Numerical Investigation of 3-D Supersonic Jet Flows using Large Eddy Simulation

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Supersonic jet flows are studied using 3-D large eddy simulation. The farfield noise generated by the jets is investigated by a computational aeroacoustics methodology that couples the near field unsteady flow field data computed by 3-D LES with a surface integral acoustic method for noise prediction. Since the engines of modern advanced commercial airliners and most military aircraft operate with jets that exhaust at supersonic speed, predicting supersonic jet noise accurately has become one of the keys to designing new low noise emission engines that are suitable for future more restrictive noise regulations. In order to accurately simulate jets at supersonic speeds we employ large eddy simulation and a characteristic filter approach. This approach has low dissipation and is suitable for incorporation into existing solvers based on compact differencing scheme. The results using characteristic filters combined with shock detectors are satisfactory in capturing shocks and resolving turbulent fluctuations. In this study, both perfectly expanded and underexpanded unheated jets are investigated with and without using characteristic filters. Comparisons of the jet mean flow, turbulent statistics, and jet aeroacoustics results with other numerical and experimental data of jets at similar flow conditions were done and reasonable agreement is observed.

I. Introduction

Aircraft produced noise comes from several places, such as the airframe (e.g., landing gears, flaps, slats) and power plants (e.g., turbine blade, jet flows), etc. One of the major sources of noise produced by aircraft is jet noise. Basically, it is caused by a mixing process between a high speed flow and the surrounding air, and hence called mixing noise. Due to the instability of the shear layer near the nozzle lip, a turbulent mixing layer is generated from the lip and propagates downstream. This mixing process increases its intensity from the nozzle exit to a maximum level, and then decays as the average exhaust velocity falls. Some theories based on acoustic analogies1,2 have been developed in the past to estimate the noise radiated by turbulent flows. For supersonic shock free jets, in addition to the mixing noise, Mach waves contribute another source of noise. McLaughlin et al.3, 4 has shown the noise generated by Mach waves is dominated by large scale turbulent structures for high Reynolds number supersonic jets. These large structures convect supersonically relative to the surrounding fluid based on the ambient speed of sound, and they contribute high noise levels near the downstream axial direction. Away from this region, the acoustic intensity contributed by Mach waves quickly diminishes and mixing noise becomes dominant.5

When a supersonic jet engine operates at an off-design condition, a quasi-periodic shock cell structure forms downstream of the nozzle exit. This shock cell structure is composed of a series of shock and expansion waves. As the turbulence passes through these shock cells, an intense noise, called (broadband) shock cell noise, is generated. This shock associated noise has been investigated experimentally6–9 and theoretically10–12.

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in the past. From the above experimental studies, it is found that the broadband shock associated noise is highly directional. The waves preferentially propagate in the upstream direction, towards the jet nozzle, and may even damage the aircraft structure. This phenomenon results in a measurable increase in the overall sound pressure levels (OASPL). For angles closer to the downstream jet axis the shock associated noise contributes less to the OASPL than the turbulent mixing noise, but is still visible in the acoustic spectra.\(^9\) In addition, imperfectly expanded supersonic jets may generate discrete sounds, called screech tones, dominating all other noise sources in the forward direction. Powell\(^{13}\) proposed that the production of screech tones is controlled by a feedback loop. The upstream propagating waves (known as broadband shock noise), generated by the interactions between the turbulent eddies and shock cells, interact with the nozzle lip, and then excite the jet shear layer, closing the resonance loop. This loop mechanism is a time dependent process, and cannot be predicted by Reynolds averaged approaches. Since shock noise and the screech tones are generated by the same mechanism, experiments\(^{11}\) indeed show they are closely related. Tam et al.\(^{14}\) suggest that screech tones can be considered a special case of broadband shock noise.

Traditional jet noise predictions rely heavily on experiments. With the recent improvements in the processing speed of supercomputers, the application of direct numerical simulation (DNS), large eddy simulation (LES), and computational aeroacoustics (CAA) to the prediction of jet noise is becoming feasible. DNS solves for all the relevant length scales of turbulence and does not use any turbulence models. Due to the wide range of time and length scales in high Reynolds number turbulent flows, the application of DNS to industrially relevant jets is still infeasible. LES, which computes the large scales directly and models the effect of the small (subgrid) scales, yields a cheaper alternative. Therefore, in this paper, LES is used to study high Reynolds number supersonic jets. With the near field data computed by the LES approach, the far field noise can then be estimated using integral acoustic methods, such as the Ffowcs Williams-Hawking method or Lighthill’s acoustic analogy.

Some previous studies of supersonic jet simulations using the DNS or LES approach are summarized below. The first DNS turbulent jet simulation was done by Freund et al.\(^{15}\) They simulated a Reynolds number 2,000, Mach 1.92 supersonic turbulent jet. The computed sound pressure levels show a good agreement with experimental data. The first use of LES as an investigative tool for supersonic jet noise prediction was carried out by Mankbadi, et al.\(^{16}\) They performed an axisymmetric LES computation of a fully expanded Mach 1.5 supersonic jet with the Smargorinsky subgrid scale (SGS) model. They used LES to capture the time-dependent flow structures of a Reynolds number 1.27 million supersonic jet and applied Lighthill’s theory\(^3\) to calculate the far field noise. DeBonis and Scott\(^{17}\) used LES with the Smargorinsky SGS model to study a cold Mach 1.4 fully expanded jet with a Reynolds number of 1.2 million. Their simulation accurately captures the physics of the turbulent flow and the time-averaged velocity field agrees with the experimental data. Al-Qadi and Scott\(^{18}\) used a sixth-order compact scheme and a TVD type characteristic filter developed by Yee et al.\(^{19}\) to study a 3-D underexpanded rectangular jet. The results demonstrate that the schemes used are capable of resolving the unsteady complex near field features. Viswanathan et al.\(^{20}\) investigated the flow field and the noise generated by round and beveled nozzles. A perfectly expanded Mach 1.9 jet at two temperature ratios of 1.0 and 2.7 is examined for the round nozzle. Their LES results correctly predict the shift of the peak OASPL as the temperature ratio increases. Shur et al.\(^{21}\) used LES to simulate two off-design jets using the monotone integrated large eddy simulation (MILES) approach with numerical shock capturing schemes. The increased sound due to the presence of the shock cells is captured. The peak frequency of the far-field sound was captured with a good agreement in the upstream direction. Bodony et al.\(^{22}\) used LES to compare a fully expanded jet and an underexpanded jet. They found Mach waves contribute significantly to the near field pressure of the underexpanded jet. Loh and Hultgren\(^{23, 24}\) used LES to investigate the near field noise of a Mach 1.4 fully expanded circular jet. Although the results are in qualitative agreement with the experimental data, further improvements on flux limiter and SGS model parameters are considered to reduce numerical damping.

The direct calculation of the radiated sound from turbulent flows and shock cells requires the accurate representation of the turbulent flow and the shock structures. Therefore, a high order nondissipative scheme is preferred (especially in simple geometries) and the reasons are as follows. First, the sound field is usually several orders of magnitude smaller in amplitude than the aerodynamic field. Second, low order schemes require an extremely fine grid resolution to convect and preserve the strength of vortical flow fields accurately. However, a second order method is still used in LES computations with unstructured grids needed for complicated geometries.\(^{25}\) On the other hand, when the flow fields contain shocks, it is necessary to add numerical dissipation to keep the scheme stable. Traditional shock capturing schemes, such as total variation
diminishing (TVD), monotone upstream-centered scheme for conservation law (MUSCL), or (weighted) essentially non-oscillatory (ENO/WENO) schemes, are usually too dissipative and not suitable for turbulence simulation and CAA. Tenaud et al.\textsuperscript{26} investigated several shock capturing schemes for performance of basic interactions between acoustic, vorticity, and entropy waves in a DNS framework. They showed that the second-order TVD scheme exhibits a strong numerical diffusion, while the third-order MUSCL scheme behavior is very sensitive to the limiter parameters. The generic ENO scheme (third-order) performs better than both TVD and MUSCL schemes, although it exhibits a slightly diffusive behavior. In another study, Lee et al.\textsuperscript{27} showed that the use of a sixth-order ENO scheme in the entire computational domain leads to a significant damping of the turbulent fluctuations. Therefore, to achieve the goal of estimating supersonic jet noise, a low dissipative and efficient shock capturing scheme (characteristic filter) proposed by Yee et al.\textsuperscript{19} is adopted in this study.

In the current paper, we continue our previous work.\textsuperscript{28} The characteristic filters have been incorporated into our 3-D LES solver. In the following sections, we briefly describe the LES methodology, characteristic filter, acoustic method, case and grid information. Finally, both the near field turbulent statistics, and the far field noise are presented.

II. Numerical Methods

A. 3-D LES methodology

We briefly describe our LES methodology used in this study. More detailed information can be found in Uzun.\textsuperscript{29} This 3-D LES solver was originally developed for the study of subsonic turbulent jet noise at high Reynolds numbers. Further, Lew et al.\textsuperscript{30} extended the solver to investigate the noise generated by subsonic hot jets. Lo et al.\textsuperscript{28} extended the solver for supersonic jets. It is an unsteady, Favre-filtered, compressible Navier-Stokes solver. Due to its high frequency resolution and nondissipative features, the sixth-order compact method developed by Lele\textsuperscript{31} is used for spatial discretization. For the time advancement scheme, the classical forth-order Runge-Kutta method is adopted. There are some inherent disadvantages for the compact scheme. It is known that the compact scheme can produce high frequency spurious modes, which arise from the boundary conditions, unresolved scales, and mesh non-uniformities, etc., and cause numerical instabilities. In order to damp out these high frequency modes and keep the numerical scheme stable, the sixth-order tri-diagonal spatial filter proposed by Gaitonde and Visbal\textsuperscript{32} is used in the 3-D LES solver. This spatial filtering operation is applied on the computational domain only once per time step (i.e. a full Runge-Kutta step), and the filter coefficient, $\alpha_f$, is set to 0.47 in this study.

This 3-D LES solver can use an eddy-viscosity SGS model (such as the classical or a localized dynamic Smagorinsky (DSM) model) to dissipate the turbulent energy. Although the DSM model can alleviate the need to specify the Smagorinsky constant for a specific problem, it usually requires much more (about 40\%) computational time. A spatial filter can also provide the same effects as the subgrid scale model. A discussion of the two methods can be found in reference.\textsuperscript{33} For the current study the spatial filter is used as the SGS model.

Tam and Dong’s\textsuperscript{34} radiation and outflow boundary conditions are implemented on the side and outflow boundaries, respectively. At the inflow plane, Whitfield and Janus\textsuperscript{35} characteristic boundary conditions are used. In order to prevent unwanted reflections caused by strong vortices passing through the outflow boundary, a sponge zone\textsuperscript{36} is attached at the outflow boundary to damp out all possible incoming waves. In the actual jet flow, the nozzle lip can reflect and scatter acoustic waves into the jet’s initial shear layer and excite the mean flow. Since the current simulations do not include the physical nozzle, in order to mimic this function the vortex ring forcing approach proposed by Bogey et al.\textsuperscript{37} is used in this study. This is done by putting a vortex ring at approximately one jet radius downstream of the inflow boundary to generate randomized velocity perturbations. Randomly generated streamwise ($v_x'$) and radial ($v_r'$) velocity perturbations are added to the local velocity components ($v_x_0$, $v_r_0$) as
\begin{align}
v &= v_x_0 + v_x', \quad (1) \\
v_r &= v_r_0 + v_r'. \quad (2)
\end{align}

For the detailed formulations of these perturbation velocities, the reader can refer to Uzun.\textsuperscript{29} Studies regarding the effect of this inflow forcing can be found in Lew et al.\textsuperscript{38} and Bogey and Bailly\textsuperscript{39} for subsonic jets and Lo et al.\textsuperscript{28} and Lo\textsuperscript{40} for supersonic jets.
Since the current simulations do not include an explicit nozzle, some special treatments on the inflow boundary must be specified. A hyperbolic tangent velocity profile used by Freund\textsuperscript{41} is specified on the inflow boundary as
\[ \frac{\bar{u}(r)}{U_j} = \frac{1}{2} \left( 1 - \tanh \left[ \frac{r_0}{4\theta_0} \left( \frac{r}{r_0} - 1 \right) \right] \right) \]
where \( r = \sqrt{y^2 + z^2}, \) \( r_0 = 1, \) \( U_j \) is the jet centerline velocity, and \( \theta_0 \) is the inflow momentum thickness. Here, \( \theta_0 \) is used to control the thickness of the inflow shear layer. A larger value of \( \theta_0 \) implies a thicker shear layer. \( \theta_0 = 0.09r_0 \) has been used by Bodony and Lele\textsuperscript{22} and Lew et al.\textsuperscript{42} for their subsonic jet studies, and they were able to achieve good agreement with experiments for turbulent statistics and farfield noise.

For laboratory jets, however, the measured value of \( \theta_0 \) is usually an order of magnitude or more smaller \(( \sim 10^{-3}r_0)\textsuperscript{43} \) compared to that used in LES and DNS of jets. For the inflow density profile, we also adopt the formula from Freund,\textsuperscript{41}
\[ \frac{\bar{\rho}(r)}{\rho_j} = (1 - \frac{\rho_\infty}{\rho_j}) \frac{\bar{u}(r)}{U_j} + \frac{\rho_\infty}{\rho_j} \]
For the underexpanded jet, a hyperbolic tangent profile similar to equation (4) is used to specify the inflow boundary as
\[ \sqrt{\frac{\bar{U}(r)}{\bar{U}_j}} \]
where \( \bar{U} \) is the jet centerline velocity, and \( \theta_0 \) implies a thicker shear layer.

Depending on the type of the shock capturing scheme, there are two filter numerical fluxes that can be used to compute \( L_f \). The formulation of the filter numerical flux based on the TVD or MUSCL scheme from Yee et al.\textsuperscript{19} is
\[ \tilde{D}^{\text{TVD/MUSCL}}_{i+1/2} = \frac{1}{2} R_{i+1/2} \Phi_{i+1/2} \]
On the other hand, the filter numerical flux based on the ENO/WENO scheme from Garnier et al.\textsuperscript{44} is
\[ \tilde{D}^{\text{ENO/WENO}}_{i+1/2} = R_{i+1/2} \Phi_{i+1/2} \]
The \( R_{i+1/2} \) is the right eigenvector matrix of the flux Jacobian at the face \( i + 1/2 \) evaluated by Roe’s\textsuperscript{45} approximate average state. \( \Phi_{i+1/2} \) can be expressed as
\[ \phi_{i+1/2}^\dagger = \kappa \theta_{i+1/2}^{\dagger} \phi_{i+1/2}^{\dagger}, \]
where \( \kappa \) is a problem dependent parameter with a range \( 0.03 \leq \kappa \leq 2 \). In this study, \( \kappa \) is set to 1. \( \theta_{i+1/2}^{\dagger} \) is the Harten switch, and \( \phi_{i+1/2}^{\dagger} \) is the nonlinear dissipation evaluated from the TVD, MUSCL, or ENO/WENO scheme. More detailed information can be found in Lo et al.\textsuperscript{46,47}
C. Shock detectors

Our numerical experiments\textsuperscript{47} (such as shock/density interaction) show that applying the characteristic filter in the entire domain may still dissipate too many turbulent fluctuations. This effect is more significant for the low order filter such as the TVD filter. In addition, the high-order compact spatial filter cannot be applied in the shock or high gradient regions, otherwise, spurious oscillations can result in a numerical instability. In order to address these problems, a shock detector can be used as a sensor to switch between the compact spatial filters and the characteristic filters. Defining the shock detector function, $\Omega_i$, and the threshold parameter, $\sigma$, then the ACM filter is applied locally in the region where a threshold criterion is exceeded, given by $\Omega_i > \sigma$. A single value of $\sigma = 0.01$ is used in this work, and numerical experiments\textsuperscript{28} show this value works well. In the shock regions the spatial filter is turned off by setting the filter parameter, $\alpha_f$, to 0.5. Outside the shock regions, the spatial filter gradually increases from second order to sixth order.

In this study, the shock detector proposed by Ducros et al.\textsuperscript{48} is used. This sensor is expressed as

$$\Omega_i = \frac{p_{i+1} - 2p_i + p_{i-1}}{p_{i+1} + 2p_i + p_{i-1}} \frac{(\text{div}(\vec{u}))^2}{(\text{div}(\vec{u}))^2 + (\text{rot}(\vec{u}))^2},$$

where the first part is the Jameson sensor,\textsuperscript{49} and $p$ and $\vec{u}$ are the pressure and velocity vector, respectively. A more sophisticated shock detection scheme, not considered in this study, can be found in reference.\textsuperscript{50}

D. Surface integral acoustic method

The Ffowcs Williams-Hawkings (FWH) surface integral acoustic method\textsuperscript{51, 52} is used to study far-field noise of supersonic fully expanded and underexpanded jets. The basic idea is briefly described as follows. The near field variables computed by the LES are stored at a certain control surface every few time steps. The far field pressure fluctuation can then be computed by performing a surface integral of those stored variables. For details regarding the formulations of the FWH method, the reader is referred to Lyrintzis.\textsuperscript{52}

III. Test Cases and Grid Information

Two test cases\textsuperscript{28} are used herein. The first one is a perfectly expanded supersonic jet taken from Tanna et al.\textsuperscript{53} According to their experimental test matrix this jet is designated SP62. The second one is an underexpanded jet used by Bodony et al.\textsuperscript{54} and designated as UE22 here. This jet has an exit Mach number 1.95 and the corresponding fully expanded Mach number is 2.2. A similar experiment was conducted by Seiner and Norum.\textsuperscript{5} Both jets are unheated, and the information, such as the jet exit Mach number ($M_j$), exit to ambient pressure ratio ($P_j/P_\infty$), exit to ambient temperature ratio ($T_j/T_\infty$), and the Reynolds number based on jet exit diameter ($Re_D$) are summarized in the following table.

<table>
<thead>
<tr>
<th>Test Case</th>
<th>$M_j$</th>
<th>$P_j/P_\infty$</th>
<th>$T_j/T_\infty$</th>
<th>$Re_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully expanded</td>
<td>1.95</td>
<td>1.00</td>
<td>0.568</td>
<td>336,000</td>
</tr>
<tr>
<td>Underexpanded</td>
<td>1.95</td>
<td>1.47</td>
<td>0.568</td>
<td>394,000</td>
</tr>
</tbody>
</table>

The computational domain is a simple 3-D rectangular box. It consists of both the physical part and the sponge zone part. The physical domain extends to approximately $65r_0$ in the streamwise direction, and $-20r_0$ to $20r_0$ in the transverse $y$ and $z$ directions. Beyond the streamwise location of $65r_0$ is the sponge zone, and the entire domain is terminated at $80r_0$ in the streamwise direction. The number of grid points in the $x$, $y$, and $z$ directions are 292, 128, and 128, respectively. The grid is concentrated near the shear layer regions and has a minimum grid spacing of $\Delta y = \Delta z \sim 0.035r_0$. In the streamwise direction, a constant grid spacing, $\Delta x = 0.1r_0$, is used near the inflow plane. For $x > 2r_0$, a constant grid stretching parameter ($\Delta x_{i+1}/\Delta x_i$), 1.0068, is used to gradually increase the grid spacing. This grid is designated g2 grid in this study. A streamwise plane of the g2 grid is shown in figure 1. Note that in the figure, every 3$^{rd}$ grid point is shown.

A constant time step, $\Delta t = 0.01$ is used in the simulation. Based on the minimum grid spacing and maximum eigenvalue, the corresponding CFL number is about 0.2. As mentioned in discussion equation
(3), the value, $\theta_0$, is used to control the inflow shear layer thickness. In this study, a value of $\theta_0 = 0.06r_0$ is used, and the shear layer is resolved by about 13 grid points for the current grid. Figure 2 shows the inflow velocity profiles with different shear layer thicknesses with $\theta_0 = 0.09r_0$ and $\theta_0 = 0.06r_0$. The effects of different inflow shear layer thickness are studied by Lo et al.$^{28}$ It was found that the $\theta_0 = 0.06r_0$ results are closer to experiments. For comparison, a value of 0.02$r_0$ was used by Frenud$^{55}$ in this subsonic, low Reynolds number jet DNS. The initial condition is set to a uniform jet started from the inflow plane to the outflow plane. Numerical experiments show that it takes about 20,000 time steps to let the initial transients exit the domain. We start to collect the flow statistics after 30,000 iterations. All the statistical results are collected over 150,000 iterations to ensure that converged statistics are obtained in the far downstream regions. All the simulations are carried out on BigBen which was decommissioned on March 31, 2010 at the Pittsburgh Supercomputering Center and BigRed at Indiana University. Due to the limitation of the transposition scheme, only 128 processors were used with the $g2$ grid, and the average computational time is about 8 to 10 days.

IV. Results

A. Supersonic fully expanded jet

1. Near field statistics

It is well known that the length of a jet potential core is dependent on its exit Mach number. In order to understand the near field data and compare with other experimental results that have different operating conditions, a rescaling of the $x$ axis is necessary. Here, we adopt the Witze$^{56}$ correlation and this rescaling approach has also been used by Bodony and Lele.$^{22}$ In the Witze correlation, the $x$ axis is given by

$$W = \kappa(x - x_c)/r_0,$$

where $\kappa = 0.08(1 - 0.16M_j)(\rho_\infty/\rho_j)^{0.22}$. The potential core length, $x_c/r_0$, is computed first, and then $x/r_0$ is shifted axially. Finally, the data is rescaled using the factor $\kappa$. Here, the length of potential core is defined by the location where the jet centerline velocity reduces to 95% of the inflow jet velocity. Through this rescaling, the differences in the compressibility or Mach number which affect the length of the potential core can be accommodated.

Figure 3 shows the variations of the time averaged streamwise velocity scaled by the Witze correlation. The LES results by Bodony et al.$^{54}$ are also added for comparison. The experimental results are taken from Lau et al.$^{57}$ Panda and Seasholtz,$^{58}$ and Eggers.$^{59}$ All these jets are fully expanded and unheated.

Figure 1. The cross section of the $g2$ grid on the $z = 0$ plane. (Every 3rd grid point is shown).

Figure 2. Variations of inflow velocity profile for different shear layer thicknesses $\theta_0$. 

Figure 3. Variations of the time averaged streamwise velocity scaled by the Witze correlation.
The temperature ratio ($T_j/T_\infty$) for Lau et al., Panda $M_j = 1.4$, Panda $M_j = 1.8$, and Eggers are 1.0, 0.73, 0.61, and 0.5, respectively. The potential core lengths for Bodony et al., Lau et al., Panda $M_j = 1.4$, Panda $M_j = 1.8$, and Eggers are $19.03r_0$, $20.57r_0$, $19.03r_0$, $24.83r_0$, and $28.81r_0$, respectively. We present results computed using the TVD and WENO ACM filters. The results computed with the spatial filter only (shown in orange color) and the spatial filter with the DSM model (shown in pink color) are also included for comparison. The potential core lengths for the spatial filter, DSM model, TVD, and WENO filters are $19.75r_0$, $20.50r_0$, $21.89r_0$, and $19.75r_0$, respectively. Unless explicitly specified, all the cases use the same forcing mode, e.g. $rf4$. Here the $rf4$ denotes removing the first four modes and using the remaining twelve modes. For more detailed information, reader can refer to Lo et al.\textsuperscript{47} Generally speaking, all the LES results decay faster than the Mach 1.8 and 2.22 experimental jets. The WENO ACM filter shows a better agreement with the experiments than the TVD ACM filter in the downstream regions (e.g. $W > 0.5$). In the upstream region, both ACM cases are not distinguishable. The TVD ACM result is very close to the spatial filter result. However, both ACM cases show a slower centerline streamwise velocity decay than the DSM case and match the experiments better.

![Figure 3](image1.png)  
**Figure 3.** Variations of scaled time averaged streamwise velocity along the centerline for different ACM filters.

![Figure 4](image2.png)  
**Figure 4.** Variations of time averaged $U_{rms}$ along the centerline for different ACM filters.

Figures 4 and 5 show the time averaged streamwise and vertical turbulence intensities along the centerline using different ACM filters. The WENO filter predicts lower peak streamwise turbulence intensities than the TVD and spatial filters. This trend is consistent with the streamwise velocity variations shown in figure 3. For example, the lower the peak $U_{rms}$ the slower the centerline streamwise velocity decay. The TVD filter predicts higher peak $U_{rms}$ values than other cases, and has about the same peak value with the experiment. For the vertical turbulence intensities, all the cases match the experiments except the TVD case, which predicts a slightly lower peak $V_{rms}$. However, we still expect the peak $U_{rms}$ and $V_{rms}$ to be overpredicted, because of the higher centerline velocity decay rates of our LES results.

Figure 6 shows the instantaneous contours of divergence of velocity (grey) and vorticity magnitude (color). As shown in the figure, Mach waves propagate into the farfield at about $30^\circ \sim 35^\circ$ relative to the downstream axis. These Mach waves are generated by turbulent eddies, and each eddy convects downstream supersonically and acts like a supersonic source, creating Mach waves. These Mach waves contribute high noise levels in the farfield and result in a peak OASPL at observation angles of $30^\circ \sim 35^\circ$. It should be noted that all observation angles are measured with respect to the downstream axis.

2. Farfield acoustics

The Ffowcs Williams-Hawkingss surface integral acoustic method is used to predict the farfield jet noise. Three different control surfaces used to store the near field LES data are investigated, and they are designated (from large to small) $f1$, $f2$, and $f3$. Figure 7 shows these three control surfaces. For clarity, each face is shown
with a different color. These three control surfaces pass through grid points. Each surface starts about one jet radii downstream and extends approximately 7.15r₀, 5r₀, and 3r₀ above and below the jet at the inflow boundary in the transverse directions for f₁, f₂, and f₃, respectively. They extend streamwise until near the end of the physical domain and the total streamwise length is about 59r₀. The sizes of each end surface located at 60r₀ are approximately 30r₀×30r₀, 28r₀×28r₀, and 24r₀×24r₀ for f₁, f₂, and f₃, respectively. By looking the contour plots and the control surfaces, the smallest control surface appears to be capable of enclosing almost all the nonlinear regions.

The flow data at each FWH surface are collected every 5 time steps over a period of 50,000 time step during the LES run. There are more than 80 points per period for a Strouhal number of 0.5 based on jet exit quantities. The average computational time of a FWH run is about 12 hours with 172 processors on Bigben and BigRed. Since the LES grid has different resolutions at different locations (e.g. more grid points near the shear layer and less grid points in the farfield and downstream regions), the maximum resolvable frequencies for each surfaces are different. Assuming six points per wavelength, the maximum resolvable Strouhal numbers, St, are about 0.62, 0.83, and 1.02 for f₁, f₂, and f₃, respectively.

Figures 8 to 10 compare the OASPL computed by different ACM filters using the f₁, f₂, and f₃ control surfaces, respectively. The observer locations are at 144r₀ counted from the nozzle exit, and no end surface is used at x = 60r₀. The effect of including the end surface was studied in Lo et al.²⁸ and found to have a small effect at low observation angles. The experiments done by Tanna⁵³ and Seiner and Norum,⁶ and the LES results computed by Bodony and Lele⁵⁴ are included for comparison. The CRAFT LES result is computed by our FWH solver with near field LES data provided by the CRAFT commercial CFD software.⁶⁰ This jet also corresponds to Tanna’s SP62 case.

From figure 8, the WENO filter predicts about 2dB lower peak OASPL than the spatial filter at Θ = 40°. At other angles, both cases predict about the same pressure levels. The TVD filter shows a similar peak OASPL as the WENO and spatial filters at Θ = 35°. However, at higher angles, the TVD filter predicts lower OASPL than the other cases. For example, it underpredicts the OASPL by about 4 dB at Θ = 70° and 5 dB at Θ = 90° compared with Tanna’s experimental data. There are some discrepancies (about 5°) for the peak OASPL angles between the experimental data and the current LES results. In addition, the current LES predicts lower OASPL at shallow angles than the experiments. The reasons for these discrepancies are currently unknown and need further investigation. For the control surface f₂, with results shown in figure 9, the WENO filter predicts about the same OASPL as the spatial filter at all observation angles. All the current LES results have the same peak OASPL at Θ = 35°. The CRAFT LES shows a peak value at about Θ = 33°. As in figure 8, the TVD filter predicts lower OASPL at higher observation angles (e.g. Θ > 45°) than the other cases. For the control surface f₃, figure 10 shows that the TVD filter predicts higher peak
Figure 7. Three different control surfaces \((f_1, f_2, f_3)\) used for FWH surface integral acoustic method.

Figure 8. Variations of OASPL for different ACM filters using \(f_1\) surface at \(R = 144r_0\).

Figure 9. Variations of OASPL for different ACM filters using \(f_2\) surface at \(R = 144r_0\).

Figure 10. Variations of OASPL for different ACM filters using \(f_3\) surface at \(R = 144r_0\).
Figure 11. Variations of 1/3 octave SPL for different ACM filters and surfaces at $\Theta = 30^\circ$, $R = 144r_0$.

Figure 12. Variations of 1/3 octave SPL for different ACM filters and surfaces at $\Theta = 45^\circ$, $R = 144r_0$.

OASPL than the WENO and spatial filters by about 2 dB. However, this peak value quickly drops to about 113 dB at $\Theta = 90^\circ$, which is about 4 dB lower than Tanna’s experiments. Comparing with figures 8 and 9, all the curves shift up by about 1 to 2 dB at all observation angles. Besides this, the trends computed by the different control surfaces are very similar.

Figures 11 to 15 show the 1/3 octave pressure spectra for jet SP62 at several different observation angles. The results computed by different ACM filters and control surfaces are presented. The experimental data from Tanna and the CRAFT LES results are also included for reference. As the plots show each control surface has its own maximum grid resolvable frequency and face $f_3$ resolves higher Strouhal numbers than the other two faces. Each pressure spectra curve exhibits a sharp drop-off as it passes the related cut off frequency. At $\Theta = 30^\circ$, our LES results are about 2 to 6 dB lower than the experiments in the low frequency region ($St \leq 0.5$). This phenomenon is consistent with the OASPL shown in figures 8 to 10. The CRAFT result is similar to ours, but it resolves higher frequencies at this observation angle. At $\Theta = 45^\circ$, the CRAFT LES results are about 2 to 5 dB lower than the experiment in low frequency regions. All the current LES results match the experiments very well at this observation angle except the results computed by the TVD filter, which predicts about 3 to 4 dB higher noise levels than the experiments. At $\Theta = 60^\circ$, the current LES predicts similar results with the experiments, but the CRAFT LES results are about 2 to 6 dB lower than the experiments for $St < 0.5$. At $\Theta = 75^\circ$ and $\Theta = 90^\circ$, our LES results match the experiments well for $St \leq 0.3$, but overpredict about 2 to 6 dB compared with Tanna’s experiments for $St > 0.3$. For the same control surface, the WENO filter predicts about the same pressure spectra as the spatial filter. However, the spectra computed by the TVD filter show a quick drop off near the high frequency regions. This situation may be caused by the highly dissipative feature of the TVD filter. It could damp out all the small turbulent fluctuations in the near field.

B. Supersonic underexpanded jet

1. Near field statistics

Due to the pressure imbalance between the jet exit and ambient conditions, a series of shock and expansion waves is generated inside the potential core. To resolve these shocks, both the global and local filtering approaches are used in this section. The local filtering approach provides numerical dissipation only in the shock regions. For the shock free regions, the characteristic filter is turned off and the spatial filter is used instead. It should be mentioned that we do not apply the DSM model in those shock free regions for the local ACM cases. In order to detect the shock locations, Ducros’ shock detector is adopted. In addition, a constant threshold parameter, $\sigma = 0.01$, is used for the local ACM cases, as discussed in section II C.
Figure 13. Variations of 1/3 octave SPL for different ACM filters and surfaces at $\Theta = 60^\circ$, $R = 144r_0$.

Figure 14. Variations of 1/3 octave SPL for different ACM filters and surfaces at $\Theta = 75^\circ$, $R = 144r_0$.

Figure 15. Variations of 1/3 octave SPL for different ACM filters and surfaces at $\Theta = 90^\circ$, $R = 144r_0$.

Figure 16. Variations of time averaged centerline Mach number near the inflow plane for different ACM filters.
with the fully expanded jet cases, for numerical efficiency, the characteristic filter is only applied at the end of each full Runge-Kutta time step.

Figures 16 and 17 show the time averaged centerline Mach number variations near the inflow plane and through the entire computational domain computed by the different ACM filters. Near the inflow plane, both local ACM cases match exactly with the DSM result. The global TVD filter is the most dissipative, and the predicted peak Mach numbers are lower than the other cases. The result computed by the global WENO filter is similar to the local ACM and DSM results, except that the peak values are slightly underpredicted. In figure 17, both the local ACM cases are basically identical and very similar to the DSM result in the downstream regions. For $x > 20r_0$, the global ACM cases show a slower centerline Mach number decay than the local ACM cases. This slower decay phenomenon makes their results closer to the experiments than the local ACM cases in the downstream regions. However, this does not mean that the global filtering approach is more accurate than the local filtering approach, because the former is more dissipative in the shock cell regions than the latter. The global TVD filter shows the slowest centerline Mach number decay in the downstream regions, but its oscillation amplitudes are smaller than for the other cases.

Figure 18 shows the time averaged centerline pressure distributions computed by the different ACM filters. As with the centerline Mach number distributions, both local ACM cases match exactly with the DSM result. Comparing with the experiments shown in this figure, the pressure oscillations predicted by LES start to diminish at $x \sim 20r_0$, and then completely disappear at $x \sim 40r_0$. Both local ACM cases and the global WENO case predict nine visible oscillations, but only eight oscillations are visible for the global TVD case. However, the experimental jet has about eleven visible oscillations.

Figures 19 and 20 show the time averaged streamwise and vertical turbulence intensities along the centerline using different ACM filters. The local TVD and local WENO cases predict similar streamwise turbulence intensity profiles with the DSM result. This trend is consistent with the centerline Mach number distributions shown in figure 17. All these three cases have the peak $U_{rms}$ values at $x \sim 32r_0$. The WENO filter predicts a similar peak $U_{rms}$ value at $x \sim 38r_0$. The TVD filter has a slightly higher peak $U_{rms}$ value at $x \sim 43r_0$ and less visible oscillations for $x \leq 43r_0$ than the other cases. Generally speaking, using a more dissipative ACM filter shifts the peak $U_{rms}$ value downstream. For the vertical turbulence intensities, both local ACM cases have similar profiles and their peak values are slightly lower than for the DSM case. The TVD filter predicts a lower $V_{rms}$ than other cases. Similar trends are observed for the fully expanded jet shown in figure 5.

A plot of the instantaneous shock locations computed from Ducros’ shock detector is shown in figure 21. To visualize the jet, the 3-D translucent Mach one iso-surface and some red spots are shown in this figure. These red spots are the shock locations detected by a shock detector, and the ACM filter is applied in these
Figure 19. Variations of time averaged $U_{rms}$ along the centerline for different ACM filters.

Figure 20. Variations of time averaged $V_{rms}$ along the centerline for different ACM filters.

Figure 21. Instantaneous Mach one iso-surface cut through by a 2-D Mach number contour of the UE22 jet computed by the local ACM approach. Note that red spots are the instantaneous shock locations detected by the shock detector.
regions. For shock free regions, we apply the spatial filter and turn off the ACM filter. It should be noted that these red spots are not fixed in time. Instead, their locations change with time. Numerical experiments show that the red spots mostly appear in the region of $15r_0 \leq x \leq 30r_0$, and the oscillations in these areas can even cause the instantaneous local Mach number exceed 4.

Figure 22. Variations of OASPL for different ACM filters using $f_3$ surface at $R = 146.4r_0$.

Figure 23. Variations of 1/3 octave SPL for different ACM filters and forcing modes at $\Theta = 30^\circ$, $R = 146.4r_0$.

Figure 24. Variations of 1/3 octave SPL for different ACM filters and forcing modes at $\Theta = 45^\circ$, $R = 146.4r_0$.

Figure 25. Variations of 1/3 octave SPL for different ACM filters and forcing modes at $\Theta = 60^\circ$, $R = 146.4r_0$.

2. Farfield acoustics

Figure 22 shows the overall sound pressure levels computed by different ACM filters using $f_3$ control surface and the $rf_4$ forcing mode. No end surface is used at $x = 60r_0$, and the observation angles are located at $146.4r_0$ counted from the nozzle exit. The corresponding experiments are performed by Norum and Seiner and represented by the diamond symbols. This experimental jet has a jet exit Mach number of 2.0 and an exit to ambient pressure ratio of 1.45. In addition, the two fully expanded experimental jets
Figure 26. Variations of 1/3 octave SPL for different ACM filters and forcing modes at $\Theta = 75^\circ$, $R = 146.4r_0$. Figure 27. Variations of 1/3 octave SPL for different ACM filters and forcing modes at $\Theta = 90^\circ$, $R = 146.4r_0$. 

The circle symbols represent a perfectly expanded Mach 2.0 jet done by Norum and Seiner$^{61}$ and the square symbols represent a Mach 1.95 jet (SP62) done by Tanna.$^{53}$ A logarithmic scaling approach$^{62,63}$ is used to scale Tanna’s results from 144$r_0$ to 146.4$r_0$. From the experimental results, due to the presence of the shock cells, this underexpanded jet has higher OASPL than the related fully expanded jets especially for $\Theta \geq 40^\circ$. For example, this underexpanded jet is about 6 to 8 dB and 10 to 12 dB higher in OASPL than the fully expanded jets at $\Theta = 60^\circ$ and $\Theta = 90^\circ$. The local WENO filter matches the experiments very well except at $\Theta \leq 35^\circ$, where it underpredicts about 2 dB from the experiments. The WENO filter matches the experiments well for $\Theta \leq 45^\circ$, but underpredicts about 2 to 3 dB from the experiments for $\Theta > 45^\circ$. Similar to the fully expanded jet cases shown in figures 8 to 10, the TVD filter predicts a much lower OASPL than the other LES results and experiments for $\Theta \geq 40^\circ$. This underprediction of the OASPL may be due to the high dissipation of the TVD filter.

Figures 23 to 27 show the 1/3 octave pressure spectra for jet UE22 at several different observation angles computed by different ACM filters. Only the results computed by face f3 are shown, because from previous numerical experiments this face can resolve a higher Strouhal number than the other two faces. The experimental results taken from Norum and Seiner$^{61}$ are included for comparison. These experimental data correspond to the case which has a jet exit Mach number of 2.0 and an exit to ambient pressure ratio of 1.45. At $\Theta = 30^\circ$, the results computed by the local WENO filter and DSM model compare well with the experiments for $St \geq 0.2$. In the low frequency region ($St < 0.2$), all the LES results are about 2 to 6 dB lower than the experiments. The WENO filter predicts a level about 2 to 3 dB higher than the experiments at $St \sim 0.5$. Similar to the fully expanded jet cases, the spectra computed by the TVD filter show a quick drop-off in the high frequency regions. At $\Theta = 45^\circ$, all the current LES results match the experiments very well. Similar to the results at $\Theta = 30^\circ$, the spectra predicted by the TVD filter drop faster than the WENO and local WENO filter for $St \geq 0.5$. This quick drop of the pressure levels corresponds to the underprediction of the OASPL by about 2 to 4 dB compared to the experiments at this observation angle as shown in figure 22. The local WENO filter matches the experiments better than other cases in the high frequency region ($St > 0.5$). Again, the DSM model and the local WENO filter predict very similar spectra for the same forcing mode, e.g. rf4. At $\Theta = 60^\circ$, most of the LES results still match the experiments well. The TVD filter predicts levels about 2 to 6 dB lower than the experiments for $St \leq 0.5$, and the OASPL results computed by the TVD filter at this angle are about 8 to 10 dB lower than the experiments. The WENO and local WENO filters show about 1 to 2 dB lower levels than the experiments for $St \leq 0.1$. At $\Theta = 75^\circ$ and $\Theta = 90^\circ$, the TVD filter underpredicts the pressure level by about 2 to 8 dB from the experiments for $St \leq 0.5$. After $St > 0.5$, their spectra quickly drop off. This underprediction of the SPL by the TVD filter corresponds to a drop-off in the OASPL by about 8 to 10 dB from the experiments at
these two angles. The WENO and local WENO filter predict about 0 to 2dB lower noise spectra than the experiments for $St \leq 0.4$. For $St > 0.4$, the results computed by the local WENO filter and DSM model match the experiments better than the other cases.

3. Fine grid results

In order to have a better resolution of the shock cells and preserve the wave strength in the entire potential core, a finer grid is generated and designated as $g7$. The number of grid points in the $x$, $y$, and $z$ directions are 700, 128, and 128 respectively. Grid $g7$ has the same constant stretching parameter in the transverse direction near the inflow plane as the grid $g2$. The corresponding minimum grid spacing in the shear layer is $\Delta y = \Delta z \sim 0.035r_0$. This constant stretching parameter starts to decrease gradually at around $x = 40r_0$, and finally reaches a constant value at $x = 80r_0$.

In the streamwise direction, as with the $g2$ grid, a constant grid spacing of $\Delta x = 0.1r_0$ is used from $x = 0$ to $x = 2r_0$. Then the streamwise grid stretching starts from $x = 2r_0$, and a constant stretching parameter $(\Delta x_{i+1}/\Delta x_i) = 1.00085$, is used until $x = 60r_0$. After $x = 60r_0$, a constant stretching parameter, 1.002, is used until the end of the domain is reached. The variations of the $x$ direction grid spacing for both the $g2$ and $g7$ grids are shown in figure 28. As shown in the figure, the $g7$ grid has $\Delta x \sim 0.14r_0$ at $x = 60r_0$. However, at the same location, the $g2$ grid has $\Delta x \sim 0.49r_0$. Figure 29 shows a cross sectional plane of the $g7$ grid. Note that every $3^{rd}$ grid point is shown in the figure. This domain has the same transverse dimension as the $g2$ grid, but the streamwise dimension is extended to $98r_0$. In addition, the potential core regions have more grid points, and the grid clustering regions extend longer than the $g2$ grid.

![Figure 28. Variations of $x$ direction grid space for different grids.](image)

![Figure 29. The cross section of the $g7$ grid on the $z = 0$ plane. (Every $3^{rd}$ grid point is shown).](image)

Figures 30 and 31 show the time averaged centerline Mach number variations near the inflow plane and through the entire computational domain computed by different grids. It should be noted that all the cases presented in this section use $rf0$ forcing mode, e.g. all forcing mode are included. As shown in figure 30, near the inflow plane, the results computed by the $g2$ and $g7$ grids are identical for cases using the same ACM filter. In figure 31, the TVD filter with the $g2$ grid shows a slower centerline Mach number decay and seems to match the experiments better than the WENO filter for $x > 20r_0$. However, the result computed by TVD filter with the $g7$ grid shows a similar centerline Mach number decay with the WENO ACM cases for $x > 20r_0$. On the other hand, the grid effects on the WENO filter seem to be insignificant.

Figures 32 and 33 show the time averaged centerline pressure variations near the inflow plane and through the entire computational domain computed by different grids. Similar to the Mach number distributions near the inflow plane, the results computed by the $g2$ and $g7$ grids are identical for the same ACM filter. In figure 33, both TVD cases show smaller oscillation amplitudes than the WENO cases, and they predict about eight visible pressure oscillations. For the WENO cases, about nine pressure oscillations are visible.
Figure 30. Variations of time averaged centerline Mach number near the inflow plane for different grids.

Figure 31. Variations of time averaged centerline Mach number for different grids.

Figure 32. Variations of time averaged centerline pressure near the inflow plane for different grids.

Figure 33. Variations of time averaged centerline pressure for different grids.
Figures 34 and 35. Variations of time averaged turbulence kinetic energy along the centerline and lip line for different grids.

Generally speaking, the results using the $g^7$ grid seem to not have significant improvements, at least in the number of pressure oscillations. Figure 34 shows the time averaged turbulence kinetic energy (TKE) along the centerline for different grid resolutions. For the $g^2$ grid, the WENO filter predicts a much higher TKE than the TVD filter for $x \leq 42r_0$. This higher levels of TKE in the early shock regions could cause a faster centerline Mach number decay rate in the downstream shock free region as shown in figure 31. After $x > 42r_0$ the TVD filter shows higher TKE levels than the WENO filter. For the $g^7$ grid, the WENO filter still predicts a higher TKE than the TVD filter for $x \leq 35r_0$, but the difference is smaller than that predicted by the $g^2$ grid. After $x > 35r_0$ the TVD filter has a higher TKE than the WENO filter. For the same filter, the $g^7$ grid predicts higher TKE than the $g^2$ grid. Figure 35 shows the TKE along the lip line ($r = r_0$) for different grid resolutions. The phenomena are similar to the centerline TKE distributions.

V. Conclusions

Two different supersonic jets were investigated by 3-D LES methodology. The first one is a Mach 1.95 fully expanded jet. The second one is an underexpanded jet with an exit Mach number of 1.95 and an exit to ambient pressure ratio of 1.47. In order to capture shocks that appear in the supersonic flows, the TVD and WENO characteristic filters were incorporated into our sixth-order compact solver. The nearfield turbulent statistics computed with different ACM filters are presented. For the farfield noise, three different FWH control surfaces were used, and the noise computed by different ACM filters was compared with the experiments. The local filtering approach was used in underexpanded jets and the results were compared with the global filtering approach.

For the fully expanded jets, the WENO filter predicts slower centerline velocity decay and matches the experiments better than the other LES cases. It also predicts similar streamwise and vertical turbulence intensity distributions with the spatial filter case. The TVD filter predicts slightly higher peak $U_{rms}$ and lower $V_{rms}$ than the other cases. The ACM filters underpredict the OASPL by about 2 to 6 dB for $\Theta \leq 35^\circ$. The TVD filter shows lower OASPL than the other cases at high observation angles. However, the WENO filter predicts about the same OASPL as the spatial filter and matches the experiments well for $\Theta \geq 35^\circ$. In addition, using a smaller control surface gives higher OASPL at all observation angles and resolves higher Strouhal numbers in the pressure spectra.

For the underexpanded jets, the local TVD and WENO filters predict almost identical centerline Mach number and pressure distributions with the DSM approach. These results match the experiments well near the inflow plane, but start to deviate at $x \sim 20r_0$. All the LES cases predict about 8 to 9 visible pressure
oscillations inside the potential core. However, the experimental jet has about eleven visible oscillations. The global TVD filter is the most dissipative. It underpredicts the peak Mach number and pressure values in the potential core. However, the downstream Mach number decay is closer to the experiments than other cases on the g2 grid. For the farfield noise, the local WENO filter and the DSM model predict similar OASPL at all observation angles and their results match the experiments well. The global TVD filter underpredicts the OASPL by 2 to 10 dB for all angles, and has a quick drop-off in OASPL at high observation angles.

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References


