Numerical Simulation of Supersonic Jet Flows and their Noise

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Large-eddy simulations (LES) of a circular supersonic fully expanded jet and an underexpanded jet are performed. Both jets operate at a design Mach number 1.95, and the corresponding fully expanded Mach number for the underexpanded jet is 2.20. Since the simulations do not include the nozzle geometry explicitly, the effects of the inflow conditions (such as the forcing modes and shear layer thickness) on the near field statistics and far field noise are investigated. The near field data are collected from the LES and then the far field noise is computed by the Ffowcs Williams-Hawkins (FWH) surface integral method. The results show that removing the lower forcing modes increases the centerline velocity decay rate, the peak turbulence intensities, and the overall sound pressure level. However, reducing the inflow shear layer thickness has the opposite effect and achieves a better agreement with the experiments in both near field statistics and farfield noise. Similar trends hold for the underexpanded jet in near field statistics; however, the number of shock cells is underpredicted and is insensitive to the inflow conditions. This may be due to contamination by spurious numerical fluctuations caused by the shocks. Further simulations including shock capturing schemes are needed.

I. Introduction

The noise radiated by jet engines has received more attention recently. Due to the strict regulations on aircraft with high jet noise emission near airports, most jet engine designers are trying to reduce the noise levels in newly designed jet engines. In order to achieve this objective, expanding the bounds of knowledge in jet aeroacoustics has become necessary. With the progress of commercial aircraft design, some of the recently developed commercial aircraft (such as the Boeing 777 and 787) are equipped with high by-pass ratio engines which can operate at supercritical pressure ratios. In addition to turbulent mixing noise, which is generated by instabilities of the shear layer near the jet lip, shock associated noise can be generated when a supersonic jet engine operates at an off-design condition. This (broadband) shock associated noise is generated by the interactions between the turbulence and the shock cells which occur downstream of the nozzle exit. Therefore, accurate prediction of supersonic jet noise has become an important step in the design of new low noise emission engines.

Traditional jet noise studies rely heavily on experiments. With the advances of fast supercomputers, the application of direct numerical simulation (DNS) to jet noise prediction is becoming possible. DNS solves for the dynamics of all the relevant length scales of turbulence and does not use any turbulence models. However, due to the wide range of time and length scales in high Reynolds number turbulent flows, the application of DNS to industrially relevant jets is still infeasible. Large eddy simulation (LES), which computes the large scales directly and models the effect of the small (subgrid) scales, yields a cheaper alternative. As a result, LES can be used as a tool for jet noise prediction since it is capable of simulating high Reynolds number flows at a fraction of the cost of DNS. The far field noise then can be estimated using near field variables directly computed by LES by coupling with acoustic methods (such as the Ffowcs Williams-Hawkins).
Williams-Hawkins method\textsuperscript{1} or Lighthill’s acoustic analogy\textsuperscript{2}). The basic idea is briefly described as follows. The near field variables computed by the LES are stored on a certain control surface or within a volume every few time steps. After collecting enough data, the far field pressure fluctuation can be computed by performing either a surface or volume integral of those stored variables.

The LES approach has been used in supersonic jet simulations by several researchers. The first uses of LES as an investigation tool for jet noise prediction was carried out by Mankbadi, et al.\textsuperscript{3} They performed an axisymmetric LES computation of a fully expanded Mach 1.5 and Reynolds number 1.27 × 10\textsuperscript{6} jet and applied Lighthill’s theory to calculate the far field noise; however, no quantitative comparisons with the experimental results are made. Freund et al.\textsuperscript{4} used DNS with 22 million grid points to simulate a low Reynolds number 2000 supersonic fully expanded jet. Both the near field statistics and the sound pressure levels are presented. Although no comparison of the near field statistics with any experimental results is shown, the sound pressure levels match well with the experimental data at similar flow conditions. DeBonis and Scott\textsuperscript{5} used LES with the Smagorinsky subgrid scale (SGS) model to study a cold Mach 1.4 fully expanded jet with a Reynolds number of 1.2 million. Their simulation accurately captures the physics of the turbulent flow and the time averaged velocity fields agree with experimental data. Al-Qadi and Scott\textsuperscript{6} used a sixth-order compact scheme and a TVD type characteristic filter developed by Yee et al.\textsuperscript{7} to study a 3-D underexpanded rectangular jet. The results demonstrate that the schemes they used are capable of resolving the unsteady complex near field features. Bodony et al.\textsuperscript{8} used LES to simulate and compare a fully expanded jet and an underexpanded jet without using any shock capturing schemes. They found Mach waves contribute significantly to the near field pressure of the underexpanded jet. Berland et al.\textsuperscript{9} used LES to investigate the screech tones generated by a 3-D plane underexpanded jet. Their computation reproduces the screech tone generation mechanism, and shows the feasibility of the direct computation of noise involving such a feedback loop with high-order algorithms. Viswanathan et al.\textsuperscript{10} study the flow and the noise characteristics on both heated and unheated jets for round and beveled nozzles. To avoid the requirement of the massive computational resources needed by the LES approach for the near wall region of the boundary layer, a two-step RANS/LES methodology\textsuperscript{11} is adopted, which uses a RANS calculation to provide a nozzle boundary layer then LES is performed in a second step for the external plume on a coarse grid. Their simulations cover a wide range of jet velocities, i.e. from Mach 0.4 to Mach 1.9, and the predicted noise levels are comparable with the experimental results, within about 2 to 3 dB. In addition, Mach wave radiation is clearly observed from the contours of pressure-time derivative, indicating the source of noise. Li and Gao\textsuperscript{12} use an unsteady Reynolds averaged approach with the dispersion-relation-preserving (DRP) scheme to investigate the near-field screech of Mach 1.17 to 1.60 underexpanded jets. To capture the shocks the variable stencil Reynolds number method is utilized. The shock cell structures and the radial density profile compare well with the experiments. In addition, the difference between the near field sound pressure levels of their simulations and the experiments are within 4 dB.

Our 3-D LES methodology has demonstrated its success in the prediction of subsonic jet noise.\textsuperscript{13–17} However, the ability of our method to predict supersonic jet flows and the related noise fields has not been validated. Therefore, the objective of the current study is to investigate the capability of our 3-D LES methodology for supersonic jet simulations. The following sections briefly describe the LES numerical method, surface integral acoustic method, case and grid information, and results of our near-field statistics and far-field noise.

II. Numerical Methods

A. 3-D LES methodology

We briefly describe the LES methodology that we used in this study. The 3-D LES code was originally developed by Uzun.\textsuperscript{13} The code is designed for the study of subsonic turbulent jet noise at high Reynolds numbers. It is an unsteady, Favre-filtered, compressible Navier-Stokes solver. The sixth-order compact method developed by Lele\textsuperscript{18} is used for spatial discretization on the internal points. For the points on the boundary, however, a third-order one-side compact scheme is used, and the points next to the boundaries are computed by a fourth-order compact scheme. The time advancement scheme is the classical fourth-order Runge-Kutta method. In order to prevent spurious reflections, Tam and Dong’s radiation and outflow boundary conditions\textsuperscript{19,20} are implemented on the boundaries. In addition, a sponge zone is attached at the outflow boundary to prevent unwanted reflections caused by strong vortices passing through the outflow boundary. It is known that the compact scheme can produce high-frequency spurious modes which arise from the boundary conditions, unresolved scales, and mesh non-uniformities, and cause numerical instabilities.
In order to eliminate these spurious modes and keep the scheme stable, a sixth-order tri-diagonal implicit spatial filter proposed by Gaitonde and Visbal\textsuperscript{21} is used in this study. We apply this spatial filter once after each full Runge-Kutta time step, and the filter coefficient, $\alpha_f$, is set to 0.47.

This LES solver can use an eddy-viscosity subgrid scale (SGS) model (such as the classical or a localized dynamic Smagorinsky (DSM) model) to dissipate the turbulent energy. Although the DSM model can alleviate the need to specify the Smagorinsky constant for a given problem, using it usually requires about 40\% more computational time than the only using spatial filter. A spatial filter can also emulate the effects of a subgrid scale model, acting as an implicit SGS model. A discussion of the two methods can be found in reference\textsuperscript{22}. For the current study both DSM and a spatial filter are used as SGS models.

Since the current simulations do not include an explicit nozzle, some special treatments on the inflow boundary must be specified. A hyperbolic tangent velocity profile used by Freund\textsuperscript{23} is specified on the inflow boundary as

$$ \bar{u}(r) = \frac{1}{2} \left\{ 1 - \tanh \left[ \frac{r_0}{4\theta_0} \left( \frac{r}{r_0} - 1 \right) \right] \right\} $$

(1)

where $r = \sqrt{y^2 + z^2}$, $r_0 = 1$, $U_j$ is the jet centerline velocity, and $\theta_0$ is the inflow momentum thickness. Here, $\theta_0$ is used to control the thickness of the inflow shear layer. A higher value of $\theta_0$ implies a thicker shear layer. In order to investigate the influence of the initial shear layer thickness, two values of $\theta_0$ (0.09$r_0$ and 0.06$r_0$) are used in this study. The former value has been used by Bodony and Lele\textsuperscript{24} and Lew et al.\textsuperscript{16} and they were able to achieve good agreement with the experiments on turbulent statistics and farfield noise. For laboratory jets, however, the measured value of $\theta_0$ is usually an order of magnitude or more smaller ($\sim 2 \times 10^{-2} r_0$)\textsuperscript{25} compared to that used in LES and DNS of jets. For the inflow density profile, we also adopt the formula from Freund,\textsuperscript{23}

$$ \bar{\rho}(r) = \left( 1 - \frac{\rho_\infty}{\rho_j} \right) \bar{u}(r) + \frac{\rho_\infty}{\rho_j} $$

(2)

For the underexpanded jet, a hyperbolic tangent profile similar to equation (1) is used to specify the inflow pressure profile.

In the actual jet flow, the nozzle lip can reflect and scatter acoustic waves into jet’s initial shear layer and excite the mean flow. Since the current simulations do not include the nozzle, in order to mimic its function and promote natural transition from an initially quasi-laminar annular shear layer, the vortex ring forcing approach proposed by Bogey and Bailly\textsuperscript{26} is used in this study. This is done by putting a vortex ring at approximately one jet radius downstream of the inflow boundary to generate randomized velocity perturbations. The streamwise ($v'_x$) and radial ($v'_r$) velocity perturbations are then added to the local velocity components. However, velocity perturbations in the azimuthal direction are not added. The expressions for $v'_x$ and $v'_r$ are

$$ v'_x = \alpha U_{x,ring} U_0 \sum_{n=0}^{n_{\text{modes}}} \varepsilon_n \cos(n\Theta + \varphi_n) $$

(3)

$$ v'_r = \alpha U_{r,ring} U_0 \sum_{n=0}^{n_{\text{modes}}} \varepsilon_n \cos(n\Theta + \varphi_n) $$

(4)

where $\Theta = \tan^{-1}(y/z)$, $\varepsilon_n$ and $\varphi_n$ are randomly generated numbers that satisfy $-1 < \varepsilon_n < 1$ and $0 < \varphi_n < 2\pi$. $U_0$ is the mean jet centerline velocity at the inflow boundary, and the total number of forcing modes, $n_{\text{modes}} + 1$, is 16. The parameter that determines the amplitude of the forcing is $\alpha$ and it is set to 0.007 in this study. The mean axial ($U_{x,ring}$) and radial ($U_{r,ring}$) velocity components of the vortex ring are given as

$$ U_{x,ring} = 2\frac{r_0}{r} \frac{r - r_0}{\Delta_0} \exp \left[ -\ln(2) \left( \frac{\Delta(x,r)}{\Delta_0} \right)^2 \right] $$

(5)

$$ U_{r,ring} = -2\frac{r_0}{r} \frac{x - x_0}{\Delta_0} \exp \left[ -\ln(2) \left( \frac{\Delta(x,r)}{\Delta_0} \right)^2 \right] $$

(6)

where $r = \sqrt{y^2 + z^2} \neq 0$, $\Delta_0$ is the minimum grid spacing in the shear layer, and $\Delta(x,r)^2 = (x - x_0)^2 + (r - r_0)^2$. The center of the vortex is located at $x_0$ which is $r_0$ in our case. Studies regarding the effect of this inflow forcing on subsonic jets can be found by Lew et al.\textsuperscript{27} and Bogey and Bailly.\textsuperscript{28}

Because both the compact scheme and the spatial filter require all the data points along a given grid line to solve the linear system, a transposition strategy\textsuperscript{13} is used for the parallelization. Initially, the grid
is partitioned along the $z$ direction, and the derivatives and the filtering are computed in both $x$ and $y$ directions. Then the data are transposed for the computation in the $z$ direction. Once the computation along the $z$ direction is finished, the data is transposed back to the initial configuration.

B. Surface integral acoustic method

The Ffowcs Williams-Hawkings (FWH) surface integral acoustic method\textsuperscript{1,29} is used to study far-field noise of supersonic fully expanded jets. Due to the lack of experimental data, no noise calculations were made for the current underexpanded jet cases. For a stationary permeable control surface, the disturbance pressure at an observer coordinate, $\vec{x}$, and time, $t$, is composed of the thickness noise, loading noise, and quadrupole noise. The quadrupole noise is neglected in this methodology.\textsuperscript{13} For details regarding the formulations of each pressure term and numerical implementation of the FWH method, the reader is referred to Uzun.\textsuperscript{13}

III. Test Cases and Grid Information

A. Test cases

We consider two test cases, one a perfectly expanded jet and the other an underexpanded jet. The fully expanded supersonic jet test case is from Tanna et al.\textsuperscript{30} This test case is designated SP62 and is named according to their experimental test matrix. The underexpanded jet case is similar to the experiment provided by Seiner and Norum.\textsuperscript{31,32} This jet has an exit Mach number ($M_j$) 1.95 and the corresponding fully expanded Mach number is 2.2. Both jets are unheated i.e. the jet stagnation temperature is the same as the ambient temperature. The information on jet Mach number ($M_j$), pressure ratio ($P_j/P_\infty$), temperature ratio ($T_j/T_\infty$), and the Reynolds number ($Re_D = \rho_j U_j D_j/\mu_j$) based on the jet centerline quantities and the diameter are summarized in table 1.

<table>
<thead>
<tr>
<th></th>
<th>$M_j$</th>
<th>$P_j/P_\infty$</th>
<th>$T_j/T_\infty$</th>
<th>$Re_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully expanded</td>
<td>1.95</td>
<td>1.00</td>
<td>0.568</td>
<td>336,000</td>
</tr>
<tr>
<td>Underexpanded</td>
<td>1.95</td>
<td>1.47</td>
<td>0.568</td>
<td>394,000</td>
</tr>
</tbody>
</table>

B. LES grid and FWH control surface

The physical domain covers $(x, y, z) = (65r_0, \pm 20r_0, \pm 20r_0)$. Beyond the streamwise location of 65$r_0$, a sponge zone with a dimension $(x, y, z) = (15r_0, \pm 20r_0, \pm 20r_0)$ is attached. Two grid resolutions ($292 \times 128 \times 128$ and $368 \times 160 \times 160$) are used in this study. Due to limited of computational resources, most of the simulations are performed on the coarse grid unless explicit mentioned. This coarse grid has been used by Lew et al.\textsuperscript{16,17} to study subsonic hot and isothermal jets with very good results. Figure 1 shows the x-y cross sectional plane of the coarse computational grid. Note that in the figure every 3rd grid point is shown.

In order to perform the FWH surface integral acoustic method, the needed flowfield variables are stored on a control surface as shown in figure 2. This surface starts about one jet radii downstream and extends approximately 7.5$r_0$ above and below the jet at the inflow boundary in the $y$ and $z$ directions. It extends streamwise until near the end of the physical domain and the total streamwise length is about 59$r_0$. The size of the end surface located at 60$r_0$ is approximately $30r_0 \times 30r_0$. The dimension of this control volume is sufficiently large to enclose all of the non-linear region. It should be noted that the main assumption of the surface integral acoustics methods is that the control surface must be placed outside the non-linear flow region. Otherwise, the quadrupole sources must be included in the FWH method and computationally expensive volume integrals are needed for the computation. This control surface is also adopted by Lew et al.\textsuperscript{16,17}

IV. Results

Our simulations are carried out on Bigben at the Pittsburgh Supercomputing Center and Big Red at Indiana University. Based on the minimum grid spacing and the maximum eigenvalue, the corresponding CFL number used in the simulation is between 0.15 to 0.3. All the statistical results are collected over
150,000 iterations to obtain converged statistics. The reason for so many steps to get converged statistics is because the Mach number drops to about 0.34 near the end of physical domain, which lowers the convergence rate. The averaged computational time required for a 4 million grid points case, including gathering near field statistics, is about 1 week by using 128 processors. The fine grid has about 9.4 million grid points and requires 4 weeks computational time on 80 processors.

A. Fully expanded jet

1. Effect of Forcing Modes

First, we investigate the effect of which forcing modes are included on the supersonic fully expanded jet. The inflow momentum thickness, $\theta_0$, is set to 0.09$r_0$, and the initial shear layer is resolved by about 13 grid points. The minimum grid spacing in the initial shear layer is 0.05$r_0$. Designating $rfx$ for each of the different forcing strategies, we compare the results of four different cases (e.g. $x = 0, 4, 6, 8$). For example, $rf0$ means using all forcing modes, $rf4$ means removing the first four modes and using the remaining twelve modes, and so on.

Figure 3 shows the variation of the mean streamwise velocity along the centerline for each of the forcing mode cases. The LES results by Bodony et al.\textsuperscript{8} are also added for comparison. All the LES results use 0.09$r_0$ inflow momentum thickness and it is marked in the plot legend. The experimental results are from Lau et al.\textsuperscript{32} Panda and Seasholtz\textsuperscript{34} and Eggers.\textsuperscript{35} All the experimental jets are fully expanded and unheated. The temperature ratio ($T_j/T_\infty$) for Lau et al., Panda $M_j = 1.4$, Panda $M_j = 1.8$, and Eggers are 1.0, 0.73, 0.61, and 0.5, respectively. Generally speaking, all the LES results decay faster than the Mach 1.8 and 2.22 experimental jets. However, it is still difficult to compare the results with each other. As the plot shows, due to the differences in the jet Mach numbers, the potential core lengths are not the same for each of the cases. In order to understand the near-filed data and compare with other experimental results that have different operating conditions, a rescaling of the $x$ axis is necessary. Here, we adopt a procedure, called the Witze\textsuperscript{36} correlation, which is also used by Bodony and Lele.\textsuperscript{24} Through this rescaling, the differences in the compressibility or Mach number which affect the length of the potential core can be accommodated. In the Witze correlation, the $x$ axis is given by $W = \kappa(x - x_c)/r_0$, where $\kappa = 0.08(1 - 0.16M_j)(\rho_\infty/p_j)^{0.22}$. The potential core length, $x_c/r_0$, is computed first, and then $x/r_0$ is shifted axially. Then the data is rescaled using the factor $\kappa$. Here, the length of potential core is defined by the location where the jet centerline velocity reduces to 95% of the inflow jet velocity.

Figure 4 shows the rescaled mean streamwise velocity along the centerline. It is clear that all the LES results decay faster than the experiments. The $rf0$ forcing case decays more slowly than the other forcing cases. As we continue to remove the modes (from $rf4$ to $rf8$) the decay rate increases. It is noted that our $rf0$ results are almost the same as Bodony's results even though the methods are different. They use the same compact method, but a cylindrical grid with $(r \times \theta \times x) = (128 \times 32 \times 256)$ and a different inflow forcing method. A qualitatively comparison of the centerline velocity decay rate in the far downstream region of the fully expanded jets is shown in figure 5. The velocity decay rate can be computed from the slope e.g. $d(U_j/U_c(x))/dx(x/2r_0)$. The $rf0$ case has a predicted decay rate value of 0.124. However, both Panda's Mach 1.8 and Eggers Mach 2.22 jets have a decay rates of 0.080.

The axial and radial turbulence intensities ($U_{rms}$ and $V_{rms}$) along the axial centerline are shown in figures 6 and 7. From the plot, an axial shift in the downstream direction and an increase of the peak intensity are observed as the lower modes are removed. Comparing with figure 3, an earlier peak intensity corresponds to a shorter potential core. The potential core length increases if we continue removing the lower forcing modes. The trends of our turbulent intensities is similar to the subsonic jet cases done by Lew et al.,\textsuperscript{27} but they have a roughly constant peak turbulence level. On the other hand, Bogey and Bailly\textsuperscript{37} observed a reduction in the peak centerline turbulence levels when the lower modes are removed. Figure 8 shows the rescaled axial centerline turbulence intensities. The trends of the LES jets are consistent with the experiment; however, we believe obtaining the same peak $U_{rms}$ value between the $rf0$ case and the experiment is just a coincidence. As reported by Lau et al.,\textsuperscript{33} the peak $U_{rms}$ and $V_{rms}$ decrease with increasing jet exit Mach number. Therefore, all of our LES results overpredict the peak intensity.

The effect of the inflow forcing modes can be summarized as follows. Removing the lower forcing modes causes an increase in both the jet centerline decay rate and the peak centerline turbulence level. In addition, it also shifts the location of the maximum $U_{rms}$ and $V_{rms}$ downstream. However, the length of the potential core decreases as we remove the lower forcing modes. Our overprediction of the jet centerline decay rate may possibly be attributed to two reasons: the inflow layer thickness and shocks which appear near the end of the potential core. The effect of shear layer thickness is described in the following section. The
presence of shocks can be identified by looking at an instantaneous centerline streamwise velocity (figure 9) or instantaneous contour plot of the divergence of velocity (figure 10). As shown in figure 9 for case rf8, large gradient regions (or shocks) appear in the circled region. Figure 10 shows the same case along a constant $z = 0$ plane. The shocks appear in the regions where the divergence of velocity is large and negative (e.g. at $x \sim 22r_0$). Since the current simulations do not use a shock capturing scheme, the spurious numerical oscillations caused by the shocks can propagate into flow field and contaminate the turbulence fluctuations. This may result in an increase of the peak turbulence level and centerline jet decay rate. It should be mentioned that the spatial filter used in this study is unable to damp out the numerical oscillations caused by the shocks.

2. Effect of shear layer thickness

Results using 0.06$r_0$ inflow momentum thickness are presented. About 13 grid points are used to resolve the initial shear layer, and the minimum grid spacing in the initial shear layer is 0.035$r_0$. Figure 11 shows the scaled mean streamwise velocity along the centerline. The cases using the dynamic Smagorinsky (DSM) model, which increases the computational time by about 40%, is also shown in the figure. As report by Bogey and Bailly, decreasing the initial shear layer thickness reduces the centerline velocity decay rate, and fluctuation levels. A similar phenomena can be observed in the figure. Both of the cases using 0.06$r_0$ inflow momentum thickness have lower centerline velocity decay rates than the cases with thicker inflow momentum thickness. However, adding the DSM model slightly increases the velocity decay rate compared to the case without it. A fine grid case is added in the same figure to compare with the coarse grid cases. Due to the finer resolution, it has a slower centerline velocity decay rate than all the coarse grid cases.

Figures 12 and 13 compare the effects of a thinner shear layer on the mean axial and radial turbulence intensities along the centerline. The cases using 0.06$r_0$ inflow momentum thickness have lower peak turbulence intensity levels compared to the 0.09$r_0$ cases. Adding the DSM model slightly increases the peak intensity values in both figures. This phenomenon corresponds to a faster centerline velocity decay rate, which is consistent with the results shown in figure 11. However, we still expect the peak $U_{\text{rms}}$ and $V_{\text{rms}}$ to be overpredicted, because of the higher centerline velocity decay rates.

3. Farfield noise

We gathered flow field data on the FWH control surfaces at every 5 time steps over a period of 50,000 time steps during our LES run. The total acoustic sampling period corresponds to a time scale in which an ambient sound travels about 5.8 times the streamwise domain length of the FWH control surface. To perform the surface integral, this control surface is divided into 172 blocks for parallel computing and the averaged computational time is about 12 hours using 172 processors on Bigben. Based on the grid resolution around the FWH control surface, and assuming 6 points per wavelength to accurately resolve an acoustic wave, the maximum resolvable frequency corresponds to a Strouhal number 0.62. Here, the Strouhal number is defined by $Sr = fD_j/U_j$.

Figure 14 shows the overall sound pressure levels (OASPL) for the fully expanded jet case and the experimental data, where the FWH prediction was computed without the end surface at 59$r_0$. We compute the OASPL along an arc with a distance of $R = 144r_0$ from the nozzle exit, and the observer angle, $\Theta$, is measured relative to the jet centerline axis. This farfield distance and observer angle are used by Tanna et al.

Other LES results are also put in the same figure for comparison. The OASPL result done by Bodony and Lele is computed by Kirchhoff surface integral method, and the result by Morris et al. is computed by FHW method. In addition, the case by Morris et al. has the same temperature ratio and jet Mach number as ours, but the Reynolds number (based on the jet diameter) is 100,000. It must be mentioned that these LES results are taken from Bodony and Lele, which is scaled to a distance of 200$r_0$. In order to compare with current cases, same approach is used to scale to a distance of 144$r_0$.

As we can see from the figure, Bodony’s case match very well with the experiment. Morris et al. overpredict the peak OASPL by about 10 dB. Removing the lower forcing mode increases the OASPL at all observer angles. This phenomenon is consistent with the increase of the peak turbulence intensity and the report by Lew et al. All the 0.09$r_0$ cases predict larger OASPL for $\Theta \geq 40^\circ$. The peak value is shifted about 5° higher compared to Tanna’s data. Cases rf4 to rf8 have about the same peak OASPL with the experiment but case rf0 underpredicts the peak value by about 2 dB. Reducing the inflow shear layer thickness (e.g. 0.06$r_0$ cases) gives a better agreement with the experiment for $\Theta \geq 40^\circ$. However, the peak value is underpredicted about 4 dB and 2 dB for cases with and without the DSM model, respectively.

Figure 15 shows the overall sound pressure levels (OASPL) for the fully expanded jet cases where the end
surface at 59r₀ has been included in the FWH computation. All the trends are similar to the cases without using the end surface; however, the noise levels at small observer angles are slightly higher and match the experiment a little better.

B. Underexpanded jet

Due to the shocks present in the potential core and downstream regions for underexpanded jets, numerical experiments show that the compact scheme plus the spatial filter used for the fully expanded jet are unable to maintain a stable computation. To overcome this problem, the DSM model is used to provide more numerical dissipation and keep the solver stable. Since the current simulations do not use any shock capturing schemes, this approach only works for cases with very weak pressure ratios, e.g., \( P_j/P_∞ = 1.47 \). In addition, since our simulations do not include the actual nozzle in the computations, we found that the Tam and Dong’s radiation boundary does not work for underexpanded jets at inflow plane. This is because the potential core has a higher pressure than the ambient, and a solid wall may be needed to confine the flow to a specific region. In order to solve this problem, we replace Tam and Dong’s radiation boundary conditions at the inflow plane with a characteristic boundary conditions by Whitfield and Janus, and we found this boundary condition works well for current cases.

Figures 16 and 17 compare the variation of mean pressure and Mach number along the centerline. The experimental result is taken from Seiner and Norum, which has a design Mach number of 2.0 and a pressure ratio \( (P_j/P_∞) \) of 1.45. From figure 16, our LES results have about the same pressure oscillation amplitude as the experiment when \( x \leq 20r₀ \), but this amplitude diminishes quickly for \( x > 20r₀ \) and then completely disappears at \( x \sim 35r₀ \). The reason that Bodony’s LES results predict smaller oscillation amplitudes may be due to his grid (about 1 million points) being coarser than ours. Both cases with 0.09r₀ and 0.06r₀ inflow momentum thickness predict about 9 visible shock cells. On the other hand, the experimental data has about 12 visible shock cells. The effect of the inflow momentum thickness on the number of the shock cells seems insignificant. However, its effect becomes pronounced at \( x \sim 35r₀ \). As shown in figure 17 the 0.06r₀ case has a slower Mach number decay which is closer to the experiment than the 0.09r₀ case.

The reason that all the LES results predict fewer shock cells than the experiment may be due to spurious numerical oscillations caused by shocks appearing in the potential core. Figure 18 shows an instantaneous contour plot of divergence of velocity for the case with 0.06r₀ inflow momentum thickness. As shown in the plot, shocks appear in the regions where the divergence of velocity has a large negative value. Expansion waves are shown as areas of positive divergence of velocity. The shock cells are revealed by alternating regions of expansion and compression waves. In the downstream region eddy shocklets are apparent. Since all the LES simulations do not use shock capturing scheme, the spurious numerical oscillations generated by the shocks can propagate into the flow field, and contaminate or magnify the physical oscillations. This phenomenon may be the reason that causes the potential core of our LES jet to break up earlier than in the experiment. To verify this further simulations that include a shock capturing scheme, such as the characteristic filter approach, are needed.

V. Conclusion and Future Work

Large-eddy simulations of a circular supersonic fully expanded jet and an underexpanded jet are performed. The effect of variations of the inflow conditions (such as the forcing modes and the shear layer thickness) are investigated. The near field statistics show that removing the lower forcing modes increases the potential core length, centerline velocity decay rate, peak turbulence intensities, and far-field OASPL. However, reducing the inflow shear layer thickness has the opposite effect and achieves better agreement with the experiments in both near field statistics and far-field noise. For the fully expanded jet we also observed that shocks may form at the end of potential core that generate spurious numerical oscillations that contaminate the physical fluctuations. Generally speaking, however, our subsonic 3-D LES methodology is capable of simulating supersonic fully expanded jets without need of significant modifications. The effects of inflow conditions that we investigated for the fully expanded jets are insignificant for the shock cell structure of underexpanded jets. Due to these shock cell structures within the potential core, the contamination from spurious numerical oscillations becomes more significant. Our underprediction of the number of shock cells may be caused by this problem. Further simulations including shock capturing schemes are needed to investigate this issue.

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VI. Acknowledgements

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References


Figure 1. The cross section of the computational grid on the $z = 0$ plane. (Every 3rd grid point is shown).

Figure 2. Control surface used for the Ffowcs Williams-Hawkings surface integral method.
Figure 3. Mean axial centerline velocity for fully expanded jets.

Figure 4. Mean axial centerline velocity (scaled by Witze’s correlation) for fully expanded jets.
Figure 5. Mean axial centerline velocity decay rate for fully expanded jets.

Figure 6. Mean axial centerline turbulence intensity for fully expanded jets.
Figure 7. Mean radial centerline turbulence intensity for fully expanded jets.

Figure 8. Mean axial centerline turbulence intensity (scaled by Witze’s correlation) for fully expanded jets.
Figure 9. Instantaneous axial centerline velocity for fully expanded jet.

Figure 10. Instantaneous divergence of velocity of fully expanded jet along \( z = 0 \) plane.
Figure 11. Mean axial centerline velocity (scaled by Witze’s correlation) for fully expanded jets.

Figure 12. Mean axial centerline turbulence intensity for fully expanded jets.
Figure 13. Mean radial centerline turbulence intensity for fully expanded jets.

Figure 14. Overall sound pressure variation for fully expanded jets at $R = 144r_0$ without end surface.
Figure 15. Overall sound pressure variation for fully expanded jets at $R = 144r_0$ with end surface.

Figure 16. Mean axial centerline pressure distribution for underexpanded jets.
Figure 17. Mean axial centerline Mach number distribution for underexpanded jets.

Figure 18. Instantaneous divergence of velocity of underexpanded jet along $z = 0$ plane.