A STUDY OF SOUND GENERATION FROM TURBULENT HEATED ROUND JETS
USING 3-D LARGE EDDY SIMULATION

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To my family for their love and support
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post-processing were also carried out on Purdue University’s 120 processor Sun Fire 6800 supercomputer.
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**Table 3.2** Several turbulent flow results for our cold/isothermal and hot jet LES. \((U)\) and \((B)\) implies Uzun’s [59] and Bodony’s [23] LES, respectively. Zaman’s [87] results are correlations based on experiments.

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NOMENCLATURE

Roman Symbols

$C_{sgs}$  Subgrid-scale model constant in the original Smagorinsky model

$C_l$  Compressibility correction constant in the subgrid-scale model

$c$  Speed of sound

$D_j$  Jet nozzle diameter

$\mathbf{e}_\phi$  Unit vector in the $\phi$ direction of the spherical coordinate system

$\mathbf{e}_r$  Unit vector in the $r$ direction of the spherical coordinate system

$\mathbf{e}_\theta$  Unit vector in the $\theta$ direction of the spherical coordinate system

$e_t$  Total energy

$f$  Arbitrary variable; frequency

$\overline{f}$  Large scale component of variable $f$

$f_{sg}$  Subgrid-scale component of variable $f$

$\mathbf{F}, \mathbf{G}, \mathbf{H}$  Inviscid flux vectors in the Navier-Stokes equations

$\mathbf{F_v, G_v, H_v}$  Viscous flux vectors in the Navier-Stokes equations

$G(\mathbf{\bar{x}, \bar{x}'}, \Delta)$  Filter function

$J$  Jacobian of the coordinate transformation from physical to computational domain

$M$  Mach number
$M_j$ Mach number at jet nozzle exit

$M_r$ Reference Mach number

$N$ Number of grid points along a given spatial direction

$p$ Pressure

$Pr$ Prandtl number

$Pr_t$ Turbulent Prandtl number

$q, Q$ Vector of conservative flow variables

$q_i$ Resolved heat flux vector

$q', \text{RHS}(q; t)$ Right-hand side of the governing equations

$Q_i$ Subgrid-scale heat flux vector

$Q_{\text{target}}$ Target solution in the sponge zone

$Re$ Reynolds number

$Re_D$ Reynolds number based on jet diameter

$r_o$ Jet nozzle radius

$r_{1/2}$ Half velocity radius

$r$ Radial direction in cylindrical coordinates

$S$ Control surface

$\tilde{S}_{ij}$ Favre-filtered strain rate tensor

$Sr$ Strouhal number

$t$ Time
\( t_n \) Time step \( n \)

\( T_{ij} \) Subgrid-scale stress tensor

\( T \) Temperature

\( T_r \) Reference temperature

\( T_{ij} \) Lighthill stress tensor

\( T_{ij}^m \) Mean component of \( T_{ij} \)

\( T_{ij}^l \) Component of \( T_{ij} \) linear in velocity fluctuations

\( T_{ij}^n \) Component of \( T_{ij} \) non-linear in velocity fluctuations

\( T_{ij}^s \) Entropy component of \( T_{ij} \)

\( \bar{U}, \bar{V}, \bar{W} \) Contravariant velocity components

\( U_c \) Jet centerline velocity as a function of streamwise distance

\( U_o \) Jet centerline velocity at nozzle exit

\( U_r \) Reference velocity

\( \bar{u} = (\bar{u}, \bar{v}, \bar{w}) \) Mean velocity vector

\( u \) Velocity component in the \( x \) direction of Cartesian coordinates

\( (u, v, w) \) Velocity vector in Cartesian coordinates

\( u_i \) Alternate notation for \( (u, v, w) \)

\( v \) Velocity component in the transverse direction of Cartesian coordinates

\( v_r \) Velocity component in the radial \( (r) \) direction of cylindrical coordinates
$v_\theta$ Velocity component in the azimuthal ($\theta$) direction of cylindrical coordinates

$v_x$ Velocity component in the axial ($x$) direction of cylindrical coordinates

$V$ Integration volume

$V_g$ Acoustic group velocity

$w$ Velocity component in the $z$ direction of Cartesian coordinates

$(x_s, y_s, z_s)$ Location in Cartesian coordinates of source term used in Tam & Dong’s boundary conditions

$(x, y, z)$ Cartesian coordinates

$x$ Streamwise direction in both Cartesian and cylindrical coordinates

$y$ Transverse direction in Cartesian coordinates

$z$ Transverse direction in Cartesian coordinates

$x_i$ or $\tilde{x}$ Alternate notation for $(x, y, z)$

Greek Symbols

$\alpha_f$ Filtering parameter of the tri-diagonal filter

$\alpha$ Parameter that controls the strength of the vortex ring forcing

$\gamma$ Ratio of the specific heats of air

$\Delta$ Local grid spacing or eddy viscosity length scale

$\delta_{ij}$ Kronecker delta

$\delta_\omega$ Vorticity thickness of shear layer

$\Delta t$ Time increment
\( \Delta \xi \)  
Uniform grid spacing along the \( \xi \) direction in the computational domain

\( \theta \)  
Azimuthal direction in cylindrical coordinates; angle from downstream jet axis

\( \mu \)  
Molecular viscosity

\( \mu_r \)  
Reference viscosity

\( \nu \)  
Kinematic viscosity

\( \rho \)  
Density

\( \rho_r \)  
Reference density

\( \sigma_{ij} \)  
Normalized Reynolds stress components

\[
\sigma_{xx} = \frac{(v_x' v_x')}{U_c^2}
\] 
Normalized Reynolds normal stress in the axial \( (x) \) direction of cylindrical coordinates

\[
\sigma_{rr} = \frac{(v_r' v_r')}{U_c^2}
\] 
Normalized Reynolds normal stress in the radial \( (r) \) direction of cylindrical coordinates

\[
\sigma_{\theta\theta} = \frac{(v_\theta' v_\theta')}{U_c^2}
\] 
Normalized Reynolds normal stress in the azimuthal \( (\theta) \) direction of cylindrical coordinates

\[
\sigma_{rx} = \frac{(v_r' v_x')}{U_c^2}
\] 
Normalized Reynolds shear stress in cylindrical coordinates

\( \tau \)  
Retarded time

\( \tilde{\phi}_i \)  
Spatially filtered variable at grid point \( i \)

\( \chi \)  
Parameter that controls the strength of the sponge zone damping term

\( \Psi_{ij} \)  
Resolved shear stress tensor

\( (\xi, \eta, \zeta) \)  
Generalized curvilinear coordinates
Other Symbols

\( (\cdot)_i' \)  Spatial or time derivative at grid point \( i \)

\( (\cdot)_\eta \)  Spatial derivative along the \( \eta \) direction

\( (\cdot)_{\xi} \)  Spatial derivative along the \( \xi \) direction

\( (\cdot)_\zeta \)  Spatial derivative along the \( \zeta \) direction

\( \bar{\cdot} \)  Mean quantity

\( \overline{(\cdot)} \)  Spatially filtered quantity

\( \tilde{(\cdot)} \)  Favre averaged quantity

\( (\cdot)' \)  Perturbation from mean value; acoustic variable

\( (\cdot)_{\infty} \)  Ambient flow value

\( (\cdot)_o \)  Flow value at jet centerline on the nozzle exit

\( \{ \} \)  Time averaging operator

\[ \frac{\partial}{\partial x} \cdot \frac{\partial}{\partial y} \cdot \frac{\partial}{\partial z} \]  Partial spatial derivative operators in Cartesian coordinates

\[ \frac{\partial}{\partial \xi} \cdot \frac{\partial}{\partial \eta} \cdot \frac{\partial}{\partial \zeta} \]  Partial spatial derivative operators in computational domain

\[ \frac{\partial}{\partial t} \]  Partial time derivative operator

Abbreviations

BGK  Bhatnagar-Gross-Krook

CAA  Computational Aeroacoustics

DNS  Direct Numerical Simulation

FWH  Ffowcs Williams - Hawkings
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<th>Full Form</th>
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<tr>
<td>LAA</td>
<td>Lighthill’s Acoustic Analogy</td>
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<td>LBM</td>
<td>Lattice Boltzmann Method</td>
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<td>LES</td>
<td>Large Eddy Simulation</td>
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<tr>
<td>OASPL</td>
<td>Overall Sound Pressure Level</td>
</tr>
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<td>RANS</td>
<td>Reynolds Averaged Navier-Stokes</td>
</tr>
<tr>
<td>RHS</td>
<td>Right-Hand Side of the Navier-Stokes Equations</td>
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<tr>
<td>rms</td>
<td>Root Mean Square</td>
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<tr>
<td>SGS</td>
<td>Subgrid-Scale</td>
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<tr>
<td>SPL</td>
<td>Sound Pressure Level</td>
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ABSTRACT


Improvements in computing speed over the past decade have made Large Eddy Simulations (LES) amenable to the study of jet noise. The study of turbulent hot jets is required since all jet engines fitted on aircraft operate at hot exhaust conditions. The primary goal of this research was to further advance the science of jet noise prediction with a specific emphasis on heated jets using 3-D LES. For the 3-D LES methodology, spatial filtering is used as an implicit subgrid scale (SGS) model in place of an explicit SGS model, such as the classical Smagorinsky or Dynamic Smagorinsky models. To study the far-field noise, the porous Ffowcs Williams-Hawkings (FWH) surface integral acoustic formulation is employed. Results obtained for the heated jets in terms of jet development are in good agreement with other LES results and experimental data. The predicted overall sound pressure level (OASPL) values for heated jets exhibited the same trend as experimental data. The levels were over-predicted by approximately 3 dB, which was deemed satisfactory. An investigation of noise sources for heated jets was also performed within the framework of Lighthill’s acoustic analogy. It is discovered that when a high-speed is jet heated, significant cancellations occur between shear and entropy noise sources compared to an unheated high speed jet. This could explain why a high speed heated jet is quieter than an unheated jet at the same ambient Mach number.

High-order compact finite difference schemes along with high-order filters are used extensively in LES, especially for aeroacoustics problems, since these schemes have very high accuracy and spectral-like resolution as well as low-dispersion and diffusion errors. Due to the implicit nature of compact schemes, one technique of parallelization is based on the data transposition strategy. However, such transposition strategy is near impossible to
apply to jets with complex geometries. Hence, an alternative parallelization methodology based on the Schur complement technique was proposed to address the decomposition deficiency of the transposition strategy. Good scalability with a nearly linear was obtained for the 3-D Schur complement up to 1,024 processors on a CRAY XT3 supercomputer. The 3-D Schur complement is slightly slower compared to the transposition scheme by about 10% on a CRAY XT3. On a cluster with ethernet connection between compute nodes, the Schur complement was faster than the transposition scheme by approximately 20%. The computational overhead associated with the Schur complement matrices may be significant, and offset the two fold reduction in the communication time in some instances when compared to the transposition strategy. Nonetheless, the Schur complement is robust and has been able to handle a massive grid size of 2 billion grid points which ran on 4,096 processors.

Kinetic based methodologies such as the Lattice-Boltzmann Method (LBM) have been used extensively to model complex fluid flow phenomena. The LBM was used to study the far-field noise generated from a Mach 0.4 unheated turbulent axisymmetric jet. A commercial code based on the LBM kernel is used to simulate the turbulent flow exhausting from a pipe. Near-field flow results such as jet centerline velocity decay rates and turbulence intensities are in agreement with experimental results and results from comparable LES studies. The predicted far field sound pressure levels are within 2 dB of published experimental results. Weak unphysical tones are present at high frequency in the computed radiated sound pressure spectra. These tones are believed to be due to spurious sound wave reflections at boundaries between regions of varying voxel resolution and do not affect the overall levels significantly. The LBM appears to be a viable approach, comparable in accuracy to LES, for the problem considered. The main advantages of this approach over Navier-Stokes based finite difference schemes may be a reduced computational cost, ease of including the nozzle in the computational domain, and ease of investigating nozzles with complex shapes such as lobed mixers and chevrons.
1. INTRODUCTION

1.1 Motivation

Aviation is a vital part of the development and expansion of the economy. Over the past several years, airports have implemented stricter regulations for aircraft noise emissions. Aircraft noise may be detrimental to communities surrounding airports. For example, Hygge et al. [1] reported that primary schoolchildren who live in the vicinity of airports and are routinely exposed to aircraft noise may exhibit deficits in reading perception, long-term memory and speech perception. The same study reports a decrease in impairment when noise is lowered below a threshold value, or when the airport is closed. In the United States, NASA established the goal in 1997 of reducing the perceived noise levels of future subsonic aircraft by a factor of two (10 EPNdB) by 2007, and by a factor of four (20 EPNdB) by 2022 [2]. This goal is challenging due to the fact that the underlying sound generation mechanisms that cause aircraft noise are not very well understood and, therefore, cannot yet be fully controlled or optimized. Jet noise is believed to be the dominant contributor to aircraft noise at takeoff. Noise reduction at the source requires a deep understanding of the turbulent flow processes responsible for the generation of sound radiated in the surrounding environment. Jet noise, however, remains one of the most elusive problems in aeroacoustics due to the complexity of the flow-generated sound processes. A better understanding of the sound generation mechanisms in turbulent subsonic jets is needed.

1.2 Background

Continuing advances in high performance computing make the application of advanced computational techniques to jet noise predictions feasible. The most expensive approach
involves Direct Numerical Simulation (DNS) of the flow-field and the sound field. The

dynamics of all the relevant turbulent length scales are simulated and thus no form of
turbulence modeling is used. Freund et al. [3] were the first to study noise from a turbulent
jet using DNS. They simulated a Reynolds number 2,000, Mach 1.92 supersonic turbulent
jet. The computed overall sound pressure levels (OASPL) were compared to experimental
data and found to be in good agreement or within 3 dB with that of similar Mach number
jets. Later, Freund et al. [4] also used DNS to simulate a Reynolds number 3,600, subsonic
turbulent jet with a Mach number of 0.9 using roughly 25 million grid points without the
presence of a jet nozzle. The computed mean flow field and radiated sound field were in
excellent agreement to a similar laboratory experiment performed by Stromberg et al. [5].
Unfortunately, due to the wide range of time and length scales present in turbulent flows
and because of the limitations of current computational resources, DNS is still restricted
to low Reynolds number flows as shown in the examples above. As a rule of thumb for
DNS, the number of grid points necessary scales with the Reynolds number as $Re^{9/4}$ and
the computational cost scales as $Re^3$. Consider a Reynolds number $Re_D = 100,000$
turbulent jet which is the Reynolds number used in this study. Based on the number of
grid points used by Freund and for his 3-D DNS jet the estimated number of grid points
needed to resolve all relevant length scales would approximately be 470 million grid points.
This estimate does not include the nozzle in the simulation. If a nozzle was included, the
required number of grid points needed could be an order of magnitude more. The enormous
magnitude in terms computational resources required to solve such a DNS problem are very
expensive at this time.

In contrast with DNS, Large Eddy Simulations (LES) involve the direct computation
of the large scales, and models are used to capture the effects of the small scales or the
subgrid scales. This methodology reduces the computational costs relative to DNS. It is
assumed that the large scales in turbulence are generally more energetic compared to the
small scales, and are directly affected by the boundary conditions. In contrast, the small
scales are more dissipative, weaker, and tend to be more universal in nature. Most turbulent
jet flows that occur in experimental or industrial settings are at Reynolds numbers greater
than 100,000. In principal, it is possible to use LES as a tool for jet noise predictions, since this approach is capable of simulating high Reynolds number flows, at a fraction of the cost of DNS.

The Reynolds Average Navier-Stokes (RANS) method has also been applied to the study of jet noise. In this method, the Navier-Stokes equations are averaged through either time or space and relies on turbulence models to model all relevant scales in turbulence. The fidelity of jet noise predictions through RANS is still a topic of ongoing research (See [6–9]), since jet noise phenomena are inherently unsteady and involve multi-scales. Nonetheless, the computational expense of RANS is significantly lower than that of LES or DNS methods by an order of a magnitude.

Large Eddy Simulations have two important uses in the simulation of complex turbulent flows of engineering interest. Detailed simulations can be used to test and validate lower order models such as the Reynolds Average Navier-Stokes (RANS) $\kappa - \epsilon$, algebraic stress, and full Reynolds stress models. It can provide data that would be impossible otherwise to obtain experimentally, and which are at Reynolds numbers much higher than can be reached using DNS [10]. Large Eddy Simulations can also be used as an engineering tool to answer practical questions. Although LES still remains an expensive alternative (see reference [11]), it will likely be the tool of choice for computing complex flows for the forseeable future. In the context of Computational Aeroacoustics (CAA), the first use of LES as an investigative tool for jet noise prediction was carried out by Mankbadi, et al. [12]. They performed a simulation of a low Reynolds number supersonic jet and applied Lighthill’s analogy [13] to calculate the far-field noise. Lyrintzis and Mankbadi [14] were the first to use Kirchhoff’s method with LES to compute the far-field noise. Uzun et al. [15] used LES coupled with the Ffowcs Williams-Hawkins surface integral acoustic method to compute the far-field radiated noise of high Reynolds number turbulent subsonic jets. Similar numerical experiments [16–21] have been carried out by investigators at higher Reynolds numbers, and were also found to be in good agreement with experimental results. An excellent review article detailing the current status of LES for jet noise prediction can be found in Reference [22].
From a practical standpoint, it is desirable to study hot jets closely since jets fitted on all aircraft operate at hot exhaust conditions and at high Reynolds numbers. However, most LES jet studies that have been carried out to date consist of either cold or isothermal jets [15–21]. Only recently have LES simulations for hot jets have been performed. Bodony & Lele [23], for example, performed two LES simulations with different hot jet temperature ratios but at low Reynolds numbers of $Re_D = 13,000$ and $Re_D = 27,000$. Their results are consistent with the experimental observations of Tanna [24] and Bridges & Wernet [25]. However, they found some discrepancies in their overall sound pressure level (OASPL) results due to limited grid resolution. Andersson et al. [26] studied a $M_j = 0.75$, $Re_D = 50,000$ hot jet and the results obtained were found to be in good agreement with the experimental data of Jordan et al. [27, 28]. However, recent hot jet experiments by Viswanathan [29] suggest the presence of a Reynolds number effect. In terms of the acoustic spectra, the increased at lower frequencies with heating was attributed to a Reynolds number effect and not the contribution of dipole noise as hypothesized by Tester & Morfey [30]. He later suggested that a critical Reynolds number of at least $Re_D = 400,000$ is needed in order to avoid effects tied with low Reynolds numbers.

In computational aeroacoustics (CAA), high-order, low dispersion and low dissipation numerical schemes are used to accurately resolve the nonlinear near-field region (turbulent) and the far-field region (acoustic field). Examples of high-order numerical schemes are the implicit compact spatial differencing scheme proposed by Lele [31] and the compact spatial filter proposed by Visbal and Gaitonde [32]. Unsteady jet noise simulations that utilize high order numerical schemes for LES and DNS must be performed using parallel computers due to their large memory and storage requirements. This is unlikely to change in the foreseeable future. Efficient parallelization of high order numerical schemes such as the one proposed by Lele is not trivial. One such parallelization methodology is based on the transposition strategy, as explained in detail in Chapter 2. The transposition methodology has disadvantages for implementation to jets that exhibit complicated geometries. To address this problem, a multiblock approach can be used [33, 34]. In a multiblock approach, a complex domain is divided into smaller, more manageable domains and high-order com-
Pact differencing and filtering schemes are applied in each block. One strategy is to use grid point overlap, while maintaining sixth order differencing across block boundaries. Information is exchanged between neighboring blocks through the overlap points during the simulation [34]. However, up to 30% of the total number of grid points of the computational domain could consist of grid point overlaps for a large (more than one hundred) number of blocks. Alternatively, single-sided compact differencing schemes can be applied near and at the boundaries of each block [35]. This methodology undoubtedly is not as accurate as centered compact differencing or filtering scheme in the interior of the domain and thus might introduce numerical errors. A more robust and efficient parallelization methodology is required to overcome the substantial communication overhead and grid augmentation associated with overlaps, while maintaining the desirable characteristics of the compact differencing and filtering schemes throughout the solution domain.

The Schur complement method, which is also known as the sub-structuring technique has been widely used in structural mechanics to solve large-scale systems with limited memory computers for over 30 years [36]. It is essentially a domain decomposition approach whereby there is no grid overlap between domains. Instead each domain shares a common interface and information is exchanged at interfaces during a simulation. Despite the existence of high-memory systems coupled with low memory prices, the Schur complement finds applications in different areas of computational mechanics requiring parallel computing [37–39]. To the best of the author’s knowledge, the parallel Schur complement algorithm was proposed to solve a system of equations based on compact-like schemes by Eliasson [40, 41]. In this study, Eliasson solved a 2-D Vlasov-Maxwell equation for a plasma with mobile magnetized electrons and ions. Although the problem is two-dimensional, a Schur complement algorithm was applied only along one direction. An alternate parallel Schur complement method is needed with better scalability than the one proposed by Eliasson. The proposed Schur complement method should offer scalability for large-scale computational platforms, and follow closely the methodology proposed by Kocak and Akay [42] who used the Schur complement method in conjunction with a low order two-dimensional Finite Element Method (FEM).
In general, numerical simulations rely on solutions of the macroscopic Navier-Stokes equations. Recent advances have been made in kinetic based methodologies such as the lattice-Boltzmann method (LBM). These methods have been shown to be accurate for the simulation of complex fluid phenomena [43]. Whereas the Navier-Stokes equations maybe solved to obtain the macroscopic properties of the fluid explicitly, the LBM involves the solution of the time history lattice-Boltzmann equation (LBE) by explicitly tracking the development of particle distribution functions either at the mesoscopic or the microscopic scale. Through the use of the Chapman-Enskog expansion [44], the LBE has been shown to recover the compressible Navier-Stokes equation at the hydrodynamic limit [43, 45, 46].

The conserved variables such as density, momentum and internal energy are obtained by performing a local integration of the particle distribution. The LBM has recently been applied to a number of engineering problems, including flows over airfoils and cylinders [47–49], flow over rectangular cavities [50, 51], and most recently flows in the micro-scale regime [52].

1.3 Objectives of Present Study

The main goal of the present study is to further develop methods to calculate the sound generated by compressible turbulent hot jets through the coupled use of a 3-D LES methodology [53] to compute the near-field in conjunction with an acoustic integral technique [54] to predict the far-field radiated sound. The specific objectives are to:

1. Determine the accuracy of 3-D LES methods to capture the flow physics of compressible subsonic turbulent heated jets. Near-field flow results are compared and contrasted to other numerical and experimental observations.

2. Use the permeable Ffowcs Williams-Hawkings surface integral acoustic method to predict the far-field sound from heated jets and again compare the far-field acoustic results to existing numerical and experimental data.
3. Investigate the noise sources from turbulent heated jets and their contribution to the far-field within the framework of Lighthill’s acoustic analogy.

4. Propose an alternate parallelization methodology based on the Schur complement technique to alleviate the substantial communication overhead of the transposition strategy currently employed in 3-D LES codes. Extend its applicability to various computing platforms, multiple processors and grid domains.

5. Propose an alternate investigative tool for the prediction of jet noise through use of the lattice-Boltzmann method (LBM). A commercially available software based on the LBM kernel called PowerFLOW is used to investigate the noise generated from a weekly compressible turbulent isothermal jet with the presence of a nozzle lip in the computational domain.

1.4 Organization of the Thesis

This thesis is organized as follows. Chapter 2 describes the governing equations, numerical methods and some discussion of the parallelization of the 3-D LES code. Chapter 3 presents the near-field results of turbulent hot jets and compares them to other available numerical and experimental data. Chapter 4 discusses the far-field noise results using the Ffowcs Williams-Hawkings equation and Lighthill’s acoustic analogy. Implementation of an alternate parallelization methodology based on the Schur complement is presented in Chapter 5. Chapter 6 details the application of the lattice-Boltzmann method (LBM) to study a low-Mach number turbulent round jet through a circular nozzle.

Parts of this work were published as conference papers listed in References [55], [56], [57] and [58].
2. 3-D LARGE EDDY SIMULATION METHODOLOGY

2.1 Introduction

The 3-D LES methodology briefly described in this chapter was developed by Uzun [15, 59] as an initial platform to study the noise radiated from a high Reynolds number, subsonic turbulent jet. A more detailed description of this 3-D LES methodology is available in Reference [59]. This chapter also describes an alternate parallelization strategy based on the Schur complement technique.

2.2 Description of 3-D LES Methodology

2.2.1 Governing Equations

For LES, the turbulent field is decomposed into a large-scale or resolved-scale component ($\tilde{f}$) and a small-scale or subgrid-scale component ($f_{sg}$). Hence, for an arbitrary variable $f$,

$$ f = \tilde{f} + f_{sg}. \quad (2.1) $$

A filtering operation is applied to $f$ so that it maintains only the large-scale information, $\tilde{f}$. This filtering operation is defined as a convolution integral operated on $f$ as follows

$$ \tilde{f}(\tilde{x}) = \int_{V} G(\tilde{x}, \tilde{x}', \Delta) f(\tilde{x}') d\tilde{x}' \quad (2.2) $$

where $G(\tilde{x}, \tilde{x}', \Delta)$ is some spatial filter. Thus, the filtering operation removes the information of the small-scale structures and the resulting governing equations contain only the large-scale turbulent motions, while the effect of the small-scales on the resolved scales can be modeled by using a subgrid-scale (SGS) model such as the classical Smagorinsky model [60] or the more sophisticated and expensive dynamic Smagorinsky model proposed by Germano et al. [61].
The 3-D LES methodology is based on the compressible form of the Navier-Stokes equations. Hence, the large-scale component is written in terms of a Favre-filtered variable

\[ \tilde{f} = \frac{\rho f}{\tilde{\rho}}. \]  

(2.3)

The Favre-filtered, compressible, non-dimensionalized continuity, momentum, and energy equations are written in conservative form and are expressed as follows

\[ \frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial \tilde{\rho} \tilde{u}_i}{\partial x_i} = 0, \]  

(2.4)\[ \frac{\partial \tilde{\rho} \tilde{u}_i}{\partial t} + \frac{\partial \tilde{\rho} \tilde{u}_i \tilde{u}_j}{\partial x_j} + \frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial}{\partial x_j}(\Psi_{ij} - T_{ij}) = 0, \]  

(2.5)\[ \frac{\partial \tilde{e}_i}{\partial t} + \frac{\partial \tilde{u}_i(\tilde{e}_i + \tilde{p})}{\partial x_i} - \frac{\partial}{\partial x_j}(\tilde{u}_j(\Psi_{ij} - T_{ij}) + \frac{\partial}{\partial x_i}(q_i + Q_i) = 0. \]  

(2.6)

In the momentum equation, the resolved shear stress tensor is given by the expression

\[ \Psi_{ij} = \frac{2\tilde{\mu}}{Re}(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}). \]  

(2.7)

whereas the Favre-filtered strain rate tensor is given by

\[ \tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_j}{\partial x_i} + \frac{\partial \tilde{u}_i}{\partial x_j}\right). \]  

(2.8)

In the energy equation, the total energy is defined as

\[ \tilde{\epsilon}_i = \frac{1}{2}\tilde{\rho} \tilde{u}_i \tilde{u}_i + \frac{\tilde{p}}{\gamma - 1}, \]  

(2.9)

and the resolved heat flux is

\[ q_i = -\left[\frac{\tilde{\mu}}{(\gamma - 1)M_r^2RePr}\right] \frac{\partial \tilde{T}}{\partial x_i}. \]  

(2.10)

The temperature \( \tilde{T} \) is obtained from using the filtered pressure and density via the ideal gas relation

\[ \tilde{p} = \frac{\tilde{\rho} \tilde{T}}{\gamma M_r^2}, \]  

(2.11)

Sutherland’s law is used for the molecular viscosity

\[ \frac{\tilde{\mu}}{\mu_r} = \left(\frac{\tilde{T}}{T_r}\right)^{3/2} \left(\frac{\gamma}{T_r} + S\right). \]  

(2.12)
The Sutherland constant, $S$, is set to $110^\circ K$, while the reference temperature at the center-line is $T_r = 286^\circ K$, and the molecular viscosity, $\mu_r$, is set as the jet centerline temperature viscosity.

Due to the filtering operation, additional terms appear in the momentum and energy equations, i.e. the subgrid-scale stress tensor and subgrid-scale heat flux expressed as

$$T_{ij} = \bar{\rho}(\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j). \quad (2.13)$$

$$Q_i = \bar{\rho}(\bar{u}_i \bar{T} - \bar{u}_i \bar{T}). \quad (2.14)$$

The single-block 3-D LES code was developed by Uzun et al. [53, 62] and it includes both the classical [60] and a localized dynamic [63] Smagorinsky (DSM) subgrid-scale model. However, the modeling of the subgrid-scale stress tensor still raises some fundamental issues as discussed by Bogey & Bailly [64, 65]. Eddy-viscosity models such as the classical Smagorinsky subgrid-scale model [60] and the localized dynamic subgrid-scale model (DSM) [61, 63] might dissipate the turbulent energy through a wide range of scales up to the larger ones, which should be dissipation free at sufficiently high Reynolds numbers [66]. In addition, since the eddy-viscosity has the same functional form as the molecular viscosity, the effective Reynolds number is reduced in the simulated flows [67]. References [64] and [68] provide a thorough analysis and discussion of the shortcomings of the eddy viscosity subgrid-scale model on jet flows. An alternative to the use of an explicit eddy-viscosity model is the use of spatial filtering for modeling the effects of the subgrid-scales. This approach minimizes the amount of dissipation on the smaller resolved scales. Using this alternative, the turbulent energy is only dissipated when it is transferred from the larger scales to the smaller scales discretized by the grid [64, 68]. Hence, for the jet simulated here, we set $T_{ij} = 0$ and $Q_i = 0$. In place of an explicit SGS model, a spatial filter [32] was used as an implicit SGS model to damp the turbulent energy.

Considering a near sonic jet, the unsteady, Favre-filtered, compressible, non-dimensional LES equations were solved. Transformation from curvilinear coordinates to a uniform grid in computational space is needed. The transformed governing equations can be written as

$$\frac{1}{J} \frac{\partial Q}{\partial t} + \frac{\partial}{\partial \xi} \left( \frac{F - F_y}{J} \right) + \frac{\partial}{\partial \eta} \left( \frac{G - G_y}{J} \right) + \frac{\partial}{\partial \zeta} \left( \frac{H - H_y}{J} \right) = 0. \quad (2.15)$$
Here $t$ is the time, $\xi$, $\eta$, and $\zeta$ are the corresponding generalized coordinates in computational space, and $J$ is the Jacobian of the coordinate transformation from the physical space to computational space, which is expressed as

$$J = \frac{1}{x_\xi(y_\eta z_\zeta - y_\zeta z_\eta) - x_\eta(y_\xi z_\zeta - y_\zeta z_\xi) + x_\zeta(y_\xi y_\zeta - y_\eta z_\xi)}.$$  \hfill (2.16)

In Equation (2.15) the bold face variables are the vector quantities and are expressed as

$$\mathbf{Q} = \begin{bmatrix} \ddot{\rho} \\ \ddot{\rho}u \\ \ddot{\rho}v \\ \ddot{\rho}w \\ \ddot{\varepsilon}_t \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \ddot{\rho}\mathbf{U} \\ \ddot{\rho}u\mathbf{U} + \xi_x \dddot{\rho} \\ \ddot{\rho}v\mathbf{U} + \xi_y \dddot{\rho} \\ \ddot{\rho}w\mathbf{U} + \xi_z \dddot{\rho} \\ (\dddot{\varepsilon}_t + \dddot{\rho})\mathbf{U} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \ddot{\rho}\mathbf{V} \\ \ddot{\rho}u\mathbf{V} + \eta_x \dddot{\rho} \\ \ddot{\rho}v\mathbf{V} + \eta_y \dddot{\rho} \\ \ddot{\rho}w\mathbf{V} + \eta_z \dddot{\rho} \\ (\dddot{\varepsilon}_t + \dddot{\rho})\mathbf{V} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \ddot{\rho}\mathbf{W} \\ \ddot{\rho}u\mathbf{W} + \zeta_x \dddot{\rho} \\ \ddot{\rho}v\mathbf{W} + \zeta_y \dddot{\rho} \\ \ddot{\rho}w\mathbf{W} + \zeta_z \dddot{\rho} \\ (\dddot{\varepsilon}_t + \dddot{\rho})\mathbf{W} \end{bmatrix}.$$  \hfill (2.17)

$$\mathbf{F}_v = \begin{bmatrix} F_{v1} \\ F_{v2} \\ F_{v3} \\ F_{v4} \\ F_{v5} \end{bmatrix}, \quad \mathbf{G}_v = \begin{bmatrix} G_{v1} \\ G_{v2} \\ G_{v3} \\ G_{v4} \\ G_{v5} \end{bmatrix}, \quad \mathbf{H}_v = \begin{bmatrix} H_{v1} \\ H_{v2} \\ H_{v3} \\ H_{v4} \\ H_{v5} \end{bmatrix},$$  \hfill (2.18)

$$\begin{bmatrix} F_{v1} \\ F_{v2} \\ F_{v3} \\ F_{v4} \\ F_{v5} \end{bmatrix} = \begin{bmatrix} 0 \\ \xi_x (\Psi_{xx} - T_{xx}) + \xi_y (\Psi_{xy} - T_{xy}) + \xi_z (\Psi_{xz} - T_{xz}) \\ \xi_x (\Psi_{xy} - T_{xy}) + \xi_y (\Psi_{yy} - T_{yy}) + \xi_z (\Psi_{yz} - T_{yz}) \\ \xi_x (\Psi_{xz} - T_{xz}) + \xi_y (\Psi_{yz} - T_{yz}) + \xi_z (\Psi_{zz} - T_{zz}) \\ \ddot{u} F_{v2} + \ddot{v} F_{v3} + \ddot{w} F_{v4} - \xi_x (q_x + Q_x) - \xi_y (q_y + Q_y) - \xi_z (q_z + Q_z) \end{bmatrix}.$$  \hfill (2.19)
\[
\begin{bmatrix}
G_{v1} \\
G_{v2} \\
G_{v3} \\
G_{v4} \\
G_{v5}
\end{bmatrix} =
\begin{bmatrix}
0 \\
\eta_x(\Psi_{xx} - T_{xx}) + \eta_y(\Psi_{xy} - T_{xy}) + \eta_z(\Psi_{xz} - T_{xz}) \\
\eta_x(\Psi_{xy} - T_{xy}) + \eta_y(\Psi_{yy} - T_{yy}) + \eta_z(\Psi_{yz} - T_{yz}) \\
\eta_x(\Psi_{xz} - T_{xz}) + \eta_y(\Psi_{yz} - T_{yz}) + \eta_z(\Psi_{zz} - T_{zz}) \\
\tilde{u}G_{v2} + \tilde{v}G_{v3} + \tilde{w}G_{v4} - \eta_x(q_x + Q_x) - \eta_y(q_y + Q_y) - \eta_z(q_z + Q_z)
\end{bmatrix}
\]

(2.20)

\[
\begin{bmatrix}
H_{v1} \\
H_{v2} \\
H_{v3} \\
H_{v4} \\
H_{v5}
\end{bmatrix} =
\begin{bmatrix}
0 \\
\zeta_x(\Psi_{xx} - T_{xx}) + \zeta_y(\Psi_{xy} - T_{xy}) + \zeta_z(\Psi_{xz} - T_{xz}) \\
\zeta_x(\Psi_{xy} - T_{xy}) + \zeta_y(\Psi_{yy} - T_{yy}) + \zeta_z(\Psi_{yz} - T_{yz}) \\
\zeta_x(\Psi_{xz} - T_{xz}) + \zeta_y(\Psi_{yz} - T_{yz}) + \zeta_z(\Psi_{zz} - T_{zz}) \\
\tilde{u}H_{v2} + \tilde{v}H_{v3} + \tilde{w}H_{v4} - \zeta_x(q_x + Q_x) - \zeta_y(q_y + Q_y) - \zeta_z(q_z + Q_z)
\end{bmatrix}
\]

(2.21)

where \( \mathbf{Q} \) is the vector of conservative flow variables, \( \mathbf{F}, \mathbf{G}, \) and \( \mathbf{H} \) are the inviscid flux vectors, \( \mathbf{F}_v, \mathbf{G}_v, \) and \( \mathbf{H}_v \) are the viscous flux vectors. \( \tilde{U}, \tilde{V}, \tilde{W} \) are given by

\[
\tilde{U} = \xi_x \tilde{u} + \xi_y \tilde{v} + \xi_z \tilde{w}.
\]

(2.22)

\[
\tilde{V} = \eta_x \tilde{u} + \eta_y \tilde{v} + \eta_z \tilde{w}.
\]

(2.23)

\[
\tilde{W} = \zeta_x \tilde{u} + \zeta_y \tilde{v} + \zeta_z \tilde{w}.
\]

(2.24)

Note that \( T_{ij} \) and \( Q_i \) are set to zero. Furthermore, \( \xi_x, \xi_y, \xi_z, \eta_x, \eta_y, \eta_z, \zeta_x, \zeta_y, \zeta_z \) are the grid transformation metrics. To ensure metric cancellation for a general 3-D curvilinear grids when high-order spatial discretization schemes are used, the code uses the following “conservative” form of evaluating the metric expressions [32]

\[
\xi_x / J = (y_\eta z)_\xi - (y_\xi z)_\eta,
\]

\[
\eta_x / J = (y_\xi z)_\eta - (y_\xi z)_\xi,
\]

(2.25)

\[
\zeta_x / J = (y_\xi z)_\eta - (y_\eta z)_\xi.
\]
\[ \xi_y/J = (z_n x)_{\xi} - (z_x x)_{\eta}, \]
\[ \eta_y/J = (z_x x)_{\xi} - (z_{\xi} x)_{\xi}, \]
\[ \zeta_y/J = (z_{\xi} x)_{\eta} - (z_x x)_{\xi}, \]

\[ \xi_z/J = (x_{\eta} y)_{\xi} - (x_{\xi} y)_{\eta}, \]
\[ \eta_z/J = (x_{\xi} y)_{\xi} - (x_{\xi y})_{\xi}, \]
\[ \zeta_z/J = (x_{\xi y})_{\eta} - (x_{\xi y})_{\xi}. \]

The grid filter width, \( \Delta \) is given as

\[ \Delta = \left( \frac{1}{J} \right)^{1/3}. \] (2.28)

### 2.2.2 Numerical Methods

As mentioned in the previous section, the 3-D LES code solves the governing equations in computational space where the grid spacing is uniform. The spatial derivatives at the interior grid points away from the boundaries are computed using a non-dissipative sixth-order compact scheme proposed by Lele [31]

\[ \frac{1}{3} f'_{i-1} + f'_i + \frac{1}{3} f'_{i+1} = \frac{7}{9\Delta\xi}(f_{i+1} - f_{i-1}) + \frac{1}{36\Delta\xi}(f_{i+2} - f_{i-2}). \] (2.29)

Here, \( f'_i \) is the approximation of the first derivative of \( f \) at point \( i \) in the \( \xi \) direction, and \( \Delta\xi \) is the grid spacing in the \( \xi \) direction which is uniform. For the points next to the boundaries, \( i = 2 \) and \( i = N - 1 \), the following fourth-order central compact scheme used

\[ \frac{1}{4} f'_1 + f'_2 + \frac{1}{4} f'_3 = \frac{3}{4\Delta\xi}(f_3 - f_1). \] (2.30)

\[ \frac{1}{4} f'_{N-2} + f'_{N-1} + \frac{1}{4} f'_N = \frac{3}{4\Delta\xi}(f_N - f_{N-2}). \] (2.31)
Finally, for the points on the left and right boundary, i.e. \( i = 1 \) and \( i = N \), the following one-sided third-order compact scheme is used

\[
f'_1 + 2f'_2 = \frac{1}{2\Delta \xi}(-5f_1 + 4f_2 + f_3). \tag{2.32}
\]

\[
f'_N + 2f'_{N-1} = \frac{1}{2\Delta \xi}(5f_N - 4f_{N-1} - f_{N-2}). \tag{2.33}
\]

In order to eliminate numerical instabilities that can arise from the boundary conditions, unresolved scales, and mesh non-uniformities, the sixth-order tri-diagonal spatial filter proposed by Visbal and Gaitonde [32] is employed for the interior grid points

\[
\alpha_f \overline{f}_{i-1} + \overline{f}_i + \alpha_f \overline{f}_{i+1} = \sum_{n=0}^{3} \frac{a_n}{2} (f_{i+n} + f_{i-n}), \tag{2.34}
\]

where the \( a_n \) coefficients are defined as

\[
a_0 = \frac{11}{16} + \frac{5\alpha_f}{8} \quad a_1 = \frac{15}{32} + \frac{17\alpha_f}{16} \quad a_2 = -\frac{3}{16} + \frac{3\alpha_f}{8} \quad a_3 = \frac{1}{32} - \frac{\alpha_f}{16}. \tag{2.35}
\]

The parameter \( \alpha_f \) satisfies the inequality given by \(-0.5 < \alpha_f < 0.5\). A higher value value of \( \alpha_f \) implies a less dissipative filter. Setting \( \alpha_f = 0.5 \) implies no filtering effect. In the 3-D LES code, the filter coefficient is set to \( \alpha_f = 0.47 \). Now, for the points next to the left-hand side boundary, i.e. \( i = 2, 3 \), the following sixth-order, one-sided right-hand side stencil is used [32]

\[
\alpha_f \overline{f}_{i-1} + \overline{f}_i + \alpha_f \overline{f}_{i+1} = \sum_{n=1}^{7} a_{n,i} f_n \quad i = 2, 3, \tag{2.36}
\]

where

\[
\begin{align*}
a_{1,2} &= \frac{1}{64} + \frac{31\alpha_f}{32} & a_{2,2} &= \frac{29}{32} + \frac{3\alpha_f}{16} & a_{3,2} &= \frac{15}{64} + \frac{17\alpha_f}{32}, \\
a_{4,2} &= -\frac{5}{16} + \frac{5\alpha_f}{8} & a_{5,2} &= \frac{15}{64} - \frac{15\alpha_f}{32} & a_{6,2} &= -\frac{3}{32} + \frac{3\alpha_f}{16}, \\
a_{7,2} &= \frac{1}{64} - \frac{\alpha_f}{32},
\end{align*}
\]

and

\[
\begin{align*}
a_{1,3} &= -\frac{1}{64} + \frac{\alpha_f}{32} & a_{2,3} &= \frac{3}{32} + \frac{13\alpha_f}{16} & a_{3,3} &= \frac{49}{64} + \frac{15\alpha_f}{32},
\end{align*}
\]
\[ a_{4,3} = \frac{5}{16} + \frac{3\alpha_f}{8} \quad a_{5,3} = \frac{-15}{64} + \frac{15\alpha_f}{32} \quad a_{6,3} = \frac{3}{32} - \frac{3\alpha_f}{16}, \]  
\[ a_{7,3} = \frac{-1}{64} + \frac{\alpha_f}{32}. \]  

A similar procedure is applied for the points near the right boundary point, \( i = N \)

\[ \alpha_f \overline{f}_{i-1} + \overline{f}_i + \alpha_f \overline{f}_{i+1} = \sum_{n=0}^{6} a_{n-N,i} \overline{f}_{n-N} \quad i = N - 2, N - 1, \]  

where

\[ a_{n-N,i} = a_{n+1,N-i+1} \quad i = N - 2, N - 1 \quad n = 0, 6. \]

The boundary points, \( i = 1 \) and \( i = N \) are left unfiltered. This spatial filter may be used as an implicit SGS model since both the classical Smagorinsky and localized Dynamic Smagorinsky models are ignored.

### 2.2.3 Boundary Conditions

Tam and Dong’s [69] radiation and outflow boundary conditions were implemented. This boundary condition was originally developed in 2-D and was recently extended to 3-D by Bogey and Bailly [70]. The radiation boundary conditions in spherical coordinates are given by

\[ \frac{1}{V_g} \frac{\partial}{\partial t} \begin{pmatrix} \rho \\ u \\ v \\ w \\ p \end{pmatrix} + \left( \frac{\partial}{\partial r} + \frac{1}{r} \right) \begin{pmatrix} \rho - \bar{\rho} \\ u - \bar{u} \\ v - \bar{v} \\ w - \bar{w} \\ p - \bar{p} \end{pmatrix} = 0, \]  

and are applied to the lateral boundaries of the computational domain shown in Figure 2.1. Here, \( \rho, u, v, w, p \) are the local primitive flow variables on the boundary, \( \bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{p} \) are the local mean flow properties, \( V_g \) is the acoustic group velocity expressed as

\[ V_g = (\bar{u} + \bar{c}) \cdot e_r = \bar{u} \cdot e_r + \sqrt{c^2 - (\bar{u} \cdot e_\theta)^2 - (\bar{u} \cdot e_\phi)^2}. \]  

From the above equation, \( \bar{c} \) is the local mean sound velocity vector defined as the vector from the acoustic source location, \( (x_s, y_s, z_s) \), to the boundary point at which \( V_g \) is being
computed. This vector is the local mean sound speed velocity vector. $e_r, e_\theta, e_\phi$ denote the unit vectors in $r$, $\theta$ and $\phi$ directions of the spherical coordinate system. These unit vectors can be expressed in terms of Cartesian coordinates as

$$e_r = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$  
$$e_\theta = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta),$$  
$$e_\phi = (-\sin \phi, \cos \phi, 0).$$  

(2.43)

The acoustic group velocity, $V_g$, is the same as the wave propagation speed, and is equal to the projection of the vector sum of the local mean sound velocity and local mean flow velocity onto the sound propagation direction. It is assumed that in the far-field, the outgoing acoustics disturbances are propagating in the radial direction relative to the acoustic source [59].

The position vector, $r$ is obtained through

$$r = \sqrt{(x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2},$$  

(2.44)

where $x, y, z$ are the coordinates of the boundary point, and $x_s, y_s, z_s$ are the coordinates of the acoustic source location. The source location is usually chosen as the end of the potential core of the jet and in this case it is set to $(x_s, y_s, z_s) = (10r_o, 0, 0)$ in the 3-D LES code (where $r_o$ is one jet radius). The derivative along the $r$ direction is expressed in terms of the derivatives in the Cartesian coordinate system as follows

$$\frac{\partial}{\partial r} = \nabla \cdot e_r = \sin \theta \cos \phi \frac{\partial}{\partial x} + \sin \theta \sin \phi \frac{\partial}{\partial y} + \cos \theta \frac{\partial}{\partial z},$$  

(2.45)

and $\nabla$ is the gradient operator in the Cartesian coordinate system. On the outflow boundary, however, where entropy and vorticity waves in addition to the acoustic waves cross, the above formulation of radiation Tam and Dong’s is not suitable. On the outflow, the following formulation is used [70]

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla (\rho - \bar{\rho}) = \frac{1}{c^2} \left( \frac{\partial p}{\partial t} + \bar{\mathbf{u}} \cdot \nabla (p - \bar{p}) \right),$$

$$\frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla (u - \bar{u}) = -\frac{1}{\bar{\rho}} \frac{\partial (p - \bar{p})}{\partial x},$$
\[
\frac{\partial v}{\partial t} + \mathbf{u} \cdot \nabla (v - \bar{v}) = -\frac{1}{\bar{\rho}} \frac{\partial (p - \bar{p})}{\partial y}, \tag{2.46}
\]
\[
\frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla (w - \bar{w}) = -\frac{1}{\bar{\rho}} \frac{\partial (p - \bar{p})}{\partial z},
\]
\[
\frac{1}{V_g} \frac{\partial p}{\partial t} + \frac{\partial (p - \bar{p})}{\partial r} + \frac{(p - \bar{p})}{r} = 0.
\]

For a more in-depth discussion on the numerical implementation methodology of this boundary condition in the 3-D LES code, refer to Uzun [59].

In addition, a sponge zone [71] was included in the end of the computational domain to dissipate the vortices present in the flow field before they reach the outflow boundary. This is done so that unwanted reflections from the outflow boundary are suppressed. Grid stretching as well as explicit filtering are applied along the streamwise direction in the sponge zone to dissipate the vortices before they exit the outflow boundary. Uzun [59] reports that for the sponge zone the combination of explicit filtering and Tam and Dong’s outflow boundary conditions were found to be stable and did not cause any problems. In the sponge zone, the turbulent flow field is forced towards a target solution with the use of a damping term added to the right-hand side of the governing equations

\[
\frac{\partial \mathbf{Q}}{\partial t} = RHS - \mathcal{X}(x)(\mathbf{Q} - \mathbf{Q}_{target}). \tag{2.47}
\]

where the damping term \( \mathcal{X}(x) \) is expressed as

\[
\mathcal{X}(x) = \mathcal{X}_{max} \left( \frac{x - x_{phy}}{x_{end} - x_{phy}} \right)^3. \tag{2.48}
\]

Here, \( RHS \) is the right hand side of the governing equations, \( x \) is the streamwise coordinate in the sponge zone, \( x_{phy} \) is the streamwise coordinate of the end of the physical domain and \( x_{end} \) is the streamwise coordinate of the of the end of the sponge zone. \( \mathbf{Q} \) as before, is the vector containing the conservative variables, \( \mathbf{Q}_{target} \) is the target solution in the sponge zone, and \( \mathcal{X}(x) \) is function that determines the strength of the damping term. In the 3-D LES code, the damping amplitude, \( \mathcal{X}_{max} \) is set to 1.0 and the target solution is specified as the self-similar solution of an isothermal incompressible round jet which can be found in Pope [72].
2.2.4 Vortex Ring Forcing

To excite the mean flow into turbulence, randomized perturbations in the form of induced velocities from a vortex ring [73] were added to the velocity profile at a short distance (approximately one jet radius) downstream from the inflow boundary. This was done to ensure the break up of the potential core. The length of the potential core here was determined by the location where the jet centerline velocity is reduced to 95% of the inflow jet velocity, \( U_c(x_c) = 0.95U_j \). The streamwise and radial velocity components of the vortex ring \((v_x,v_r)\) were added to the local velocity components \((v_{xo},v_{ro})\) as shown by the formulation below

\[
v_x = v_{xo} + \alpha U_{xring} U_o \sum_{n=0}^{n_k} \epsilon_n \cos(n\Theta + \varphi_n)
\]

\[
v_r = v_{ro} + \alpha U_{rring} U_o \sum_{n=0}^{n_k} \epsilon_n \cos(n\Theta + \varphi_n)
\]

where \(\Theta = \tan^{-1}(y/z)\), \(\epsilon_n\) and \(\varphi_n\) are randomly generated numbers that satisfy \(-1 < \epsilon_n < 1\) and \(0 < \varphi_n < 2\pi\). \(U_o\) is the mean jet centerline velocity at the inflow boundary. The parameter that determines the amplitude of the forcing is \(\alpha\) and a value of \(\alpha = 0.007\) was imposed. Finally, the parameter of interest is the number of modes given by \(n_k\). Velocity perturbations in the azimuthal direction are not added. \(U_{xring}\) and \(U_{rring}\) are the mean non-dimensional streamwise and radial velocity components induced by the vortex ring and are given by

\[
U_{xring} = 2\frac{r_o}{r} \frac{r - r_o}{\Delta_o} \exp\left(-\ln(2) \left( \frac{\Delta(x,y)}{\Delta_o} \right)^2 \right)
\]

\[
U_{rring} = -2\frac{r_o}{r} \frac{x - x_o}{\Delta_o} \exp\left(-\ln(2) \left( \frac{\Delta(x,y)}{\Delta_o} \right)^2 \right)
\]

where \(r = \sqrt{y^2 + z^2} \neq 0\), \(\Delta_o\) is the minimum grid spacing in the shear layer, and \(\Delta(x,y)^2 = (x - x_o)^2 + (r - r_o)^2\). The location where the center of the vortex ring is located is \(x_o\) and for our case it is set at \(x_o = r_o\). An approximate location is shown in Figure 2.1. The radius of the vortex ring is \(r_o\) and is set equal to the initial jet radius.
2.2.5 Time Advancement

The standard fourth-order explicit Runge-Kutta scheme is used for time advancement. The governing equations can be expressed in the following form

\[
\frac{\partial \mathbf{q}}{\partial t} = \text{RHS}(\mathbf{q}; t), \tag{2.53}
\]

where \( \mathbf{q} \) is the vector of the conservative variables. Following Mitchell et al. [74], the first step of the Runge-Kutta time advancement scheme is an Euler predictor

\[
\mathbf{q}' = \text{RHS}(\mathbf{q}^n; t_n)
\]

\[
\tilde{\mathbf{q}} = \mathbf{q}^n + \frac{\Delta t}{2} \mathbf{q}'
\]

\[
\hat{\mathbf{q}} = \mathbf{q}^n + \frac{\Delta t}{6} \mathbf{q}'
\]

followed by an Euler corrector

\[
\mathbf{q}' = \text{RHS} \left( \tilde{\mathbf{q}}; t_n + \frac{\Delta t}{2} \right)
\]

\[
\tilde{\mathbf{q}} = \mathbf{q}^n + \frac{\Delta t}{2} \mathbf{q}'
\]

\[
\hat{\mathbf{q}} = \tilde{\mathbf{q}} + \frac{\Delta t}{3} \mathbf{q}'
\]

followed by a leapfrog predictor

\[
\mathbf{q}' = \text{RHS} \left( \hat{\mathbf{q}}; t_n + \frac{\Delta t}{2} \right)
\]

\[
\tilde{\mathbf{q}} = \mathbf{q}^n + \Delta t \mathbf{q}'
\]

\[
\hat{\mathbf{q}} = \tilde{\mathbf{q}} + \frac{\Delta t}{3} \mathbf{q}'
\]

and concluded by a Milne corrector as follows

\[
\mathbf{q}' = \text{RHS}(\hat{\mathbf{q}}; t_n + \Delta t)
\]

\[
\mathbf{q}^{n+1} = \hat{\mathbf{q}} + \frac{\Delta t}{6} \mathbf{q}'. \tag{2.57}
\]

This time integration scheme requires 2 temporary arrays, \( \tilde{\mathbf{q}}, \hat{\mathbf{q}} \) and the solution vector at time step \( n \), \( \mathbf{q}^n \) to be stored.
2.3 Parallelization Methodology

This section provides details regarding the parallelization methodology, i.e. the parallelization scheme implemented in the 3-D LES code. The following subsection deals with the original parallelization technique which is based on the transposition scheme. In Chapter 5, an alternate parallelization method based on the Schur complement is described.

Consider once again the non-dissipative sixth-order compact scheme proposed by Lele [31], i.e. Equation (2.29) and the sixth-order tri-diagonal spatial filter proposed by Visbal and Gaitonde [32] Equation (2.34). These compact schemes are implicit and require a solution of a linear system of equations along grid lines. Care must be taken when crafting parallel methodologies when one decomposes the domain of the computational grid. One such method of parallelizing a solver that implements compact schemes is the transposition strategy [59]. For example, assume that the computational domain has the shape shown in Figure 2.2. The computational grid is first partitioned into non-overlapping single blocks along the $z$-direction, i.e. the top diagram of Figure 2.2. Then the spatial derivatives and filtering are computed in the $x$ and $y$ directions, independently from all other blocks. In order to apply the compact scheme and filter along the $z$-direction, a data transposition is performed to re-align the blocks in the $y$-direction (bottom diagram of Figure 2.2). Then the processors apply the compact and filtering schemes along the $z$-direction. When the computations along the $z$-direction are complete, another transposition is applied back to the initial configuration to send the newly computed information back to the original configuration for further computations. The transposition strategy yields the best approximation when utilizing the implicit compact and filtering schemes since it uses every grid point along a given direction. However, the transposition scheme has disadvantages. Firstly, it can only be used for simple geometries or single-block cases and applying the transposition technique to complex geometries is very difficult. Secondly, the maximum number of processors that the transposition scheme can use is restricted to the number of grid points in a particular direction. For example, if there are 128 grid points in the $z$ direction, then only a maximum of 128 processors can be used. Finally, due to the transposition process,
large amounts of data have to be sent between processors resulting in a large communication overhead. For the transposition strategy, the computational resources that are spent on the communication between processors was initially reported to be 40% of the simulation process [75] on the IBM SP2 machine Cloud at the Rosen Center for Advanced Computing (RCAC), Purdue University.

Recently however, using the Cray XT3 machine BigBen at Pittsburgh Supercomputer Center (PSC), the measured communication time of the transposition scheme was reported to be only 16% of the simulation process. The stark difference in measured communication resources between Cloud and BigBen may not be too surprising since the former machine was introduced in 2001, whereas the latter was put into production in mid 2006. The interconnect hardware on Cloud is based on the MX2 technology and has a peak transfer rate of 2.4 GB/s [76]. On BigBen or Cray XT3, the interconnect is based on its proprietary HyperTransport™ technology and has a peak transfer rate of 6.4 GB/s [77]. Hence, the communication transfer rate on the Cray XT3 is roughly 2.5 times faster compared to the IBM SP2 and thus the reduction in communication resources used for the single-block code. However, there are still many compute clusters with communication hardware that are still slow since installing a high speed network can be as high as half the cost of the entire compute cluster. Though a communication resource usage of 16% is acceptable for clusters with high speed communication networks, the transposition scheme still suffers one main setback: it is difficult or near impossible to include a nozzle geometry requiring a multiblock grid using this method. Hence, to alleviate the difficulty of including a nozzle, a domain decomposition methodology based on the parallel Schur complement was proposed.
Figure 2.1. Boundary conditions used in the 3-D LES code.

Figure 2.2. Schematic of the transposition scheme. The upper schematic shows the first step of the transposition scheme. The lower schematic shows the second step. The transposition strategy returns to its final step in upper schematic after all grid points are differentiated and filtered by the compact scheme.
3. NEAR-FIELD TURBULENT FLOW RESULTS

3.1 Introduction

This chapter discusses the test cases used for the compressible hot jet simulations. In addition to hot jet results, we also simulate two cold or isothermal jet cases for comparison. Near-field results are then presented for each of these test cases and compared to similar LES results in the literature and experimental data.

3.2 Test Cases and Setup

Table 3.1 summarizes the parameters for the heated and unheated jet test cases that are considered. The test cases are appropriately named according to the experimental test matrix of Tanna et al. [78]. Four jet test cases are considered, i.e. two unheated (cold) and two heated cases. Hence, we would like to see the effects of heating on our jet while keeping the ambient Mach number fixed using our 3-D LES methodology. In previous studies, (see References [15,54,59,68,79,80]) we have only used our LES methodology at a fixed ambient Mach number of $M_\infty = 0.9$ for jet noise prediction without any heating, i.e. case SP07. Thus, cases SP03 ($M_\infty = 0.5$ and $T_j/T_\infty = 1$) and SP23 ($M_\infty = 0.5$ and $T_j/T_\infty = 1.76$) were chosen as an opportunity to use our 3-D LES methodology to determine if we can adequately predict jet flow physics and the far-field noise at a different Mach number for both heated and unheated conditions. Furthermore, there is available LES [23,81] and experimental data [25,78,82] for this test case, i.e. SP23, in the literature to compare with.

Here, $M_j = U_j/a_j$ is the jet Mach number where $U_j$ is the centerline jet velocity and $a_j$ is the centerline jet speed of sound. $M_\infty = U_j/a_\infty$ is the acoustic Mach number where $a_\infty$ is the ambient speed of sound. $N_x$, $N_y$, and $N_z$ simply correspond to the number...
of grid points in the $x$, $y$ and $z$ directions respectively. All test cases each have a total of approximately 4.8 million grid points whereby the number of grid points in the $x$, $y$ and $z$ directions are $N_x = 292$, $N_y = 128$ and $N_z = 128$, respectively. Figure 3.1 shows the, $x$-$y$, cross sectional plane of the computational domain. The physical part of the computational domain extends to approximately $60r_o$ in the streamwise direction and $-20r_o$ to $20r_o$ in the transverse $y$ and $z$ directions. Beyond the streamwise location of $60r_o$ is the sponge zone. The physical domain length of $60r_o$ was chosen due to two reasons. Firstly, Uzun et al. [15] reported that the Reynolds stresses achieve their full asymptotic self-similar state if a domain length of at least $45r_o$ was used. Secondly, in order to capture the Overall Sound Pressure Levels (OASPL) adequately at the shallow angles, a domain length of at least $55r_o$ is required based on the recommendations of Uzun et al. [15] and Shur et al. [83]. Figures 3.2 through 3.4 show the cross sectional grid planes for stations $x = 15r_o$, $x = 30r_o$ and $x = 50r_o$, respectively. As observed from Figures 3.1 through 3.4, grid stretching is applied in all three directions in which more grid points are clustered near the shear layer in order to resolve the relatively high velocity gradients there. The region of $0 \leq x/r_o \leq 2$ is the ‘buffer’ region for the vortex ring forcing and is sensitive to grid stretching. An investigation yielded that our 3-D LES code would become unstable if grid stretching was employed in the forcing region. Hence, no grid stretching is used and an equal grid spacing of $\Delta x = 0.1r_o$ is applied from the inflow until $x = 2r_o$. Beyond the forcing region, however, a stretching parameter of $1.0068$ is used based on the recommendation of Shur et al. [83]. They indicate that gradual stretching in the turbulent region of the jet is important as too much grid stretching causes false sound generation due to grid induced deformation of the vortices. Based on the minimum grid spacing and jet Mach number, the time resolution was determined to be $\Delta t = 0.15$ for test cases SP07 and SP46 and $\Delta t = 0.11$ for test cases SP03 and SP23, respectively.

The Reynolds number is defined as $Re_D = \rho_j U_j D_j / \mu_j$ where $\rho_j$, $U_j$ and $\mu_j$ are the jet centerline density, velocity and viscosity at the inflow, respectively. $D_j$ is simply the jet diameter. The Reynolds numbers specified above for all jets correspond to the experimental conditions of Tanna et al. [78]. In the previous section, we mentioned that a vortex ring is
used to excite the mean flow. The vortex ring used here contains a total of 16 azimuthal jet modes of forcing. Bogey and Bailly [84] performed a simulation with all modes present and later removed the first four modes and found that the jet was quieter with the latter case and matched experimental results better. Hence, based on their results, the first four azimuthal modes of forcing are not included in the forcing. The forcing amplitude is set to $\alpha = 0.007$. Further details regarding the impact of the vortex ring on the 3-D LES jet can be found on References [84] and [79].

We consider a hyperbolic tangent velocity profile used by Freund [85] on the inflow boundary given by

$$\tilde{u}(r) = \frac{1}{2} U_j \left[ 1 - \tanh \left( b \left( \frac{r}{r_o} - \frac{r_o}{r} \right) \right) \right],$$

(3.1)

where $r = \sqrt{y^2 + z^2}$, $r_o = 1$, and $U_j$ is the jet centerline velocity. The parameter that controls the thickness of the shear layer is $b$. In the code, this parameter is set to $b = 2.8$. A higher value of $b$ implies a thinner shear layer. As a comparison, Freund [85] used a value of $b = 12.5$ for his 3-D jet DNS. All jet cases considered here employs a thicker shear layer compared to Freund’s jet. For laboratory jets however, the measured value of $b$ is usually an order of magnitude or higher compared to that used in LES and DNS of jets. For all cases considered here, there are approximately 10 grid points in the initial jet shear layer. For unheated or isothermal jets (SP03 and SP07), the Crocco-Buseman relation is used for the density profile on the inflow boundary

$$\tilde{\rho}(r) = \rho_j \left( 1 + \frac{\gamma - 1}{2} M_r^2 \frac{\tilde{u}(r)}{U_j} \frac{1 - \tilde{u}(r)}{U_j} \right)^{-1},$$

(3.2)

where $M_r = 0.9$. For the heated jet cases however (SP23 & SP46), the inlet density profile is also adopted from Freund [85]

$$\tilde{\rho}(r) = \left( \rho_j - \rho_\infty \right) \frac{\tilde{u}(r)}{U_j} + \rho_\infty,$$

(3.3)

where $\tilde{u}(r)/U_j$ is the mean streamwise velocity on the inflow boundary normalized by the jet centerline velocity, $\rho_j$ is the density at the jet centerline and $\rho_\infty$ is the free stream density. The ratio $\rho_\infty/\rho_j$ determines whether or not the jet is hot or cold. A value lower than unity implies a cold jet, whereas a value greater than unity implies a hot jet.
3.3 Turbulent Flow Results for Heated & Unheated Jets

In terms of computational resources utilized, test case SP07 and SP46 took approximately 25 days of runtime to reach 200,000 time steps using 32 processors on a Sun Fire F6800 server in order to obtain reasonably converged statistics. Test case SP03 and SP23 however took approximately 38 days of run time (300,000 time steps) to achieve reasonably converged statistics using the same machine and number of processors. The reason why converged statistics take more time to obtain for cases SP03 and SP23 is due to the relatively slow convective speeds of the mean flow [59].

Table 3.2 shows several one point statistical results for the heated and unheated jets. These results are compared to the recent LES data from Bodony and Lele [86] and the experimental correlations from Zaman [87]. First, some definitions used in Table 3.2. The second column of Table 3.2 indicates the slope of the mean streamwise velocity decay, \( C = \left( \frac{d}{d(x/D_j)} \frac{U_o}{U_c} \right) \). In other words, it is a measure of how fast the mean axial centerline velocity decays. The third column of Table 3.2 is the slope of the half-velocity radius normalized by the initial jet radius, \( A = \left( \frac{d(r_{0.5}/r_o)}{d(x/r_o)} \right) \). The half-velocity radius, \( r_{1/2} \) at a particular downstream location is defined as the radial location where the mean streamwise velocity is one-half the jet mean centerline velocity. In essence, \( A \) is an indicator of how fast the jet grows. The fourth last column of Table 3.2 is the slope of the mean streamwise mass flux normalized by the mass flux at the jet exit, \( K = \left( \frac{d}{d(x/D_j)} \frac{M}{M_e} \right) \). Finally, the last column in Table 3.2 is the end of the potential core location normalized by the initial jet radius. The length of the potential core here is defined when the jet centerline velocity reduces to 95% of the inflow jet velocity, \( U_c(x_c) = 0.95U_j \). In addition to the results presented in Table 3.2, a simulation that was previously performed by Uzun [59] included for comparison. The simulation performed by Uzun [59] has the same computational domain size and is a Mach, \( M_j = 0.9 \), isothermal jet with a Reynolds number of \( Re_D = 100,000 \). The most notable difference in terms of simulation setup by Uzun is the number of grid points. Uzun used a total of 12 million grid points with a grid decomposition of \( N_x = 470 \), \( N_y = 160 \) and \( N_z = 160 \). Furthermore, the Dynamic Smagorinsky Model (DSM) was also used.
Figure 3.5 shows the jet centerline decay rate, $C$ (defined previously), in the far down-stream region for both heated and unheated jets. The data plotted are in its raw form and has not been re-scaled in the abscissa. Overall, we note that the trends predicted by our LES are in good agreement with the experimental results of Bridges and Wernet [25]. The results are also tabulated in Table 3.2. Our decay rate results are also compared with the experimental correlation proposed by Zaman [87] and to Bodony and Lele’s [86] recent LES. Zaman shows that the mean centerline decay rate for compressible jets is a weak function of the Mach number, but scales according to the ratio of the ambient density and jet exit density. From the experimental literature [72,87–90], the decay rate has been found to range from $0.155 \leq C \leq 0.185$ for unheated jets. For SP03, the value of $C = 0.171$ is acceptable and is close to the LES simulation of Bodony & Lele but is slightly higher than the experimental correlation of Zaman. The predicted value of $C = 0.168$ for SP07 compares well with the experimental correlation by Zaman and also falls within the range of the experimental data. For SP07(U), the decay rate is closer to experimental the correlation probably due to the combination of higher grid resolution and the influence of the dynamic subgrid scale model. The decay rate from Bodony and Lele however is slightly higher than the experimental results and our results. If we keep the ambient jet Mach number, $M_{\infty}$, constant and heat the jet, the mean axial centerline decay rate increases. This is reflected when we compare SP07 and SP46. The predicted decay rate, $C$, for SP46 is also within agreement with the experimental correlation of Zaman. In addition, the decay rate for SP23 also compares well with the experimental Zaman’s correlation. Most importantly though is that we are observing similar trends for our decay rate when compared to the LES of Bodony and Lele and also from experimental correlations, i.e. heated jets decay faster compared to an unheated jet. It also noted that our LES results are closer to Bodony and Lele’s LES data for low Mach number jets (SP03 and SP23) compared the higher Mach number jet (SP07 and SP46).

In addition to the parameters defined in Table 3.2, we adopt the procedure used by Bodony and Lele whereby the axial coordinates, i.e. $x/r_o$, are shifted axially to aid in the presentation of near-field data over a range of operating conditions so that differences in
compressibility or Mach number which affect the length of the potential core can be accommodate \cite{82,86,91}. The procedure adopted by Bodony and Lele is called the Witze \cite{91} correlation and is given by \( W = \kappa(x - x_c)/r_o \) where \( \kappa = 0.08(1 - 0.16M_J)(\rho_\infty/\rho_j)^{0.22}. \) Thus \( x_c/r_o \) is computed first and then \( x/r_o \) is shifted axially. Then the data is re-scaled using the factor \( \kappa. \) The Witze correlation here will be used to present the data for the mean centerline decay and axial turbulence intensities.

Figure 3.6 shows the variation of the mean streamwise velocity along the centerline for isothermal jet SP03. Here the Witze shift is used on the abscissa to take into account the potential core length and jet centerline Mach number. There is close agreement between the current LES SP03 and experiment and the LES of Bodony and Lele. Figure 3.7 shows the mean streamwise velocity decay for case SP07. The other LES results correspond to Bodony & Lele and from Bogey and Bailly \cite{73}. The DNS data are from Freund \cite{85}. The experimental results are from Bridges and Wernet \cite{25}, Tanna et al. \cite{92} (extracted from Bodony and Lele \cite{86}) and Jordan et al \cite{27}. We note that the LES jets decay slightly faster when compared to the laboratory jets of Tanna, Bridges and Wernet and Jordan et al. This is probably due to the initial turbulent shear layers issuing from the laboratory jets as opposed to our transitional jet. Nonetheless, our LES results show closer agreement to experiments of Tanna compared to Freund’s and Bodony and Lele’s data. However, the agreement of Bogey and Bailly compared to experiments is excellent. Figure 3.8 shows the jet centerline decay for heated jet SP23. We again note the closer agreement of the current LES to experiments compared to Bodony and Lele’s data. Figure 3.9 shows the jet centerline decay for heated jet SP46. Likewise we see that the heated laboratory jets decay slower compared to our LES heated jets. In Bridges & Wernet’s \cite{25} technical report, they mention that the data for the mean streamwise velocity decay along the centerline for SP46 showed some problems beyond \( x/r_o = 20 \) or \( W = 1. \) They were not able to find an explanation for this behavior. Hence, the good collapse of our data from \( W = 1 \) onwards is only fortuitous. Likewise, case SP46 from Bodony and Lele decays faster compared to our LES and to the experiments of Bridges and Wernet. As a note, we could not find velocity centerline decay data for Tanna’s hot jet experiments. Finally, to put effects of heating into
perspective, Figure 3.10 shows the decay rate of three LES jets SP07, SP23 and SP46. We see that regardless of Mach number, as the jet is heated the decay rate increases. SP03 was not plotted here for clarity. Furthermore, the decay rate of SP03 is close to the value of SP07.

Figure 3.11 shows the streamwise variation of the half-velocity radius normalized by the jet radius for heated and unheated jets using our LES methodology. From Figure 3.11, we can see that for all test cases, i.e. SP03, SP07, SP23 and SP46, the streamwise half-velocity radius exhibits quasi-linear growth sufficiently far downstream from the jet inflow. The third column in Table 3.2 shows the streamwise half-velocity growth rate, \( A \), for the current hot jet and previously simulated isothermal jet. The range of data for unheated incompressible jets from the experimental literature \[72\] is reported to be \( 0.086 \leq A \leq 0.096 \). Hence, the isothermal jets (SP03 and SP07) falls well within the experimental range and satisfactory. No correlation was available from Zaman’s \[87\] report for the half-velocity growth rate. Noting that the decay rate, \( C \), is higher for our heated jet when compared to an unheated jet, we see that the same trend for the half-velocity growth rate. Hence, a heated jet grows faster than an unheated jet keeping \( M_\infty \) fixed comparing SP07 and SP46. Tam and Ganesan \[93\] showed through linear stability analysis that it is the jet density ratio that determines its growth rate, which is consistent with our results. We have not found experimental values for the half-velocity growth rate of heated jets as of yet. Note that the half-velocity growth rate value reported by Bodony and Lele for their jets are higher which is consistent with the higher \( C \) values reported in Table 3.2.

Figure 3.12 shows the streamwise variation of the mass flux, \( M \), normalized by the mass flux through the jet nozzle, \( M_e \), for our heated and unheated jets. Sufficiently far downstream from the jet nozzle we see linear growth of the jet entrainment. The last column in Table 3.2 shows the entrainment rate values for both unheated and heated jets. The value of \( K = 0.23 \) and \( K = 0.24 \) for cases SP03 and SP07, respectively, agrees quite well with the experimental correlation value of Zaman though slightly lower. Uzun’s jet SP07(U) with \( K = 0.267 \) has the closest agreement with experimental correlations. Bodony and Lele’s jets slightly over-predict the experimental correlations but not alarm-
ingly. Noticing from the higher growth rates reported for our heated jets, we see the same
trend repeated here for the mass flux rate, $K$. Nonetheless, the LES values reported for our
hot jets agree rather well with the correlations of Zaman and the recent LES of Bodony and
Lele.

The end of the potential core for cold jets and SP03 and SP07 are 11.85$r_o$ and 13.74,
respectively. Uzun’s SP07(U) reports a potential core length of 11$r_o$. This is shown in the
last column of Table 3.2. Using Zaman’s [87] experimental correlation, a value of 12.5$r_o$ is
obtained. From the experimental literature for unheated jets, values of approximately 10$r_o$,
12$r_o$ and 14$r_o$ were reported by Raman et al. [94], Jordan et al. [27] and Arakeri et al. [95],
respectively. The potential lengths reported by Bodony and Lele are also in close agreement
with our results. Now, when the jet is heated though, the potential core length shortens as
shown in the fourth column of Table 3.2. Again, based on Zaman’s correlation, a value of
9.42$r_o$ and 7.6$r_o$ and is obtained for SP23 and SP46, respectively. This observation was
also reported by Bodony and Lele [86] and the recent LES results of Andersson [96] when
their jets were heated as compared to an unheated jet. Andersson reported a potential core
length value of approximately 8$r_o$ for his $M_\infty = 0.75$ jet with $T_j/T_\infty = 2$ whereas Jordan
et al. [27] reported a potential core length of about 10$r_o$ for their $M_\infty = 0.75$ jet with
$T_j/T_\infty = 2$. Hence, the trends obtained by the current simulations agree well with both
the experimental and LES observations.

Figures 3.13 through 3.16 shows the axial centerline turbulence intensities for unheated
and heated jets shifted axially using Witze [91] correlation. From Figure 3.13 we note the
good agreement with the experiment of Bridges and Wernet in terms of peak location for
case SP03. However, we under-predict the decay from $W > 1$ compared to experiments.
There are measured ‘spikes’ in the experimental streamwise turbulence intensities at $W \simeq
-0.25$, $W \simeq 0.8$ and $W \simeq 1.25$, respectively. No physical explanation was given by
Bridges and Wernet on these measured ‘spikes’. For Figure 3.14, case SP07 compares well
in terms of location and peak value of the axial turbulence intensities with the experimental
results of Jordan et al. [27] but not in terms of overall streamwise variation, i.e. the data
do not collapse at $-0.5 \lesssim W \lesssim -0.25$. However, there is reasonable collapse in the
range of $0 \lesssim W \lesssim 0.75$ when compared to the experiments Bridges and Wernet [82]. The turbulence intensities for SP23 are shown in Figure 3.15. The experimental conditions from Jordan et al. are quite close the conditions of SP23, i.e. $M_\infty = 0.75$ and $T_j/T_\infty = 2$, and we use them here as a comparison. The peak value of our SP23 LES closely corresponds to the values obtained by Jordan et al. and Bridges and Wernet and is satisfactory. Figure 3.16 on the other hand, shows the centerline axial turbulence intensities for the heated jet test case SP46. This case compares well to the experiments of Bridges and Wernet [82] in terms of peak location and peak value. Beyond the peak location however, the axial velocity fluctuations on the centerline decays slightly faster than for the laboratory jets of Bridges & Wernet and Jordan et al. Bodony and Lele suggest that after the end of the potential core, the organized motions are no longer supported and they decay rapidly, giving substantial amounts of energy to the mean flow of the jet. Hence, in conclusion, our LES results for the axial root mean square velocity fluctuations compare reasonably well with the experimental literature.
Table 3.1 List of test cases. All physical domains correspond to 
\((x, y, z) = (60r_0, \pm 20r_0, \pm 20r_0)\).

<table>
<thead>
<tr>
<th>Test Case</th>
<th>(M_j)</th>
<th>(M_{\infty})</th>
<th>(N_x \times N_y \times N_z)</th>
<th>(Re_D)</th>
<th>(T_j / T_{\infty})</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP03</td>
<td>0.50</td>
<td>0.50</td>
<td>(292 \times 128 \times 128)</td>
<td>200,000</td>
<td>1.00</td>
</tr>
<tr>
<td>SP07</td>
<td>0.90</td>
<td>0.90</td>
<td>(292 \times 128 \times 128)</td>
<td>200,000</td>
<td>1.00</td>
</tr>
<tr>
<td>SP23</td>
<td>0.38</td>
<td>0.50</td>
<td>(292 \times 128 \times 128)</td>
<td>223,000</td>
<td>1.76</td>
</tr>
<tr>
<td>SP46</td>
<td>0.55</td>
<td>0.90</td>
<td>(292 \times 128 \times 128)</td>
<td>200,000</td>
<td>2.70</td>
</tr>
</tbody>
</table>
Table 3.2 Several turbulent flow results for our cold/isothermal and hot jet LES. (U) and (B) implies Uzun’s [59] and Bodony’s [23] LES, respectively. Zaman’s [87] results are correlations based on experiments.

<table>
<thead>
<tr>
<th>Test Case</th>
<th>$C_r \left( \frac{d}{d(x/D_i)} \frac{U_r}{U_i} \right)$</th>
<th>$A_r \left( \frac{d}{dx} \frac{r_{o,z}}{r_{o}} \right)$</th>
<th>$K_r \left( \frac{d}{d(x/D_i)} \frac{M}{M_{e}} \right)$</th>
<th>$x \div r_o \cdot (0.95U_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP03</td>
<td>0.171</td>
<td>0.10</td>
<td>0.23</td>
<td>11.85</td>
</tr>
<tr>
<td>SP03 (B)</td>
<td>0.174</td>
<td>0.115</td>
<td>0.30</td>
<td>8</td>
</tr>
<tr>
<td>Zaman [87]</td>
<td>0.16</td>
<td>-</td>
<td>0.26</td>
<td>12.5</td>
</tr>
<tr>
<td>SP07</td>
<td>0.168</td>
<td>0.096</td>
<td>0.24</td>
<td>13.74</td>
</tr>
<tr>
<td>SP07 (U)</td>
<td>0.161</td>
<td>0.092</td>
<td>0.267</td>
<td>11</td>
</tr>
<tr>
<td>SP07 (B)</td>
<td>0.179</td>
<td>0.106</td>
<td>0.30</td>
<td>10</td>
</tr>
<tr>
<td>Zaman [87]</td>
<td>0.16</td>
<td>-</td>
<td>0.26</td>
<td>12.5</td>
</tr>
<tr>
<td>SP23</td>
<td>0.208</td>
<td>0.097</td>
<td>0.32</td>
<td>8.96</td>
</tr>
<tr>
<td>SP23 (B)</td>
<td>0.244</td>
<td>0.112</td>
<td>0.39</td>
<td>7.50</td>
</tr>
<tr>
<td>Zaman [87]</td>
<td>0.212</td>
<td>-</td>
<td>0.37</td>
<td>9.42</td>
</tr>
<tr>
<td>SP46</td>
<td>0.273</td>
<td>0.107</td>
<td>0.461</td>
<td>8.68</td>
</tr>
<tr>
<td>SP46 (B)</td>
<td>0.323</td>
<td>0.122</td>
<td>0.477</td>
<td>7.35</td>
</tr>
<tr>
<td>Zaman [87]</td>
<td>0.263</td>
<td>-</td>
<td>0.46</td>
<td>7.60</td>
</tr>
</tbody>
</table>
Figure 3.1. The cross section of the computational grid on the \( z = 0r_o \) plane. (Every 3\(^{rd}\) grid point is shown).

Figure 3.2. The cross section of the computational grid on the \( x = 15r_o \) plane. (Every other grid point is shown).
Figure 3.3. The cross section of the computational grid on the $x = 30r_o$ plane. (Every other grid point is shown).

Figure 3.4. The cross section of the computational grid on the $x = 50r_o$ plane. (Every other grid point is shown).
Figure 3.5. Mean axial velocity centerline variation for both heated and unheated jets. Experimental results of Bridges & Wernet are also plotted.

Figure 3.6. Mean axial velocity centerline variation for isothermal jet SP03 with the $x$-axis shifted using the Witze correlation.
Figure 3.7. Mean axial velocity centerline variation for isothermal jet SP07 with the $x$-axis shifted using the Witze correlation.

Figure 3.8. Mean axial velocity centerline variation for heated jet SP23 with the $x$-axis shifted using the Witze correlation.
$W = k(x - x_c)/r_o$

$U_c(x) / U_j$ -0.5 0 0.5 1 1.5 2

0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1

0.95U_o

Figure 3.9. Mean axial velocity centerline variation for heated jet SP46 with the $x$-axis shifted using the Witze correlation.

$U_o(x)/U_c$

0 5 10 15 20 25 30

0 1 2 3 4 5 6 7 8

LES SP07, Isothermal Jet, $M_j = 0.9$
LES SP23, Hot Jet $T_j/T_a = 1.76$, $M_j = 0.38$
LES SP46, Hot Jet $T_j/T_a = 2.70, M_j = 0.55$

Hot jet, SP46
Slope: $C = 0.273$,
Zaman’s experiment (1998):
slope $C = 0.263$

Hot jet, SP23
Slope: $C = 0.208$,
Zaman’s experiment (1998):
slope $C = 0.212$

Isothermal jet, SP07
Slope: $C = 0.168$,
Zaman’s experiments (1998):
slope $C = 0.155$

Figure 3.10. Mean axial velocity centerline decay rate for unheated and heated jets.
Figure 3.11. Streamwise variation of the half-velocity radius normalized by the jet radius for both isothermal and heated jets.

Figure 3.12. Streamwise variation of mass flux normalized by the mass flux at the nozzle for test case SP07 and SP46.
Figure 3.13. Centerline axial turbulence intensity for isothermal jet SP03.

Figure 3.14. Centerline axial turbulence intensity for isothermal jet SP07.
$W = \frac{k(x - x_c)}{r_o}(v_x' \text{rms}) / U_o$}

Figure 3.15. Centerline axial turbulence intensity for heated jet SP23.

$W = \frac{k(x - x_c)}{r_o}(v_x' \text{rms}) / U_o$

Figure 3.16. Centerline axial turbulence intensity for heated jet SP46.
4. FAR-FIELD AEROACOUSTICS

4.1 Introduction

The previous chapter discussed the near-field results of two isothermal and two heated jets. This chapter discusses the far-field aeroacoustic results using two methodologies. The first is based on the Ffowcs Williams-Hawkings (FWH) surface integral acoustic method and the second is based on Lighthill’s acoustic analogy (LAA). Within the far-field predictions computed by LAA, we also examine the contribution of the nominal noise source terms to the far-field. This chapter also includes an additional isothermal jet test case.

4.2 Ffowcs Williams-Hawkings Surface Integral Acoustic Method

The aeroacoustic analysis is done after the LES computations are finalized. To couple the two, the porous Ffowcs Williams-Hawkings [97, 98] formulation is utilized to study the far-field jet noise as suggested by Lyrintzis [99] and Lyrintzis and Uzun [54]. For simplicity, a continuous stationary control surface around the turbulent jet is used. The observers are also assumed to be stationary. The formulation for the disturbance pressure is as follows

\[ p'(\tilde{x}, t) = p'_T(\tilde{x}, t) + p'_L(\tilde{x}, t) + p'_Q(\tilde{x}, t), \quad (4.1) \]

where

\[ 4\pi p'_T(\tilde{x}, t) = \int_S \left[ \frac{\rho_\infty U_n}{r} \right]_{ret} dS, \quad (4.2) \]

\[ 4\pi p'_L(\tilde{x}, t) = \frac{1}{c_\infty} \int_S \left[ \frac{L_r}{r} \right]_{ret} dS + \int_S \left[ \frac{L_r}{r^2} \right]_{ret} dS, \quad (4.3) \]

and

\[ U_i = \frac{\rho u_i}{\rho_\infty}, \quad (4.4) \]

\[ L_i = p'\delta_{ij}n_j + \rho u_iu_n. \quad (4.5) \]
Here, \( p'_T(\vec{x}, t) \) is known as the thickness noise, \( p'_L(\vec{x}, t) \) is the loading noise and \( p'_Q(\vec{x}, t) \) is the quadrupole noise pressure term that includes all sources outside the control surface. The quadrupole noise pressure term is neglected in this methodology. \((\vec{x}, t)\) are the observer coordinates and time, \( r \) is the distance from the source on the surface to the observer, \( c_\infty \) and \( \rho_\infty \) are the ambient speed of sound and ambient density, respectively. A time derivative is indicated with a dot over a variable and the subscript \( r \) and \( n \) implies a dot product of the vector with the unit vector in the radiation direction \( \vec{r} \) and in the surface normal direction \( \vec{n} \), respectively. \( dS \) is an elemental surface to be integrated over, and the subscript \( ret \) implies the evaluation of the integrand at the retarded time, \( \tau = t - r/c_\infty \).

For simplicity, a continuous stationary control surface around the turbulent jet is used. For details regarding the numerical implementation of the Ffowcs Williams-Hawkings method, the reader is referred to Uzun [59]. As a note, all data here is presented alongside the Lighthill computations. The far-field noise prediction using Lighthill’s acoustic analogy is discussed in detail in the next section. Due to the nature of our curvilinear grid, the control surface is shaped as in Figure 4.1. The (blue) control surface starts about one jet radii downstream and is situated at approximately 7.5\( r_0 \) above and below the jet at the inflow boundary in the \( y \) and \( z \) directions. It extends streamwise until the end near of the physical domain at which point the cross stream extent of the control surface is approximately 30\( r_0 \). Hence, the total streamwise length of the control surface is 59\( r_0 \). We show results for an open control surface. A open control surface here is defined where there is no surface at the end of the physical domain, i.e. \( x = 60r_0 \). Based on our grid resolution around our control surface and assuming that with our numerical method 6 points per wavelength are needed to accurately resolve an acoustic wave [59], the maximum frequency resolved corresponds to a Strouhal number of \( Sr \approx 1 \) for both test cases SP07 and SP46 and \( Sr \approx 1.6 \) for test case SP23, where the Strouhal number is defined as \( Sr = fD_j/U_j \). The overall sound pressure levels are computed along an arc with a distance of \( R = 144r_0 \) from the jet nozzle exit. This arc length corresponds to the distance used by Tanna et al. [78] in their experiments. The angle, \( \Theta \), however, is measured relative to the centerline jet downstream axis.
Figure 4.2 shows the overall sound pressure levels (OASPL) for SP07 LES and experimental data. Please note that all experimental and LES data from other investigators have been scaled to a common distance of $R = 144r_o$ (using a $1/R$ scaling) from the center of the jet exit. In addition to the experimental data shown, we have also included the SAE ARP 876C [100] database prediction for a jet operating at similar conditions as ours, i.e. SP07. This database prediction consists of actual engine jet noise measurements and can be used to predict overall sound pressure levels within a few dB at different jet operating conditions. As we can see the prediction agrees well with the experimental results of Tanna et al. [78]. From the LES results, test case SP07 compares well with the experimental results of Tanna and the SAE prediction within the range of $50^\circ \lesssim \Theta \lesssim 90^\circ$. Below that range the LES over predicts the OASPL values. Our data also compares rather well with the acoustic results of Bodony and Lele [23].

Figure 4.3 shows the OASPL plot for heated jet test case SP46 computed at $R = 144r_o$. We also included Tanna’s and Viswanathan’s [29] experimental data as well the SAE ARP876C prediction. In terms of the shape of the OASPL curve, we are in good agreement with the experimental results of Tanna and Viswanathan though approximately $3 \text{ dB}$ higher. The peak radiation angle reported by Tanna is located at $\Theta = 30^\circ$, whereas ours is located at approximately $32.5^\circ$. Bodony’s OASPL prediction is slightly higher than ours, but also follows the trend predicted by Tanna. The SAE ARP876C prediction is able to predict the values reported by Tanna very well, as shown. Hence, overall, our predicted OASPL are in good agreement with the experimental data. Figure 4.5, on the other hand, shows the OASPL data for test case SP23 and is compared to the experimental data of Tanna et al. Again, on average we over-predict the sound levels by approximately $3 \text{ dB}$ when compared to the results of Tanna. But nonetheless, our predicted OASPL follows the trend measured by Tanna et al. [78]. The predicted values from the SAE ARP 876C show good agreement with the measured data from Tanna as well. The computed spectra using the FWH method are discussed alongside the Lighthill results in the next section.
4.3 Noise Source Investigation using Lighthill’s Acoustic Analogy

The heated jet LES simulation results from the previous section deal only with predicting the far-field sound (via integral acoustic methodologies) and not with the root cause of it, i.e. the possible noise generating sources. In the literature however, the investigation of noise sources in unheated jets using DNS and LES data itself is quite recent. Within the framework of Lighthill’s acoustic analogy [13, 101], Freund [102] was the first to investigate noise sources in a low Reynolds number $Re_D = 3,600$, $M_j = 0.9$ cold jet using his recent DNS data. One conclusion that Freund reported was that the contributions from the shear, self and entropic noise sources are highly correlated at small angles to the jet axis, and not statistically independent, as often assumed. However, Freund also noted that better parametrization of Reynolds-number effects on the probable noise sources needs to be carried out since his DNS jet was set in the low-Reynolds-number limit. It is in this regard that LES offers an attractive alternative whereby the jet simulations can be simulated at higher Reynolds numbers with a fraction of the cost of DNS, as mentioned earlier. Hence, in the realm of LES, Uzun et al. [53] were the first to use simulation data coupled with Lighthill’s acoustic analogy to investigate the noise sources in a $M_j = 0.9$, $Re_D = 400,000$ isothermal jet. In their investigation, they found that significant cancellations occur among the noise generated by the individual components of the Lighthill stress tensor for a high Reynolds number isothermal jet. In addition, Bogey and Bailly [103] used causality methods to study noise sources of several unheated jets with different Reynolds numbers. The investigation of noise sources in turbulent hot jets using LES data, however, is fairly limited. To the best of our knowledge, the only use of LES data to study noise sources in turbulent heated jets was recently performed by Bodony and Lele [104]. Again, through the use of Lighthill’s acoustic analogy, their results indicate that when compared to an unheated high speed jet, significant phase cancellation exists between the momentum and entropy sources in the near-field and that additional cancellation occurs in the far-field. Bodony and Lele then suggest that the significant cancellations between these two sources in the far-field is a probable explanation as to why a high speed heated jet is quieter when
compared to an unheated jet while keeping the jet velocity constant. However, it must be noted that Bodony & Lele’s [104] heated jets were run for high Mach number jets of Mach 0.9 and 1.5. They did not perform low speed heated or unheated jet computations when they performed their Lighthill analysis.

With that in mind, the aim of this chapter is to investigate the contribution of possible noise sources from the near field to the far-field of two turbulent heated and unheated, high Reynolds number jets using LES data. As mentioned, an additional unheated jet test case is included in this study which is a Mach 0.5 jet with a Reynolds number of $Re_D = 200,000$. The conditions of this low speed unheated jet correspond to set point SP03 in Tanna’s [78] experimental test matrix. Set point SP03 was chosen as a compliment to test case SP23. Hence, SP23 has the same acoustic Mach number as SP03 ($M_\infty=0.5$) but at a heated condition. The noise source investigation will be conducted within the framework of Lighthill’s [13, 101] acoustic analogy for all cases.

4.3.1 Brief Formulation

We begin by considering Lighthill’s [13] equation which is written as

$$\frac{\partial^2 \rho'}{\partial t^2} - a_\infty^2 \frac{\partial^2 \rho'}{\partial x_j \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j},$$

(4.6)

where the Lighthill stress tensor is given by

$$T_{ij} = \rho u_i u_j + (p - a_\infty^2 \rho) \delta_{ij} - \sigma_{ij}.$$  

(4.7)

Here, $a_\infty$ is the ambient speed of sound, $\sigma_{ij}$ is the viscous stress and $\rho'$ is the fluctuation density. In Lighthill’s pioneering paper [13], he argues that the viscous stress term in the source term can be neglected and was confirmed by Colonius and Freund [105]. Specifically, Colonius and Freund showed that the viscous terms in the Lighthill stress tensor has hardly any contribution to the far-field even for a $Re_D = 2,000$ jet. Hence for this study, the viscous stress term is neglected. It is important to note that the double divergence of $T_{ij}$ on the right hand side of Equation (4.6) serves only as a nominal acoustic source term, and it should not be interpreted as a true acoustic source. Furthermore, $T_{ij}$ is not unique, i.e.
recasting the equations with respect to pressure gives a different source term. In Lighthill’s original formulation of his equation, all effects aside from propagation in a homogenous stationary medium, such as refraction, self-modulation of sound due to non-linearity and attenuation due to thermal action are lumped into the right hand side [106]. It is also understood that most of $T_{ij,ij}$ does not radiate into the far-field. However, what the right hand side of Equation (4.6) does provide is an exact connection between the near field turbulence and the far-field noise and thus serves only as a nominal acoustic source.

Following the standard Reynolds decomposition employed by Freund [102], the Lighthill stress tensor, $T_{ij}$, can be split into

$$T_{ij} = T_{ij}^m + T_{ij}^l + T_{ij}^n + T_{ij}^s,$$  \hspace{1cm} (4.8)

whereby each of the individual components are given as,

$$T_{ij}^m = \rho \ddot{u}_i \ddot{u}_j + (\bar{p} - a_\infty^2 \bar{\rho}) \delta_{ij}, \hspace{1cm} (4.9)$$

$$T_{ij}^l = \rho \ddot{u}_i \dot{u}_j' + \rho \dot{u}_i \ddot{u}_j', \hspace{1cm} (4.10)$$

$$T_{ij}^n = \rho \dot{u}_i' \dot{u}_j', \hspace{1cm} (4.11)$$

$$T_{ij}^s = (p' - a_\infty^2 \rho') \delta_{ij}. \hspace{1cm} (4.12)$$

Here, $T_{ij}^m$ is the mean component and by definition, does not make noise. $T_{ij}^l$ is a component that is linear in velocity fluctuations and is called the shear noise since this component consists of turbulent fluctuations interacting with the sheared mean flow. $T_{ij}^n$ is a component that is quadratic in velocity fluctuations and is called self noise since this component involves the turbulent fluctuations interacting with themselves. Finally, $T_{ij}^s$ is the entropy component and aptly named the entropy noise and shows the degree to which the pressure and density deviate from the isentropic relation in the turbulent flow. To compute the far-field sound, Lighthill assumed that the source generating mechanism is compact and in a unbounded flow coupled with the free-space Green’s function and Fraunhofer’s approximation, the far-field pressure fluctuations can be written as

$$p - p_\infty = (\rho - \rho_\infty) a_\infty^2 \approx \frac{1}{4\pi} \iiint_V \frac{(x_i - y_i)(x_j - y_j)}{|x - y|^3} \frac{1}{a_\infty^2} \frac{\partial^2}{\partial t^2} T_{ij} \left(y, t - \frac{|x - y|}{a_\infty}\right) dy. \hspace{1cm} (4.13)$$
4.3.2 Setup and Computational Details

The noise sources from all four jets including the isothermal case were computed in this study. The shape of the (red) integration volume is similar to that of the FWH surface shown in Figure 4.1 with the exception that it is smaller in size in the lateral direction. The crosswise extent of the integration volume is roughly $7r_o$ and opens up to $22r_o$. This crosswise length was chosen since Uzun et al. [53] showed that the majority of the noise sources for a high speed jet are confined within a crosswise extent of roughly 5 to 6 jet radii along the entire streamwise domain. The streamwise length of the integration volume is $59r_o$, and this plays a crucial role in the ability to capture the effective quadrupole noise sources in the computational domain. When Uzun et al. [53] used a domain length of $32r_o$ for their Lighthill computations, they reported spurious noise levels in their OASPL directivity for observer angles $\Theta > 80^\circ$ ($\Theta$ measured relative to jet centerline downstream axis). They suggest that the sudden truncation of the domain creates spurious dipole noise sources as the quadrupole sources pass through the downstream surface. Bodony and Lele [104] used a domain length of approximately $55r_o$ for their Lighthill analysis and reported no spurious noise levels in their OASPL directivity.

For each test case, the five primitive variables, $q = [\rho, \mathbf{u}, p]^T$, were saved every ten time steps over a duration of 40,000 time steps during the simulation. This resulted in roughly 430 GB of data saved in double precision unformatted. Due to this large data size, a total of 1,140 processors was used to compute Lighthill’s volume integral with a total run time of 5 hours on the Lemieux supercomputer at the Pittsburgh Supercomputing Center (PSC). Based on the spatial grid resolution of the Lighthill control volume, the highest resolvable Strouhal number for test cases SP07 and SP46 is 2 whereas for test cases SP03 and SP23 it is 2.7. Similar to the FWH methodology, the far-field sound is calculated along an arc with a radial distance of $R = 144r_o$ with the observer angle, $\Theta$, measured relative to the jet centerline downstream axis. Refer to Reference 53 for details regarding the numerical methods used for the Lighthill analysis.
4.3.3 Results

Referring to Figure 4.2 once more, we see that the far-field noise predicted by the acoustic analogy for jet SP07 is in good agreement with the experimental results of Viswanathan and Tanna. In addition, the comparison between FWH and Lighthill is also very good. However, the Lighthill prediction seems to give a better prediction, i.e. closer to experiments, for observation angles, $20^\circ < \Theta < 40^\circ$, compared to the FWH results. It is also important to note that we do not see any spurious noise levels for observer angles $\Theta > 80^\circ$ as was reported by Uzun et al. [53] due to a longer integration domain used in this study. Figure 4.3 shows the OASPL directivity for the first heated jet SP46. Again, here we note the reasonable comparison between our Lighthill results and the experiments of Tanna [78] and Viswanathan [29]. Like the FWH results, the Lighthill computations over predict the laboratory results by approximately 3 dB which is acceptable. Figure 4.4 shows the OASPL of the low-speed unheated jet of SP03. Compared to Tanna’s SP03 data, the Lighthill computations show good agreement for observation angles $20^\circ < \Theta < 80^\circ$. Beyond $\Theta = 80^\circ$, we under-predict the laboratory measurements indicating that the LES computation is probably missing finer scales near the jet axis. Our computational results are also in good agreement with the LES-Kirchhoff predictions of Bodony and Lele [104] and the empirical prediction of the SAE ARP876C database. A FWH computation was not performed for SP03. Finally, Figure 4.5 shows the OASPL values for the second heated jet test case SP23. Trend wise, the computed Lighthill results agree well with laboratory experiments of Tanna but over predict again by approximately 3 dB. Note that at the peak radiation level at $\Theta \simeq 30^\circ$, the Lighthill results are slightly lower compared to the FWH prediction and are closer to experiments. Thus, it seems so far that for all three jets considered here, Lighthill’s acoustic analogy does a slightly better job in predicting the far-field noise near the peak radiation angle compared to the FWH method.

Next we look at the individual noise source components of the Lighthill stress tensor. Figure 4.6 shows the OASPL contribution from $T_{ij}$ and its individual components $T_{ij}^l$, $T_{ij}^n$ and $T_{ij}^s$ to the far-field noise for isothermal jet SP07. Even for an isothermal jet, the
entropic part of the noise source is significant near the jet axis where the observation angle is small, but becomes insignificant in the nozzle region, i.e. large angles. Our observation follows that of Uzun et al. [53] for the low angles but differs for the near nozzle region. Our results show a continuous decay but Uzun et al. show spurious levels in the entropy noise for angles $\Theta > 100^\circ$. The shear ($T_{ij}^l$) and self ($T_{ij}^n$) noise are greater than the total noise for angles $\Theta < 45^\circ$ while the entropy noise is greater compared to the total noise for $\Theta < 15^\circ$. The shear noise, however, is seen here to have a more bi-directional character with an extinction near $\Theta = 80^\circ$. The shear noise shape computed here confirms the theory proposed by Ribner [107], i.e. the sound intensity should vary proportional to a factor of $I \sim \cos^4 \Theta + \cos^2 \Theta$ for shear noise. This bi-directional behavior was also reported by Freund [102] using DNS and by Uzun et al. [53].

Figure 4.7 shows a plot similar to that of Figure 4.6 but for the first heated test case SP46. The most noticeable difference between Figures 4.7 and 4.6 is that for case SP46 the entropy noise is greater compared to the total noise and the self noise is significantly lower compared to the total noise. More specifically, the entropy noise is louder than the total noise for observation angles $\Theta < 60^\circ$ as opposed to SP07’s $\Theta < 45^\circ$. The shear noise again shows a similar trend as in SP07, i.e. the sudden extinction at $\Theta \simeq 80^\circ$ and then an increase. Figure 4.8 shows the noise source components for the unheated jet case SP03. Unlike its high speed counterpart, i.e. SP07, the entropic noise sources that radiate into the far-field seem to be negligible compared to the momentum type sources with the self noise being the most dominant over all observer locations. The shear noise term seem to be only important only for obtuse angles (upstream) and for shallow angles. Again, we see the bi-directional shape of the shear noise term. Finally, Figure 4.9 shows the noise source components for the second heated jet SP23. In this particular case, there is a stark contrast in the directivity behavior compared to the previous three jets. For this set point, $M_j = 0.38$ and $T_j/T_\infty = 1.76$, the most dominant source is the entropy noise. The self and shear noise contributions appear to be insignificant for a low speed heated jet. Likewise, the directivity pattern of the shear noise follows that of the previous three jets.
To obtain a clearer representation of the effect of heating, we re-plot all the noise sources but with all the jet test cases together. Figure 4.10 shows the total noise for all three jets. The corresponding experimental data are plotted as well. Comparing the SP07 and SP46 data from the experiments, the effect of heating the jet actually makes the jet quieter for the same jet acoustic Mach number. Our simulations capture the same behavior albeit for angles $\Theta < 45^\circ$. For angles greater than $45^\circ$, however, test case SP46 is slightly noisier. For the low speed jets, the behavior is reversed, i.e. we see that overall the low speed heated jet is now louder by approximately 5 dB compared to its unheated counterpart. Although, the difference in the total noise level between our LES-Lighthill results are slightly larger compared to experiments, we nonetheless capture the experimental trend. Figures 4.11 through 4.13 show the effect of heating on each of the sources for all three jets. From Figures 4.11 and 4.12, we see that the effect of heating actually decreases the shear and self noise if the Mach number is kept constant. In fact, the decrease is more pronounced for the low speed jets by about 25 dB compared to the high speed jets. We must bear in mind that all noise source levels could probably be lower since as we have seen, the LES results over predict the experimental measurements by approximately 3 dB. Nonetheless, the trends are captured well here. The entropy noise source on the other hand, i.e. Figure 4.13, is amplified when the jet is heated. The increase in the entropy noise comes as no surprise since entropy fluctuations are related to temperature variations. Test case SP03 overall shows the lowest levels of noise where the self noise dominates the directivity. One can also infer from Figure 4.13 that the effects of compressibility become important on the entropic source term for an unheated jet when the Mach number is increased.

From the OASPL plots, it is clear that some of the noise components are louder and more intense than the total noise at some observation angles, which suggests that cancellations and synergism among the sources are taking place. Freund [102] suggests that the cancellations among the noise generating components must be correlated and defined the following correlating coefficients,

\[ C_{ln} = \frac{\overline{pl pn}}{p_{rms}^l p_{rms}^n}, \quad C_{ls} = \frac{\overline{pl ps}}{p_{rms}^l p_{rms}^s}, \quad C_{ns} = \frac{\overline{pn ps}}{p_{rms}^n p_{rms}^s}, \quad (4.14) \]
where the pressure terms are the fluctuating pressure history from each source and the superscripts $l$, $n$ and $s$ indicate the shear, self and entropy noise components, respectively. Figures 4.14 through 4.16 show how each of the correlation coefficients behave for an unheated and heated jets along an arc in the far-field at $R = 144r_o$. Looking at the first correlation coefficient, $C_{ln}$, we see that by heating the jet there appear to be less cancellations among the shear and self noise terms. In other words, a high speed unheated jet has strong cancellations among the shear and self noise terms. From Figure 4.14 we can deduce that cancellations dominate the shear and self terms at nearly all observation angles. It is also important to note that Uzun et al. [53] reported the same observation for their unheated jet SP07. In addition, there appears to be more cancellations for SP03 compared to SP23. Figure 4.15 shows the correlation of $C_{ls}$ and from here we observe that over all observation angles the shear and entropy noise terms contain significant cancellations in the far-field when a $M_\infty = 0.9$ jet is heated. The cancellation is strongest at $\Theta = 5^\circ$ with $C_{ls} \approx -0.9$ for SP46 as opposed to $C_{ls} \approx -0.3$ for SP07 at the same angle. This observation could probably explain why a heated jet is quieter compared to an unheated jet from high Mach numbers, i.e. $M_\infty > 0.7$. Bodony and Lele [104] performed a similar analysis and also reported significant cancellations between the momentum (shear) and entropy terms in their heated jet compared to a similar unheated case. As a side note, it is important to point out that Bodony and Lele did not rely on the correlation coefficient used above but instead deduced their findings (cancellations or amplifications) based on the phase spectra for each noise source term. The plots of the coefficients used here gives an alternate representation of the behavior amongst the sources in terms of strength and spatial directivity. Coming back to Figure 4.15, based on the observation that SP46 has more cancellations compared to SP07, it was hoped that more cancellations would appear for SP03 compared to SP23 indicating that SP03 is quieter. However, this is not the case. Instead, SP23 has more cancellations compared to SP03 for angles $5^\circ < \Theta < 90^\circ$ and shows amplification for observation points upstream of the jet. The strength of cancellations for low speed jets is not as drastic as the one shown by the high speed jets which could indicate that the interaction between the shear and entropic sources play a crucial role in
determining the far-field characteristics for high speed jets. Our assumption that there are more cancellations for SP03 compared to SP23 is confirmed from Figure 4.16. In this plot, cancellations appear throughout all observation angles for SP03 compared to SP23. In fact the SP23 show small synergism (amplifications) among the self and entropic noise source terms for all observation angles. Likewise, one could deduce that for low speed jets, the interplay between self and entropic sources this time play a more dominant role in determining the far-field noise. In addition, there is slightly more cancellation among the self and entropy noise for SP07 compared to SP46 for angles $\Theta < 65^\circ$ but then SP46 registers more cancellations compared to SP07 for angles $\Theta > 65^\circ$.

Next we focus on spectra. Figures 4.17 through 4.19 show the 1/3-Octave pressure spectra for the unheated jet SP07 for observation angles $\Theta = 30^\circ$, $\Theta = 60^\circ$ and $\Theta = 90^\circ$, respectively. The experimental data by Tanna et al. [78] and Viswanathan [29] are also plotted as a reference. All spectra presented herein are in 1/3-Octave band format to facilitate the comparisons with experiments. At $\Theta = 30^\circ$ (Figure 4.17), the entropy noise registers the lowest energy across the spectrum compared to shear and self noise. However, the entropy noise surpasses the total noise at the higher frequency spectrum, i.e. for $Sr > 1.6$. In addition, the shear noise is more intense than the total noise for all frequencies. The self noise, however, is lower than the total noise in the low frequency region, i.e. for $Sr < 0.3$. An interesting observation is that at high frequencies, all noise components register higher sound pressure levels compared to the experimental data. The fact that the total noise is lower than the individual noise sources suggests that there are significant cancellations amongst the spectral noise components, as we have seen in Figures 4.14 through 4.16. At the observation angle of $\Theta = 60^\circ$ (Figure 4.18), the entropy noise is now lower compared to total and other noise components suggesting that the entropy noise source is negligible at this angle. In addition, the shear noise spectra overall is now lower compared to the total noise. An interesting note is that the frequency where the maximum SPL occurs shifts from $Sr = 0.3$ to $Sr = 0.6$ indicating that there is more high frequency content as an observer moves toward the near nozzle region. Moving on to $\Theta = 90^\circ$, i.e. Figure 4.19, we now see that the shear and entropy noise are lower compared to the total
noise suggesting that the majority of the noise in the near nozzle region is due to the entropy term. In addition, for all three observation angles the computed spectra are in reasonable agreement with the two experimental measurements of Tanna et al. and Viswanathan.

Figures 4.20 through 4.22 show spectra for the first heated jet SP46, i.e. $M_\infty = 0.9$ with $T_j / T_\infty = 2.7$. The corresponding experimental data are plotted as well. At $\Theta = 30^\circ$, all the noise source components are louder than the total noise for $Sr > 0.5$. At this angle also, the dominant noise source is the entropy term. It is interesting to note that the comparison of the total noise with experiments are in good agreement especially in the low frequency portion of the spectra. The least dominant term here is the self noise. For the spectra at observation angle $\Theta = 60^\circ$, we still observe that the entropy noise term is the dominant source but compared to $\Theta = 30^\circ$, its intensity is lowered. In addition, this time the shear and self noise terms are lower compared to the total noise spectra. Finally, at the $\Theta = 90^\circ$ observation angle, we see that the shear noise term does not contribute much and again the self and entropy sources are probably the main contributors of noise at this angle. Again, we observe some of the noise sources being more intense and some lower than the total noise suggest the presence of significant cancellations amongst the spectra. In brief, we note that as we progress from the shallow angles to the sideline direction ($\Theta \simeq 90^\circ$), the shear noise contribution decreases but the entropy noise source dominates for this heated jet.

Figures 4.23 through 4.25 show the far-field noise spectra obtained from the Lighthill computation for isothermal jet SP03 at observation angles $\Theta = 30^\circ$, $\Theta = 60^\circ$ and $\Theta = 90^\circ$, respectively. Referring to Figure 4.8, we see that $\Theta = 30^\circ$, the momentum sources (self and shear) dominate the far-field and this is reflected in the spectra of Figure 4.23. Here there is hardly any contribution to the far-field from the entropic term. The shear noise term drops in amplitude across the frequency spectra eventually reaching the level of the entropic noise source term at $\Theta = 90^\circ$. This level is also shown in reflected in the OASPL plot given in Figure 4.8. For all the spectra shown for SP03, the Lighthill computations give reasonable agreement up until the grid frequency cut-off compared the experimental data of Tanna. Figures 4.26 through 4.28 show the spectral characteristics for
the second heated jet SP23 \((M_{\infty} = 0.5, T_j/T_{\infty} = 1.76)\) for observation angles \(\Theta = 30^\circ\), \(\Theta = 60^\circ\) and \(\Theta = 90^\circ\), respectively. The prevailing theme we see is that for this low speed heated jet, the entropy noise dominates across the frequency spectrum. The shear and self noise sources hardly contribute and this has already been seen in the OASPL plots in Figures 4.9, 4.14, 4.15 and 4.16.

To observe the effects of heating on high speed jets, Figures 4.29 through 4.31 show the spectral comparisons between SP07 and SP46. SP23 has been left out since this jet condition is different in terms of ambient jet Mach number compared to SP07 and SP46. With the exception of \(\Theta = 30^\circ\), the total noise spectra of SP46 is slightly louder compared to SP07. This is no surprise because if we look closely at Figure 4.10, the OASPL of SP46 is also slightly higher compared to its unheated counterpart. The experiments, however, show that at observation angle \(\Theta = 30^\circ\) SP46 is quieter compared to SP07. Hence, Figures 4.32 through 4.34 show the comparison between SP07 and SP46 for each noise source component at observation angle \(\Theta = 30^\circ\) in the far-field distance of \(R = 144r_o\). For the shear noise term, \(T_{ij}^l\), the unheated jet spectra is louder compared to its heated jet counterpart for \(Sr < 0.6\) but is then less intense after that. For the self noise term, i.e. Figure 4.33, the heated jet noise is consistently lower compared to SP07 by approximately 8 dB across the frequency spectrum. Thus the effects of heating, while keeping the ambient Mach number fixed, lowers the self noise source for a \(M_{\infty} = 0.9\) jet. In other words, the intensity of the turbulent fluctuations interacting among themselves is lessened when the jet is heated. Hence, in addition to the added cancellations among the noise sources for heated jets (see Figure 4.15), the reduction in the self noise source could explain why a high speed heated jet is less noisy compared to an unheated jet. This observation is also supported by the findings of Bodony and Lele [104]. The entropy source term shows an increased intensity level across the spectrum for heated jets. Figures 4.35 through 4.37 again show a similar comparison but at \(\Theta = 60^\circ\). This time, the shear and self noise terms for SP46 are consistently lower compared to SP07 signifying that the shear noise source is now more intense for an unheated jet as we progress towards the near nozzle region. The entropy noise source terms for the heated jet are still higher for SP46 compared to SP07.
For all plots here the spectral shape of the noise sources follow the experimental results reasonably well. Finally, Figures 4.38 through 4.40 show the spectral characteristics at $\Theta = 90^\circ$. The shear noise terms are reduced significantly by approximately 15 dB for both SP07 and SP46 but the heated jet shear noise term is still lower compared to the unheated jet. Again, the self noise source for the heated jet is lower across the frequency spectrum compared to when it is unheated. The entropy noise source for the heated jet SP46 shows an increased level compared to SP07 throughout the spectrum.

For the low speed jets, Figures 4.41 through 4.43 show the comparison of total noise spectra between jets SP03 and SP23 at $R = 144r_o$ and observation angles $\Theta = 30^\circ$, $\Theta = 60^\circ$ and $\Theta = 90^\circ$, respectively. At observation angle $\Theta = 30^\circ$, the total noise for SP03 is lower compared to that of heated jet SP23 but only up to Strouhal numbers 1. Beyond $Sr = 1$, the sound pressure level of both jets are almost the same. The same observation can be seen for observation angles $\Theta = 60^\circ$ and $\Theta = 90^\circ$ though the match-up of SPL seem to occur at roughly $Sr = 1.5$. Hence, one could deduce that a low-speed heated jet is louder compared to an unheated jet mainly due to an increased energy content in the low-frequency region below $Sr = 1$. Comparing our Lighthill computations to experiments, shows that our predictions are consistent with observation angle $\Theta = 30^\circ$ compared to experiments. However, our Lighthill predictions here not do not show the expected trend for angles $\Theta = 60^\circ$ and $\Theta = 90^\circ$, respectively. In fact, they show the opposite trend compared to experiments. Figures 4.44 through 4.46 show the spectra for shear, self and entropy noise sources at observation angle $\Theta = 30^\circ$. Here the momentum noise source of the low speed isothermal jet is higher compared to its unheated counterpart. Not surprisingly, the entropy noise source spectral level is higher for SP23 compared to SP03 with a higher energy content in the low frequency region. Spectra for the noise sources at observation locations of $\Theta = 60^\circ$ (Figures 4.47 through 4.49) and $\Theta = 90^\circ$ (Figures 4.50 through 4.52) follow the same trend shown for $\Theta = 30^\circ$, i.e. the momentum sources are higher for the unheated case compared to the heated jet except for the entropy noise source term.
Figure 4.1. The control surface used for the Ffowcs Williams-Hawkins surface integral method and control volume for Lighthill’s acoustic analogy.
Figure 4.2. Overall sound pressure level variation for unheated jet SP07 at $R = 144r_o$ from the nozzle exit.

Figure 4.3. Overall sound pressure level variation for heated jet SP46 at $R = 144r_o$ from the nozzle exit.
Figure 4.4. Overall sound pressure level variation for heated jet SP03 at $R = 144r_o$ from the nozzle exit.

Figure 4.5. Overall sound pressure level variation for heated jet SP23 at $R = 144r_o$ from the nozzle exit.
Figure 4.6. Overall sound pressure level variation of the noise from $T_{ij}$ and its components for SP07 at $R = 144r_o$ from the nozzle exit.

Figure 4.7. Overall sound pressure level variation of the noise from $T_{ij}$ and its components for SP46 at $R = 144r_o$ from the nozzle exit.
Figure 4.8. Overall sound pressure level variation of the noise from $T_{ij}$ and its components for SP03 at $\mathcal{R} = 144 r_o$ from the nozzle exit.

Figure 4.9. Overall sound pressure level variation of the noise from $T_{ij}$ and its components for SP23 at $\mathcal{R} = 144 r_o$ from the nozzle exit.
Figure 4.10. Overall sound pressure level variation of total noise, $T_{ij}$, for all jets at $R = 144r_o$ from the nozzle exit.

Figure 4.11. Overall sound pressure level variation of shear noise, $T_{ij}^l$, for all jets at $R = 144r_o$ from the nozzle exit.
Figure 4.12. Overall sound pressure level variation of self noise, $T_{ij}^n$, for all jets at $R = 144r_o$ from the nozzle exit.

Figure 4.13. Overall sound pressure level variation of entropy noise, $T_{ij}^s$, for all jets at $R = 144r_o$ from the nozzle exit.
Figure 4.14. Correlation amongst the shear and self noise components, $C_{ln}$, for all jets in the far-field at $R = 144r_o$ from the nozzle exit.

Figure 4.15. Correlation amongst the shear and entropy noise components, $C_{ls}$, for all jets in the far-field at $R = 144r_o$ from the nozzle exit.
Figure 4.16. Correlation amongst the self and entropy noise components, $C_{ns}$, for all jets in the far-field at $R = 144r_o$ from the nozzle exit.

Figure 4.17. Spectra of the noise from $T_{ij}$ and its components for SP07 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.
Strouhal Number, $Sr = f D_j / U_j$

1/3-Octave SPL (dB/Sr)

0 0.5 1 1.5 2 2.5 ... noise, $T_{ij}$, SP07, $\Theta = 60^\circ$

shear noise, $T_{ij}^s$, SP07, $\Theta = 60^\circ$

self noise, $T_{ij}^n$, SP07, $\Theta = 60^\circ$

entropy noise, $T_{ij}^e$, SP07, $\Theta = 60^\circ$

Tanna’s Exp., SP07, $\Theta = 60^\circ$

Viswanathan’s Exp., SP07, $\Theta = 60^\circ$

Cutoff, $Sr = 2$

Figure 4.18. Spectra of the noise from $T_{ij}$ and its components for SP07 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.

Strouhal Number, $Sr = f D_j / U_j$

1/3-Octave SPL (dB/Sr)

0 0.5 1 1.5 2 2.5 ... noise, $T_{ij}$, SP07, $\Theta = 90^\circ$

shear noise, $T_{ij}^s$, SP07, $\Theta = 90^\circ$

self noise, $T_{ij}^n$, SP07, $\Theta = 90^\circ$

entropy noise, $T_{ij}^e$, SP07, $\Theta = 90^\circ$

Tanna’s Exp., SP07, $\Theta = 90^\circ$

Viswanathan’s Exp., SP07, $\Theta = 90^\circ$

Cutoff, $Sr = 2$

Figure 4.19. Spectra of the noise from $T_{ij}$ and its components for SP07 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Strouhal Number, $Sr = \frac{f D_j}{U_j}$

1/3-Octave SPL (dB/Sr)

0 0.5 1 1.5 2 2.5 3

Figure 4.20. Spectra of the noise from $T_{ij}$ and its components for SP46 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.

Strouhal Number, $Sr = \frac{f D_j}{U_j}$

Cut-off, $Sr = 2$

Figure 4.21. Spectra of the noise from $T_{ij}$ and its components for SP46 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.
Strouhal Number, $Sr = f D_j / U_j$

1/3-Octave SPL (dB/Sr)

0 0.5 1 1.5 2 2.5 3

25 30 35 40 45 50 55 60 65 70 75 80 85 90

Figure 4.22. Spectra of the noise from $T_{ij}$ and its components for SP46 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.

Cut-off, $Sr = 2$

Strouhal Number, $Sr = f D_j / U_j$

95

90

85

80

75

70

65

60

55

50

45

40

35

30

25

0

0.5

1

1.5

2

2.5

3

total noise, $T^{ij}$, SP46, $\Theta = 90^\circ$

shear noise, $T_{ij}^{\text{sl}}$, SP46, $\Theta = 90^\circ$

self noise, $T_{ij}^{\text{sn}}$, SP46, $\Theta = 90^\circ$

entropy noise, $T_{ij}^{\text{sn}}$, SP46, $\Theta = 90^\circ$

LES-FWH, SP46, $\Theta = 90^\circ$

Tanna’s Exp., SP46, $\Theta = 90^\circ$

Viswanathan’s Exp., SP46, $\Theta = 90^\circ$

Cut-off, $Sr = 2.7$

Strouhal Number, $Sr = f D_j / U_j$

90

85

80

75

70

65
60
55
50
45
40
35
30
25
20
15
10
5
0
0.5
1
1.5
2
2.5
3

total noise, $T^{ij}$, SP03, $\Theta = 30^\circ$

shear noise, $T_{ij}^{\text{sl}}$, SP03, $\Theta = 30^\circ$

self noise, $T_{ij}^{\text{sn}}$, SP03, $\Theta = 30^\circ$

entropy noise, $T_{ij}^{\text{sn}}$, SP03, $\Theta = 30^\circ$

Tanna’s Exp., SP03, $\Theta = 30^\circ$

Viswanathan’s Exp., SP03, $\Theta = 30^\circ$

Cut-off, $Sr = 2.7$

Strouhal Number, $Sr = f D_j / U_j$

Figure 4.23. Spectra of the noise from $T_{ij}$ and its components for SP03 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.24. Spectra of the noise from $T_{ij}$ and its components for SP03 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.25. Spectra of the noise from $T_{ij}$ and its components for SP03 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.26. Spectra of the noise from $T_{ij}$ and its components for SP23 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.27. Spectra of the noise from $T_{ij}$ and its components for SP23 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.28. Spectra of the noise from $T_{ij}$ and its components for SP23 at $\Theta = 90^\circ$, $R = 144r_0$ from the nozzle exit.

Figure 4.29. Spectra of the total noise, $T_{ij}$, for SP07 and SP46 at $\Theta = 30^\circ$, $R = 144r_0$ from the nozzle exit.
Figure 4.30. Spectra of the total noise, $T_{ij}$, for SP07 and SP46 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.31. Spectra of the total noise, $T_{ij}$, for SP07 and SP46 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.32. Spectra of the shear noise, $T_{ij}^l$, for SP07 and SP46 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.33. Spectra of the self noise, $T_{ij}^n$, for SP07 and SP46 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.
Strouhal Number, $Sr = f D_j / U_j$

1/3-Octave SPL (dB/Sr)

Figure 4.34. Spectra of the entropy noise, $T_{ij}^s$, for SP07 and SP46 at $\Theta = 30^o$, $R = 144r_o$ from the nozzle exit.

Figure 4.35. Spectra of the shear noise, $T_{ij}^l$, for SP07 and SP46 at $\Theta = 60^o$, $R = 144r_o$ from the nozzle exit.
Figure 4.36. Spectra of the self noise, $T_{ij}^n$, for SP07 and SP46 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.37. Spectra of the entropy noise, $T_{ij}^s$, for SP07 and SP46 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.38. Spectra of the shear noise, $T_{ij}^l$, for SP07 and SP46 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.39. Spectra of the self noise, $T_{ij}^n$, for SP07 and SP46 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.40. Spectra of the entropy noise, $T_{ij}^s$, for SP07 and SP46 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.41. Spectra of the entropy noise, $T_{ij}$, for SP03 and SP23 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.42. Spectra of the entropy noise, $T_{ij}$, for SP03 and SP23 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.43. Spectra of the entropy noise, $T_{ij}$, for SP03 and SP23 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.44. Spectra of the shear noise, $T_{ij}^{l}$, for SP03 and SP23 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.45. Spectra of the self noise, $T_{ij}^{n}$, for SP03 and SP23 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.46. Spectra of the entropy noise, $T_{ij}^s$, for SP03 and SP23 at $\Theta = 30^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.47. Spectra of the shear noise, $T_{ij}^l$, for SP03 and SP23 at $\Theta = 60^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.48. Spectra of the self noise, $T_{ij}^{\text{nu}}$, for SP03 and SP23 at $\Theta = 60^\circ$, $\mathcal{R} = 144r_o$ from the nozzle exit.

Figure 4.49. Spectra of the entropy noise, $T_{ij}^{\text{sf}}$, for SP03 and SP23 at $\Theta = 60^\circ$, $\mathcal{R} = 144r_o$ from the nozzle exit.
Figure 4.50. Spectra of the self noise, $T_{ij}^l$, for SP03 and SP23 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.

Figure 4.51. Spectra of the entropy noise, $T_{ij}^n$, for SP03 and SP23 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
Figure 4.52. Spectra of the entropy noise, $T_{ij}^s$, for SP03 and SP23 at $\Theta = 90^\circ$, $R = 144r_o$ from the nozzle exit.
5. ALTERNATE PARALLELIZATION STRATEGY USING THE SCHUR COMPLEMENT

5.1 Introduction

As discussed in Chapter 2, the current disadvantage of the transposition strategy is mainly restricted to the inclusion of the nozzle geometry in the computational domain. Hence, this chapter gives a brief description of the proposed parallel Schur complement method followed by an application to the compact scheme used in our LES code.

5.2 Application of the Schur Complement to Compact Schemes

In the Schur complement method, a domain is first decomposed into non-overlapping sub-domains and a global solution is achieved via a coupled solution of these sub-domains. Neighboring sub-domains share common grid points along the interface (See Figure 5.1). For simplicity, each sub-domain is assigned a single processor. After applying a compact finite difference scheme (or filter) to each sub-domain, the system of equations for a domain with $N$ sub-domains has the following form:

\[
\begin{bmatrix}
K_{11} & 0 & 0 & K_{1\Gamma} \\
K_{22} & 0 & K_{2\Gamma} & \\
\vdots & \ddots & \ddots & \ddots \\
0 & 0 & K_{N\Gamma} & K_{N\Gamma}
\end{bmatrix}
\begin{bmatrix}
f'_1 \\
f'_2 \\
\vdots \\
f'_N
\end{bmatrix}
= 
\begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_N
\end{bmatrix},
\]

(5.1)

where $K_{ii} (i = 1, \ldots, N)$, $K_{\Gamma\Gamma}$, and $K_{\Gamma\Gamma} = K_{\Gamma\Gamma}^T$ denote the stiffness (coefficient) matrices of the sub-domains, the interfaces $\Gamma$, and coupling between sub-domains and the interfaces,
respectively. The sub-domain vectors are \( f_i' \) and \( f_i \), and the interface vectors are \( f_{i\Gamma} \) and \( f_{\Gamma} \), respectively. The interface equations are solved before solving the solution of each sub-domain. Hence, the interface equation of the system can be written in a short form as

\[
(K_{\Gamma\Gamma} - G_{\Gamma\Gamma}) f_{\Gamma}' = f_{\Gamma} - g_{\Gamma}. 
\] (5.2)

where

\[
G_{\Gamma\Gamma} = \sum_{i=1}^{N} K_{i\Gamma} A_{i\Gamma}, \quad g_{\Gamma} = \sum_{i=1}^{N} K_{\Gamma i} A_i. 
\] (5.3)

and

\[
A_{i\Gamma} = K_{ii}^{-1} K_{i\Gamma}, \quad A_i = K_{ii}^{-1} f_i. 
\] (5.4)

The system of equations given in Equation (5.2) is known as the Schur complement equation. The dimension of the Schur complement matrix depends on the number of unknowns on the interfaces, where the number of interface unknowns increases with the number of sub-domains [42]. Figure 5.1 shows a 1-D grid partitioned into four sub-domains. To assemble the Schur complement matrix in parallel, contributions to \( G_{\Gamma\Gamma} \) and \( g_{\Gamma} \) are first computed by each sub-domain processor without message passing, i.e. sub-domains 1 through 4. Then, the interface matrix coefficient, \( (K_{\Gamma\Gamma} - G_{\Gamma\Gamma}) \), and the source vector, \( f_{\Gamma} - g_{\Gamma} \), are assembled via message passing.

In Kocak and Akay’s paper [42], the parallel Schur complement algorithm was written with a finite element flavor in mind. Nevertheless, the algorithm is equally applicable to implicit compact differencing and filtering schemes. The Message-Passing Interface (MPI) libraries coupled with the Fortran 90 programming language are used throughout this work. Again, consider a one-dimensional grid partitioned into four sub-domains \((N = 4)\) shown in Figure 5.1. There are a total of 19 grid points and each sub-domain has 4 grid points. In addition, each sub-domain shares a common grid point at the interface, i.e. sub-domains 1 and 2 share grid point index 17, sub-domains 2 and 3 share grid point index 18 and finally sub-domains 3 and 4 share grid point index 19. Some of the compact scheme equations are repeated here from the Chapter 2 as a refresher. Equation (2.29) is used to estimate the sixth-order spatial derivatives at the interior grid points. For the points next to the
boundaries, i.e. $i = 2$ and $i = N_g - 1$, the following fourth-order central compact scheme is used

$$\frac{1}{4} f'_1 + f'_2 + \frac{1}{4} f'_3 = \frac{3}{4\Delta\xi} (f_3 - f_1).$$  \hspace{1cm} (5.5)

$$\frac{1}{4} f'_{N_g-2} + f'_{N_g-1} + \frac{1}{4} f'_{N_g} = \frac{3}{4\Delta\xi} (f_{N_g} - f_{N_g-2}).$$  \hspace{1cm} (5.6)

Finally, for the points on the left and right boundary, i.e. $i = 1$ and $i = N_g$, the following one-sided third-order compact scheme is used

$$f'_1 + 2 f'_2 = \frac{1}{2\Delta\xi} (-5 f_1 + 4 f_2 + f_3).$$  \hspace{1cm} (5.7)

$$f'_{N_g} + 2 f'_{N_g-1} = \frac{1}{2\Delta\xi} (5 f_{N_g} - 4 f_{N_g-1} - f_{N_g-2}).$$  \hspace{1cm} (5.8)

Applying the compact scheme by Lele, i.e. Equations (2.29) and (5.5) through (5.8), to the domain in Figure 5.1, the resulting coefficient matrices in Equation (5.1) are

\[
K_{11} = \begin{bmatrix}
1 & 2 & 0 & 0 \\
1/4 & 1 & 1/4 & 0 \\
0 & 1/3 & 1 & 1/3 \\
0 & 0 & 1/3 & 1
\end{bmatrix}, \quad K_{22} = K_{33} = \begin{bmatrix}
1 & 1/3 & 0 & 0 \\
1/3 & 1 & 1/3 & 0 \\
0 & 1/3 & 1 & 1/3 \\
0 & 0 & 1/3 & 1
\end{bmatrix}, \quad K_{44} = \begin{bmatrix}
1 & 1/3 & 0 & 0 \\
1/3 & 1 & 1/3 & 0 \\
0 & 1/4 & 1 & 1/4 \\
0 & 0 & 2 & 1
\end{bmatrix}, \quad K_{1\Gamma} = K_{\Gamma 1}^T = \begin{bmatrix}
0 & 0 & 0 & 1/3 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
K_{2\Gamma} = K_{\Gamma 2}^T = \begin{bmatrix}
1/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/3 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad K_{3\Gamma} = K_{\Gamma 3}^T = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/3
\end{bmatrix}, \quad K_{4\Gamma} = K_{\Gamma 4}^T = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/3 \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]  \hspace{1cm} (5.9)
and finally, the assembled Schur complement coefficient matrix in Equation (5.2) is found to be

\[
K_{\Gamma \Gamma} - G_{\Gamma \Gamma} = \begin{bmatrix}
\frac{548}{2145} & \left(-\frac{1}{165}\right) & 0 \\
\left(-\frac{1}{165}\right) & \frac{14}{35} & \left(-\frac{1}{165}\right) \\
0 & \left(-\frac{1}{165}\right) & \frac{1429}{3610}
\end{bmatrix}
\] (5.10)

In the Schur complement matrix, i.e. Equation (5.10), the off-diagonal terms or the interface coupling coefficients are equal and are, \(-1/165 = -0.00606\) for the case of four points in each sub-domain. Table 5.1 shows the effect of increasing the number of grid points in each sub-domain keeping the total number of sub-domains fixed at four. For the case of four points in each sub-domain, the Schur complement matrix must be solved directly via inversion or by the popular Thomas algorithm since the matrix is tridiagonal. However, as the number of points in each sub-domain increases, we see that the coupling coefficients decrease. There is a dramatic decrease as the number of points increases. Thus, the Schur complement matrix becomes strongly diagonally dominant as the number of points in each sub-domain increases. In the case of 64 points in each sub-domain, the Schur complement matrix, in a machine precision sense, is a diagonal matrix. Hence, if the coupling coefficients are small enough compared to the diagonal terms, matrix inversion or some other algorithm may not be required and the interface variables can be approximated by dividing the right hand side of Equation (5.1) with the diagonal terms of the Schur complement matrix. Eliasson [40] also reported the same behavior of the coupling coefficients (as the number of sub-domain points increase) when he used the Schur complement method to solve the Vlasov-Maxwell equation. In his methodology, he uses Jacobi iteration to solve the Schur complement matrix. However, when the coupling coefficients were small enough, i.e. approaching machine zero, Jacobi iteration was not applied and the Schur complement matrix was solved directly. This behavior of the coupling coefficient is particularly advantageous when extending the parallel Schur complement to 3-D calculations as this would save considerable amounts of computation time. On the other hand, if
we increase the number of sub-domains, keeping the total number of grid points fixed, the numerical value of the coupling coefficients will increase.

The filtering formula is given in Equation (2.34). For the points next to the left and right-hand side boundary, i.e. \( i = 2, 3 \) and \( i = N - 2, N - 1 \), a sixth-order, one-sided stencil is used and is described by Equations (2.36) through (2.39) are used to filter to points next to the boundary. The boundary points, however, are left unfiltered. See Reference [32] for more details regarding the formulation of this spatial filtering for the near boundary points. The Schur complement coupling coefficients for the tridiagonal spatial filter are shown in the last column of Table 5.1. As can be seen, the decrease in the coupling coefficient for the filtering scheme is not as dramatic compared to its compact differencing scheme counterpart. A significant decrease occurs when the number of sub-domain points increases from 32 to 64. It is worth mentioning that in the general numerical methodology of LES, filtering is applied before the end of each time step advancement. The compact differencing scheme, however, is applied several times depending on the number of stages that are present in an explicit time advancement scheme. Therefore, even if direct inversion of the Schur complement matrix is required for the filtering scheme, the computational cost will not be as substantial compared to the compact scheme since it is only applied once and not several times for each time step.

5.3 Linearized Euler Equation (LEE) Test Cases

This section details some work and results of a 1-D and 2-D LEE test case utilizing the Schur complement. The 1-D and 2-D LEE solutions are then compared to analytical solutions [108] with excellent agreement.

5.3.1 One Dimensional Test Case

The governing 1-D Linearized Euler Equation (LEE) is given by

\[
\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} = 0,
\]  
(5.11)
where

\[
\begin{pmatrix}
\rho \\
u \\
p
\end{pmatrix}, \quad \begin{pmatrix}
M\rho + u \\
Mu + p \\
Mp + u
\end{pmatrix}
\]

(5.12)

where \(\rho\) is the density, \(u\) is the velocity in the \(x\) direction, \(p\) is the pressure and \(M\) is the Mach number, respectively. Subsonic non-reflecting boundary conditions are prescribed at the inlet and outlet. The inlet boundary conditions are given by

\[
\frac{\partial \rho}{\partial t} = \frac{\partial p}{\partial t}, \quad \frac{\partial u}{\partial t} = \frac{\partial p}{\partial t}, \quad \frac{\partial p}{\partial t} = \frac{M - 1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial p}{\partial x} \right)
\]

(5.13)\(\frac{\partial }{\partial t}\) \(\frac{\partial }{\partial x}\)

while the outlet boundary conditions are specified as

\[
\frac{\partial \rho}{\partial t} = \frac{\partial p}{\partial t} - M \left( \frac{\partial \rho}{\partial x} - \frac{\partial p}{\partial x} \right), \quad \frac{\partial u}{\partial t} = \frac{\partial p}{\partial t}, \quad \frac{\partial p}{\partial t} = -\frac{M + 1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} \right)
\]

(5.16)\(\frac{\partial }{\partial t}\) \(\frac{\partial }{\partial x}\)

The 1-D Linearized Euler Equation above is solved on a mesh consisting of 131 equally spaced grid points with \(x \in [0, 20]\). The mesh is split into four non-overlapping domains similar to the one shown in Figure 2. Hence, each sub-domain has 32 grid points \((N_{Px} = 32)\). The free stream Mach number is set at \(M = 0.5\) and \(\Delta t = 0.005\). The filtering coefficient is set to \(\alpha_f = 0.47.\) For the initial conditions, a modified aeroacoustic benchmark problem \([33, 108, 109]\) is specified by a Gaussian entropy disturbance given as

\[
\rho = 1 + 0.1 \exp \left[ \frac{-(x - x_c)^2 \ln(2)}{2} \right], \quad p = 1, \quad u = 1
\]

(5.19)(5.20)(5.21)

where \(x_c = 5.\) The compact scheme proposed by Lele \([31]\) is used for spatial discretization, i.e. Equations (2.29) and (5.5) through (5.8). For time advancement, the standard fourth-order Runge-Kutta method is used.
The results obtained with the parallel Schur complement method are compared to an analytical solution given by

$$\rho(x, t) = 1 + 0.1 \exp \left[ \frac{-(x - Mt - x_c)^2 \ln(2)}{2} \right].$$  \hspace{1cm} (5.22)

Figure 5.2 shows the initial density and pressure waveform along the x-axis. As time advances, the density waveform travels from left to right. Figure 5.3 shows the density waveform at $t = 20$. We notice that the solution of the parallel Schur method matches very well when compared to the analytical solution. The waveform eventually reaches the end of the domain and leaves. Table 5.2 shows the root mean square (R.M.S) error for the 1-D LEE test case for different number of sub-domain points. We also report the R.M.S. error obtained from a serial 1-D LEE code. Here, $N_x$ is the total number of grid points, $N_{P_x}$ is the number of points per sub-domain and R.M.S. is simply the root mean square error. Keep in mind that the number of points per sub-domain, $N_{P_x}$, does not include the interface point. The number of domains is kept at four. The corresponding coupling coefficients for the three cases studied can be found in Table 5.1.

Overall, the R.M.S error decreases as the total number of grid points and points per sub-domain increases. We also performed a test whereby only the diagonal terms of the Schur complement matrix were inverted. The R.M.S. error changed slightly for the first case of 67 points where the computed R.M.S error is $2.01 \times 10^{-4}$. The remaining two cases for the R.M.S. error were different only after the 13th decimal. We also note the difference in the R.M.S. errors reported between the serial 1-D LEE version and the parallel Schur complement when compared to the exact solution. The largest difference is reported for the first test case of $N_x = 67$ points. It is important to note that using 67 grid points is rather coarse for a 1-D LEE problem such as the one here. However, there is only a small difference in the R.M.S. errors for the last two cases and is close to the serial results. The small differences in R.M.S. errors could be a direct result of the linear algebra performed to solve the Schur complement matrix. Also notice that the R.M.S errors do not decrease following a sixth-order accurate scale as the number of grid points is doubled. This is due to the fact that we did not keep the CFL number constant, i.e. we reduced $\Delta x$ but kept $\Delta t$ constant. Nonetheless, an overall error in the order of $10^{-4}$ and $10^{-5}$ is satisfactory.
Hence, the Schur complement method has been applied to a simple 1-D LEE test case and the solution compares very well to its corresponding analytical solution.

### 5.3.2 Two-Dimensional Test Case

In this section, the second test case chosen is a solution of a 2-D LEE. The chosen benchmark corresponds to Category (3) in Reference [108]. The LEE governing equation in 2-D can be written as

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0,$$

where

\[U = \begin{pmatrix} \rho \\ u \\ v \\ p \end{pmatrix}, \quad E = \begin{pmatrix} M_x \rho + u \\ M_x u + p \\ M_x v \\ M_x p + u \end{pmatrix}, \quad F = \begin{pmatrix} M_y \rho + v \\ M_y u \\ M_y v + p \\ M_y p + v \end{pmatrix},\]

and \(\rho, u, p\) are defined in the previous section, \(v\) is the velocity in the \(y\) direction, and \(M_x\) and \(M_y\) are the Mach numbers in the \(x\) and \(y\) directions, respectively. Tam and Webb's [110] non-reflecting boundary conditions are used on the top, bottom and right boundaries and are given by

$$\frac{\partial \rho}{\partial t} = -M_x \frac{\partial \rho}{\partial x} + \frac{\partial p}{\partial t} + M_x \frac{\partial p}{\partial x},$$

(5.25)

$$\frac{\partial u}{\partial t} = -M_x \frac{\partial u}{\partial x} - \frac{\partial p}{\partial x},$$

(5.26)

$$\frac{\partial v}{\partial t} = -M_x \frac{\partial v}{\partial x} - \frac{\partial p}{\partial y},$$

(5.27)

$$\frac{\partial p}{\partial t} = -V(\theta) \left( \cos \theta \frac{\partial p}{\partial x} + \sin \theta \frac{\partial p}{\partial y} + \frac{p}{2r} \right).$$

(5.28)
On the left boundary however, a subsonic inflow boundary condition is specified and is given by

\[
\frac{\partial \rho}{\partial t} = -V(\theta) \left( \cos \theta \frac{\partial \rho}{\partial x} + \sin \theta \frac{\partial \rho}{\partial y} + \frac{\rho}{2r} \right),
\]
\[
\frac{\partial u}{\partial t} = -V(\theta) \left( \cos \theta \frac{\partial u}{\partial x} + \sin \theta \frac{\partial u}{\partial y} + \frac{u}{2r} \right),
\]
\[
\frac{\partial v}{\partial t} = -V(\theta) \left( \cos \theta \frac{\partial v}{\partial x} + \sin \theta \frac{\partial v}{\partial y} + \frac{v}{2r} \right),
\]
\[
\frac{\partial p}{\partial t} = -V(\theta) \left( \cos \theta \frac{\partial p}{\partial x} + \sin \theta \frac{\partial p}{\partial y} + \frac{p}{2r} \right),
\]

where

\[ V(\theta) = M \cos \theta + (1 - M^2 \sin^2 \theta)^{1/2}, \]

and \((r, \theta)\) are the polar coordinates for the boundary points centered at the location of the acoustic source. For this 2-D case, the origin \((0,0)\) is chosen as the acoustic source location whereby the domain size is set as \((x, y) \in [-100, 100] \times [-100, 100]\). Only the symmetry test is performed in this section whereby entropy and acoustic sources are specified as initial conditions,

\[
\rho = \exp \left[ -\ln(2) \frac{x^2 + y^2}{9} \right],
\]
\[
u = 0,
\]
\[
v = 0,
\]
\[
p = \exp \left[ -\ln(2) \frac{x^2 + y^2}{9} \right].
\]

For this study we will compare the pressure wave form obtained from the Schur complement to the analytical solution. The analytical solution for the pressure wave form is given by

\[
p(x, y, t) = \frac{1}{2\alpha_1} \int_0^\infty \exp \left( \frac{\xi^2}{4\alpha_1} \right) \cos(\xi t) J_0(\xi \eta) \xi d\xi,
\]

where \(\alpha_1 = (\ln 2)/9, \eta = \sqrt{(x - Mt)^2 + y^2}\) and \(J_0(\ )\) is the Bessel function of order zero. Analytical solutions for the density and velocities can be found in Hardin et al. [108]. Figure 5.4 shows the computational grid used for the 2-D LEE problem along with its partitioning in the x direction. All points on the x and y axes are equally spaced. The
Mach numbers are set to $M_x = 0.5$ and $M_y = 0$ with $\Delta t = 0.5$. Figure 5.6 shows the pressure waveform solution computed by the Schur complement and compared to the analytical solution at $t = 33$ for Case $A$. We note the very good agreement between the numerical and exact solution in Figure 5.6.

Table 5.3 shows the R.M.S error and speed-up tests for the 2-D LEE problem with two different number of grid points in each direction. $N_x$ and $N_y$ are the number of grid points in the $x$ and $y$ directions, respectively. In addition, $N_{bx}$ and $N_{by}$ are the number blocks in the $x$ and $y$ directions, respectively, and $N_{procs}$ is the total number of processors used. Keep in mind that the size of the Schur complement matrix is dependent on the number of interfaces in a particular direction. As an observation, test cases $B$ and $E$ have only one interface in the $x$ and $y$ directions. In addition, the coupling coefficient computed for $N_{Px} = 65$ is very close to the coupling coefficient for $N_{Px} = 64$ shown in Table 1. All runs were performed on a departmental Linux cluster whereby each node has two AMD 1.4 GHz processors and each node is connected via regular ethernet (see Acknowledgements).

Based on the R.M.S. errors in Table 5.3, we note that the agreement between the Schur complement and the exact solution is very good. We also show the results from a serial 2-D LEE code. For each case, as the number of points is roughly doubled in each direction, the R.M.S. error decreases by approximately four times for each corresponding block topology. As in the 1-D LEE test case, the CFL number is not constant, which is why we are not seeing a sixth-order reduction in the R.M.S. errors. In addition, the block arrangements hardly had an effect on the R.M.S. errors. We observe that the R.M.S errors for the 2-D LEE parallel Schur complement are slightly higher than its serial counterpart. Again, we believe that these small errors are introduced when the Schur complement matrix is solved numerically. However, the speed-ups obtained between different block topologies are interesting. A speed-up per-time step of roughly 30% is gained for a block arrangement of $2 \times 2$ compared to a $4 \times 1$ using the same number of grid points. Gropp et al. [111] reported a similar observation when they parallelized a 2-D Poisson problem using a second order finite difference scheme. This can be explained by examining the number of points along the shared interface between each block. For Case $A$, a buffer size of 131 points
along with the four primitive variables (131 × 4) needs to be sent and received between each block. If a 2-D decomposition is done as in Case B, half that amount is needed and thus the increase in performance. Cases D and E also report similar observations in terms performance though slightly lower. A speed-up of roughly 2 times is reported between cases A and C. For the speed-up analysis, Case A was taken as a reference for the coarse grid setup and Case D for the finer grid setup. As a note, timing is measured from the first time step to the last. The timing measured here does not include the start-up time which is found to be rather small (on the order of 2% of the simulation time).

We also used the same methodology whereby only the diagonal elements of the Schur complement matrix were inverted for the above cases. Here, we did not see any gain in speed-up using this methodology since this 2-D problem is rather computationally inexpensive, but we expect to see a performance gain for 3-D applications. Overall, the comparison between the computed 2-D LEE results via the Schur complement and the exact solution are very good. With the encouraging results obtained for the 1-D and 2-D LEE test cases, the parallel Schur complement will now be applied to a full 3-D LES jet.

5.4 Jet Flow Simulation via the Schur Complement

This section details work that was done for a 3-D LES jet test case using the parallel Schur complement methodology. Results obtained are compared to previously simulated results from our single-block code [79]. The next section briefly describes the 3-D LES methodology aside from the parallelization used.

5.4.1 Computational Setup

To test our Schur complement methodology for a 3-D round jet, we use a setup similar to that of a previous simulation with the single-block code [79]. The grid setup is similar as that compared to Chapter 3. However, the physical part of the domain extends to approximately 25r_o in the streamwise direction and −15r_o to 15r_o in the transverse y and z directions. The total number of grid points used here is 287 × 128 × 128 in the x − y − z
directions respectively. This gives a total of approximately 4.7 million grid points. Figure 5.7 shows the $x-z$ cross sectional plane of the computational domain. The same hyperbolic tangent velocity profile used for the heated and unheated jet simulations is employed, i.e. Equation (3.1). However, the shear layer thickness parameter is set to $b = 3.125$. Based on the minimum grid resolution, the time step is set to $\Delta t = 0.25$. In addition, since an isothermal jet is being studied, the Crocco-Buseman relation is used at the inflow boundary, i.e. Equation (3.2). A jet centerline Mach number of 0.9 and Reynolds number $Re_D = \rho_j U_j D_j / \mu_j = 100,000$ is specified. Since this is an isothermal jet, the centerline temperature is the same as the ambient temperature. Like previous jet case for our LES jet, we remove the first four modes of forcing. This also corresponds to test case $rf4$ in Reference [79].

Table 5.4 shows the block decomposition cases used for the Schur complement. As a first test the Schur complement is first decomposed in one direction, i.e. case S16a. Figure 5.8 shows the block arrangement for the 3-D LES test case using the parallel Schur complement. A block arrangement of $N_{bx} \times N_{by} \times N_{bz} = 1 \times 1 \times 16$ is chosen mainly because we want to make a one-to-one comparison of the near-field and far-field solution between the Schur complement and single-block code. Recall that in Chapter 2, the single-block code is partitioned in this manner as well. Hence, for both the single-block and Schur complement case a total of 16 processors are used. Also, Blocks 1 through 15 have 7 points in each sub-domain along the $z$ direction not including the interface points. Block 16 however, has 8 grid points. We are aware that this will lead to a slight load imbalance, but we decided to keep the number of grid points the same between the two simulations. It is also worth mentioning that a minimum of 7 grid points is required for the spatial tri-diagonal filter used here. Performing a decomposition up to 1,024 processors or cores is necessary to validate the near and far-field results and to test scalability of the code. Figure 5.9 shows the three dimensional grid decomposition ($z = 0r_o$) for case S16b with $N_{bx} \times N_{by} \times N_{bz} = 4 \times 2 \times 2$. Note that no overlaps are shown in this setup as expected. Figures 5.10 through 5.12 show the decomposition for case S1024 at several stations along the computational grid.
5.4.2 Near and Far-field Results

This section details the near and far-field results from the Schur complement. Performance and scalability results of the Schur complement will be presented in the next section. For the purposes of validating the meanflow and far-field noise of the Schur complement against the single-block code, only cases S16a, S16b and S1024 were chosen. The simulation was run over the first 10,000 time steps for the initial transients to exit the domain. Flow statistics is then collected over the next 35,000 time steps. As a note, the procedure used here follows what was performed for the single-block case [79]. Thus, for both simulations, a total of 45,000 time steps were needed to achieve reasonably converged statistics.

Figure 5.13 shows the instantaneous dilatation, $\nabla \cdot \mathbf{u}$, contours of our jet along with the boundaries of each block for case S16a. From Figure 5.13, we note that there is a smooth transition of the dilatation contours between each block, and a close-up view in Figure 5.14 shows this very clearly. Figures 5.15 and 5.16 show the instantaneous dilatation for case S1024. Again we note the smooth dilatation contours across boundaries even for a 1,024 processor case. Furthermore, the strong pseudo-acoustic radiation angle of $\Theta \simeq 30^\circ$ (measured from the jet exit centerline axis) is visible in the dilatation contour plots. Hence, as an initial assessment, the smoothness of the contours implies that the parallel Schur complement applied at the interfaces between each block is at least solving the governing equations sufficiently without problems.

Now we will examine some one-point statistical results. Figure 5.17 shows the mean centerline velocity decay for all four jet cases. Note the good agreement of all the Schur complement cases with the single-block code. Figure 5.18 shows the inverse of Figure 5.17, i.e. the jet centerline decay rate. The measured decay rate of roughly 0.076 for all jets under-predicts the experimental correlations of Zaman [87]. The measured values for all jets is lower due to the relatively short physical domain length of $x = 25r_o$. A physical domain length of at least $60r_o$ or longer is needed for the growth rate to be within range of the experimental values (See References [15] and [55]). However, our main goal here is to make a direct comparison between the Schur complement and the single-block
methodology results and not a direct comparison with laboratory experiments. Figure 5.19 shows the jet half growth. A measured growth rate of roughly 0.076 for all jets is obtained but is again under-predicts the experimental [72] range of 0.086 ≤ A ≤ 0.096. Figure 5.20 shows the cross-sectional streamwise mean velocity profile at x = 25r_o and again the good agreement of the Schur complement compared to the single-block methodology. Figures 5.21 to 5.24 show the variation of normalized Reynolds stresses at the end of the physical domain, x = 25r_o. The normalized Reynolds stresses are defined in cylindrical coordinates as

\[ \sigma_{xx} = \frac{v'_x v'_x}{U_c^2(x)}, \quad \sigma_{rr} = \frac{v'_r v'_r}{U_c^2(x)}, \quad \sigma_{\theta\theta} = \frac{v'_\theta v'_\theta}{U_c^2(x)}, \quad \sigma_{rx} = \frac{v'_r v'_x}{U_c^2(x)} \]  

(5.39)

where \( v'_x, v'_r, v'_\theta \) are the axial, radial and azimuthal components of the fluctuating velocity, respectively, \( U_c(x) \) is the mean jet centerline velocity at a particular axial location, and the overbar denotes time-averaging. Figures 5.25 and 5.26 on the other hand show the mean axial turbulence intensities along the shear layer and centerline of the jet. From the Reynolds’ stress plots, i.e. Figures 5.21 to 5.26, the mean fluctuating components of the Schur complement show very good agreement with the single-block version further validating the new parallelization methodology. Almost all curves shown here seem to collapse on each other.

Next, we look at some preliminary results for far-field aeroacoustics of the 3-D LES Schur complement test case. We performed a study whereby several points were chosen on a control surface placed in the linear region of the flow field and projected the spectral solution to the far-field using the \( 1/R \) rule. This was done for both the single-block and Schur complement codes. Flow field data is gathered on the control points at every 5 time steps over a period of 25,000 time steps. The total acoustic sampling period corresponds to a time scale in which the ambient sound wave travels about 10 times the domain length in the streamwise direction. Based on the grid resolution around the chosen control points and assuming that 6 points per wavelength are needed to accurately resolve an acoustic wave [59], the maximum resolved frequency corresponds to a Strouhal number of \( St = 1.1 \). In addition, based on the data sampling rate, there are about 14.5 temporal points per period in the highest resolved frequency. The overall sound pressure level (OASPL) is computed
along an arc of radius of $R = 60r_o$ from the jet nozzle. As in Chapter 4, the angle $\Theta$ is measured relative to the centerline jet axis. Figure 5.27 shows the acoustic pressure spectra at $R = 60r_o$ for two different observer angles, i.e. $\Theta = 30^\circ$ and $\Theta = 60^\circ$. Again, we note the overall good agreement of the Schur complement spectra compared to the single-block results for various grid decompositions. The Schur and single-block prediction is roughly 2-3 dB higher compared to the empirical prediction of SAE ARP 876C due to the artificial vortex ring forcing as also mentioned in Chapter 4. It must be stressed that this is a rather premature conclusion to make since we did not use the FWH method to compute the far-field sound. A future study could be performed that utilizes the FWH with the Schur complement. Nonetheless, the far-field acoustic results of the Schur complement are satisfactory.

5.4.3 Performance and Scalability

The parallel Schur complement and single-block simulation was performed on a Cray XT3 BigBen, at the Pittsburgh Supercomputing Center. The single-block case took approximately 3 days to complete using 16 processors in parallel. On the other hand, a total of about 8.5 days of computing time was required for the 3-D LES Schur complement test case using 16 processors. Hence, the Schur complement case is roughly three times slower than its single-block counterpart. The rather dismal performance of the Schur complement case is not a cause for concern and for the most part not entirely surprising. In this test case, the grid was decomposed in one direction only, i.e in the $z$ direction. And due to this 1-D decomposition, approximately 73% of the simulation time was spent on the communication process, which, is why the simulation takes almost thrice as long to complete compared to the single-block code.

However, recall that a speed-up of approximately 30% was gained between a 1-D and 2-D decomposition for the 2-D LEE test case. Hence, this strongly suggests that a decomposition in three dimensions would definitely increase the performance of our 3-D parallel Schur complement. Gropp et al. [111] reported a dramatic speed-up of a factor of nearly 3
between a 3-D decomposition and a 1-D decomposition for a parallel 3-D Poisson problem using a second-order finite difference scheme. In essence, a 3-D decomposition will effectively reduce the ‘surface area’ shared between each block and thus reducing the amount of information being passed back and forth and communication time. As an example, a decomposition of $N_{bx} \times N_{by} \times N_{bz} = 4 \times 2 \times 2$ (Case S16b) effectively reduces the surface area shared by the interfaces in the $z$ direction by a factor of 8. Furthermore, this arrangement will increase the number of points per domain to 64 or more in each direction allowing the direct solution using the diagonal elements.

Indeed, after performing the 3-D decomposition, case S16b showed much improved performance over S16a by almost a factor of two. Using a profile tool called CrayPat on BigBen, we were able to analyze the percentage of resources used for each subroutine and performance of the single-block and Schur complement codes. The single-block code spends roughly 16% during each time step communicating between processors, whereas for the Schur complement this amount was reduced to only 2%. However, case S16b was found to be roughly 10% slower compared to the single block code on BigBen even with the direct solution of the Schur complement matrix. One possible explanation is the almost uneven number of grid points in each block and thus the increase in cache misses during the simulation. Due to the number of grid points in each direction for the current case ($N_x \times N_y \times N_z = 287 \times 128 \times 128$), each block will not have the same number of grid points. Some will have just slightly more. The single-block setup guarantees an exact amount and therefore less cache misses. CrayPat showed that the number of cache misses by the Schur complement code was on average 4-5 times more compared to the single-block code and thus slowing down slightly the Schur performance. Further analysis needs to be done in order to ascertain further the adverse performance of the Schur complement.

Now that the 3-D Schur complement has been validated, scalability tests can now be performed. Figure 5.30 shows the scalability plot of the single-block and Schur complement code on BigBen. The number of processors or cores used to compare the single-block and Schur complement were 16, 32, 64 and 128, respectively for the 4.7 million grid point case. The maximum number of processors was 128 due to the limitation of the transposition
scheme i.e. the number of grid points in one direction. From Figure 5.30, the transposition strategy shows better linear speed-up compared to the Schur complement. Again one of the possible reasons is the number of cache misses that are less in the single-block code compared to the Schur complement as explained previously. The performance of the Schur is still acceptable.

Uzun [15] reported super-linear speed-up for the single-block code on a Compaq Alphaserver ES45 named Lemieux which has since been retired at the Pittsburgh Supercomputing Center. Lemieux comprised of 610 nodes and two separate front end nodes. Each computational node has four 1 GHz EV68 Symmetric Multi-processing (SMP) processors and 4 Gbytes of memory. Node-to-node connection was made possible through a dual-rail Quadrics interconnect. Although Lemieux was slower in terms of processor clock speed compared to BigBen (AMD 2.6 GHz), we suspect that the dual-rail Quadrics high speed interconnect system on Lemieux must have had a significant impact on the communication for the single-block code. Since Lemieux has been retired, testing this hypothesis is not possible. Nonetheless, this underscores the importance of having a state-of-the-art high speed interconnect to achieve the highest communication efficiency between compute nodes. Figure 5.31 shows the speedup plot for the Schur complement up to 1,024 processors on BigBen. Even at 1,024 processors the Schur complement shows an efficiency of roughly 84% which is desirable.

One of the initial motivations of the Schur complement was to also harness its low communication capability for use in compute clusters that do not have high-speed interconnect. Figure 5.32 shows the speed-up plot for the single-block and Schur complement on the a compute cluster named Booster which nodes are connected via a regular gigabit ethernet switch. Booster has four AMD 2.3 GHz processors on each node with access to 4 GB RAM. The test cases used here for the Schur complement on Booster were S16b, S32 and S64. From Figure 5.32, the performance of both methodologies on Booster is mediocre with 60-75% efficiency at best. This is probably due to the slow interconnect between nodes. However, the Schur complement now shows improved performance over the transposition scheme due to the lower amount of resources used for communication com-
pared to the transposition scheme. Hence, this study shows that the Schur complement is a desirable alternative compared to the single-block code for parallel problems that utilizes high-order compact schemes and need to be run on clusters with slow interconnects.
Table 5.1 Effect of number of points in each sub-domain on the coupling coefficient of the Schur complement matrix for the compact differencing scheme and spatial filtering scheme with four sub-domains total. $N_{Px}$ = Number of points per sub-domain.

<table>
<thead>
<tr>
<th>$N_{Px}$</th>
<th>Differencing Scheme</th>
<th>Filtering Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$-6.06 \times 10^{-3}$</td>
<td>$-5.94 \times 10^{-2}$</td>
</tr>
<tr>
<td>8</td>
<td>$-1.29 \times 10^{-4}$</td>
<td>$-1.40 \times 10^{-2}$</td>
</tr>
<tr>
<td>16</td>
<td>$-5.85 \times 10^{-8}$</td>
<td>$-8.11 \times 10^{-4}$</td>
</tr>
<tr>
<td>32</td>
<td>$-1.20 \times 10^{-14}$</td>
<td>$-2.75 \times 10^{-6}$</td>
</tr>
<tr>
<td>64</td>
<td>$-5.06 \times 10^{-28}$</td>
<td>$-3.16 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

Table 5.2 Root mean square error of the density waveform from a serial code and the parallel Schur complement compared to an exact solution for 1-D LEE. Four sub-domains used throughout for the parallel Schur complement.

<table>
<thead>
<tr>
<th>$N_x$</th>
<th>$N_{Px}$ (Schur Only)</th>
<th>R.M.S. Error (Serial)</th>
<th>R.M.S. Error (Schur)</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>16</td>
<td>$9.96 \times 10^{-5}$</td>
<td>$1.94 \times 10^{-4}$</td>
</tr>
<tr>
<td>131</td>
<td>32</td>
<td>$1.19 \times 10^{-5}$</td>
<td>$1.85 \times 10^{-5}$</td>
</tr>
<tr>
<td>259</td>
<td>64</td>
<td>$1.11 \times 10^{-5}$</td>
<td>$1.43 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Table 5.3 Root mean square (R.M.S) error of the pressure waveform along the x-axis of the parallel Schur complement compared to an exact solution for the 2-D LEE. The Speed-up study is compared between the parallel Schur complement codes only.

<table>
<thead>
<tr>
<th>Case</th>
<th>N_x × N_y</th>
<th>N_Px × N_Py</th>
<th>N_bx × N_by</th>
<th>R.M.S. Error</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial</td>
<td>131 × 131</td>
<td>131 × 131</td>
<td>1 × 1</td>
<td>8.17 × 10^{-4}</td>
<td>-</td>
</tr>
<tr>
<td>A</td>
<td>131 × 131</td>
<td>32 × 131</td>
<td>4 × 1</td>
<td>9.54 × 10^{-4}</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>131 × 131</td>
<td>65 × 65</td>
<td>2 × 2</td>
<td>9.54 × 10^{-4}</td>
<td>1.36</td>
</tr>
<tr>
<td>C</td>
<td>131 × 131</td>
<td>32 × 65</td>
<td>4 × 2</td>
<td>9.73 × 10^{-4}</td>
<td>2.01</td>
</tr>
<tr>
<td>Serial</td>
<td>259 × 259</td>
<td>259 × 259</td>
<td>1 × 1</td>
<td>1.66 × 10^{-4}</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>259 × 259</td>
<td>64 × 259</td>
<td>4 × 1</td>
<td>2.12 × 10^{-4}</td>
<td>1.00</td>
</tr>
<tr>
<td>E</td>
<td>259 × 259</td>
<td>129 × 129</td>
<td>2 × 2</td>
<td>2.11 × 10^{-4}</td>
<td>1.23</td>
</tr>
<tr>
<td>F</td>
<td>259 × 259</td>
<td>64 × 129</td>
<td>4 × 2</td>
<td>2.12 × 10^{-4}</td>
<td>1.89</td>
</tr>
</tbody>
</table>

Table 5.4 3-D LES jet block partitioning cases. Grid points allocation is N_x × N_y × N_z = 287 × 128 × 128 or 4.7 million. N_procs, N_bx, N_by and N_bz are the total number of processors or cores and number of blocks in the x, y and z directions, respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>N_procs</th>
<th>N_bx × N_by × N_bz</th>
</tr>
</thead>
<tbody>
<tr>
<td>S16a</td>
<td>16</td>
<td>1 × 1 × 16</td>
</tr>
<tr>
<td>S16b</td>
<td>16</td>
<td>4 × 2 × 2</td>
</tr>
<tr>
<td>S32</td>
<td>32</td>
<td>8 × 2 × 2</td>
</tr>
<tr>
<td>S64</td>
<td>64</td>
<td>4 × 4 × 4</td>
</tr>
<tr>
<td>S128</td>
<td>128</td>
<td>8 × 4 × 4</td>
</tr>
<tr>
<td>S256</td>
<td>256</td>
<td>16 × 4 × 4</td>
</tr>
<tr>
<td>S512</td>
<td>512</td>
<td>8 × 8 × 8</td>
</tr>
<tr>
<td>S1024</td>
<td>1024</td>
<td>16 × 8 × 8</td>
</tr>
</tbody>
</table>
Figure 5.1. Schematic for 1-D grid.

Figure 5.2. Initial density waveform for the 1-D LEE test case. Case for 131 grid points.
Figure 5.3. Density waveform at non-dimensional time of $t = 20$.

Figure 5.4. 2-D computational grid with $4 \times 1$ arrangement for $N_x \times N_y = 131 \times 131$ (Every other grid point shown).
Figure 5.5. 2-D computational grid with a $2 \times 2$ block decomposition for $N_x \times N_y = 131 \times 131$ (Every other grid point shown).

Figure 5.6. Pressure waveform solution along the $x$-axis at $t = 33$ for Case $\mathcal{A}$, $N_x \times N_y = 131 \times 131$. 
Figure 5.7. The cross section of the computational grid on the $y = 0r_o$ plane. (Every $3^{rd}$ and $4^{th}$ grid point shown in $z$ and $x$, respectively).

Figure 5.8. Block arrangement of the 3-D LES test case used for the parallel Schur complement ($N_{bx} \times N_{by} \times N_{bz} = 1 \times 1 \times 16$).
Figure 5.9. The cross section of the computational grid on the $z = 0r_o$ plane for 3-D decomposition of Case S16b. (Every 4th grid point shown in $y$ and $x$, respectively).

Figure 5.10. The cross section of the computational grid on the $z = 0r_o$ plane for 3-D decomposition of Case S1024.
Figure 5.11. The cross section of the computational grid on the $x = 10r_o$ plane for 3-D decomposition of $N_{bx} \times N_{by} \times N_{bz} = 16 \times 8 \times 8$.

Figure 5.12. The cross section of the computational grid on the $x = 20r_o$ plane for 3-D decomposition of Case S1024.
Figure 5.13. Instantaneous dilatation contours of the 3-D LES iso-thermal jet for 1-D decomposition of 16 blocks. The black square box is a close-up area and is shown in the next figure.

Figure 5.14. Close-up view of the square box shown in the previous figure.
Figure 5.15. Instantaneous dilatation contours of the 3-D LES iso-thermal jet for the 3-D decomposition of 1,024 blocks. The black square box is a close-up area and is shown in the next figure.

Figure 5.16. Close-up view of the square box shown in the previous figure.
Figure 5.17. Mean centerline decay for four test cases.

Figure 5.18. Mean centerline decay for four test cases. This figure is the inverse of the previous figure.
Figure 5.19. Mean half growth rate for all test cases.

Figure 5.20. Mean cross-sectional streamwise velocity profile at station \( x = 25r_o \). Radius is normalized by the streamwise jet velocity half-radius.
Figure 5.21. Normalized Reynolds stress profiles, $\sigma_{xx}$, for all cases at station $x = 25r_o$.

Figure 5.22. Normalized Reynolds stress profiles, $\sigma_{rr}$, for all cases at station $x = 25r_o$. 
Figure 5.23. Normalized Reynolds stress profiles, $\sigma_{\theta\theta}$, for all cases at station $x = 25r_o$.

Figure 5.24. Normalized Reynolds stress profiles, $\sigma_{rx}$, for all cases at station $x = 25r_o$. 
Figure 5.25. Mean streamwise turbulence intensities for all cases along the shear layer $r = r_o$.

Figure 5.26. Mean streamwise turbulence intensities for all cases along the jet centerline axis.
Figure 5.27. Overall sound pressure levels for all cases at an observer distance of \( R = 60r_o \) from jet nozzle exit. The ‘ext.’ stands for extrapolated value from the computational domain to the far-field.

Figure 5.28. Far-field pressure spectra for all cases at \( R = 60r_o \) and \( \Theta = 30^\circ \).
Figure 5.29. Far-field pressure spectra for all cases at $R = 60r_o$ and $\Theta = 60^\circ$.

Figure 5.30. Speed-up comparisons between the single-block code and 3-D parallel Schur complement performed on BigBen.
Figure 5.31. Speed-up study for parallel Schur complement from 16 cores to 1,024 cores on BigBen.

Figure 5.32. Speed-up comparisons between the single-block code and 3-D parallel Schur complement performed on Booster.
6. NOISE PREDICTION OF A SUBSONIC TURBULENT ROUND JET USING THE LATTICE-BOLTZMANN METHOD

6.1 Introduction

The use of the lattice-Boltzmann method for acoustics is most recent, with most applications for flows with Mach numbers below \( M_\infty = 0.2 \). Recent studies pertaining to aeroacoustics include those by Ricot et al. [50] and Crouse et al. [112] on fundamental sound propagation studies, Crouse et al. [113, 114] on automobile interior noise, and Seror et al. [115] on the noise generated by a complete aircraft landing gear. Yu and Girimaji [116] applied an LBM-LES technique to study several low aspect-ratio rectangular turbulent jets. They reported good agreement with experimental data in terms of flow statistics, but their study did not include far-field sound. A cursory survey of the literature suggests that the use of LBM has not yet been attempted for the study of jet noise.

The aim of this study was to investigate the far-field noise of a nearly incompressible turbulent jet (\( M_j = 0.4 \)) using the lattice-Boltzmann method. The near-field flow physics and far-field noise simulations were performed using the commercial code PowerFLOW 4.0c [117], which is based on the LBM kernel. The next section gives a brief overview of the lattice-Boltzmann methodology followed by a case setup, some results and closing remarks.

6.2 Brief Description of the Lattice-Boltzmann Methodology

The lattice-Boltzmann equation has the following form [43, 44, 113, 118]

\[
\begin{align*}
    f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) &= -\frac{\Delta t}{\tau} (f_i(x, t) - f_i^{eq}(x, t)),
\end{align*}
\]

where the distribution function \( f_i(x, t) \) yields the number density of kinetic particles at position, \( x \), with a particle velocity \( c_i \) in the \( i \) direction at time \( t \). The left hand side of
Equation (6.1) computes the particle advection from one center cell to another whereas the right hand side of Equation (6.1), known as the collision operator, represents the relaxation of the particles. The Bhatnagar-Gross-Krook [118] (BGK) approximation is used to relax the equilibrium distribution function $f_{i}^{eq}(x, t)$. The relaxation time $\tau$ however, is related to the kinematic viscosity, $v$, such that $\tau = (v + 0.5)/T$, where $T$ is the lattice temperature. This relation is also commonly referred to as Single Relaxation Time (SRT). The conservative macroscopic variables such as density, momentum, and energy are obtained through the zeroth, first and second order moments of the distribution function, i.e.

$$\rho(x, t) = \sum_{i} f_{i}(x, t).$$  \hspace{1cm} (6.2)

$$\rho u(x, t) = \sum_{i} c_{i} f_{i}(x, t).$$  \hspace{1cm} (6.3)

$$\rho \theta(x, t) = \frac{1}{2} \sum_{i} (c_{i} - u)^{2} f_{i}(x, t).$$  \hspace{1cm} (6.4)

The pressure is obtained using the equation of state for an ideal gas with the assumption that the gas constant is unity. This can be expressed as $p = \rho T$. In addition, the LBM approach recovers the compressible, viscous Navier-Stokes equation in the hydrodynamic limit [46] for wavelengths $\lambda \gg \Delta x$ and frequencies $f \ll \Delta t/\tau$ [113]. To recover the macroscopic hydrodynamics, $f_{i}^{eq}(x, t)$ must be chosen in such a way that the essential conservation laws are satisfied and the resulting macroscopic equations are Galilean invariant. In a three-dimensional model, one common choice is the D3Q19 model [48, 119]. The ‘D3’ refers to the number of spatial dimensions and in this case the velocity space is in three dimensions. ‘Q19’ refers to the number of velocity directions and this case 19 velocity states. Figure 6.1 shows the 19-state velocity model coupled with the equilibrium distribution function give by

$$f_{i}^{eq} = \rho w_{i} \left[ 1 + \frac{c_{i} \cdot u}{T} + \frac{(c_{i} \cdot u)^{2}}{2T^{2}} - \frac{u^{2}}{2T} + \frac{(c_{i} \cdot u)^{3}}{6T^{3}} - \frac{c_{i} \cdot u}{2T^{2}} u^{2} \right],$$  \hspace{1cm} (6.5)

where $w_{i}$ are the weighting parameters that have a value of 1/18 in the 6 coordinate directions, 1/36 in the 12 bi-diagonal directions and 1/3 for the zero velocity particle. The lattice temperature, $T$, is set to 1/3 for isothermal simulations. The LBM code used in this study has been shown to be both second order accurate in both time and space [112, 120].
To account for the presence of solid boundaries in the simulation, the no-slip boundary condition was imposed by utilizing a particle ‘bounce back’ process on a solid surface [119, 121]. In addition, an improved volumetric boundary scheme [119, 121] for arbitrary geometries has been devised and implemented to accurately control and govern the momentum flux across the boundary. Further details regarding the handling of solid geometries can be found in references [49, 119, 121].

To account for the unresolved turbulent scales, an eddy viscosity turbulence model was used. Specifically, PowerFLOW uses the two-equation $k-\varepsilon$ Renormalization Group (RNG) turbulence model to compute the turbulence viscosity with the addition of a swirl corrector to model part of the large scale structures. This methodology is also commonly referred to as Very Large Eddy Simulation (VLES). Preliminary simulations with the turbulence model yielded a laminar jet, i.e. the plume exiting the pipe did not exhibit break-up beyond the potential core. Thus no turbulence model was used. The simulations were carried on using an under-resolved grid and no subgrid scale model. This procedure has been argued to be analogous to an LES [64, 65, 122].

The main advantage in utilizing the PowerFLOW-LBM package is in its ease of handling complicated geometries. Jacobians are not needed to compute grid metrics reducing computational costs. PowerFLOW employs an adaptive type meshing technique instead of body-fitted meshing. This feature is an essential advantage when applied to jets with complicated geometries such as scalloped mixers and chevrons. It also facilitates the inclusion of nozzles in the computational domain. The more general but distinct advantages of LBM over the conventional Navier-Stokes solvers include the linearity of the convection operator in Equation (6.1) due to the kinetic nature of the LBE method [51]. In addition to particle convection and collisions, the use of multi-scale expansions allows the recovery of the nonlinear macroscopic advection process. The strain rate from the non-equilibrium distribution function are obtained directly and alleviate the need to solve the Poisson term (e.g. strain and rotation rate tensor) in the coarse grained Navier-Stokes equations, which often presents numerical difficulties in terms of accuracy for finite-difference based algo-
LBM features ease of parallelization for large to massive supercomputing architectures due to its simple formulation.

The most notable disadvantage of the LBM implementation used in the present study is that it does not recover flow physics correctly for cases with high Mach numbers ($M > 0.4$). This limitation is due to the discrete velocity model (D3Q19) is insufficient to span the particle phase space so that important moments that account for the thermal energy transfer are not be correctly recovered [120]. Furthermore, the collision integral assumes a low-speed approximation. Efforts have been made to extend the LBM to higher Mach number flows. Recently, Sun and Hsu [124] used an LBM technique to study a shock tube problem and obtained good results compared to the Riemann solution. Shan et al. [120] and Chen et al. [125] have laid a theoretical groundwork to efficiently extend the LBM to higher Mach numbers and arbitrary Knudsen numbers. Recently, Li et al. [126, 127] devised a modified Boltzmann equation and applied it to a 2-D aeroacoustic benchmark problem. They obtained good results and showed that their methodology is valid up to a Mach number of 0.9.

6.3 Computational Procedures

In order to evaluate the effectiveness of LBM for jet noise simulations, comparisons were made with an established methodology. Simulations of comparable jet flows were performed using both LBM and the LES methods.

6.3.1 Lattice-Boltzmann Methodd

A pipe with a length of $L = 0.508$ meters and a diameter of $D_j = 0.0508$ meters is considered as a part of the computational domain, starting at $x = 0$. The pipe diameter was chosen to match the jet diameter of the experiments of Tanna [24]. The centerline of the pipe is along the $x$-axis, with and the $y$- and $z$- axes along the vertical and horizontal transverse directions, respectively. The addition of a nozzle in the computational domain was intended to eliminate the need for an artificial forcing mechanism to trip the flow.
Artificial forcing techniques used in many LES simulations (See References [79,128]) may give rise to spurious far-field sound. As discussed later, however, available computational resources did not allow a very fine grid within the pipe.

The computational domain was partitioned into several variable resolution (VR) regions in order to tailor the grid as needed to resolve the flow details while reducing computational costs. This methodology is similar to the grid stretching techniques typically employed in many other CFD methods. Figure 6.2 shows a side view of the entire computational domain. Successive VR regions are concentric and cylindrical, as shown in Figure 6.3. The outermost bounding region is rectangular. As illustrated in Figure 6.2, it is important to provide at least four jet diameters of spacing between each VR region radially and in the streamwise direction. An initial simulation with no streamwise spacing between VR regions resulted in periodic oscillations, termed ‘VR tones’ in the far-field pressure spectra. These tones had sound pressure levels as high as 15 dB and appeared to bias the overall spectral makeup. Streamwise spacing between VR regions actually eliminated these tones in the lower frequency portion of the spectrum although similar tones were present in the high frequency range. Each grid cell is called a ‘voxel’. Each VR region represents one grid resolution level and the VRs cascade outwards from the fine resolution region towards the coarse resolution region. The voxel cell size between each successive VR region differs by a factor of two. The domain includes a total of 20 million voxels. The smallest voxels, of size approximately $8 \times 10^{-4}$ meters (0.8 mm), are located in the shear layer region of the pipe. The voxel size corresponds to approximately $\Delta r/D_j \simeq 0.016$ which is considered very coarse for wall bounded flow studies. The ratio needed to resolve the duct boundary layers is at least one order of magnitude less without the implementation of a wall model. This was deemed prohibitively expensive. The first grid point off the wall is at $\Delta r^+ \simeq 10$. Although the adopted cell size did not resolve the boundary layer details, it was sufficiently small to supply physical jet inflow conditions without the need for artificial forcing techniques.

The entire simulation domain includes a total of 11 VR regions with a domain size set of $(x, y, z) = (250D_j, \pm 125D_j, \pm 125D_j)$. These values, which may seem large at first,
were chosen to minimize reflections of the propagating sound waves and convected jet plume flow structures back into the near-field. To the same end, coarse VR regions further away from the jet dissipate outgoing traveling sound waves and thus act as ‘sponge’ zones. Non-reflecting type boundary conditions are applied at the inlet lateral and outlet boundaries [129]. The region where the physical properties are measured and stored is roughly \((x, y, z) = (27.5D_j, \pm 5D_j, \pm 5D_j)\). This region has a funnel type shape, as shown in Figure 6.4. The inner most funnel shaped VR region is the measurement region for near-field flow statistics. In the core VR region, virtual probes are placed on the jet axis centerline and nozzle lip-line along the streamwise direction for calculations of the turbulent spectral content. The location of the probes is indicated in Figure 6.4. Additional probes are placed sufficiently far away from the non-linear turbulent region to compute far-field acoustics. The far-field acoustic measurement probes are placed \(15r_o\) above the centerline jet axis. Figure 6.5 shows the voxel setup for the entire domain. Note the very coarse voxels in the outer VR region that are larger than the pipe dimensions. Figure 6.6 shows a close-up view of the voxel distribution in the pipe region, whereas Figure 6.7 shows the voxel distribution at the pipe exit.

The mean density and temperature gradients imposed in the nozzle were the ambient conditions. To initialize the velocity at the entrance of the pipe, a top-hat velocity profile is specified with a jet centerline velocity of \(U_j = 100\) m/s. When the simulation is evolved, the jet centerline velocity at the exit of the pipe reaches approximately 130 m/s or \(M_j = U_j/a_j \approx 0.4\) due to the formation of a thick boundary layer. As previously mentioned, no turbulence model was used for this test case, and an under-resolved DNS (uDNS) was assumed. The kinematic viscosity needed to be adjusted to reasonably resolve the turbulent scales with the available voxel resolution. In this case, fixing the kinematic viscosity of around \(1.1 \times 10^{-3}\) m\(^2\)/s resulted in a jet Reynolds number of \(Re_D = (U_j D_j)/v_j \approx 6,000\).
6.3.2 Large-Eddy Simulation

The setup used for the LES case was very similar to one presented in Chapter 3. The Mach number was also $M_j = 0.4$ with $Re_D = 6,000$. The total number of grid points used was roughly 5 million with a grid point allocation consisting of $N_x = 292$, $N_y = 128$ and $N_z = 128$ in the $x$, $y$ and $z$ directions, respectively. The physical part of the computational domain extends to approximately $60r_o$ in the streamwise direction and $-20r_o$ to $20r_o$ in the transverse $y$ and $z$ directions. The sponge zone was added beyond the streamwise location of $60r_o$. Based on the minimum grid spacing and the ambient Mach number, the time resolution was determined to be $\Delta t = 0.01 r_o/a_\infty$. The first four azimuthal modes of forcing out of sixteen were removed. The forcing amplitude was kept at $\alpha = 0.007$. As for the previous LES setup there was no nozzle-lip in the simulation domain, and a hyperbolic tangent velocity profile was enforced at the inflow boundary. Considering an isothermal jet, the Crocco-Busemann density was accordingly specified as a function of the jet Mach number and inlet velocity profile.

6.4 Results

The physical time scaling or time step for the LBM is $1.45 \times 10^{-6}$ seconds. A total of 400,000 time steps was required to achieve reasonably converged statistics for flow analysis. In terms of computational resources used, this test case took approximately two days of runtime using 128 processors in parallel on a Dell Xeon cluster called Mammouth. In comparison, the LES simulations took approximately 3.5 days of run time using 128 processors on the same machine.

6.4.1 Near-field Flow Variables

Figure 6.8 shows a close-up view of the instantaneous velocity magnitude filled contours of the jet. This is a slice along the jet centerline or on the $z = 0$ plane. The shear layers appear to be relatively thick. The shear layer becomes unstable at a location ap-
proximately two jet diameters from the exit. Figure 6.9 shows a contour plot of the mean axial velocity on the \( z = 0 \) plane. The mean streamwise velocity on the jet centerline is plotted in Figure 6.10. The variation of the streamwise velocity with distance, \( U_j/U_c(x) \), agrees with the velocity decay data measured by Bridges and Wernet [25] for a Mach 0.5, \( Re_D = 860,000 \) free jet. Here, \( U_j \) is the jet exit velocity and \( U_c(x) \) is the local mean streamwise velocity on the centerline. The decay rate for the LES data is slightly faster than that measured by Bridges and Wernet, but the difference is small. Figure 6.11 shows the inverse quantity \( U_c(x)/U_j \). The decay slope of 0.155 from the LBM is close to the experimental correlation of 0.16 reported by Zaman [87]. The LES simulation yielded a slope of 0.163 which seems to be closer to the experimental data despite the absence of a nozzle. Although not shown in the Figure 6.11, the measured decay rate from Bridges and Wernet’s experiments is close to 0.15. The potential core length, \( x_c/r_o \), is defined as the location where the jet mean centerline velocity is reduced to 95% of the inflow jet velocity, \( U_c(x_c) = 0.95U_j \). In the present simulation, a potential core length of 12\( r_o \) and 11.4\( r_o \) was obtained for the LBM and LES, respectively. This value is within the range of core lengths, between 10 and 14 jet radii, typically observed in laboratory experiments.

Figure 6.12 shows a contour plot of the mean axial turbulence intensity for the simulated jet. The axial intensity near the nozzle lip is small, which indicates a nearly laminar exit shear layer. Figures 6.13 and 6.14 shows the axial turbulence intensities along the centerline and lip-line of the jet for both LBM and LES. The simulation results are also compared to recent experimental measurements of Laurendeau \textit{et al.} [130, 131] for a \( M_j = 0.3 \) isothermal jet. Qualitatively, the trends of the axial turbulence intensities are consistent with those of an axisymmetric turbulent jet; the peak R.M.S fluctuation for the lip-line occurs earlier and is greater compared to the centerline peak fluctuation. The decay rate downstream of the peak intensity is slightly lower for the LBM than for the LES. The peak value and the axial location of the computed lip-line turbulence intensity are in agreement with experimental observations. The simulation results are much lower than the experimental data over the range \( 0 \lesssim x/r_o \lesssim 9 \). The lower values may be due to the fact that the simulated exit shear layer is still laminar or transitional. The LES result feature a
2% turbulent intensity near the nozzle exit. This could be a consequence of the vortex ring excitation of the mean flow. Recall that there is no forcing used for the LBM case. The Reynolds number of the simulated jet, 6,000, is much smaller than the Reynolds number of the measured jet, which was 300,000, due to the need for an increased viscosity. The experiments were performed using a converging nozzle rather than a straight pipe. Nonetheless, according to Laurendeau [132], the high turbulence intensities measured along the lip-line in the experiments could be due to insufficient flow seeding and on the possible influence of the method used to trip the boundary layer. Nevertheless the high intensity at the nozzle exit appears to be very plausible. A recent high resolution, well-resolved LES study performed by Uzun and Hussaini [11] shows that the axial turbulence intensity at the jet exit could be as high as 14%, depending on the distance to the wall. The centerline mean axial turbulence intensity, on the other hand is in better agreement than for the experiments of Laurendeau et al. [131]. The difference is not as pronounced as for the lip-line, and the agreement is good in the range of $10 \lesssim x/r_o \lesssim 15$. Again, note a slight under-prediction compared to the experiments over the region from the nozzle exit to approximately $9r_o$. No experimental data was collected beyond $x = 15r_o$ due to the limited size of the PIV window.

Figures 6.15 through 6.18 show the axial velocity spectra at four different locations ($x = 20r_o$, $25r_o$, $30r_o$ and $35r_o$) downstream of the jet along the nozzle lip-line and center-line. The velocity spectra figures show that the jet development is indeed broadband. This is further substantiated by the fact that a portion of the spectra decays with frequency according to Kolmogorov’s well known -5/3 law, indicating that part of the spectra falls in the inertial subrange (equilibrated turbulence) before dropping-off at higher frequencies. Based on the spatial grid resolution, the maximum Strouhal number ($Sr = fD_j/U_j$) for adequate resolution in this nonlinear region is approximately $Sr = 3$. Although not shown, the axial velocity spectra for locations upstream of the break-up of the potential core, around $x = 5r_o$, did not exhibit similar -5/3 decay. Instead, there was a concentration of energy in the low frequency region and a transfer of energy indicated by a sharp drop in the spectra to higher frequencies. The apparent roll-off in the LES spectra in Figure 6.15
is probably due to the relatively coarse grid. Nonetheless, the inertial range was captured in both simulations. Laboratory measurements of axial pressure spectra along an incompressible free shear layer performed by George et al. [133] indicate the development of the turbulence-mean-shear contribution and decays according to a -11/3 power law in the inertial subrange. Figure 6.19 shows the pressure spectrum in the shear layer at \( x = 5r_o \) well upstream of the break-up of the potential core. A -11/3 spectral decay rate as that measured by George et al. is apparent in the LBM simulation results. The decay is quite clear over the range \( 2 \ll Sr \ll 5 \). There is a build-up of energy starting around \( Sr = 6 \), followed by a drop-off following the Kolmogorov’s \(-11/3\) decay rate before rolling-off strongly at around \( Sr = 11 \). There are noticeable tones in the pressure spectra shown in Figure 6.19. These so-called VR tones are believed to be artifacts caused by spurious acoustic reflections at the boundaries between different VR regions. Each VR region is annular in shape. Standing waves and ‘duct modes’ [134] of resonance may therefore be present if there is any reflection at the boundary. Further work is needed to precisely identify the modes involved. Preliminary studies have suggested appropriate suppression methods. These tones do not seem to contaminate the underlying spectrum, as evidenced by the \(-11/3\) decay rate. Downstream of the potential core however, the pressure spectrum should shift in theory from a decay rate of \(-11/3\) to a decay rate of \(-7/3\) in the inertial subrange, indicating turbulence-turbulence interaction. The \(-7/3\) decay rate was also measured by George et al. [133] and is once again reflected in the simulation, as shown in Figure 6.20. The near-field pressure spectra for the LES is again plotted alongside the LBM computations at location \( x = 20r_o \) in the shear layer. The LES results seems to capture part of the inertial range within the frequency range of \( 0.5 \ll Sr \ll 0.8 \). Although \( x = 10r_o \) is located slightly upstream of the potential core, LBM results nonetheless indicates turbulence-turbulence interaction for a broad range of frequencies in the subrange compared to the spectra at \( x = 20r_o \) and \( x = 30r_o \), respectively. From the simulation, the sharpest roll-off seems to occur at around \( Sr = 10 \). Overall, based on the spectral characteristics of the near field velocity and pressure, the LBM simulation have many of the same features as the LES results.
6.4.2 Far-field Acoustics

A total of 15 probes are placed at approximately $15r_o$ from the centerline of the jet, as shown in Figure 6.4. Acoustic data were collected at every time step over a duration equivalent to 360,000 time steps. Based on the grid resolution at the probe location and assuming at least 12 voxels are needed to resolve one acoustic wavelength, the maximum resolvable frequency is roughly 4,000 Hz or about $Sr = 1.6$. The amplitude of the radiated sound spectra was extrapolated using a $1/R$ correction to a far-field distance of $R = 144r_o$ to replicate the microphone location of the measurements by Tanna et al. [78]. This far-field extrapolation methodology is crude, and yields only a first cut approximation of the far-field sound propagation amplitude. A more sophisticated methodology such which includes Doppler effects such the Ffowcs Williams-Hawkings (FWH) [54, 97, 98] surface integral acoustic method needs to be used and will be pursued in a future study.

As a note, the distance $R$ is measured from the jet centerline exit whereas $\Theta$ is measured relative to the jet exit centerline axis.

For the LES, the porous Ffowcs Williams-Hawkings [97, 98] (FWH) surface integral acoustic method was used to calculate the far-field radiated sound. The integral method follows the description of Lyrintzis and Uzun [54]. For simplicity, a continuous stationary control surface around the turbulent jet was used. For details regarding the numerical implementation of the Ffowcs Williams-Hawkings method, the reader is referred to Uzun [59]. The control surface starts about one jet radius downstream, and is located at approximately $7.5r_o$ above and below the jet at the inflow boundary in the $y$– and $z$– directions. It extends streamwise until the near end of the physical domain at which point the cross stream extent of the control surface is approximately $30r_o$. Hence, the total streamwise length of the control surface is $59r_o$. Results were obtained for an open control surface. A open control surface here is defined where there is no surface at the end of the physical domain, i.e. $x = 60r_o$. Flow field data are gathered on the control surface every 5 time steps over a period equivalent to 55,000 time steps. Based on the grid resolution around our control surface and assuming that with our numerical method 12 points per wavelength are needed.
to accurately resolve an acoustic wave [59], the maximum frequency resolved corresponds to a Strouhal number of $Sr = fD_j/U_j \simeq 1.8$. The overall sound pressure levels are computed along an arc with a distance of $R = 144r_o$ from the jet nozzle exit. The angle $\Theta$, is measured relative to the centerline jet axis.

The directivity pattern along an observation arc at $R = 144r_o$ is shown in Figure 6.21. The overall sound pressure levels (OASPL) for the $M_j = 0.4$ jet is plotted as a function of the angle $\Theta$. The simulation results are compared to the experimental data of Tanna et al. [78] and the more recent data of Laurendeau et al. [131]. Here, SP02 simply refers to set point 2 in Tanna’s experimental test matrix. The results of Laurendeau are adjusted based on distance and Lighthill’s $U^8$ power law. In addition to the experimental data shown, the SAE ARP 876C [100] database prediction for a jet operating at similar conditions are included. This database prediction consists of actual engine jet noise measurements and can be used to predict overall sound pressure levels within a few dB at different jet operating conditions. Hence, this prediction model is empirical. From Figure 6.21, the LBM computation agree reasonably well with the experimental data. The LES data show an over-prediction of approximately 2 to 3 dB compared to Tanna’s experiment. Likewise, the LES data is higher compared to the LBM results. The vortex ring inflow forcing is believed to be the probable cause of the over-prediction of the LES compared to the laboratory jet. Nonetheless, there seem to be good agreement at some observation angles, whereas at some angles the simulation either over-predicts or under-predicts the data by roughly 1 to 3 dB, depending on which experiments one chooses to observe. The computed far-field noise using LBM seems to yield encouraging results.

Figures 6.22 through 6.24 show the far-field spectra at a distance of $R = 144r_o$ with observer angles of $\Theta = 45^\circ$, $60^\circ$, and $75^\circ$, respectively. Tanna did not have spectra available for observation angle $\Theta = 30^\circ$. Again, the cut-off frequency based on spatial grid resolution is $Sr = 3$. As shown in Figures 6.22 through 6.24 there are weak but distinct tones present in the spectrum at $Sr \simeq 2$ and $Sr \simeq 4.5$ due to the VR tones. As for the axial pressure spectra shown in Figure 6.19, these weak VR tones do not appear to severely contaminate the overall spectrum. The computed spectra using LBM ignoring these spurious
tones are acceptable. From the spectral plots, it appears that the LES-FWH methodology over-predicts Tanna’s experiment in the low frequency region whereas the LBM result under-predicts the experiments in the high frequency region.
Figure 6.1. D3Q19 LBM model (Image scanned from Reference 48).

Figure 6.2. Variable resolution (VR) region setup for the LBM test case.
Figure 6.3. A different view of the entire VR domain setup. Note that each VR region are concentric cylinders except for the outer most boundary.

Figure 6.4. A close-up view of the VR region ($z = 0$ plane) where the jet pipe is installed. Cross like symbols indicate probes/microphones locations in the near-field region for flow data, and far-field region for acoustic data.
Figure 6.5. Voxel setup for entire computational region. Section on the $z = 0$ plane.

Figure 6.6. A close-up view in the $z = 0$ plane of the voxel/cell concentration near in the pipe region.
Figure 6.7. Voxel setup at the pipe exit. Section taken at $x = 0.0508$ m plane

Figure 6.8. Instantaneous snapshot of velocity magnitude flow field from LBM on the $z = 0$ plane.
Figure 6.9. Mean streamwise velocity contours from LBM.

Figure 6.10. Mean streamwise velocity decay along the jet centerline axis for numerical simulations compared to the experiments of Bridges & Wernet [25].
Figure 6.11. Mean streamwise velocity decay along the jet centerline axis for both LBM and LES.

Figure 6.12. Mean streamwise turbulence intensity contours from LBM.
Figure 6.13. Normalized root mean square axial velocity vs. distance. Experimental data are from Laurendeau [131].

Figure 6.14. Normalized root mean square axial velocity vs. distance. Experimental data are from Laurendeau [131].
Figure 6.15. Streamwise velocity spectra at station \( x = 20r_o \) located along the shear and centerline of the jet for LBM and LES.

Figure 6.16. Streamwise velocity spectra at station \( x = 25r_o \) located along the shear and centerline of the jet for LBM.
Figure 6.17. Streamwise velocity spectra at station $x = 30r_o$ located along the shear and centerline of the jet for LBM and LES.

Figure 6.18. Streamwise velocity spectra at station $x = 35r_o$ located along the shear and centerline of the jet for LBM.
Figure 6.19. Spectral content of pressure along the shear layer at $x = 5r_o$.

Figure 6.20. Spectral content of pressure along the shear layer for LBM at $x = 10r_o$, $x = 20r_o$ and $x = 30r_o$, respectively. The LES data is for location $x = 20r_o$. 
Figure 6.21. Overall sound pressure level directivity at $R = 144r_o$ with the observation angle $\Theta$ measured relative to the jet centerline axis. (*) indicate that the data from Laurendeau is adjusted based on distance $R = 144r_o$ and Lighthill’s $V^8$ power law.

Figure 6.22. One third octave sound pressure level in the far-field at $\Theta = 45^\circ$, $R = 144r_o$. 
Strouhal Number \( \left( \text{Sr} = \frac{f D_j}{U_j} \right) \)

Frequency, \( f \) (Hz)

1/3-Octave SPL (dB/Sr)

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LBM, \( \Theta = 60^\circ \)
LES-FWH, \( \Theta = 60^\circ \)
Tanna’s Exp., \( \Theta = 60^\circ \)

LBM cut-off, \( \text{Sr} = 1.6 \)
LES-FWH cut-off, \( \text{Sr} = 1.8 \)

Figure 6.23. One third octave sound pressure level in the far-field at \( \Theta = 60^\circ, R = 144r_o \).

Strouhal Number \( \left( \text{Sr} = \frac{f D_j}{U_j} \right) \)

Frequency, \( f \) (Hz)

1/3-Octave SPL (dB/Sr)

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LBM, \( \Theta = 75^\circ \)
LES-FWH, \( \Theta = 75^\circ \)
Tanna’s Exp., \( \Theta = 75^\circ \)

LBM cut-off, \( \text{Sr} = 1.6 \)
LES-FWH cut-off, \( \text{Sr} = 1.8 \)

Figure 6.24. One third octave sound pressure level in the far-field at \( \Theta = 75^\circ, R = 144r_o \).
7. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

7.1 Conclusions

The flowfield and radiated sound of heated and unheated jets was studied using a 3-D Large Eddy Simulation (LES) methodology. The 3-D LES methodology developed by Uzun et al. [53], based on high-order compact finite difference schemes for spatial differentiation was used. The code also used a high-order compact spatial filter to damp unwanted numerical instabilities. Time advancement was performed using the standard explicit 4th-order 4-stage Runge-Kutta method. The 3-D LES method included the dynamic Smagorinsky model (DSM). However, for all jet LES simulations the DSM was not used; the compact spatial filter acts as an implicit subgrid-scale model. This approach is commonly referred to as the Implicit Large Eddy Simulation or ILES. Since the nozzle geometry was not modeled explicitly, a hyperbolic tangent velocity profile was specified at the inflow. Random excitation in the form of a vortex ring was enforced close to the jet exit to break-up the potential core. Tam and Dong’s radiation and inflow boundary conditions were used on the boundaries of the LES domain. In addition, a sponge zone was attached at the end of the physical domain to fully dissipate the outgoing non-linearities of the turbulent jet plume. The far-field sound was computed from the LES data using the stationary porous Ffowcs Williams-Hawkings (FWH) surface integral acoustic method and through the use of Lighthill’s acoustic analogy (LAA). Parallelization of the LES methodology is achieved using the transposition strategy. The 3-D LES code was written in FORTRAN90 and the Message Passing Interface (MPI).

All jets tested here had a Reynolds number of roughly 200,000 and jet centerline Mach numbers ranging between 0.38 and 0.9. All cases utilized a curvilinear grid of almost 5 million grid points. One point statistics such as potential core lengths, half growth rates,
jet mean centerline decay, mass fluxes and turbulence intensities were found to be in good agreement with laboratory experiments. The mean flow data such as the centerline decay and streamwise turbulent intensities were re-scaled on the abscissa to take into account the jet density ratio, centerline jet Mach number and potential core length. This re-scaling technique is known as the Witze shift. The current LES results show that at least for a shock-free compressible jet, the jet decay rate grows as the jet temperature is increased and that the decay rate is a weak function of the Mach number. Comparison to the LES computations of Bodony and Lele [86] were also in good agreement and in most respects our results were better. The LES computations performed by Bodony and Lele utilized a cylindrical grid, the dynamic Smagorinsky model and sponge zone type boundary conditions on all boundaries. The number of grid points used by Bodony and Lele was roughly one million. Any of these factors could explain the differences observed. Despite the absence of an explicit subgrid-scale model the near-field results of cold and heated jets were in good agreement with experimental data. Further analysis is still needed to in order to determine if the ILES approach is still valid for higher order moments of turbulence.

The far-field spectra and overall sound pressure levels (OASPL) computed using FWH and LAA were in most cases 1-3 dB over-predicted compared to the experimental data of Tanna et al. and Viswanathan. It is believed that the over-prediction stems from the use of the artificial vortex ring forcing to excite the mean flow. The FWH control surface used was an open control surface. Computing the Lighthill volume integral required the use of 1,140 processors in parallel due to the large size of instantaneous LES data, i.e. 430 GB. The close spectral agreement between FWH and LAA validates the use of FWH for far-field sound predictions at a lesser computational cost than simulations over the entire domain. The use of the LAA provided some insight can be gained into the noise source mechanisms for heated and unheated jets. The computational results confirmed that a heated Mach 0.9 jet is quieter than a comparable unheated jet. However, a Mach 0.5 heated jet is noisier than to an unheated jet with the same ambient Mach number. Lighthill’s acoustic analogy yielded that significant cancellations occurred between the momentum (shear noise) and entropy sources for a Mach 0.9 jet probably explaining why a high speed heated jet is more
quiet than to its unheated counterpart. For a Mach 0.5 jet, less cancellations were noted between the shear and entropy sources. On the other hand, strong synergism between the self and entropy sources was observed, which probably explains why a lower speed heated jet is somewhat noisier than a unheated one.

To offset the limitations of the transposition scheme, an alternate parallelization strategy based on the Schur complement was proposed and developed. Limitations of the transposition scheme include a large communication overhead, difficulty in applying it to nozzle geometries and a fixed maximum number of processors based on the number of grid points in a particular direction. The Schur complement is a domain decomposition strategy similar to the multiblock method, but with no overlap between blocks. Instead, each block shares a common interface; information is shared and passed along such interface to eventually generate the Schur complement matrix. The application of the Schur complement to a tri-diagonal compact scheme yielded an interesting features. As the number of interior grid points in each block was increased, the Schur complement matrix became diagonally dominant, which is trivially simple to solve. The Schur complement approach was applied to a 1-D and 2-D Linearize Euler Equations (LEE) aeroacoustic benchmark problem. Numerical results obtained were in very good agreement with a serial version of the LEE, and available analytical solution. The Schur complement was applied to a 3-D LES jet by first decomposing the 3-D grid along one direction, mimicking the transposition strategy. The 1-D decomposition of the Schur complement was almost three times slower than the transposition strategy. A 3-D decomposition significantly improved the performance of the Schur complement, but was 10% slower compared to the single-block code. It was believed that a large number of cache misses for the case of the Schur complement method than for the single-block code may have caused the slight drop in performance. In addition, the number of grid points per block was not equal due to the way the grid was chosen. Further investigations are needed to in order to improve the performance of the 3-D Schur complement code. Nonetheless, near-field and preliminary far-field results using the 3-D Schur complement were in very good agreement with the single-block computations.
A numerical simulation of a Mach 0.4 unheated jet through a uniform pipe with a circular nozzle was performed using the lattice-Boltzmann method (LBM). Turbulent flow near-field statistics were in good agreement with experimental data and parallel LES results. The centerline mean axial turbulence intensity agreed reasonably well with the recent experimental results of Laurendeau et al. [131]. The peak R.M.S. velocity fluctuations of this study agreed well with the results of Laurendeau et al. [131]. The intensities in the shear layer region from the simulations was under-predicted probably due to the laminar nature of the exiting boundary layer. To resolve this problem, a greater Reynolds number and a finer grid resolution should be applied. It was also found that the LBM yielded results that are analogous to an LES. The decay of the axial velocity and pressure spectra follows Kolmogorov’s $-5/3$ and $-7/3$ (turbulence-turbulence interaction for pressure) power law, respectively for both cases. The computed overall sound pressure levels in the far-field agreed with experimental results from Tanna et al. [78] and Laurendeau et al. [131]. The computed far-field spectra agree well with the experiments of Tanna. The LBM results were comparable to the LES-FWH computations. Weak but distinct tones appeared in the high frequency portion of the spectra. These tones are probably an artifact of the ‘explosion’ and ‘coalescing’ of the distribution functions at the VR boundary. This process creates a standing wave pattern that bounces between VR boundaries thus creating a tone like feature in the spectra. These pressures of these tones did not seem to bias the underlying spectrum.

7.2 Recommendations for Future Work

7.2.1 Further Analysis of Heated Jets

Further analysis needs to be performed to clarify the effects of heating. The quantities of interest are the mean centerline density fluctuations, the Reynolds stresses (as done for the Schur complement validation), the near-field energy and dissipation spectra, the contributions to the energy-spectrum balance and spectral energy-transfer rate. Bodony and Lele [86] reported a stronger correlation between the mean centerline density fluctuations and the far-field radiated sound pressure for heated jets than previously believed. Bodony
and Lele’s [86] peak density fluctuations over-predicted the experimental data of Panda and Seasholtz [135] by a factor of two or four. The over-prediction of the density fluctuation might cause the over-prediction of the far-field sound in heated jets. One aspect of the present study that should be re-visited is the vortex ring inflow forcing. For a Mach 0.9 unheated jet, removal of the first four modes seemed to yield the best result [79, 84]. This approach may no longer be the best for low Mach number and/or heated jets. The need for improved inflow forcing techniques are needed, for example the use of a preliminary nozzle flow simulation or the inclusion of the nozzle geometry in the computations. Two-point space-time correlations of the near-field flow would also be interesting to examine to learn the spatial and temporal scales present in heated and unheated jets.

The computation of the far-field sound through the use of the porous Ffowcs Williams-Hawkings (FWH) surface integral method utilized an open control surface. An open control surface implies that contributions from the back-end of the control surface were not taken into account in the far-field calculation. This was done because Uzun et al. [53] and Lew et al. [79] reported spurious noise levels in the overall sound pressure levels when a closed control surface was used. The spurious noise levels should diminish if the stream wise length of the control surface is at greater than or equal to $60r_o$. It is argued that when a closed control surface is used an effective line of dipoles are created when quadrupoles cross the control surface. Recall that the FWH method used includes only the thickness and loading terms. The quadrupole term was ignored due its high computational cost. It is possible that the addition of the quadrupole term may reduce the spurious noise levels due to natural balancing of the three FWH components have been proposed. Alternatives to the inclusion of the costly quadrupole term in the FWH have been proposed. Shur et al. (using a shorter control surface) [83,136] replaced $\rho$ in equation 4.5 with $p'/c_\infty^2$ resulting in the OASPL and directivity in better agreement with experiments. This methodology could be applied in a future study. Morfey and Wright [137] extended a modified aeroacoustic analogy by Goldstein [138] that is formulated for bounded domains. Their results show that the method is valid for jets and boundary layers where the control surface cuts through sheared mean flow.
Further analysis of noise sources is needed. Lighthill’s acoustic analogy offered insight of heated and unheated jet sound generation. In addition to the pressure correlations, the cross correlation spectra would also be an interesting feature to study as it will highlight cancellations and synergisms that occur across a wide spectrum. Further decomposition of the Lighthill stress tensor by decomposing the density in the momentum stress tensor (equation 4.7) to $\rho = \bar{\rho} + \rho'$ was suggested by Bodony and Lele [139]. Even by decomposing the density term, Bodony and Lele arrived at roughly the same conclusion in terms of further source cancellations between momentum and entropic sources for heated jets. For the entropic source term however, Bodony and Lele used Lilley’s [140] decomposition

$$p' - a^2_{\infty} \rho' = -\frac{\gamma - 1}{2} \rho u_k u_k + a^2_{\infty} \int \frac{\partial}{\partial x_k} \left[ \rho u_k \left( \frac{h_{\infty} - h_s}{h_{\infty}} \right) \right] \, dt,$$  

(7.1)

where $h_s$ and $h_{\infty}$ are the stagnation and free stream enthalpies, respectively. Bodony and Lele show that the above decomposition gives a better representation of the entropic sources for heated jets. A future study could use equation 7.1 shown above for the entropy term.

Alternate forms of the momentum sources could also be studied. Instead of performing a Reynolds type decomposition, the momentum stress of the Lighthill tensor can be recast in a form that emphasizes the vorticity as a source of sound. Powell [141] first proposed the theory of vortex sound. Howe [142] and Doak [143] later proposed modifications to Powell’s theory and put it on a more firm mathematical footing. The use of vortex sound for noise source studies in free shear layers is rather recent. Cabana et al. [144] recast the momentum source of the Lighthill tensor to stress the importance of vorticity as a sound source. The specific Lighthill momentum source decomposition that Cabana et al. studied (in vector form) is given by

$$\nabla \cdot (\nabla \cdot (\rho u \otimes u)) = \rho u \cdot \nabla \Theta + \frac{\rho \Theta^2}{2} + 2 \Theta u \cdot \nabla \rho + \rho \nabla^2 \left( \frac{u^2}{2} \right) + \nabla \left( \frac{u^2}{2} \right) \cdot \nabla \rho$$

$$+ \rho u \cdot \nabla \times \omega + u \cdot \nabla \rho \times \omega - \rho \omega \cdot \omega + u \cdot ((\nabla \rho) \omega)$$

$$+ (u \otimes \nabla \rho) : \nabla u,$$  

(7.2)

where $\Theta = \nabla \cdot u$ and $\omega = \nabla \times u$. Using equation 7.2, Cabana et al. showed that terms IV, VI and VIII were the strongest sources radiating to the far-field compared to other terms.
for a 2-D DNS temporal mixing layer. Note that terms VI, VII and VIII are an expansion from

\[ \nabla \cdot (\rho \omega \times \mathbf{u}). \]  (7.3)

which is the vortex sound source. Contributions from each of the terms in Equation (7.2) would be interesting to study for a 2-D spatially developing mixing layer and then for a full 3-D LES jet.

The various decompositions and interpretations of the acoustic analogy however, are not immune to criticism [145]. The solution of the inverse problem of finding the sound generation mechanisms given a radiated sound pressure field is not unique. Therefore there is a certain amount of speculation in analogy based source models. Through the acoustic analogy, noise sources are thought be equivalent to quadrupoles. It is argued that quadrupoles (as of yet) offer no ideas to the design of quieter jet engines. Even the FWH itself is another form of Lighthill’s acoustic analogy when ‘solid’ boundaries are present. Tam et al. [146] argued that there is experimental evidence of two sound sources in shock-free compressible turbulent jets, i.e. the fine-scale turbulence and large scale structures. The only numerical study that seem to have validated the two-source noise model is the recent study by Bogey and Bailly [103] via causality methods (essentially two-point correlations). Tam et al. then suggest that to design a quieter jet, one needs to suppress the large scale structures of the flow that radiate to the far-field. Unfortunately, Tam et al. still offer no direct methodology on how to achieve this.

### 7.2.2 Enhancements to the Parallel Schur Complement

Additional scalability tests need to be performed to assess the performance of the Schur complement. The parallel Schur complement scheme should be tested for a jet LES using more than 1,024 processors. There are two methods of achieving this. The first is to keep the number of grid points of a particular case fixed while increasing the number of processors, as was done in the present study. The second method is to double the number of grid points as the number of processors is doubled, thus keeping the memory requirements
per processor constant. Performance improvements of the Schur complement over the single-block code also still needs be investigated. One possible approach is to reduce the number of cache misses by utilizing a grid that can be decomposed into equal-size domains. In terms of far-field noise, FWH should be used instead of the crude extrapolation technique used in Chapter 6, on Section 6.4.2.

Once the performance issue(s) are addressed, it would also interesting to see how well the Schur complement will scale up to more than 10,000 processors. The two most notable machines that have more than 10,000 cores are Ranger located at the Texas Advanced Computing Center (TACC), Texas and Kraken located at the National Institute for Computational Sciences (NICS), Tennessee. Both of these supercomputers have slightly more than 60,000 cores and are part of the Teragrid network. Testing the Schur complement code up to more than 10,000 cores is important as it may give an idea on how to program code to run on hundreds of thousands of cores. The use of a two-tier level parallelization technique such as combining the Schur complement and transposition scheme may be desirable to simulate jets with complex geometries on hundreds of thousands of processors. With the petascale barrier recently broken by IBM’s Roadrunner [147], it may only be a matter of time until jet noise simulations with several hundred million grid points or even a few billion grid points will be routinely performed.

7.2.3 Complex Nozzle Geometries and High Mach Number Jets through LBM-LES

The anomaly of the VR tones did not seem to contaminate the overall spectra from the LBM noise jet study. However, these VR tones still needs to be addressed and eliminated. One possible solution is to specify a perfectly matched layer (PML) between the boundaries of two VR regions. Preliminary studies coupling perfectly matched layers and LBM by Najafi yazdi and Mongeau [134] have recently been performed and shown to be very promising.

The preliminary results from LBM-LES jet noise simulation seemed to yield reasonable near and far-field results. The far-field needs to be computed with an FWH type method-
ology in place of inverse square law extrapolation. The simulation of jets with complex or realistic nozzle geometries and at high Mach numbers, i.e. Mach 0.6 to 2 is the next logical step. The simulation of jets with complex nozzle geometries is already underway. Figure 7.1 shows the grid setup for a chevron nozzle in PowerFLOW. The finest grid resolution which is located close to the nozzle wall is $\Delta r_{\text{min}} \approx 4 \times 10^{-3} D_j$. The finest grid resolution is still considered relatively coarse since the Reynolds number specified is $Re_D = 100,000$ with a Mach number of $M_j = 0.5$. The smallest cell or grid size should be on the order of at least $10^{-4}$. Figure 7.2 shows the instantaneous streamwise velocity magnitude of the chevron jet, which looks realistic. Note that in Figure 7.2 the chevron jet shear layers appear to break-up roughly one diameter downstream of the exit. In an effort to demonstrate the LBM’s capability of handling complex geometries one avenue of active jet noise research is microjets for noise suppression. It has been shown experimentally that adding microjets can decrease the noise levels by 1-2 dB [148, 149]. It was also found that a microjet-chevron jet combination reduces the noise levels by 3-4 dB [150]. It is argued that because the measured turbulence intensities are reduced the far-field sound is lower. This is where numerical simulations can add valuable insights to this study. Figures 7.3 and 7.4 show the instantaneous velocity magnitude for a round jet and round jet-microjet combination, respectively using LBM-LES. It can be inferred from the figures that there is enhanced mixing for the microjet case compared to the baseline. The diameter of the microjet is $D_{mj} = 0.02D_j$ and the finest resolution which is located at the exit of the microjet nozzle is $\Delta r_{\text{min}} \approx 1 \times 10^{-3} D_j$. The results shown here are preliminary and further work is still ongoing. It is important to point out the computational cost of these simulations. The microjet test case with slightly more than 100 million cells took four days of runtime on 128 processors. The only other LES simulation of a microjet-round jet combination is by Huet et al. [151]. Details of Huet’s et al. work are not known. If a conventional LES simulation (Navier-Stokes) were to be performed, it may take weeks or even months due to the complexity of the geometry and the small time step (assuming explicit time stepping is used).
Preliminary studies using PowerFLOW with a hybrid LBM-finite difference (LBM-FD) scheme have been performed by Li et al. [152] and Nie et al. [153] to extend the LBM to high Mach number flows. Li et al. performed an LBM-FD computation on an 3-D DLR transonic aircraft with reasonable agreement to experiments. Nie et al. on the other hand performed a 2-D inviscid simulation of a $\theta = 15^\circ$ wedge with a free stream Mach number of $M_\infty = 1.8$. The measured shock angle was $\beta = 51^\circ$ which under-predicts the theoretical value of $\beta = 51.4^\circ$. Nie et al. also computed the transonic flow over an RAE 2822 airfoil with a Mach number of $M_\infty = 0.73$. Although the computed shock was slightly dissipated on the suction-side of the airfoil compared to experiments, the shock location nonetheless was predicted correctly.
Figure 7.1. Grid layout for the chevron jet. Every other cell is shown.

Figure 7.2. Instantaneous streamwise velocity for chevron jet at Mach 0.5 using LBM-LES.
Figure 7.3. Instantaneous velocity magnitude for round jet with $M_j = 0.5$. Isosurface velocity of 90 m/s is shown.

Figure 7.4. Instantaneous velocity magnitude for round jet with 18 micro-jets. Main jet Mach number is $M_j = 0.5$. Isosurface velocity of 90 m/s is shown.
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