Analysis of Modified Compressibility Corrections for Turbulence Models

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Two modified compressibility corrections are introduced based on the notion that the reduction in the growth rate in compressible free shear flows levels off at a certain turbulence Mach number, as seen in experiments and direct numerical simulation (DNS). The OVERFLOW code is used to assess Sarkar’s compressibility correction and two modifications when used with the SST turbulence model. Test cases include an axisymmetric jet, a 3-D jet with crossflow, a generic boundary layer, and a generic mixing layer. The modified corrections do not disturb the beneficial behavior of the standard correction observed for the axisymmetric jets. For the 3-D cases, which have a higher Mach number, the modified corrections show significant improvement over the Sarkar and uncorrected models. The compressible boundary layer test case shows that there is still an underprediction of the wall skin friction coefficient with the compressibility corrections, but this can be solved by using a simple modification in the turbulence model, as is already done in OVERFLOW. Finally, analysis of the compressible mixing layer shows that the modified corrections exhibit the correct trend in limiting the reduction of the growth rate, but do not match the level of reduction seen with experimentally-obtained results.

Nomenclature

\( a \) = speed of sound
\( CC\text{-}A \) = modified compressibility correction 1
\( CC\text{-}B \) = modified compressibility correction 2
\( C_f \) = skin friction coefficient
\( D \) = nozzle exit diameter
\( \varepsilon_d \) = dilatation dissipation
\( \varepsilon_s \) = solenoidal dissipation
\( k \) = kinetic energy of turbulence fluctuations
\( H(x) \) = Heaviside step function
\( M \) = Mach number
\( M_c \) = convective Mach number
\( M_{rms} \) = root mean square Mach number
\( M_t \) = turbulence Mach number
\( R_{1/2} \) = jet half radius
\( Re \) = Reynolds number
\( U \) = velocity in x-direction
\( U_{jet} \) = jet centerline velocity in x-direction at nozzle exit plane
\( U_{\infty} \) = freestream velocity in x-direction
\( Vel \) = velocity magnitude
\( \delta \) = shear layer thickness
\( \gamma \) = specific-heat ratio
\( \nu \) = kinematic viscosity
\( \theta \) = momentum thickness
\( \omega \) = specific dissipation rate

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I. Introduction

OVERFLOW\(^1\) is a computational fluid dynamics (CFD) code used for the prediction of aerodynamics and heat transfer of the Space Shuttle and future Constellation launch vehicles. The work presented here is carried out to support computational analysis of the Orion spacecraft, which will bring humans into orbit, to the moon, and eventually to Mars. A key component of the spacecraft is the Launch Abort System (LAS), which is intended to separate the Crew Module (CM) from the rest of the rocket in case of an emergency during ascent. The LAS consists of the four Abort Motors (AM), four Jettison Motors (JM), and eight Attitude Control Motors (ACM), as shown in Fig. 1. Understanding the interaction between the rocket exhaust plumes, the freestream, and the capsule is important in the design and analysis for the spacecraft. For example, an impinging jet might create large localized pressure and temperature peaks. In addition, the shock cells caused by an underexpanded jet present near the capsule surface may cause large changes in pressure coefficients. All these characteristics will impact the flight mechanics of the vehicle.

![Figure 1. Diagram of Launch Abort System (LAS) showing Abort Motors (AM), Jettison Motors (JM), and Attitude Control Motors (ACM). Courtesy of NASA.](image)

A lot of previous work investigating supersonic free shear flows has been undertaken. Eggers\(^3\) performed some of the first baseline experiments to examine mean-flow and turbulence quantities in supersonic jets. Birch and Eggers\(^4\) compiled many of the early experiments performed for free shear flows into the “Langley curve,” which is commonly used to calibrate turbulence models. It was found that the spread rate in mixing layers decreases with increasing convective Mach number.

Improvements in computing power have increased the popularity of direct numerical simulation (DNS), large eddy simulation (LES), and hybrid approaches for modeling or simulating turbulent flows. While these methods show promise for the future, their application to complex flowfields, such as that of the LAS, is not feasible due to the grid size constraints. Thus, the development of Reynolds-averaged Navier Stokes (RANS) models is still of interest. The DNS of fundamental flows can, however, aid in the development of turbulence models, which provide closure to the RANS equations. When modeling jets using RANS solvers, turbulence models are critically important. Choosing a turbulence model determines the important parameters in the jet, such as the location of the shocks or the extent of the potential core. Sarkar et al.\(^5\) showed that additional terms are needed in the turbulence model to capture the increase in growth rate with increasing Mach number observed in free shear flows. Other formulations\(^6,7\) were developed in the early 1990’s and subsequently used with most popular turbulence models. However, validation for these models typically occurs only for benchmark flows, and the performance for more complex flowfields is largely untested. There is a lack of validation at high Mach numbers such as those experienced by the LAS. The authors’ previous paper\(^8\) showed that while the standard corrections work well for the axisymmetric Mach 2 jet, they show a deficiency in mixing for a 3-D jet with crossflow at higher Mach numbers. The goal of the current project is to evaluate the performance of two modified compressibility corrections.

II. Test Cases

In order to assess the modified compressibility corrections, several test cases are computed. An axisymmetric case frequently utilized for testing turbulence models is selected. Then, a 3-D jet in angled crossflow is used to assess further parameters of interest which could not be studied using axisymmetric jets. Experimental data for these cases were measured using particle image velocimetry (PIV) and pressure probes in the flow field. Generic test cases for a boundary layer and mixing layer are also examined.
A. Eggers Jet

The Eggers\textsuperscript{3} jet refers to a 1966 experiment characterizing supersonic turbulent jets. A Mach 2.22 isothermal jet with nozzle exit diameter of 1.01” (0.026 m) was exhausted into ambient air, with a pressure ratio of 11. The experiment supplies velocity profiles in the near- and far-field of the jet. This experiment provides a standard test case for supersonic jet validation.

B. Glenn Jet

The Glenn jet experiment (Wernet et al.\textsuperscript{9}), performed at NASA Glenn Research Center, was used to acquire data for a hot supersonic jet in subsonic crossflow. It was performed as part of the aerodynamics analysis and design process for the LAS. While actual LAS flight conditions were not attainable, the jet conditions used here were at the highest possible pressures and temperatures attainable in the test facility, which was stagnation pressure and temperature of 410 psi (2.8×10\textsuperscript{6} Pa) and 1350 °R (750 K), respectively. A survey of available literature showed that no other experiments had been performed at similar conditions. The nozzle was placed in a wind tunnel with cold freestream air at Mach 0.3. The air was heated using a natural gas combustor and exhausted through the conical nozzle at an angle of 25 degrees relative to the freestream. The nozzle exit design Mach number was 3.0, but due to the underexpansion, Mach numbers exceeding four are seen downstream. Experimental data was obtained with PIV at various locations downstream of the nozzle exit normal to the freestream, producing two dimensional velocity vector fields.

III. Turbulence Modeling

A. Turbulence Model

Choosing the correct turbulence model is critical for a good computational prediction of the flowfield. The OVERFLOW code contains various algebraic, one-equation, and two-equation turbulence models. A comparison of the performance of various turbulence models for jet flows is given in the authors’ previous work\textsuperscript{8}. This study will only examine Menter’s Shear Stress Transport (SST) model\textsuperscript{10}. This model uses the Boussinesq approximation to incorporate the contribution of the Reynolds stresses through an eddy viscosity into the RANS equations. The SST model solves two partial differential equations for the turbulence kinetic energy, $k$, and the specific dissipation rate, $\omega$. The model is essentially a hybrid between the $k$-$\epsilon$ and $k$-$\omega$ models, where a function is used to blend between the models based on distance to the nearest wall.

B. Current Compressibility Corrections

Current turbulence models have been designed for low-speed, isothermal flows. Future research should strive to develop a new model that is general for all types of flows. However, at this time, the practical approach is to modify existing models for more complicated flows. There are various corrections available (e.g., temperature, rotation and curvature), but this study will focus only on the compressibility correction.

The compressibility correction is devised to deal with additional effects seen for higher Mach number flows, specifically, the effects of compressibility on the dissipation rate of the turbulence kinetic energy. For free shear flows, this is exhibited as the decrease in growth rate in the mixing layer with increasing Mach number\textsuperscript{4}. Standard turbulence models do not account for this Mach number dependence, and thus a compressibility correction is used. For compressible flows, two extra terms, known as the dilatation dissipation, $\varepsilon_d$, and the pressure-dilatation occur in the turbulence kinetic energy equation. The pressure-dilatation term is usually neglected because its contributions have been shown to be small\textsuperscript{11}. The dilatation dissipation term is included in addition to the solenoidal, or incompressible, dissipation, $\varepsilon_s$. In the formulation of the $k$-$\omega$ and SST models, the extra term also occurs in the specific dissipation rate equation. Thus, the effect is that the growth rate of turbulence is inhibited when the correction is active. Sarkar\textsuperscript{5} modeled the ratio of the dilatation dissipation to the solenoidal dissipation, $\varepsilon_d/\varepsilon_s$, as a function of the turbulence Mach number, $M_t$, defined as

$$M_t = \frac{\sqrt{2k}}{a}$$  \hspace{1cm} (1)
For Sarkar’s model,\
\[
\frac{\varepsilon_d}{\varepsilon_s} = f(M_t) = M_t^2
\] (2)

The model proposed by Zeman\(^6\) is formulated as follows
\[
f(M_t) = \left\{ 1 - \exp \left[ -\frac{1}{2} (\gamma + 1)(M_t - M_{t0})^2 / \Lambda^2 \right] \right\} H(M_t - M_{t0})
\] (3)

where \(M_{t0} = 0.10 \sqrt{2 / (\gamma + 1)}\) and \(H(x)\) is the Heaviside function. Finally, Wilcox’s\(^7\) model is given by
\[
f(M_t) = (M_t^2 - M_{t0}^2) H(M_t - M_{t0})
\] (4)

The model constant \(M_{t0} = 0.25\). For a complete discussion of the compressibility corrections, see Wilcox\(^1\). It must also be noted that the implementation of the compressibility correction for the SST turbulence model in OVER-FLOW follows the suggestion by Suzen and Hoffman\(^12\), where the compressibility correction is multiplied by the SST blending function, thus effectively disabling the correction in near wall regions.

C. Modified Compressibility Corrections

In 2001, Rossman\(^13\) performed experimental analysis for mixing layers with very high convective Mach numbers, \(M_c\), defined as
\[
M_c = \frac{U_1 - U_2}{a_1 + a_2}
\] (5)

where \(U_1\) is the fast side velocity and \(U_2\) is the slow side velocity, and \(a_1\) and \(a_2\) are the sonic velocities in the streams. Fig. 2 shows the mixing layer growth rate data by Rossman. Previous data at lower \(M_c\) simply show that the growth rate decreases. However, Rossman’s experimental data (the blue boxes) clearly demonstrate that the mixing layer growth rate levels off beyond \(M_c\) of 1.0, and then remains flat for larger \(M_c\).

A similar trend is also seen for homogeneous shear flow. Blaisdell et al.\(^14\) examined the compressibility effects in turbulent homogeneous shear flow using DNS. First, it was verified that the growth rate of turbulence in a homogeneous shear flow is reduced for the compressible case when compared with the incompressible case. This result is attributed to an increase in the dissipation rate and an increase in the energy transfer caused by the pressure-dilatation term. The solenoidal dissipation remained the same as for the incompressible case, but the dilatation dissipation term increased with increasing r.m.s. Mach number, \(M_{rms} (M_{rms} \approx M_t)\). The second key observation was that the ratio of the dilatational to solenoidal dissipation rate levels off beyond \(M_{rms}\) of 0.3. This implies that the decrease in growth rate with increasing turbulence Mach number reaches a minimum value.

The DNS by Blaisdell et al.\(^14\) is used as the guideline to formulate the modified compressibility correction, which levels off the dilatation dissipation after a certain turbulence Mach number. The formulation for the first modification, CC-A, is given by
\[
f(M_t) = \left\{ \begin{array}{ll}
M_t^2 & \text{if } 0 \leq M_t \leq \sqrt{0.1} \\
0.1 & \text{if } M_t \geq \sqrt{0.1}
\end{array} \right.
\] (6)

The second formulation, CC-B, is given by
\[
f(M_t) = \left\{ \begin{array}{ll}
10M_t^4 & \text{if } 0 \leq M_t \leq \sqrt{0.1} \\
0.1 & \text{if } M_t \geq \sqrt{0.1}
\end{array} \right.
\] (7)

The reason for the fourth power on the \(M_t\) term is that this may improve predictions in the boundary layers where the turbulence Mach numbers are low. Ristorcelli\(^15\) first suggested that the dilatation dissipation scales as \(M_t^4 / Re\). This formulation simply uses the \(M_t^4\) dependence.

Fig. 3 shows the ratio of dilatation to solenoidal dissipation, as a function of turbulence Mach number, for the compressibility corrections by Sarkar, Zeman, and Wilcox. Also, the two new compressibility corrections are shown. The DNS data strongly suggest that there is a value (0.1) at which the dissipation levels off. Note that the arrows in the figure indicate the development of the simulations in time starting from different sets of initial conditions. In the region before \(M_t = \sqrt{0.1} \approx 0.3\), the Zeman and Sarkar corrections provide the most additional turbulence dissipation.
while Wilcox’s correction gives the least. To this point, i.e., \( M_t = \sqrt{0.1} \), the modified correction CC-A acts exactly the same as Sarkar’s correction. For \( M_t \) greater than \( \sqrt{0.1} \), the standard corrections continue to provide additional dissipation, while the modified corrections level off this dissipation. This achieves the goal of matching the observed limit in the dissipation ratio shown by the DNS data. These models are implemented in the OVERFLOW code and evaluated using a range of test cases.

IV. Numerical Methods

A. Code Description

The OVERFLOW code is an overset grid solver developed by NASA. It solves the time-dependent, Reynolds-averaged Navier-Stokes equations for compressible flows. The overset capability makes it useful for computing flow fields involving complex geometries. The code is formulated using a finite difference method and has various central and upwind schemes for spatial discretization built in. A diagonalized, implicit approximate factorization (ADI) scheme is used for time advancement. Other capabilities, such as local time-stepping and grid sequencing, are also available to accelerate convergence.

B. Grid Generation

The grid for the axisymmetric Eggers jet is shown in Fig. 4. Four zones are used for the nozzle, top, aft, and the nozzle lip regions. The X-axis represents the jet’s axis of symmetry. The collar grid around the nozzle lip is used to better resolve the high flow gradients present in this region. The grid spacing along the inner nozzle boundary is designed to keep the wall \( y^+ \) below one in order to adequately resolve the viscous sublayer near the wall. The grid is particularly fine in the aft region off the nozzle lip to resolve the shear layer that determines the jet development. In order to sufficiently model the freestream and eliminate upper and aft boundary effects, the domain of the plume extended at least 30 diameters in the radial and 80 diameters in the axial directions, measured from the nozzle exit plane. A grid generation study is shown in the first author’s thesis. The final Eggers grid system contains 91.3×10^3 grid points.

For the Glenn jet, grid scripts were obtained from researchers at NASA Ames Research Center. The model includes the entire wind tunnel. Similar to the grid for the axisymmetric jet, particular attention is paid to ensure sufficient resolution along the nozzle lip and the mixing region. For the final grid, the plume region contains 9×10^6 grid points, leading to a total of 29×10^6 grid points. Figure 5 shows the grid for 3-D Glenn jet in crossflow.

The grid for the boundary layer test case is a single zone grid with particular refinement near the wall to capture the viscous sublayer. The grid is shown in Fig. 6. The grid for the mixing layer, shown in Fig. 7, is also a single zone grid refined in the mixing region. The top and bottom boundaries extend out sufficiently far to avoid any interference. The domain also extends far enough downstream to ensure the development of a self-similar flowfield.

C. Boundary Conditions

For the axisymmetric jet, freestream boundary conditions are imposed at the top of the nozzle, with Mach 0.01 and standard pressure and temperature. For the Glenn jet, the freestream boundary conditions are imposed at the tunnel entrance with Mach 0.3. All walls are specified as viscous and adiabatic. The nozzle conditions for the axisymmetric jet are set as a ratio of stagnation pressure and temperature from the nozzle plenum to the freestream. For the Glenn jet, a “q file,” containing the conserved variables, is specified upstream of the throat, where the variables are extracted from the experimental conditions. The natural gas combustion products in the nozzle are modeled as a second species with specific heats ratio, \( \gamma \), of 1.32. The top and aft domain boundaries for the jets are specified as pressure outflow boundaries with a given value of static pressure.

Boundary conditions for the two-dimensional flat plate test case include a viscous adiabatic wall, standard free stream inflow conditions, and outflow. The freestream Reynolds number, which is based on the grid length unit, is set to 1×10^7. The freestream temperature is set to 180 °R (100 K) and the freestream Mach number is varied from 2 to 8. For the two-dimensional mixing layer, the velocity is fast on one side and slow the other, and there is a uniform temperature of 460 °R (256 K) across the inflow. A prescribed “q file” with a hyperbolic tangent velocity profile, along with a characteristic condition, is specified at the inlet. The far-fields on top and bottom are specified as a pressure outflow boundaries, and the outflow is a characteristic condition.
D. Solution Method

The numerical schemes used are chosen based on robustness and rate of convergence. For consistency, all test cases use the same inputs for the numerical methods. The HLLC upwind scheme proved better than the central difference and Roe’s upwind scheme for the discretization of the spatial derivatives. This is especially true for cases in which shocks are present, as the HLLC is an improved algorithm for capturing shocks with low smearing. A van Albada limiter is used for the upwind Euler terms, along with a MUSCLE scheme flux limiter for added smoothing. The spatial differencing for all convective terms uses third order accuracy. Local timestep scaling with a constant CFL number is employed, where the timestep scaling is based on the local cell Reynolds number. All cases are run with three levels of grid sequencing for improved convergence. 1,000 iterations are run on the coarse and medium grid levels each before running 6,000 iterations on the fine grid. The solution is considered converged when all residuals drop at least three orders of magnitude and momentum forces at various downstream locations are constant (change less than convergence criteria).

V. Results

A. Axisymmetric Jet

Previous studies\(^8\),\(^17\) have shown that Sarkar’s compressibility correction performs well for the axisymmetric jet by Eggers when used with the SST turbulence model. Thus, the modified corrections are first evaluated for axisymmetric jets to ensure the beneficial behavior for the cases presented previously is not disturbed. Fig. 8 shows a contour plot of turbulence Mach number for the Eggers jet (with Sarkar compressibility correction). The maximum \(M_t\) is 0.32, and the changes in the new formulation go in effect above approximately 0.3, thus it is expected that the modified corrections will not have a significant effect for this test case. The centerline velocity and normalized half radius for the Eggers jet, using the modified corrections, are shown in Fig. 9. As expected, there is almost no change in using the new corrections. All compressibility corrections show a major improvement over the uncorrected model. Since the \(M_t\) for this case is mostly below 0.3, the correction CC-A acts essentially like Sarkar’s and there is no change in the predictions. Also, there is very little difference in the results from CC-B.

B. 3-D Jet in Crossflow

The Glenn jet provides a higher Mach number test case which highlights the problems in using Sarkar’s compressibility correction. Turbulence Mach number contours (using Sarkar’s compressibility correction) for this jet are shown in Fig. 10. When comparing with Fig. 8, it is evident that the values of \(M_t\), caused by elevated Mach numbers, are significantly larger. The maximum \(M_t\) is 0.46, so it is expected that the limiter implemented in the modified corrections will have some impact.

Figure 11 presents the velocity magnitude contours. For the uncorrected model, the velocity magnitude and vortex roll-up diffuses too quickly when compared with experiment. Sarkar’s compressibility correction, on the other hand, predicts velocity magnitudes that are too large at downstream locations because turbulent mixing in the shear layer is excessively suppressed. The jet also shows too much vorticity. It appears that Sarkar’s model overcorrects by inhibiting the mixing with the freestream for too long. Both modified corrections clearly improve the predictions. The shape of the vortex roll-up is predicted exceptionally well by both modified corrections. Also, the decay of the velocity going downstream is predicted much better when compared with the other models. The velocity profiles along a \(Y = 0\) cut are shown in Fig. 12. It is clear that, when compared with Sarkar’s correction, both modifications do not inhibit turbulence growth as much. This is particularly clear for the later locations. The CC-B formulation is marginally more diffusive than the CC-A formulations. Overall, the predictions are greatly improved.

C. Compressible Boundary Layer

A generic two-dimensional flat plate test case is used for computations of a compressible boundary layer. Results are compared to the Van Driest II with Karman-Schoenherr correlation\(^18\) for Mach numbers of 2, 4, 6, and 8. In Fig. 13, the wall shear stress, \(C_f\), is plotted as a function of the Reynolds number based on momentum displacement thickness, \(Re_\theta = \frac{U_\infty \theta}{\nu}\).

First, results are validated with those by Wilcox\(^11\). Wilcox found that at Mach 4 and \(Re_\theta\) of \(10^4\), the Sarkar correction underpredicts skin friction by 18%. This compares well with the current results which reflect a 20% change.
at the same conditions. The CC-A model behaves identical to the Sarkar model. This can be explained by examining the $M_t$ levels. Table 1 shows that the maximum $M_t$ seen for the flat plate cases using Sarkar’s compressibility correction stays below 0.3. The second correction, CC-B, shows improvement over Sarkar’s model because it provides less dissipation below $M_t$ of 0.3. However, the uncorrected model still gives the best prediction. This shows that the modified corrections still exhibit a boundary layer problem like Sarkar’s correction, especially at higher Mach numbers.

Table 1. Maximum Turbulence Mach Number for Flat Plate

<table>
<thead>
<tr>
<th>$M_\infty$</th>
<th>$M_{t,max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.166</td>
</tr>
<tr>
<td>4.0</td>
<td>0.238</td>
</tr>
<tr>
<td>6.0</td>
<td>0.271</td>
</tr>
<tr>
<td>8.0</td>
<td>0.289</td>
</tr>
</tbody>
</table>

Wilcox suggests that Sarkar’s compressibility correction exhibits too little skin friction for a flat plate boundary layer, while an uncorrected $k$-$\omega$ model predicts the skin friction very accurately for all Mach numbers. Wilcox’s solution was to use a Heaviside step function to disable the correction in regions of lower $M_t$ (i.e., the boundary layer), which is shown in Eq. 4. This presents one mitigation for the compressibility correction problems in the boundary layer. However, this could reduce the effectiveness of the correction in a free shear layer somewhat. A second possible way to address this problem is incorporating the compressibility correction terms only in the $k$-$\epsilon$ part of the SST model, and leaving the near-wall $k$-$\omega$ model undisturbed, as proposed by Suzen and Hoffmann. It must be noted that this is the implementation used in OVERFLOW. The results for using this formulation are presented in Fig. 14. The first modified correction, CC-A, is not included because Fig. 13 shows no difference between it and Sarkar’s formulation. Both Sarkar’s correction and CC-B show very good agreement with the Van Driest II correlation for most of the $Re_\theta$ range.

D. Compressible Mixing Layer

The final test case assessed is the compressible mixing layer. This is the classic test case for compressible turbulence modeling. The experimental data used is from the study by Barone et al. The data come from various experiments which use velocity profiles to determine shear layer thickness. Cases for various convective Mach numbers are run, with a uniform temperature across the inflow. Table 2 summarizes the conditions used for this analysis.

Table 2. Conditions for Compressible Mixing Layer

<table>
<thead>
<tr>
<th>$M_c$</th>
<th>$U_2/U_1$</th>
<th>$M_{t,max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.8462</td>
<td>0.031</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6471</td>
<td>0.180</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5238</td>
<td>0.306</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4074</td>
<td>0.485</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3548</td>
<td>0.612</td>
</tr>
<tr>
<td>2.0</td>
<td>0.2157</td>
<td>1.224</td>
</tr>
</tbody>
</table>

The shear layer thickness, $\delta$, is defined as the difference in the locations of $y$, $\Delta y$, where $(U - U_2)^2/(U_1 - U_2)^2$ is 9/10 and 1/10. The growth rate is measured at various locations in the fully-developed region and averaged. To scale the growth rate for different velocity ratios, the relation based on classic free shear scaling is used:

$$\frac{d\delta}{dx} = C_1 \left[ \frac{U_1 - U_2}{(U_1 + U_2)/2} \right]$$

Figure 15 shows the growth rate scaled by the incompressible growth rate to give the compressibility factor, $\Phi$. The uncorrected model, which exhibits no decrease in growth rate, is not shown. Results using Sarkar’s correction (and the SST model) compare well with the results by Barone et al. when using the $k$-$\epsilon$ model. In the first region, the growth rate decreases faster than the experiment, meaning that the compressibility correction provides too much
added dissipation initially. In the region beyond $M_e$ of 0.6, the growth rate does not decrease as quickly as it should, implying that more dissipation is needed. The modified correction, CC-A, matches Sarkar’s until $M_e$ of around 0.8. Then, the modified correction levels off the growth rate. This trend is also seen in the experimental data. However, the value at which it levels off, where $\Phi$ is around 60%, is too high compared with the experimental data, which levels off around 45%.

A possible solution to this problem is limiting the dissipation ratio at a later location to better match the experimental data. Another mitigation includes using a different formulation for the dissipation ratio below the cutoff, in order to provide less dissipation initially and then increased dissipation after $M_e$ of 0.7. According to Barone et al., differences are also seen depending on the turbulence model used. For their study, the corrections perform better when used with the $k$-$\omega$ model as opposed to the $k$-$\epsilon$ model. Various turbulence models could be tested to assess the baseline turbulence model dependence. At this time, the 2006 version of the $k$-$\omega$ model used by Barone et al. is not available in the current version of OVERFLOW.

VI. Conclusions and Future Work

Two modified compressibility corrections are developed and tested for various test cases. The formulation is based on observations from DNS data that suggest that the dissipation ratio levels off above a certain turbulence Mach number for homogeneous shear flow. The following summarizes key conclusions from the compressibility corrections analysis:

- Recent experiments have shown a plateau in the mixing layer growth rate which is not reflected in current compressibility model corrections. The modified corrections aim to incorporate this trend.

- The modified formulations do not disturb the good predictions for the axisymmetric supersonic jets at the given conditions because $M_t$ is not high enough to trigger a difference.

- The predictions for the high Mach number Glenn jet are greatly improved and show very good agreement with experimental data. The modified corrections both provide the correct level of turbulence dissipation in the jet mixing region, leading to a good prediction of downstream jet development.

- For the flat plate boundary layer, the first correction does not change the behavior compared with Sarkar’s model, while the second correction slightly improves the prediction. However, neither correction completely mitigates the boundary layer problem. Thus, using a modification that disables the correction in the near-wall region is suggested when using a compressibility correction, as has been the usual practice.

- The mixing layer test case shows that Sarkar’s model compares fairly well with the experimental data until $M_e$ of around 1. For $M_e$ greater than one, it decreases the growth rate excessively, while the experiment shows it leveling off. The modified correction exhibits the correct trend in leveling off the growth rate at $M_e$ of 1, but the value of the growth rate (when the leveling-off happens) is too high.

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References

Figure 2. Normalized mixing layer growth rate data compared with previous efforts and recent experiments by Rossman. Used with permission from Rossman.\textsuperscript{13}

Figure 3. Dissipation ratio as a function of turbulence Mach number for compressibility corrections on log-log scale, along with DNS data by Blaisdell el al.\textsuperscript{14}
Figure 4. Grid topology for axisymmetric jet (showing every 4th grid line).

Figure 5. Grid topology for Glenn jet (showing every 5th grid line) along centerline showing tunnel walls (black), nozzle (red), plume region (green), and freestream (blue).
Figure 6. Grid topology for generic boundary layer (showing every 3rd grid line).

Figure 7. Grid topology for generic mixing layer (showing every 5th grid line).
Figure 8. Contours of turbulence Mach number for Eggers jet.
Figure 9. (a) Normalized centerline velocity and (b) normalized half radius for Eggers jet.
Figure 10. Contours of turbulence Mach number for Glenn jet.
Figure 11. Downstream velocity magnitude contours for Glenn jet.
Figure 12. Normalized downstream velocity magnitude profiles for Glenn jet.
Figure 13. Skin friction predictions as a function of Reynolds number based on momentum thickness, compared with Van Driest II correlation, for flat plate. Computations use standard compressibility correction implementation.

Figure 14. Skin friction predictions as a function of Reynolds number based on momentum thickness, compared with Van Driest II correlation, for flat plate. Computations use modified compressibility correction implementation.
Figure 15. Mixing layer growth as function of convective Mach number.