3-D Large Eddy Simulation for Jet Aeroacoustics *

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We present 3-D Large Eddy Simulation (LES) results for a turbulent isothermal round jet at a Reynolds number of 100,000. Our recently developed LES code is part of a Computational Aeroacoustics (CAA) methodology that couples surface integral acoustics methods with LES for the far field noise estimation of turbulent jets. The LES code employs high-order accurate compact differencing together with implicit spatial filtering and state-of-the-art non-reflecting boundary conditions. A localized dynamic Smagorinsky subgrid-scale (SGS) model is used for representing the effects of the unresolved scales on the resolved scales. A computational grid consisting of 12 million points was used in the present simulation. Mean flow results obtained in our simulation are found to be in excellent agreement with the available experimental data of jets at similar flow conditions. Furthermore, the near field data provided by LES is coupled with the Ffowcs Williams-Hawkins method to compute the far field noise. Far field aeroacoustics results are also presented and comparisons are made with another computational study.

Introduction

The turbulent jet noise problem still remains one of the most complicated and difficult problems in aeroacoustics. There is a need for substantial amount of research in this area that will lead to improved jet noise prediction methodologies and aid in the design process of aircraft engines with low jet noise emissions. With the recent improvements in the processing speed of computers, the application of Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) to jet noise prediction methodologies is becoming more feasible. The first DNS of a turbulent jet was done for a Reynolds number 2,000, supersonic jet at Mach 1.92 by Freund et al. The computed sound pressure levels were compared with experimental data and found to be in good agreement with jets at similar convective Mach numbers. Freund also simulated a Reynolds number 3,600, Mach 0.9 turbulent jet using 25 million grid points, matching the parameters of the experimental jet studied by Stromberg et al. Excellent agreement with the experimental data was obtained for both the mean flow field and the radiated sound. Such results clearly show the attractiveness of DNS to the jet noise problem. However, due to a wide range of length and time scales present in turbulent flows, DNS is still restricted to low-Reynolds-number flows in relatively simple geometries. DNS of high-Reynolds-number jet flows of practical interest would necessitate tremendous resolution requirements that are far beyond the reach of the capability of even the fastest supercomputers available today.

Therefore, as frustrating as it might seem, turbulence still has to be modelled in some way to do simulations for problems of practical interest. LES, with lower computational cost, is an attractive alternative to DNS. In an LES, the large scales are directly solved and the effect of the small scales or the subgrid scales on the large scales are modelled. The large scales are generally much more energetic than the small ones and are directly affected by the boundary conditions. The small scales, however, are usually much weaker and they tend to have more or less a universal character. Hence, it makes sense to directly simulate the more energetic large scales and model the effect of the small scales. LES methods are capable of simulating flows at higher Reynolds numbers and many successful LES computations for different types of flows have been performed to date. Since noise generation is an unsteady process, LES will probably be the most powerful computational tool to be used in jet noise research in the foreseeable future since it is the only way, other than DNS, to obtain time-accurate unsteady data.

One of the first attempts in using LES as a tool for jet noise prediction was carried out by Mankbadi et al. They employed a high-order numerical scheme to perform LES of a supersonic jet flow to capture the time-dependent flow structure and applied Lighthill’s theory to calculate the far field noise. The first application of Kirchhoff’s method in combination with LES to far field jet noise was conducted by Lyrintzis and Mankbadi. LES has also been used together
with Kirchhoff’s method\textsuperscript{7} for the noise prediction of a Mach 2.0 jet by Gamet and Estivalezes\textsuperscript{8} as well as for a Mach 1.2 jet with Mach 0.2 coflow by Choi \textit{et al.}\textsuperscript{9} with encouraging results obtained in both studies. Zhao \textit{et al.}\textsuperscript{10} did LES for a Mach 0.9, Reynolds number 3,600 jet obtaining mean flow results that compared well with Freund’s DNS and experimental data. They also studied the far field noise from a Mach 0.4, Reynolds number 5,000 jet. They compared Kirchhoff’s method results with the directly computed sound and observed good agreement. Morris \textit{et al.}\textsuperscript{21} simulated high speed round jet flows using the Nonlinear Disturbance Equations\textsuperscript{(NLDE)}\textsuperscript{11–15} in their NLDE method, the instantaneous quantities are decomposed into a time-independent mean component, a large-scale perturbation and a small-scale perturbation. The mean quantities are obtained using a traditional Reynolds Averaged Navier-Stokes\textsuperscript{(RANS)} method. As in LES, they resolved the large-scale fluctuations directly and used a subgrid-scale model for the small-scale fluctuations in their unsteady calculations. In a recent paper,\textsuperscript{16} they also report the noise from a supersonic elliptic jet. Chyczewski and Long\textsuperscript{17} conducted a supersonic rectangular jet flow simulation and also did far field noise predictions with a Kirchhoff method. Boersma and Lele\textsuperscript{18} did LES for a Mach 0.9 jet at Reynolds numbers of 3,600 and 36,000 without any noise predictions. Bogey \textit{et al.}\textsuperscript{19} simulated a Reynolds number 65,000, Mach 0.9 jet using LES and obtained very good mean flow results, turbulent intensities as well as sound levels and directivity. Constantinescu and Lele\textsuperscript{20} did simulations for a Mach 0.9 jet at Reynolds numbers of 3,600 and 72,000. They directly calculated the near field noise using LES. Their mean flow parameters and turbulence statistics were in good agreement with experimental data and results from other simulations. The peak of the power spectra of the sound waves was also captured accurately in their calculations. More recently, Bogey and Bailly’s LES\textsuperscript{21} for a Mach 0.9 round jet at Reynolds number, $Re_D = 400,000$ using 12.5 and 16.6 million grid points produced mean flow and sound field results that are in good agreement with the experimental measurements available in the literature.

In general, the LES results in the literature are encouraging and show the potential promise of LES application to jet noise prediction. Except for the study of Bogey\textsuperscript{21}, the highest Reynolds numbers reached in the LES simulations so far are still below those of practical interest. Simulations of jets at higher Reynolds numbers would be very helpful for analyzing the broad-banded noise spectrum at such high Reynolds numbers. On the other hand, none of the LES studies done so far have predicted the high-frequency noise associated with the unresolved scales. Obviously, a subgrid-scale model for noise is highly desirable. Piomelli \textit{et al.}\textsuperscript{22} Rubinstein and Zhou,\textsuperscript{23} Seror \textit{et al.}\textsuperscript{24} and Zhao \textit{et al.}\textsuperscript{25} did some initial work on the investigation of the contribution of small-scales to the noise spectrum. More research in this area that will lead to development of subgrid-scale acoustic models is needed.

With this motivation behind the current study given, we now present the two main objectives of this research:

1. Development and validation of a versatile 3-D LES code for turbulent jet simulations. A high-order accurate 3-D LES code utilizing a robust dynamic subgrid-scale (SGS) model has been developed. Generalized curvilinear coordinates are used, so the code can be easily adapted for calculations in several applications with complicated geometries. Since the sound field is several orders of magnitude smaller than the aerodynamic field, we make use of high-order accurate non-dissipative compact schemes which satisfy the strict requirements of CAA. Implicit spatial filtering is employed to get rid of the high-frequency oscillations resulting from unresolved scales. Non-reflecting boundary conditions are imposed on the boundaries of the domain to let outgoing disturbances exit the domain without spurious reflection. A sponge zone that is attached downstream of the physical domain damps out the disturbances before they reach the outflow boundary. In our recent studies,\textsuperscript{26} we focused on this first objective. Previous simulations were performed with the constant-coefficient Smagorinsky SGS model. However, the results were found to be sensitive to the choice of the Smagorinsky constant. The current version of our code has the dynamic SGS model implemented. As will be shown in the results section, an LES with the dynamic SGS model for a compressible round jet produced mean flow results in excellent agreement with experimental observations.

2. Accurate prediction of the far field noise. Even though the sound is generated by a nonlinear process, the sound field itself is known to be linear and irrotational. This implies that instead of solving the full nonlinear flow equations out in the far field for sound propagation, one can use a cheaper method instead, such as Lighthill’s acoustic analogy\textsuperscript{5} or surface integral acoustic methods such as Kirchhoff’s method\textsuperscript{7} and the porous Ffowcs Williams-Hawkings (FW-H) equation.\textsuperscript{27,28} In this paper, we will couple the near field data provided by LES with the Ffowcs Williams-Hawkings method for computing noise propagation to the far field.
Governing Equations

The governing equations for LES are obtained by applying a spatial filter to the Navier-Stokes equations in order to remove the small scales. The effect of the subgrid-scales is computed using the dynamic Smagorinsky subgrid-scale model proposed by Moin et al. for compressible flows. Since we are dealing with compressible jet flows, the Favre-filtered unsteady, compressible, non-dimensionalized Navier-Stokes equations formulated in curvilinear coordinates are solved in this study using the numerical methods described in the next section.

Numerical Methods

We first transform a given non-uniformly spaced curvilinear computational grid in physical space to a uniform grid in computational space and solve the discretized governing equations on the uniform grid. To compute the spatial derivatives at interior grid points away from the boundaries, we employ the following non-dissipative sixth-order compact scheme of Lele:

\[
\frac{1}{3} f'_{i-1} + f'_i + \frac{1}{3} f'_{i+1} = \frac{7}{9\Delta\xi} (f_{i+1} - f_{i-1}) + \frac{1}{36\Delta\xi} (f_{i+2} - f_{i-2})
\]  

(1)

where \( f'_i \) is the approximation of the first derivative of \( f \) at point \( i \) along the \( \xi \) direction, \( f_i \) denotes the value of \( f \) at grid point \( i \), and \( \Delta\xi \) is the uniform grid spacing in the \( \xi \) direction. For the left boundary at \( i = 1 \) and the right boundary at \( i = N \), we apply the following third-order one-sided compact scheme equations, respectively

\[
f'_1 + 2f'_2 = \frac{1}{2\Delta\xi} (-5f_1 + 4f_2 + f_3)
\]  

(2)

\[
f'_N + 2f'_{N-1} = \frac{1}{2\Delta\xi} (5f_N - 4f_{N-1} - f_{N-2})
\]  

(3)

For the points at \( i = 2 \) and \( i = N - 1 \) next to the boundaries, we use the following fourth-order central compact scheme formulations, respectively

\[
\frac{1}{4} f'_1 + f'_2 + \frac{1}{4} f'_3 = \frac{3}{4\Delta\xi} (f_3 - f_1)
\]  

(4)

\[
\frac{1}{4} f'_{N-2} + f'_{N-1} + \frac{1}{4} f'_N = \frac{3}{4\Delta\xi} (f_N - f_{N-2})
\]  

(5)

Spatial filtering can be used as a means of suppressing unwanted numerical instabilities that can arise from the boundary conditions, unresolved scales and mesh non-uniformities. In our study, we considered the following sixth-order tri-diagonal filter used by Visbal and Gaitonde:

\[
\alpha_f f'_{i-1} + f'_i + \alpha_f f'_{i+1} = \sum_{n=0}^{3} \frac{a_n}{2} (f_{i+n} + f_{i-n})
\]  

(6)

where

\[
a_0 = \frac{11}{16} + \frac{5\alpha_f}{8}, \quad a_1 = \frac{15}{32} + \frac{17\alpha_f}{16},
\]

\[
a_2 = -\frac{3}{16} + \frac{3\alpha_f}{8}, \quad a_3 = \frac{1}{32} - \frac{\alpha_f}{16}
\]  

(7)

The parameter \( \alpha_f \) must satisfy the inequality \(-0.5 < \alpha_f < 0.5\). A less dissipative filter is obtained with higher values of \( \alpha_f \) within the given range. With \( \alpha_f = 0.5 \), there is no filtering effect.

The standard fourth-order explicit Runge-Kutta scheme is used for time advancement. We apply Tam and Dong’s 3-D radiation and outflow boundary conditions on the boundaries of the computational domain as illustrated in figure 1. We additionally use the sponge zone method in which grid stretching and artificial damping are applied to dissipate the vortices present in the flowfield before they hit the outflow boundary. This way, unwanted reflections from the outflow boundary are suppressed. Since the actual nozzle geometry is not included in the present calculations, randomized perturbations in the form of a vortex ring are added to the velocity profile at a short distance downstream of the inflow boundary in order to excite the 3-D instabilities in the jet and cause the potential core of the jet to break up at a reasonable distance downstream of the inflow boundary. This forcing procedure has been adapted from Bogey et al.

For subgrid-scale modelling, both the standard constant-coefficient Smagorinsky as well as the dynamic Smagorinsky models have been implemented into the LES code. The main idea in the dynamic model is to compute the model coefficients as functions of space and time by making use of the information contained within the smallest resolved scales of motion. For the dynamic model, a fifteen-point explicit filter developed by Bogey and Bailly is used as the test-filter. The ratio of the test-filter width to the grid

Fig. 1 Schematic of the boundary conditions.
spacing is taken as 2. The usual practice in LES calculations is to average certain dynamically computed quantities over statistically homogeneous directions and then use these averaged quantities to compute the model coefficients. This is an ad hoc procedure that is used to remove the very sharp fluctuations in the dynamic model coefficients and to stabilize the model. Obviously, such an approach is not useful for turbulent flows for which there is no homogeneous direction. In our implementation, the dynamically computed model coefficients are locally averaged in space using a second-order three-point filter in order to avoid the sharp fluctuations in the model coefficient. No negative model coefficients are allowed. This procedure works reasonably well for the jet flows we are studying.

**LES for a Round Jet at a Reynolds Number of 100,000**

The previous LES studies that we carried out can be found in Uzun. Here, we will present results from an LES performed with the dynamic SGS model for a turbulent isothermal round jet at a Mach number of 0.9, and Reynolds number \(Re_D = \rho_j U_j D_j / \mu_j = 100,000\) where \(\rho_j, U_j, \mu_j\) are the jet centerline density, velocity and viscosity at the nozzle exit, respectively and \(D_j\) is the jet diameter. The jet centerline temperature was chosen the same as the ambient temperature.

A fully curvilinear grid consisting of approximately 12 million grid points was used in the simulation. The physical portion of the domain extended to \(60r_o\) in the streamwise direction and from \(-20r_o\) to \(20r_o\) in the transverse \(y\) and \(z\) directions, where \(r_o = D_j/2\) is the jet radius. The coarsest resolution in this simulation is estimated to be about 170 times the local Kolmogorov length scale. Figures 2 through 5 show how the grid is stretched in all three directions. Initially, the grid points are clustered around the shear layer of the jet in order to accurately resolve the shear layer. After the potential core of the jet breaks up, the grid is also stretched in the \(y\) and \(z\) directions in order to redistribute the grid points and achieve a more uniform distribution. The \(y - z\) cross-section of the grid remains the same along the streamwise direction after a distance of \(35r_o\).

It has been observed by Bogey and Bailly that the jet spreading rate and the jet decay coefficient do not reach their asymptotic values if the computational domain size is kept relatively short. Furthermore, they found out that Reynolds stresses do not reach their self-similar state either in a relatively short domain. Our aim is to fully validate the dynamic model, so it is important that the asymptotic jet spreading rate and the centerline velocity decay rate as well as the self-similar state of Reynolds stresses are reached in the simulation. This will facilitate the comparison of our numerical results with the experimental jet data at

![Fig. 2 The \(x - y\) cross-section of the grid on the \(z = 0\) plane. (Every 4th grid node is shown.)](image)

![Fig. 3 The \(y - z\) cross-section of the grid on the \(x = 5r_o\) plane. (Every 4th grid node is shown.)](image)

similar flow conditions. Hence we decided to choose a long domain length of \(60r_o\) in the streamwise direction.

In our simulation, the initial transients exited the domain over the first 20,000 time steps. We then collected the flow statistics over 150,000 time steps. This sampling period corresponds to a time length in which an ambient sound wave travels about 60 times the domain length in the streamwise direction. A total of about 123,000 CPU hours was needed for this simulation on an IBM-SP machine using 160 processors in parallel. It should be noted that the mean flow convection speeds in the far downstream region are relatively low, hence the turbulence convects slowly in the far downstream region resulting in slowly converging statistics. As a result, we had to collect data over a period of 150,000 time steps to obtain reasonably converged statistics.
For code validation, we first compared our jet’s centerline decay rate in the far downstream region with the experiments of Zaman. Zaman did a series of experiments for initially compressible jets and examined the asymptotic jet centerline velocity decay and entrainment rates. He found out that the asymptotic jet centerline velocity decay rate decreased with increasing initial jet Mach number. In figure 6, we plot the streamwise variation of the inverse of the mean jet centerline velocity normalized by the jet inflow velocity. In the far downstream region, we see that the slope of the linear fit to the curve is 0.161. From the experiments of Zaman, the corresponding experimental slope for a Mach 0.9 jet is about 0.155 which is very close to our computed value. For incompressible jets, on the other hand, experimental values ranging from 0.165 to 0.185 have been previously reported in the literature.

Furthermore, in figure 7, we show the streamwise variation of the mass flux normalized by the mass flux through the jet nozzle. In the far downstream region, we again see a linear growth. This time the slope of the line is 0.267 showing very good agreement with the experimental value of 0.26 from Zaman. The jet spreading rate, which is the slope of the half-velocity radius growth in the far downstream region depicted in figure 8, has been found to be 0.092 in our simulation. The half-velocity radius, \( r_{1/2} \) at a given downstream location is defined as the radial location where the mean streamwise velocity is one half of the jet mean centerline velocity at that location. To the best of our knowledge, no experimental value has been reported from an initially compressible jet in the
The self-similarity coordinate is taken as \( r/r_{1/2} \) where \( r \) is the radial location and \( r_{1/2} \) is the half-velocity radius. The vertical axis of the figure is normalized by the mean jet centerline velocity. From this figure, it can be observed that the profiles of our jet at the three downstream locations collapse onto each other fairly well and exhibit self-similarity which is consistent with experimental observations. Our results are also in very good agreement with the experimental profiles of Hussein et al.\(^{38}\) for an incompressible jet at \( Re_D = 95,500 \) as well as that of Panchapakesan et al.\(^{39}\) for an incompressible jet at \( Re_D = 11,000 \). The convective Mach number of the jet flow is low in the far downstream region, hence the compressibility effects are negligible. Therefore, it is safe to compare our profiles in the far downstream region with incompressible experimental data.

We also compared our Reynolds stress profiles with the experimental profiles of Hussein et al.\(^{38}\) and Panchapakesan et al.\(^{39}\) Figures 10 through 13 plot our properly scaled Reynolds stress profiles at three downstream locations and compare them with the two experiments. Again, the self-similarity coordinate is taken as \( r/r_{1/2} \). The vertical axis of the figures is normalized by the square of the mean jet centerline velocity. As can be observed from the plots, the properly scaled profiles collapse onto each other exhibiting self-similarity. Overall, the agreement is very good. It should be noted that our Reynolds stresses computed by LES are based on filtered velocities, while the actual experimental profiles are based on unfiltered velocities. Hence, some differences should be expected between LES and experiments when the Reynolds stresses are compared. As can be seen from the figures, the Reynolds normal stress \( \sigma_{xx} \) profiles match the experiment of Panchapakesan et al.\(^{39}\) better, while the other Reynolds stresses are in between the two experimental profiles. It should also be noted that the difference between the initial conditions imposed in our simulation and the actual initial conditions in experimental jets could be another possible reason for the differences between LES and experimental profiles. We have imposed randomized velocity fluctuations in the form of a vortex ring\(^{34}\) in our simulation since the actual nozzle ring was not included in the calculations. Furthermore, it can be argued that the experimental Reynolds stress profiles have been mea-
sured in the very far downstream region, usually at distances of 100 jet radii or more. It has also been observed experimentally that the different Reynolds stress components reach asymptotic self-similarity at different downstream locations, depending on initial conditions. Since our domain length of 60 jet radii is still relatively short compared to experiments, the normalized Reynolds normal stresses $\sigma_{rr}$ may not have reached their true asymptotic values. In a previous simulation that we carried out using the same dynamic SGS model and a shorter domain length of $35r_o$, we found out that the Reynolds stresses in the far downstream region were lower than the ones in the present simulation. This clearly means that the Reynolds stresses in the short domain simulation did not reach their asymptotic self-similar states. If we did a new simulation using a domain longer than $60r_o$ in the streamwise direction, it is possible that the Reynolds normal stress $\sigma_{xx}$ profiles in the far downstream region might shift upwards and come in between the two experiments. However, such a simulation would require even more grid points and many more time steps to run since the very slowly convecting mean flow in the far downstream region would cause very slowly converging flow statistics. Hence such a simulation would not be very practical.

To summarize, the mean flow properties computed using the dynamic SGS model compared very well with the experimental data. Furthermore, the comparison of Reynolds stresses in the present simulation with experiments is found to be reasonable. These findings provide us with valuable evidence towards the validity of the numerical schemes as well as the dynamic subgrid-scale model used in our LES code.

Far Field Aeroacoustics

Next, we will look at some far field aeroacoustics results obtained by coupling the near field LES data with a Ffowcs Williams-Hawkings code which has the capability to work on any general control surface geometry.

The Ffowcs Williams-Hawkings formulation for a stationary control surface, $S$ is given as follows

$$p' (\vec{x}, t) = p'_T (\vec{x}, t) + p'_L (\vec{x}, t) + p'_Q (\vec{x}, t) \quad (8)$$

where

$$4\pi p'_T (\vec{x}, t) = \int_S \left[ \frac{\rho_o \bar{U}_n}{r} \right]_{ret} dS \quad (9)$$

$$4\pi p'_L (\vec{x}, t) = \frac{1}{a_o} \int_S \left[ \frac{\bar{L}_r}{r^2} \right]_{ret} dS + \int_S \left[ \frac{L_r}{r^2} \right]_{ret} dS \quad (10)$$
and

\[ U_i = \frac{\rho u_i}{\rho_o} \]

\[ L_i = P_{ij}\hat{n}_j + \rho u_i u_n \]  

\((\hat{x}, t)\) are the observer coordinates and time, \(r\) is the distance from the source (surface) to the observer, subscript \(o\) implies ambient conditions, superscript \('\) implies disturbances (e.g. \(\rho = \rho' + \rho_o\)), \(\rho\) is the density, \(u_i\) is the velocity vector, \(a_o\) is the ambient sound speed, and \(P_{ij}\) is the compressive stress tensor with the constant \(p_o\delta_{ij}\) subtracted. The dot over a variable indicates a time derivative, a subscript \(r\) or \(n\) indicates a dot product of the vector with the unit vector in the radiation direction \(\hat{r}\) or the unit vector in the surface normal direction \(\hat{n}\), respectively. The surface integrals are over the control surface \(S\), and the subscript \(\text{ret}\) indicates evaluation of the integrands at the emission (retarded) time \(\tau = t - r/a_o\). The quadrupole noise pressure \(p_Q'(\hat{x}, t)\) denotes the quadrupoles outside the control surface and has been neglected in the current study. A more detailed description of the Ffowcs Williams-Hawkings method can be found in Lyrintzis and Uzun.40

We put a control surface around our jet as illustrated in figure 14 and we gathered flow field data on the control surface at every 10 time steps over a period of 23,000 time steps during our LES run. The total acoustic sampling period corresponds to a time scale in which an ambient sound wave travels about 9.4 times the domain length in the streamwise direction. Assuming at least 6 points per wavelength are needed to accurately resolve an acoustic wave, we see that the maximum frequency resolved with our grid spacing around the control surface corresponds to a Strouhal number of approximately 1.0. The Nyquist frequency, on the other hand, which is the maximum frequency that can be resolved with the time increment of our data sampling rate, corresponds to a Strouhal number of about 4.5. However, we choose the maximum frequency that is based on spatial resolution as our grid cutoff frequency.

We computed the overall sound pressure levels along an arc of radius 60\(r_o\) from the jet nozzle. The center of the arc is chosen as the jet nozzle exit and the angle \(\theta\) is measured from the jet axis as illustrated in figure 15. We applied the Ffowcs Williams-Hawkings method to compute the acoustic pressure signal at 36 equally spaced azimuthal points on a full circle at a given \(\theta\) location on the arc. In order to confine spurious spectral contributions to low frequencies, we multiplied the pressure history at every azimuthal location by a windowing function similar to that used by Freund.2 We ran the windowed pressure history through a 2048 point Fast Fourier Transform, and converted to polar notation to obtain the power spectral density at each frequency. We then averaged the acoustic pressure spectra over the equally spaced 36 azimuthal points and finally integrated the averaged spectra to compute the overall sound pressure level at the given \(\theta\) location. In our spectra, we observed very energetic low frequencies corresponding to \(St \leq 0.05\). It is not clear where these spurious energetic low frequencies are coming from. As pointed out by Bogey et al.19,34 who also made similar observations, these energetic spurious low frequencies might be related to the very low frequency reflections resulting from the outflow boundary which may not be damped out effectively by the sponge zone. Only 35 grid points were used to construct the sponge zone in our grid. Because of these observations, we did not include the Strouhal numbers less than 0.05 in the overall sound pressure level calculations. Only the resolved frequencies (0.05 \(\leq St < 1.0\)) were included in the integration.

Figure 16 shows our overall sound pressure levels computed along the arc and compares them with some experimental data. Table 1 summarizes the
From the overall sound pressure level plot, we see that our jet is louder than the experimental cold jets for values of 0.9 jet and the difference between the isothermal and cold jet predictions were found to be only 0.3 dB. It is also seen that there is as much as 3 dB difference between our prediction and the ARP 876C prediction at the downstream angles. For larger values of θ, the overall sound pressure levels are comparable to the experimental values of Mollo-Christensen et al.\textsuperscript{41} as well as those of Lush.\textsuperscript{42} The agreement with ARP 876C is also very good for values of θ greater than 50°.

It is known that the large scale turbulent structures in the jet flow are responsible for the noise radiation in the downstream direction while the high-frequency noise generated by the fine scale turbulence becomes dominant in the sideline and upstream direction. The differences in the overall sound pressure levels at the downstream angles could be related to the difference in the nozzle exit conditions. This is because the jet nozzle exit conditions probably have a strong influence on the generation of large scale structures in the flow, whereas the fine scale turbulence is much less affected by initial conditions and has a more universal character. Our vortex ring forcing which is clearly artificial and different than the initial conditions in the actual experiments, is apparently generating large scale structures that result in somewhat higher overall sound pressure levels in the downstream direction.

Howe\textsuperscript{44} provides the following equation for estimating the overall sound pressure level at the θ = 90° location

\[
\frac{\langle p^2(r, 90°) \rangle}{p^2_{ref}} \approx \frac{9 \times 10^{13} A}{r^2} \left[ \frac{(\rho J/\rho_0)^w}{1 - 0.1 M^2.5 + 0.015 M^{4.5}} \right]
\]

where \(r\) is the observer distance from the nozzle exit, \(\langle p^2(r, 90°) \rangle\) is the mean square acoustic pressure at the \((r, \theta = 90°)\) observation point, \(p_{ref}\) is the standard reference pressure, \(A\) is the nozzle cross-sectional area, \(M\) is the jet Mach number, \(w\) is the so-called jet density exponent, and \(\rho_J/\rho_0\) is the ratio of the jet density to the ambient density. Howe\textsuperscript{44} also provides an equation in terms of the jet Mach number to estimate the jet density exponent, \(w\). Since we have an isothermal jet, our \(\rho_J/\rho_0 = 1\) and therefore there is no need to estimate the jet density exponent for our jet. The above equation predicts an overall sound pressure level of 105.82 dB at the \(R = 60r_o, \theta = 90°\) location. This prediction is very close to our computed value of 105.57 dB as well as the ARP 876C prediction of 105.30 dB at the same location. From these findings, we see that we have captured most of the energetic frequencies in our simulation and the unresolved higher frequencies are not likely to alter the overall sound pressure levels significantly.

Finally, we compare our acoustic pressure spectra at three observation locations with the recent results of Bogey and Bailly\textsuperscript{21} for an isothermal round jet at a Reynolds number of 400,000. It should be noted here that Bogey and Bailly\textsuperscript{21} computed the spectra in the near field of the jet only, using the data directly provided by LES. The first observation point of Bogey and Bailly\textsuperscript{21} is located inside of our control surface while the other two are located outside, hence we compute the near field spectra at the latter two points using the Flows Williams-Hawkins method. On the other hand, to compare our far field spectra with their near field spectra, we adjust the \(\theta\) location along the arc such that an acoustic ray drawn between the end of the potential core \((x = 11r_o)\) and the \(\theta\) location on the arc crosses their observation point.

Figure 17 plots the spectra at the \(R = 60r_o, \theta = 25°\) location over our resolved frequency range and compares it with the corresponding near field spectra of Bogey and Bailly.\textsuperscript{21} We see that our spectra has a
than ours since it was computed in the near field of Bogey and Bailly. The spectra of two observation points, respectively and make comparisons with the spectra obtained by Bogey and Bailly.

Fig. 17  Acoustic pressure spectra at $R = 60r_o$, $\theta = 25^\circ$ in the far field and comparison with the corresponding spectra of Bogey and Bailly.\textsuperscript{21}

Fig. 18  Acoustic pressure spectra at $R = 60r_o$, $\theta = 50^\circ$ in the far field, acoustic pressure spectra at $x = 20r_o$, $r = 15r_o$ in the near field and comparison with the corresponding spectra of Bogey and Bailly.\textsuperscript{21}

Fig. 19  Acoustic pressure spectra at $R = 60r_o$, $\theta = 80^\circ$ in the far field, acoustic pressure spectra at $x = 11r_o$, $r = 15r_o$ in the near field and comparison with the corresponding spectra of Bogey and Bailly.\textsuperscript{21}

peak at around $St = 0.25$ while that of Bogey and Bailly\textsuperscript{21} is at $St = 0.3$. Other than this difference, the two spectra look qualitatively similar. The spectra of Bogey and Bailly\textsuperscript{21} has higher sound pressure levels than ours since it was computed in the near field of the jet. We also show a 4$^{th}$ order polynomial fit to our spectra in the figure. From this polynomial fit, it is observed that there is a drop of about 15 dB/St in the sound pressure level from the peak Strouhal number of 0.25 to the grid cutoff Strouhal number of 1.0. Figures 18 and 19 show our near field spectra at the other two observation points, respectively and make comparisons with the spectra obtained by Bogey and Bailly.\textsuperscript{21} The acoustic rays that originate from the end of the potential core and pass through these two near field observation points hit the far field arc at the $\theta = 50^\circ$ and $\theta = 80^\circ$ locations, respectively. We also show our far field spectra corresponding to these two near field observation points. From these figures, we see that our near and far field spectra basically have the same shape. Our spectra also become broadband at these observation points demonstrating the fact that higher frequencies are becoming more dominant. It is also interesting to note that our spectra demonstrate a slow decay after they reach their peak whereas the spectra of Bogey and Bailly\textsuperscript{21} stay relatively flat after the peak. A possible reason for this difference could be the different Reynolds numbers of the two simulations. There is almost 10 dB/St difference between our near and far field spectra in figure 18. The distance between the far field point and the end of the potential core is about 53.60$r_o$, whereas the distance between the near field point and the end of the potential core is 17.49$r_o$. The ratio of these two distances is about 3. A factor of 3 means about 10 dB/St difference according to the $r^{-1}$ decay assumption of acoustic waves. In this assumption, the acoustic wave amplitudes drop by a factor of 2 for every doubling of the distance from the source region. Similarly, the difference between our near and far field spectra in figure 19 is about 12 dB/St. In this case, the ratio of the far field observation point distance to the near field observation point distance is about 4 which also translates into 12 dB/St difference according to the $r^{-1}$ decay assumption.
Concluding Remarks

As part of a jet noise prediction methodology, we have developed and fully tested a 3-D Large Eddy Simulation code. The localized dynamic model was found to perform very well. The mean flow results obtained in our simulation for a Reynolds number 100,000 jet were in excellent agreement with experimental data. We also coupled the near field data computed by our LES code with the Ffowcs Williams-Hawkins (FW-H) formulation to compute the far field noise of the jet. Far-field aeroacoustics results were encouraging.

In our near future work, we plan to simulate the same Reynolds number 400,000 jet test case which Bogey and Bailly\textsuperscript{21} studied. This will help us fully validate our CAA methodology. We also plan to do a more detailed study of the far field acoustics of the jet using both the Kirchoff’s and the Ffowcs Williams-Hawkins (FW-H) formulations. Furthermore, we will do noise calculations using the $r^{-1}$ decay assumption which is cheaper than integral acoustic methods. We will compare far field noise computations with the surface integral methods to the results obtained by using the $r^{-1}$ decay assumption. We will also investigate the sensitivity of far field noise predictions to the position of the control surface on which aeroacoustic data is collected.

Our eventual goal in this research is to study the aeroacoustics of a high Reynolds number jet by doing a well-resolved LES. It is imperative that there exists well-documented experimental data for the LES test case chosen so that comparisons can be made with the experiment to ensure the validity of simulation results. Only then the simulation database can be used reliably to investigate the jet noise generation mechanisms and recover valuable information about the jet noise physics.

Acknowledgments

This work is part of a joint project with Rolls-Royce, Indianapolis and is sponsored by the Indiana 21st Century Research & Technology Fund. It was also partially supported by the National Computational Science Alliance under the grant CTS010032N, and utilized the SGI Origin 2000 computer systems at the University of Illinois at Urbana-Champaign. Some of the computations were performed on the IBM-SP research computer of Indiana University. We are grateful to Dr. Christophe Bogey for his help while we implemented the vortex ring forcing method and the dynamic subgrid-scale model into our code. We also thank him for providing us with data from his Reynolds number 400,000 jet simulation.

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