part of both the shear noise and the self noise spectra is more energetic than that of the total noise. The entropy noise is relatively weak over most of the frequency range when compared with the shear noise and the self noise. Figure 23 shows that at the observation angle of $30^\circ$, the
shear noise and the self noise cancel each other with a correlation coefficient of about -0.6, whereas the self noise and the entropy noise cancel each other with a correlation coefficient of about -0.3. Hence, the interaction of the shear noise, self noise and entropy noise at the 30° location results in a total noise spectrum that has reduced spectral energy levels in the high frequency range.

From figure 28, it is seen that the total noise spectrum is very similar to the self noise spectrum at the 60° location. The portion of the shear noise spectrum for Strouhal number greater than 2 has the same spectral energy levels as the self noise spectrum. Except for the low frequency region, the entropy noise is very weak over the entire frequency range. From figure 23, we now see that at the observation angle of 60°, the shear noise and the self noise cancel each other with a correlation coefficient of about -0.4, whereas the self noise and the entropy noise cancel each other with a correlation coefficient of about -0.15. Such an interaction among the noise components at the given observation point causes the total noise spectrum to be essentially the same as the self noise spectrum.

It seems appropriate at this point to compare the current far field noise spectra predictions with the results previously obtained by using the FWH method and also with some experimental noise spectra recently obtained from the NASA Glenn Research Center. Experimental noise data from a Mach 0.85 cold jet will be shown in the comparisons. The Mach number of this jet is close enough to that of our simulated jet. The estimated Reynolds numbers of the experimental jet is approximately 1.2 million, while the ratio of the jet temperature to the ambient temperature is 0.88. The far field noise spectra of the experimental jet were obtained at 40 jet diameters away from the nozzle. In order to facilitate the comparison with numerical results, the experimental noise spectra were shifted to 30 jet diameters away from the nozzle using the 1/r decay assumption of the acoustic waves. In this adjustment, the experimental SPL values were shifted upwards by approximately 2.5 dB/St. Moreover, since the Mach number of the experimental jet was not exactly 0.9, the experimental noise spectra were also adjusted for Mach 0.9 following the SAE ARP 876C guidelines. The spectra obtained from the previous Reynolds number 100,000 jet simulation will also be included in the comparisons. Figures 29 through 31 make comparisons at the observation locations of 30°, 60° and 90°, respectively. All spectra shown in these figures are curve fits to the actual data. The FWH method results for the Reynolds number 400,000 jet are shown both for the open and closed control surfaces. The reader is reminded here that the open control surface extends until the 31r_o downstream location. The slight difference in the streamwise extent of the surface and volume integrals is not expected to cause a significant difference in the comparisons. The FWH method results for the Reynolds number 100,000 jet were obtained using an open surface that extended 59r_o in the streamwise direction. At the 30° location, we see that the closed surface FWH method prediction for the Reynolds number 400,000 jet is in fairly good agreement with Lighthill’s acoustic analogy until Strouhal number 2 or so. Then, we observe higher spectral energy levels in Lighthill’s acoustic analogy prediction for higher frequencies. The open surface FWH method prediction for the Reynolds number 400,000 jet, on the
other hand, shows lower spectral energy levels at all frequencies. This is due to the fact that the relatively short open control surface cannot effectively capture the acoustic waves travelling at the shallow angles. It is interesting to note that the agreement of the shape of the Reynolds number 100,000 jet noise spectrum with the experimental spectrum until Strouhal number 1 is better than that between the Reynolds number 400,000 jet noise spectra and the experiment. The reason for this is believed to be the fact that the larger domain in the Reynolds number 100,000 LES allows a better evaluation of the lower frequencies. For the Reynolds number 400,000 jet, the Lighthill prediction seems to be showing the best qualitative agreement with the experimental noise spectrum at this observation location. The peaks of all noise spectra in the figure are seen to be in the Strouhal number 0.25 - 0.3 range. However, the experimental spectrum exhibits a much stronger decay right after the peak. The decay rate of the spectrum obtained from Lighthill’s acoustic analogy seems to be similar to the experimental spectrum decay rate in the frequency range where 1.5 < St < 3.0. Then, the Lighthill spectrum decays with a faster rate for the higher frequencies. At the 60° location, for the Reynolds number 400,000 jet, the FWH method yields almost identical results for the open and the closed control surfaces. The Lighthill prediction is also in acceptable agreement with the FWH prediction, considering the fact that the two methods are based on completely different formulations. The comparison with the experimental noise spectrum at this observation location reveals that the experimental peak is located at a lower frequency than that of the numerical predictions. Furthermore, the numerical results for the Reynolds number 400,000 jet show a faster spectrum decay rate at the higher frequencies. The decay of the Reynolds number 100,000 jet spectrum after the peak takes place at a faster rate than that observed in the Reynolds number 400,000 jet spectra as well as in the experiment. Finally, the comparison at the 90° location shows that the closed surface FWH prediction gives increased spectral energy levels relative to those given by the open surface FWH prediction. The Lighthill prediction is seen to be in between the two predictions given by the FWH method. It should be repeated here once again that from our previous analysis, the spectra of the Reynolds number 400,000 jet for observation angles in the range θ > 80° are expected to be affected by a spurious line of dipoles appearing on the outflow surface as the quadrupole sources move out of the control volume. The numerical predictions at this observation location once again reveal a spectrum decay rate that is larger than that of the experimental noise spectra. The decay of the the Reynolds number 100,000 jet spectrum takes place at a faster rate than that of the Reynolds number 400,000 jet spectra. The experimental peak is again located at a lower frequency than that of the numerical predictions. The comparison of the numerical OASPL predictions against the OASPL values of the NASA Mach 0.85 cold jet and the SAE ARP 876C54 database prediction along the far field arc is plotted in figure 32. We see OASPL differences as high as 6 dB between the numerical predictions and the NASA experimental jet. The agreement between the numerical OASPL values and the SAE ARP 876C54 prediction seems to be better.

The differences observed between the shape of the numerical and experimental noise spectra might be due to various reasons. One reason could be the mismatch of the inflow conditions in the numerical simulations with those in the actual experiment. The experiment was performed at a high enough Reynolds
number so that the jet shear layers at the nozzle exit were fully turbulent. In the numerical simulations, since it was deemed computationally too expensive to include the nozzle geometry, laminar shear layers were fed into the domain and randomized velocity fluctuations in the form of a vortex ring were imposed into the jet shear layers. Moreover, it has been observed experimentally\(^\text{61, 62}\) that high-frequency sources are located a small distance downstream of the jet nozzle and a significant portion of the noise spectrum originates from this near field of the jet. Hence, the high-frequency noise generated in the near-nozzle jet shear layer within a few diameters downstream of the nozzle exit is missing in the current simulations. The absence of the noise generated just downstream of the nozzle could be responsible for the faster decay rates in the high frequency range of the spectra in the current computations. The present findings once again emphasize the importance of correctly modelling the inflow conditions in jet noise simulations. It is also believed that the limited domain size in the simulations might influence the low frequencies.

**Concluding Remarks**

Using state-of-the-art numerical techniques, we have developed and tested a Computational Aeroacoustics (CAA) methodology for jet noise prediction. The CAA methodology has two main components. The first one is a 3-D Large Eddy Simulation (LES) code. The latest version of the LES code employs high-order accurate compact finite differencing as well as implicit spatial filtering schemes together with Tam and Dong’s boundary conditions on the LES domain boundaries. Explicit time integration is accomplished by means of the standard 4\(^{th}\)-order, 4-stage Runge-Kutta method. The localized dynamic Smagorinsky subgrid-scale model is utilized to model the effect of the unresolved scales on the resolved scales. The code also has the capability to turn off the dynamic SGS model and perform simulations by treating the spatial filter as an implicit SGS model. The second component of the CAA methodology consists of integral acoustics methods. We have developed acoustics codes that employ Kirchhoff’s and Ffowcs Williams - Hawkings (FWH) methods as well as Lighthill’s acoustic analogy.

In this paper, we presented results from an LES done for a Reynolds number 400,000 jet. A much more detailed report of this research can be found in Uzun.\(^\text{41}\) The time accurate LES data were coupled with integral acoustics methods for far field noise calculations. Far field aeroacoustics results also compared favorably with existing experimental measurements. The possible reasons for the discrepancies between numerical predictions and experiments were discussed. In our Reynolds number 400,000 jet simulations, the highest noise frequency resolved in the surface integral acoustics calculations corresponded to Strouhal number 3, while the highest frequency resolved when Lighthill’s acoustic analogy was employed corresponded to Strouhal number 4. Both of these frequencies are larger than Bogey and Bailly’s cutoff frequency of Strouhal number 2 in their recent Reynolds number 400,000 jet LES.\(^\text{33–35}\) Hence, to our best knowledge, the LES and the noise computations done for the Reynolds number 400,000 jet in this study are certainly some of the biggest calculations ever done in jet noise research. Moreover, our noise computations for the Reynolds number 400,000 jet have cutoff frequencies which are greater than the cutoff frequencies of all other jet noise LES results in the literature to date. Use of integral acoustics methods allows clustering of the majority of the grid points inside the jet

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**Fig. 31** Acoustic pressure spectra comparisons at the \( R = 60r_o, \theta = 90^\circ \) location on the far field arc.

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**Fig. 32** Overall sound pressure levels along the far field arc.
flow where non-linear noise generation takes place and rapid grid stretching outside the jet. Consequently, the maximum frequency resolved in noise computations using integral acoustics methods is higher compared to that captured in the simulations (with similar number of grid points) in which only the near field jet noise is computed using direct LES data. Finally, to the best of our knowledge, computation of the Lighthill volume integral over the turbulent near field of a turbulent jet at a reasonably high Reynolds number has been carried out for the first time in this study. The Lighthill stress tensor was decomposed into several components and the noise generated by the individual components was analyzed in detail. We found that significant cancellations occur among the noise generated by the individual components of the Lighthill stress tensor. Far field noise predictions using the FWH method on the closed control surface were found be comparable to those given by Lighthill’s volume integral. Moreover, Lighthill’s acoustic analogy was found to be about 40 times more computationally expensive than the FWH method. Hence, it is preferable to use the cheaper FWH method over the very expensive Lighthill volume integral. Hence, it is preferable to use the cheaper FWH method over the very expensive Lighthill volume integral.

Field jet turbulence and the far field noise is sought. However, if a connection between the near field jet turbulence and the far field noise is sought, then an analysis of the Lighthill source term inside the jet would be very useful. Both FWH (applied on a closed control surface) and Lighthill’s methods show increased OASPL levels for observation angles greater than $80^\circ$ on the far field arc. Such spurious effects are believed to be due to the spurious line of dipoles appearing on the outflow surface and the relatively short domain size in the streamwise direction. A longer domain will decrease the strength of the line of dipoles appearing on the outflow surface.

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References
