Locally Optimized Scheduling and Power Control Algorithms for Multi-hop Wireless Networks under SINR Interference Models

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***Abstract*— In this paper, we develop locally optimized scheduling and power control algorithms for multi-hop wireless networks under *SINR interference* models. Our scheme can be implemented in a fully distributed manner and requires only that each node solve a simple local optimization problem. Since, in our algorithms, each node operates independently of other nodes, it needs to predict the behavior of neighboring nodes when carrying out its local optimization. For such prediction, our proposed algorithms exploit the past records of neighboring nodes' scheduling and power control decisions. Through simulations, we show that our algorithms significantly outperform the state-of-the-art.**

I. Introduction

In this paper, we develop efficient scheduling and power control algorithms that support high data rates in multi-hop wireless networks. In recent years, throughput-optimal scheduling and power control algorithms that maximize the achievable throughput of multi-hop wireless networks have been extensively studied in the literature [1], [2], [3]. However, these throughput-optimal algorithms are often difficult to implement mainly because of the following reasons: first, the optimal algorithm operates in a fully centralized manner. Thus, a centralized scheduler needs to collect global information of the queue lengths from the entire network and to distribute its scheduling and power control decision back to the entire network. Both information collection and decision distribution could result in significant communication overhead. Second, these throughput-optimal algorithms require the centralized scheduler to solve a complex global optimization problem, whose complexity could exponentially increase in the network size.

There have been recent efforts to develop efficient scheduling algorithms that alleviate the communication and computation overheads. Progress has been made under restrictive classes of interference models. A commonly used model is the K-hop interference model, in which two links that are within K-hops of each other cannot simultaneously communicate, and the capacity of a link is a constant value if there is no interference [4], [5], [6], [7], [8], [9], [10]. In [4], the Maximal Matching (MM) scheduling algorithm is used under the node-exclusive interference model (the special case of the K-hop interference model with $K = 1$.) This algorithm can operate in a distributed fashion and is proven to have at least one half of the achievable throughput under the node-exclusive interference model. These features have motivated subsequent research on the distributed algorithms with provable performance [5], [6], [7], [8], [9], [10].

However, the K-hop interference models do not adequately capture the more realistic Signal-to-Interference-and-Noise-Ratio (SINR) interference models, where link capacities could vary according to the signal and the interference levels. In this paper, we consider the following two SINR interference models that are commonly used: *the linear SINR interference model* and *the logarithmic SINR interference model*. Under the linear SINR interference model, the capacity of each link is linearly proportional to the SINR value. In contrast, under the logarithmic SINR interference model, the capacity of a link is represented as a logarithmic function of the SINR value.

Scheduling algorithms under the different types of SINR interference models have been studied in the literature [3], [11], [12], [13]. In [3], the authors have proposed a simple and distributed scheduling algorithm that is an approximation to the optimal "Dynamic Routing and Power Control" (DRPC) algorithm (which is centralized and requires high computational complexity) and could run under both the linear and the logarithmic SINR interference models. In [11], for the logarithmic SINR interference model the author has proposed the "Jointly Optimal Congestion-control and Power-control" (JOCP) algorithm that is distributed and optimal under the assumption that SINR values are high. The authors in [12] and [13] have also proposed heuristic algorithms under the target SINR interference model where the capacity of a link is a constant value when the received SINR exceeds a threshold, or zero otherwise.

In this paper, we propose new distributed scheduling and power control algorithms under SINR interference models and compare their performance to the state-of-the-art in [3] and [11]. We show that our algorithms have constant complexity, consume a constant amount of computation time at each time slot, and operate in a fully distributed manner. In fact, the complexity of our algorithms is independent of the size of the network.

The rest of this paper is organized as follows. Section II illustrates the system model, including our SINR interference models, and reviews the fundamentals. Section III develops a locally-optimized scheduling algorithm under the linear SINR interference model. Section IV extends our result in Section III to the logarithmic SINR interference model. Here, the scheduling algorithm developed in Section III is generalized to have power control capability. Section V provides simulation results that compare the performance of our algorithms with the optimal performance. We also provide extensive comparison to other algorithms under different SINR interference models. We conclude in Section VI.

II. System Model

In this paper, we consider a multi-hop wireless network with N nodes and L links. All nodes and links are assumed to be static. Each link l corresponds to a transmitting node, denoted by $b(l)$, and a receiving node, denoted by $e(l)$. Let $\mathcal{N}$ and $\mathcal{L}$ denote the set of all nodes and the set of all links, respectively. An outgoing link of node i is defined as a link whose transmitter is node i. We define the outgoing link set of node i as $\mathcal{L}_{i,out} = \{l \in \mathcal{L} | \; b(l) = i\}$ and the number of outgoing links of node i as $L_{i,out}$. Similarly, an incoming link of node i is a link whose receiver is node i. We define the incoming link set of node i as $\mathcal{L}_{i,in} = \{l \in \mathcal{L} | \; e(l) = i\}$, and the number of the incoming links of node i as $L_{i,in}$.

We consider a time-slotted network. Let $P_l(t)$ denote the transmission power at the transmitter of link l at time slot t, and $\vec{P}(t) = [P_l(t), \, l \in \mathcal{L}]$ be the power assignment vector at time slot t. Let $\mathbb{P}_{i,\max}$ be the maximum power limit of each transmitting node i, such that $0 \le \sum_{l \in \mathcal{L}_{i,out}} P_l(t) \le \mathbb{P}_{i,\max}$.

We assume that the link capacity is a function of its SINR value. Let $r_l(t)$ be the capacity of link l at time slot t, and $\xi_l(t)$ be the measured SINR value at the receiver of link l at time slot t. Specifically, $\xi_l(t)$ is the ratio of the received signal power to the received interference and noise power, and is given by

$$\xi_l(t) = \frac{G_{ll} P_l(t)}{\sum_{h \in \mathcal{L} \setminus \{l\}} G_{hl} P_h(t) + \eta_l}, \tag{1}$$

where η_l denotes the time-invariant background noise at the receiver of link l, and G_{hl} denotes the wireless channel gain from the transmitter of link h to the receiver of link l. Since the nodes are static, the channel gains are assumed to be fixed and known. (We do not consider fading effects here.) We then consider two types of functional relationships between $r_l(t)$ and $\xi_l(t)$:

- **The logarithmic SINR interference model:** the capacity of link l is determined by $r_l(t) = B \log(1+\xi_l(t))$, where B denotes the fixed channel bandwidth.
- **The linear SINR interference model:** the capacity of link l is determined by $r_l(t) = B\xi_l(t)$. This model can be viewed as an approximation of the logarithmic SINR interference model, when the SINR level $\xi_l(t)$ is low.

In this paper, we will first study the linear SINR interference model due to its analytical simplicity, and then the logarithmic SINR interference model.

Remarks: In practical systems, there are often additional constraints on the feasible transmission patterns. For example, a node may not be able to receive when it is also transmitting. Also, a node may not be able to receive from multiple transmitters at the same time. Note that our model can be easily adapted to these settings. For example, if a node cannot receive when it is transmitting, we can simply set $G_{hl} = \infty$ when the transmitter of link h is the same as the receiver of link l, i.e., $b(h) = e(l)$. Similarly, if a node cannot receive from multiple transmitters simultaneously, we can set $G_{hl} = \infty$ when the receiver nodes of link l and link h are the same, i.e, $e(h) = e(l)$.

We assume that there are U users in the network whose data could travel multiple links from their source nodes to their corresponding destination nodes. Let λ_u be the data arrival rate of user u at the source node. Let $\vec{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_U]$. The capacity region under a scheduling and power control algorithm is defined as the set of vectors $\vec{\lambda}$ under which the network remains *stable*. Here, stability means that all queues remain finite. The algorithm that achieves the largest capacity region is referred to as the *throughput-optimal* scheduling and power control algorithm (or simply the optimal algorithm in the rest of the paper). It has been proven in [1] that one such optimal algorithm computes the power-assignment vector at time slot t as the solution to the following global optimization problem;

$$\vec{P}^*(t) = \arg\max_{\vec{P} \in \Pi} \sum_{l \in \mathcal{L}} r_l(\vec{P}) \, q_l(t), \tag{2}$$

where $\Pi = \{\vec{P}; 0 \le \sum_{l \in \mathcal{L}_{i,out}} P_l(t) \le \mathbb{P}_{i,\max} \;\; \forall i \in \mathcal{N}\}$, and $q_l(t)$ denotes the queue length of link l at time slot t.

As mentioned in the Introduction, this optimal algorithm is extremely difficult to implement due to the communication overhead (of collecting $q_l(t)$'s and distributing $\vec{P}^*(t)$) and the computational overhead (of solving (2)).

III. A Locally Optimized Scheduling Algorithm for the Linear SINR Interference Model

In this section, we propose a new distributed algorithm, *the locally optimized scheduling algorithm*, under the linear SINR interference model. Our algorithm may be viewed as a suboptimal approximation of the optimal algorithm (2). It significantly lowers the computation and communication overhead of the optimal algorithm.

A. Distributed Scheduler

Our goal is to develop a distributed scheduling and power control algorithm under which each node schedules its *own* resources in a distributed fashion. In other words, each node should decide *by itself* whether it should transmit or not, and, if it transmits, to which adjacent nodes and at what power level it should transmit. Note that under the linear SINR interference model, it has been proven in [2] that the optimal scheduling decision and power assignment are of the form that each node should either transmit on only one of its outgoing links with the node's maximum power, or not transmit at all. Hence, for this interference model, we need to focus only on the scheduling decision. Let $S_i(t)$ denote the scheduling decision of node i at time slot t, given by

$$S_i(t) = \begin{cases} 0, & \text{if node } i \text{ does not schedule its outgoing links,} \\ s, & \text{if node } i \text{ schedules the } s\text{-th outgoing link with power } \mathbb{P}_{i,\max} \ (s = 1, \cdots, L_{i,out}). \end{cases}$$

Since each node i has $L_{i,out}$ outgoing links to schedule, it has a total of $L_{i,out} + 1$ choices of scheduling decisions.

B. Local Optimization

We propose to develop a distributed scheduler as an approximate solution to (2) as follows. We define *the neighboring links* of node i as the links that are close enough to node i so that node i can communicate basic information, such as the queue lengths and SINR values, with these links directly. Let $\mathcal{L}_i$ be the set of all neighboring links of node i. We then introduce the notion of *local optimization* as follows:

Local Optimization: each node searches its scheduling decision $S_i = s$ that solves the following optimization problem

$$\max_{s \in \{0,1,\cdots,L_{i,out}\}} E\left[\sum_{l \in \mathcal{L}_i} r_l q_l \middle| S_i = s\right], \tag{3}$$

where the expectation is taken with respect to some empirical distribution of the other links' decisions in $\mathcal{L}_i$. Note the following:

- Each node only needs to update the queue lengths in $\mathcal{L}_i$,
- The number of terms in the summation (3) is usually much smaller than in (2),
- Each node only decides its own actions.

In reality, the capacity of each link depends not only on the scheduling decision of its transmitting node, but also on the decisions of all other nodes. However, each node does not know *a priori* the scheduling decisions of other nodes in the neighborhood. Hence, we take expectation in (3), with respect to an empirical distribution of the other links' decisions.

The locally optimal scheduling decision S_i^* of node i that maximizes the expectation of the local queue-weighted link-capacity sum $\sum_{l \in \mathcal{L}_i} r_l q_l$ in (3) is simply given by

$$S_i^* = \arg\max_{s \in \{0,1,\cdots,L_{i,out}\}} \sum_{l \in \mathcal{L}_i} q_l E\left[r_l | S_i = s\right]. \tag{4}$$

The term $\sum_{l \in \mathcal{L}_i} q_l E\left[r_l | S_i = s\right]$ in (4) can be viewed as the expected local queue-weighted link-capacity sum, provided that node i selects the scheduling decision s.

Remark: If the scheduling decisions of the nodes are determined by a centralized algorithm that solves (2), the result in [2] shows that each node should either transmit on one link at full power, or not transmit at all. One could argue that, since in our local algorithm each node makes its own scheduling decision, perhaps allowing more flexible scheduling decisions, i.e., allowing nodes to transmit at intermediate power levels or on multiple outgoing links, can further increase the optimal value of (3). However, the following proposition shows that this is not the case.

Proposition 1: Let $\vec{p}_i \in \mathbb{R}_+^{L_{i,out}}$ denote the power allocation vector for all outgoing links from node i, where $\mathbb{R}_+^{L_{i,out}}$ is the set of $L_{i,out}$-dimensional vectors with nonnegative components. Consider the following optimization problem

$$\begin{aligned} \max_{\vec{p} \in \mathbb{R}_+^{L_{i,out}}} \quad & \sum_{l \in \mathcal{L}_i} q_l E[r_l | \vec{p}_i = \vec{p}] \\ \text{subject to} \quad & \vec{1}^T \vec{p} \le \mathbb{P}_{i,\max}, \end{aligned} \tag{5}$$

where $\vec{1}$ is a $L_{i,out}$-dimensional column vector with all 1's, and the expectation is taken with respect to the distribution of neighboring nodes' scheduling decisions. Then, the solution to (3) also corresponds to an optimal solution to (5).

The proof of Proposition 1 is provided in Appendix I. Proposition 1 means that scheduling the S_i^*-th outgoing link in (4) with maximum power when $S_i^* \neq 0$ or scheduling no links when $S_i^* = 0$ is the best response for node i, given the empirical distribution of neighboring nodes' scheduling decisions.

C. Sample Average

In order to solve (4), each node i initially needs to estimate $E\left[r_l | S_i = s\right]$ for all $l \in \mathcal{L}_i$ and for $s = 0, 1, \cdots, L_{i,out}$, where the expectation is taken with respect to the empirical distribution. We assume that the decisions of other nodes are distributed according to the empirical distribution in the past. Thus, each node chooses its independent scheduling decision that would be the best response to the past.

We next describe how each node i collects the empirical distribution and obtains $E\left[r_l | S_i = s\right]$. Our algorithm runs for a fixed number of iterations in a time slot. We assume that the total length of these iterations is much smaller than the length of a time slot. Further, our algorithm operates as if the queue length remains fixed during these iterations. Then, after these fixed number of iterations, the scheduling decisions of the last iteration will be used for actual data transmission (to be described in Section III-D.) Now, consider the n-th iteration in a time slot. Let $S_i[k]$ denote the scheduling decision at a past iteration k ($k < n$), and let $P_l[k]$ and $r_l[k]$ denote the corresponding transmission power and capacity, respectively, of link l at this iteration k. Note that we have used the square bracket $[\cdot]$ for the index of an iteration within a time slot, while we have used the parenthesis $(\cdot)$ for the index of a time slot. Define *the hypothetical link capacity* at past iteration k as follows: $r_{l|i,s}[k]$ is the capacity of link l if the decision of node i is s, and the decisions of other nodes j are $S_j[k]$ ($j \neq i$). If each node i maintains $r_{l|i,s}[k]$ for all $l \in \mathcal{L}_i$, all $s = 0, 1, \cdots, L_{i,out}$, and all $k = n - W, \cdots, n$, then the node can now estimate $E[r_l | S_i = s]$ at iteration n as an empirical average over the last W iterations, i.e., $\bar{r}_{l|i,s}[n-1] \triangleq \frac{1}{W} \sum_{k=n-W}^{n-1} r_{l|i,s}[k]$. (Here, W denotes the window size of the empirical distribution.) Then, each node i

can solve (4) at iteration n as follows:

$$S_i^*[n] = \underset{s\in\{0,1,\cdots,L_{i,out}\}}{\arg\max} \sum_{l\in\mathcal{L}_i} \bar{r}_{l|i,s}[n-1]\, q_l. \tag{6}$$

The remaining question is how each node i can maintain $r_{l|i,s}[k]$. We assume that each node i knows the past link capacities of neighboring link $r_l[k]$ ($l \in \mathcal{L}_i$). We will elaborate how this information is obtained in Section III-D. Further, each node knows the channel gains between its neighboring links and the maximum transmission power of neighboring nodes in advance. We now show that $r_{l|i,s}[k]$ can be calculated based on the above information. Specifically, let l_s be the s-th outgoing link of node i ($s = 0, 1, \cdots, L_{i,out}$). Note that for ease of notation we use l_0 as an imaginary link that corresponds to the case that node i does not transmit over any links. We further define $G_{l_0 h} = 0$ for all $h \in \mathcal{L}$. Then, $r_{l|i,s}[k]$ is given as follows.

1) When $s = S_i[k]$, $r_{l|i,s}[k] = r_l[k]$ by the definition of the hypothetical link capacity.

2) When $s \neq S_i[k]$, we calculate $r_{l|i,s}[k]$ differently depending on the following two cases.

- **For link $l \in \mathcal{L}_i \setminus \mathcal{L}_{i,out}$,** we express $r_l[k]$ as follows,

$$r_l[k] = \frac{BG_{ll}P_l[k]}{\sum_{h\in\mathcal{L}\setminus\mathcal{L}_{i,out}\setminus\{l\}} G_{hl}P_h[k] + G_{l_{S_i[k]}l}\mathbb{P}_{i,\max} + \eta_l}. \tag{7}$$

  Note that the formula for $r_{l|i,s}[k]$ is similar to (7) except that $G_{l_{S_i[k]}l}$ is replaced by $G_{l_s l}$. Since each node will transmit at full power, $P_l[k] = \mathbb{P}_{b(l),\max}$ if $r_l[k] \neq 0$. Hence, we can easily compute $r_{l|i,s}[k]$ from $r_l[k]$ as follows,

$$r_{l|i,s}[k] = \begin{cases} \dfrac{BG_{ll}\mathbb{P}_{b(l),\max}r_l[k]}{BG_{ll}\mathbb{P}_{b(l),\max} - r_l[k]\mathbb{P}_{i,\max}(G_{l_{S_i[k]}l} - G_{l_s l})}, & \text{if } r_l[k] \neq 0, \\ 0, & \text{if } r_l[k] = 0. \end{cases} \tag{8}$$

- **For link $l \in \mathcal{L}_{i,out}$,** $r_{l|i,s}[k]$ is simply given by

$$r_{l|i,s}[k] = \begin{cases} \dfrac{BG_{ll}\mathbb{P}_{i,\max}}{\sum_{h\in\mathcal{L}\setminus\mathcal{L}_{i,out}} G_{hl}P_h[k] + \eta_l}, & \text{if } l = l_s, \\ 0, & \text{if } l \neq l_s. \end{cases}$$

D. Implementation Details

We now describe the locally-optimized scheduling algorithm for the linear SINR interference model. In our algorithm, each time slot consists of two phases: *a scheduling phase* and *a transmission phase*. In the scheduling phase, each node iteratively executes (6). Our approach to collect the empirical distribution of neighboring nodes is as follows. At iteration k, some nodes with non-zero scheduling decisions transmit at their maximum power to the receiving nodes. Simultaneously, all nodes measure the SINR values of each incoming link and calculate the expected link capacities, $r_l[k]$, based on the measured SINR. Next, the nodes distribute the estimated link capacities, $r_l[k]$, to neighboring nodes. It should be noted that inefficiently distributing this information could increase the communication overhead.

While there are different ways of doing this, we now elaborate one approach for distributing such information in a timely fashion at each iteration to avoid significant communication overhead. Note that the amount of information to be distributed is much smaller than that of the actual data. Thus, there exist several ways for each node to deliver the information, spending a very small amount of wireless resources. One of the possible methods is following. Suppose that the entire frequency band is divided into tiny sub-bands and that the number of these sub-bands is much larger than the number of links in each neighborhood. Then, it is easy to assign these sub-bands to each link, such that any two links in $\mathcal{L}_{i,out}$ for all $i \in \mathcal{N}$ do not share the same sub-band. Then, the transmitting node of each link emit power whose strength is proportional to the value of information, $r_l[k]$, on the sub-band assigned to the link. Since nodes know the channel gains from other nodes in the neighborhood, they simply measure the received power on each sub-band and estimate the information value. Once each node i successfully receives the measured link capacities from its neighborhood, it calculates the hypothetical link capacities $r_{l|i,s}[k]$ and saves them into its memory.

After a fixed number of iterations M are executed in

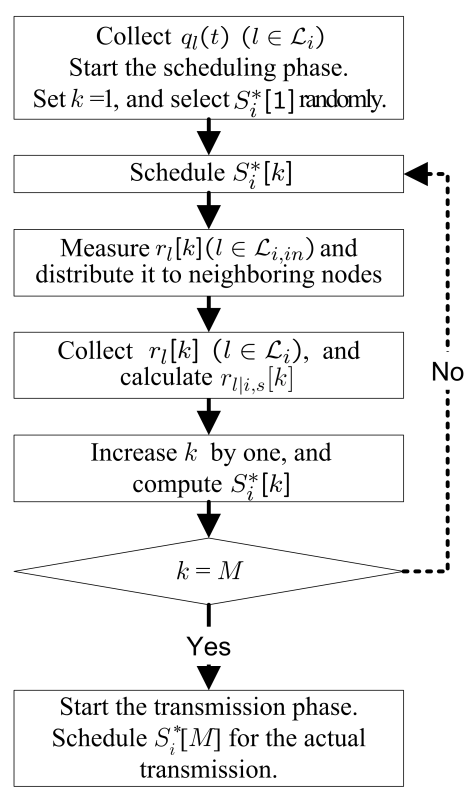


Fig. 1. The flow chart of the Locally Optimized Scheduling Algorithm (LOSA): node i's operation at each time slot

the scheduling phase, the transmission phase begins. In the transmission phase, each node i executes the decision $S_i[M]$, i.e., the decision from the last iteration of the scheduling phase. Then, the actual link capacities at each time slot t are determined by the link capacities $r_l[M]$ resulting from the decision $S_i[M]$'s, i.e., $r_l(t) = r_l[M]$. Upon completing transmission, each node updates the queue lengths of its outgoing links by

$$q_l(t+1) = \left[q_l(t) + \kappa \left(\sum_{u=1}^{U} I_l^u \lambda_u - r_l(t) \right) \right]^+, \quad (9)$$

where κ is a constant step size and

$$I_l^u = \begin{cases} 1, & \text{if data of user } u \text{ passes through link } l, \\ 0, & \text{otherwise.} \end{cases}$$

Recall that λ_u is the data rate of user u. After updating the queues, each node moves to the next time slot $t+1$. The flow chart of the locally-optimized scheduling algorithm (LOSA) is shown in Fig. 1.

In our algorithm, each node collects queue lengths and link capacities from its neighborhood, and carries out a fixed number of computations regardless of the network size. Thus, the complexity of our algorithm depend on the size of the neighborhood and the number of iterations M, not on the network size.

IV. Extension to the logarithmic SINR interference model

We now extend our algorithm to the logarithmic SINR interference model. Recall that the main idea of our algorithm is that each node makes its own scheduling and power control decision such that it is the best response (in terms of maximizing the expected queue-weighted-link-capacity sum) under the empirical distribution of neighboring nodes' actions. In Proposition 1, we showed that the best response for each node under the linear SINR interference model is of the form that a node either schedules only one outgoing link with maximum power, or schedules no link at all. The following proposition (Proposition 2) characterizes the condition on the best response for each node under the logarithmic SINR interference model when the following assumption holds.

***Assumption** 1:* For all nodes i and all links h, $G_{l_1 h} = G_{l_2 h}$ for all $l_1, l_2 \in \mathcal{L}_{i,out}$. In other words, the channel gain between two links depends only on the transmitting node and the receiving node.

***Proposition** 2:* The optimal solution to (5) is given by the solution to the following optimization problem when Assumption 1 is satisfied:

$$\max_{\vec{p} \in \mathbb{R}_0^{L_{i,out}}} \quad \sum_{l \in \mathcal{L}_i} q_l E[r_l | \vec{p_i} = \vec{p}]$$
$$\text{subject to} \quad \vec{1}^T \vec{p} \le \mathbb{P}_{i,\max}$$

where $\mathbb{R}_0^{L_{i,out}}$ is the set of $L_{i,out}$-dimensional vectors such that at most one component is nonzero and the others are zero, and the expectation is taken with respect to the distribution of neighboring nodes' scheduling decisions.

The detailed proof is provided in Appendix II. From Proposition 2, given the distribution of neighboring nodes' scheduling decisions, the best response for each node i under the logarithmic SINR interference model is either to choose one link to transmit or not to transmit at all. However, note that in Proposition 2 the best response could be to transmit at an intermediate power level instead of always at full power as in Proposition 1. Hence, we now need to take power control into account. We define the decision vector of node i at time slot t as,

$$\vec{D}_i(t) \triangleq [S_i(t), \mathbb{P}_i(t)] \quad (10)$$

for $S_i(t) \in \{1, \cdots, L_{i,out}\}$ and $\mathbb{P}_i(t) \in [0, \mathbb{P}_{i,\max}]$. The decision vector can be viewed as a generalization of the scheduling decision in the previous section. The first component of the decision vector, $S_i(t)$, denotes the scheduling decision of node i as in Section III, and the second component, $\mathbb{P}_i(t)$, denotes the transmission power of node i at time slot t. Therefore, $\vec{D}_i(t) = [s, p]$ corresponds the decision that node i schedules the s-th outgoing link with power p. Note that the case of 'scheduling no links', i.e., '$S_i(t) = 0$', can be represented by '$\mathbb{P}_i(t) = 0$'.

We now extend the local optimization problem in (3) to the logarithmic SINR interference model as follows,

Extended Local Optimization: each node i searches its decision vector $\vec{D}_i = [s, p]$ that solves the following optimization problem,

$$\max_{s \in \{1, \cdots, L_{i,out}\},\, p \in [0, \mathbb{P}_{i,\max}]} E\left[\sum_{l \in \mathcal{L}_i} r_l q_l \middle| \vec{D}_i = [s, p] \right], \quad (11)$$

where the expectation is taken with respect to some empirical distribution of the other links' decisions in $\mathcal{L}_i$. The extended local optimization also has the following features similar to (3): each node updates only local queue lengths, maximizes only the local queue-weighted-link-capacity sum, and chooses its own decision independently.

In order to find the optimal $\vec{D}_i$, we first find the locally optimal power assignment, $\mathbb{P}_{i,s}^*$, for each scheduling decision s. Next, we find the scheduling decision S_i^* that maximizes (11). Specifically, each node i finds $[\mathbb{P}_{i,1}^*, \mathbb{P}_{i,2}^*, \cdots, \mathbb{P}_{i,L_{i,out}}^*]$ such that

$$\mathbb{P}_{i,s}^* = \arg\max_{p \in [0, \mathbb{P}_{i,\max}]} \sum_{l \in \mathcal{L}_i} q_l E\left[r_l \middle| \vec{D}_i = [s, p] \right], \quad (12)$$

and then finds S_i^* such that

$$S_i^* = \arg\max_{s \in \{1, \cdots, L_{i,out}\}} \sum_{l \in \mathcal{L}_i} q_l E\left[r_l \middle| \vec{D}_i = [s, \mathbb{P}_{i,s}^*] \right]. \quad (13)$$

The locally-optimal decision vector of node i can be represented by $\vec{D}_i^* = [S_i^*, \mathbb{P}_{i,S_i^*}^*]$.

To obtain $\vec{D}_i^*$, each node needs to estimate $E[r_l | \vec{D}_i = [s, p]]$ for every link l in the neighborhood. We use the same method in Section III-C for estimating $E[r_l | \vec{D}_i = [s, p]]$. We assume that neighboring nodes' scheduling and power control decisions follow the empirical distribution from the past. Now consider the n-th iteration in a time slot. Let $\vec{D}_i[k] = [S_i[k], \mathbb{P}_i[k]]$ denote the decision vector of node i at a past iteration k. We also define the hypothetical link capacity under the logarithmic SINR interference model as follows; $r_{l|i,s,p}[k]$ is the capacity of link l when the decision vector of node i is $[s,p]$ and the decision vectors of other nodes are $\vec{D}_j[k]$ ($j \neq i$). Then, we can estimate $E[r_l|\vec{D}_i = [s,p]]$ at iteration n as an empirical average over the last W iterations, i.e.,

$$\bar{r}_{l|i,s,p}[n] \triangleq \frac{1}{W} \sum_{k=n-W}^{n-1} r_{l|i,s,p}[k]. \tag{14}$$

However, maintaining $r_{l|i,s,p}[k]$ over all values of p in the interval $[0, \mathbb{P}_{i,\max}]$ is not realistic since the required memory space is too large. We now show that each node i can maintain only $r_{l|i,s,\mathbb{P}_{i,\max}}[k]$ because the value of $r_{l|i,s,p}[k]$ for $p < \mathbb{P}_{i,\max}$ can be derived easily from $r_{l|i,s,\mathbb{P}_{i,\max}}[k]$. To see this, let l_s be the s-th outgoing link of node i. Recall that $r_{l|i,s,\mathbb{P}_{i,\max}}[k]$ is given by

$$r_{l|i,s,\mathbb{P}_{i,\max}}[k] = \tag{15}$$
$$\begin{cases} B\log\left(1+\frac{G_{ll}P_l[k]}{\sum_{h\in\mathcal{L}_i\setminus\{l,l_s\}} G_{hl}P_h[k]+G_{l_s l}\mathbb{P}_{i,\max}+\eta_l}\right), & \\ & \text{if } l \neq l_s, \\ B\log\left(1+\frac{G_{l_s l_s}\mathbb{P}_{i,\max}}{\sum_{h\in\mathcal{L}_i\setminus\{l\}} G_{hl}P_h[k]+\eta_l}\right), & \text{if } l = l_s. \end{cases}$$

Note that the formula for $r_{l|i,s,p}[k]$ is similar to (15) except that $\mathbb{P}_{i,\max}$ is replaced by p. It is then easy to see that

$$r_{l|i,s,p}[k] = \begin{cases} B\log\left(1+\dfrac{a_l[k]}{p+b_l[k]}\right), & \text{if } l \neq l_s, \\ B\log\left(1+c_l[k]p\right), & \text{if } l = l_s, \end{cases}$$

where $a_l[k] = \frac{G_{ll}P_l[k]}{G_{l_s l}}$, $b_l[k] = \frac{G_{ll}P_l[k]/G_{l_s l}}{\exp(\frac{r_{l|i,s,\mathbb{P}_{i,\max}}[k]}{B})-1} - \mathbb{P}_{i,\max}$, and $c_l[k] = \frac{\exp(\frac{r_{l|i,s,\mathbb{P}_{i,\max}}[k]}{B})-1}{\mathbb{P}_{i,\max}}$. Note that $a_l[k]$, $b_l[k]$, and $c_l[k]$ are readily obtainable because each node i is assumed to know the channel gains between its neighboring links and their transmission powers at each past iteration k.

Once each node can estimate $r_{l|i,s,p}[k]$, the power control problem (12) for each scheduling decision s at iteration n can be rewritten with the hypothetical link capacities as follows;

$$\mathbb{P}^*_{i,s}[n] = \underset{p\in[0,\mathbb{P}_{i,\max}]}{\arg\max} \sum_{l\in\mathcal{L}_i} \bar{r}_{l|i,s,p}[n-1]q_l. \tag{16}$$

The objective function of (16) is a continuous and differentiable function of p. Unfortunately, the function could have multiple local maximizers in the domain $[0, \mathbb{P}_{i,\max}]$. Thus, to solve (16), we need to check all local maximizers in the domain. Alternatively, we may approximate the solution by simply searching among K points $p_1,...,p_K$ where $p_k = \frac{(k-1)\mathbb{P}_{i,\max}}{K-1}$ and takes the point p_k that maximize (16) as an approximation of $\mathbb{P}^*_{i,s}[n]$. Once the locally-optimal power assignment $\mathbb{P}^*_{i,s}[n]$ is obtained for each scheduling decision s, we can find the locally optimal scheduling decision $S^*_i[n]$ in (13) and corresponding locally optimal power assignment $\mathbb{P}^*_i[n] = \mathbb{P}^*_{i,S^*_i[n]}[n]$.

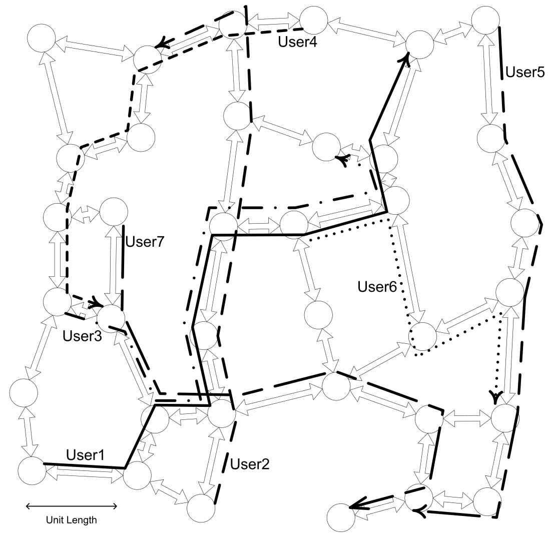


Fig. 2. The Network Topology and Data Flows

After the decision $\vec{D}_i[n] = [S^*_i[n], \mathbb{P}^*_i[n]]$ is made, each node i schedules the $S^*_i[n]$-th outgoing link with power $\mathbb{P}^*_i[n]$. Each node then measures the capacity of its incoming links and distributes $r_l[k]$ and $P_l[k]$ to its neighboring node. Each node can then compute and record $r_{l|i,s,\mathbb{P}_{i,\max}}[k]$ with $r_l[k]$ and $P_l[k]$. Note that each node i needs to collect $P_l[k]$ as well as $r_l[k]$ from the neighborhood to calculate $r_{l|i,s,\mathbb{P}_{i,\max}}[k]$ under the logarithmic SINR interference model. The rest of the extended locally optimized scheduling algorithm is the same as in III-D.

V. Simulation Results

In this section, we provide simulation results to evaluate the performance of the locally optimized scheduling algorithm (LOSA) and the extended locally optimized scheduling algorithm (exLOSA). We first study how the performance of LOSA is influenced by the size of the neighborhood, $\mathcal{L}_i$, and the number of iterations, M. We use the network topology with 36 nodes and seven users, as shown in Fig. 2. We assume that data arrivals at source nodes are at a constant rate λ. The channel gain from the transmitter of link h to the receiver of link l is given by $G_{hl} = d_{hl}^{-\alpha}$ where d_{hl} is the distance from node $b(h)$ to node $e(l)$, and α is the attenuation factor. The system parameters used for our simulations are given in Table I.

A. Linear SINR interference model

We first simulate LOSA and the optimal scheduling algorithm (OPT) under the linear SINR interference model. Note that the optimal algorithm (OPT) solves the global optimization problem (2). Let $\mathcal{L}_{i,1}$ denote the one-hop neighborhood of node i, defined as $\mathcal{L}_{i,1} = \mathcal{L}_{i,in} \cup \mathcal{L}_{i,out}$. Then, we can define the k-hop neighborhood inductively as $\mathcal{L}_{i,k} =$

TABLE I

SYSTEM PARAMETERS

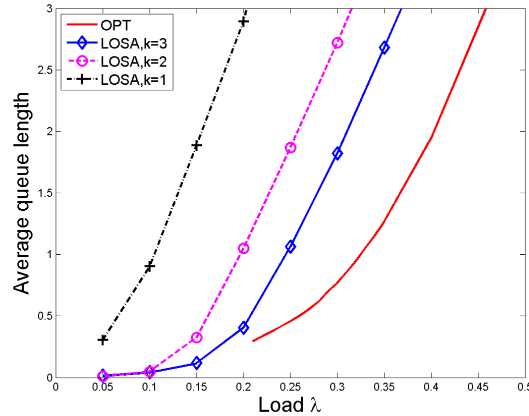
Parameters	Values
Background Noise (η_l)	0.1
Maximum Power ($\mathbb{P}_{i,\max}$)	1
Attenuation Factor (α)	4
Bandwidth (B)	1
Number of Past Records (W)	5
Number of Time Slots	2000
Step size (κ)	0.1
Unit length	1

$\cup_{l\in\mathcal{L}_{i,k-1}}(\mathcal{L}_{b(l),1}\cup\mathcal{L}_{e(l),1})$. In order to model practical constraints that nodes cannot receive while it is transmitting, and nodes cannot receive from multiple transmitters simultaneously, we set $G_{hl}=\infty$ if $b(h)=e(l)$ or $e(h)=e(l)$.

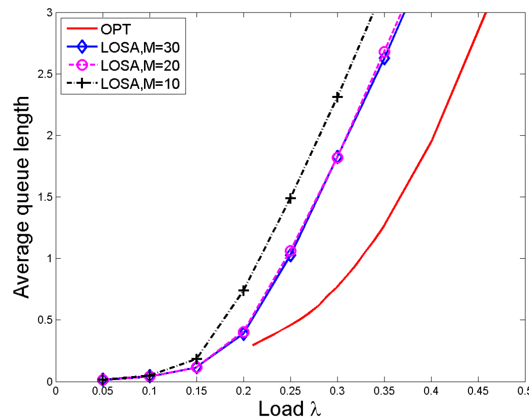
In Figs. 3(a) and 3(b), each curve illustrates the average queue length over all links for a given scheduling algorithm. Note that the scheduling algorithms that result in curves to the right can carry a larger load and therefore have better performance.

Fig. 3(a) shows the relationship between the size of neighborhood and performance. Each curve labeled with 'LOSA' corresponds to the results from LOSA where $\mathcal{L}_i$ consists of links in each k-hop neighborhood (i.e., $\mathcal{L}_i \triangleq \mathcal{L}_{i,k}$) for $k = 1,2,3$, respectively. In this simulation, we set the number of iterations in each time slot $M = 30$. From the simulation result, we can see that the algorithm using the larger neighborhood has a larger throughput.

Next, Fig. 3(b) shows the relationship between the number of iterations and performance. Each curve labeled with 'LOSA' corresponds to the results from LOSA where the total number of iterations is $M = 10, 20, 30$, respectively. We choose $\mathcal{L}_i \triangleq \mathcal{L}_{i,3}$ in this simulation. From the figure, we can observe that all values of M results into a throughput that is reasonably close to the optimal.



(a) Different k-hop neighborhood



(b) Different number of iterations M

Fig. 3. The average queue length under the linear SINR interference model as we vary (a) the size of neighborhood with $M = 30$ and (b) the number of iterations with $k = 3$

B. Logarithmic SINR interference model

We now compare the performance of exLOSA to that of the other algorithms developed under the logarithmic SINR interference model. For this comparison, we select the following two scheduling and power control algorithms from the literature: JOCP algorithm [11] and a distributed approximation algorithm , approx-DRPC, to the optimal DRPC algorithm in [3]. JOCP in [11] solves the global optimization problem (2) under the assumption that the SINR values of all links are high, i.e., $r_l(t) = B\log(1+\xi_l(t)) \approx B\log(\xi_l(t))$. Note that this high-SINR assumption excludes the need for scheduling in JOCP. Hence, the solution for JOCP will force all links to be active simultaneously at some optimized power levels. In [3], after developing the optimal DRPC algorithm (which is centralized with high computational complexity), the authors also proposed a distributed heuristic algorithm for approximating DRPC, where each node randomly decides to be a transmitter with a given probability and then decide which node it should transmit to using the feedback from adjacent receiving nodes. Finally, in order to study the improvement due to the power control capability of exLOSA, we also simulate LOSA under the logarithmic SINR interference model. In the rest of our simulations, we set $M = 30$ and $\mathcal{L}_i \triangleq \mathcal{L}_{i,3}$.

We first compare the performance of these algorithms under the same physical constraint, i.e., $G_{hl}=\infty$ if $b(h)=e(l)$ or $e(h)=e(l)$. Fig. 4 shows the average queue length of these algorithms at different loads λ. From Fig. 4, we can see that exLOSA performs significantly better than JOCP and approx-DRPC. The performance improvement over approx-DRPC is expected because our algorithm uses a more sophisticated procedure to choose transmission patterns than approx-DRPC. The performance improvement over JOCP is because JOCP is developed under the high-SINR assumption, which is often violated in practice. There are two reasons why the high-SINR assumption may be violated. The first is that an outgoing link from a node can create excessive self-interference to the incoming links. Since in JOCP the optimal solution is to operate all links simultaneously, this self-interference will lead to low-SINR values. To verify this, we intentionally modify the simulation setting such that the nodes are able to receive from multiple transmitters and to receive while

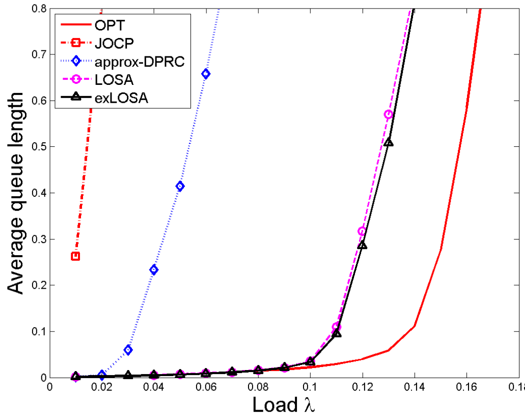


Fig. 4. The average queue length under the logarithmic SINR interference model when nodes are unable to receive while transmitting and are unable to receive from multiple transmitters simultaneously

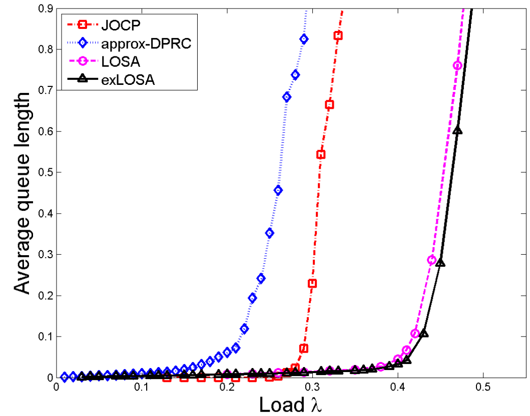


Fig. 6. The average queue length under the modified logarithmic SINR interference model ($\theta = 9$) when nodes are able to receive while transmitting and are able to receive from multiple transmitters simultaneously

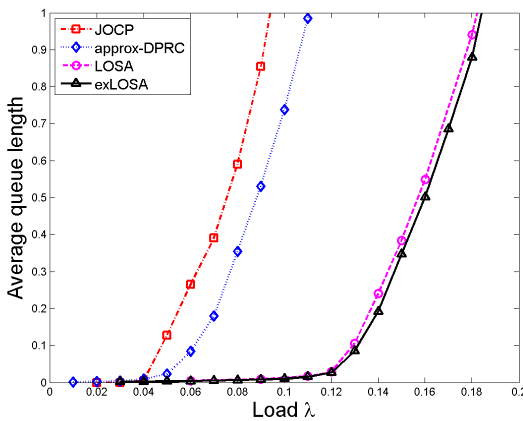


Fig. 5. The average queue length under the logarithmic SINR interference model when nodes are able to receive while transmitting and are able to receive from multiple transmitters simultaneously

it is transmitting at the same time. In particular, we alter the channel gain such that $G_{hl} = 0$ if $b(h) = e(l)$, and $G_{hl} = d_{hl}^{-\alpha}$ otherwise. Fig. 5 shows that the performance of JOCP improves under the new setting. However, even without self-interference, the performance of JOCP is impacted by the second reason, i.e., the optimized power levels of JOCP may still result into low SINR values at some links. To verify this, we modify the logarithmic SINR interference model: the capacity of link l is determined by $r_l(t) = B\log(1 + \theta\xi_l(t))$ where θ is an artificial factor. We expect that, when θ is large, the high SINR assumption would be more accurate, i.e., $r_l(t) \approx B\log(\theta\xi_l(t))$. The result when $\theta = 9$ is provided in Fig. 6. From the simulation result, JOCP now performs better than approx-DRPC. However, there is still a significant performance gap between JOCP and our algorithms. This is because, even under the modified setting, the solution of JOCP cannot preclude low-SINR on some links. These simulations suggest that exLOSA and LOSA perform well under a variety of different settings.

Another remark on the simulation results is that exLOSA performs better than LOSA, but the difference in throughput is not large. These results suggest that applying LOSA without power control is perhaps good enough for the logarithmic SINR interference model.

VI. Conclusion

In this paper, we have studied the locally optimized scheduling algorithm and its extended version for the linear and the logarithm SINR interference models, respectively. The key feature of these algorithms is that they exploit the empirical distribution of neighboring nodes' actions to predict the best response (i.e., the scheduling decision). Under the linear SINR interference model, we have shown that the best response is either to select one of the node's outgoing links to schedule with maximum power or to select no links at all. Similarly, we have also shown that the best response under the logarithmic SINR interference model is either to select one outgoing link to schedule with some positive power or to schedule no links at all. The simulation results demonstrate the performance improvement of our algorithms over the state-of-the-art distributed algorithms under different scenarios.

Appendix I
Proof of Proposition 1

To prove this result, we use the idea in [2]. We first show that, for any decision of node i that schedules more than one outgoing links with positive power, there exists another decision that schedules one less link with positive power that can achieve a larger value of the objective function. Let l_1 and l_2 be two of n outgoing links that node i schedules with positive power. Let P_0 denote the sum of the transmission power of links l_1 and l_2. We let $P_{l_1} = x$, then $P_{l_2} = P_0 - x$, where $0 \leq x \leq P_0$ and $0 \leq P_0 \leq \mathbb{P}_{i,\max}$. Each node is assumed to know the empirical distribution of the other nodes' actions in the neighborhood. In other words, node i knows the power allocation $P_l[k]$ and the link capacity $r_l[k]$ of each neighboring link l ($l \in \mathcal{L}_i$) at the k-th past sample. Then, the objective function (3) under the linear SINR interference model can be expressed as follows,

$$\begin{aligned} & E\left[\sum_{l\in\mathcal{L}_i} r_l q_l \middle| P_{l_1}=x, P_{l_2}=P_0-x\right] \\ = & \frac{B\,q_{l_1}}{K}\sum_{k=1}^{K}\frac{G_{l_1l_1}x}{I_{l_1}[k]+G_{l_2l_1}(P_0-x)+\eta_{l_1}} \\ + & \frac{B\,q_{l_2}}{K}\sum_{k=1}^{K}\frac{G_{l_2l_2}(P_0-x)}{I_{l_2}[k]+G_{l_1l_2}x+\eta_{l_2}} \\ + & \sum_{l\in\mathcal{L}_i\setminus\{l_1,l_2\}}\frac{B\,q_l}{K} \\ \times & \sum_{k=1}^{K}\frac{G_{ll}P_l[k]}{I_l[k]+G_{l_1l}x+G_{l_2,l}(P_0-x)+\eta_l}, \end{aligned} \tag{17}$$

where $I_l[k]$ is the total interference received by link l at the k-th past sample, excluding the interference from links l_1 and l_2.

The functions of the forms $\frac{ax}{b-x}$, $\frac{a-x}{b+x}$, $\frac{a}{b+x}$, and $\frac{a}{b-x}$ ($a,b \geq 0$) are all convex with respect to x. Hence, it is easy to show that (17) is strictly convex with respect to a single variable x in $[0, P_0]$. Thus, in order to maximize (17), x should be either 0 or P_0. This means that for given P_{l_s} ($s \neq 1,2$), $P_{l_1}=0$ and $P_{l_2}=P_0$, or $P_{l_1}=P_0$ and $P_{l_2}=0$. Thus, node i should schedule either link l_1 or link l_2. This result can be iteratively applied to the remaining set of outgoing links. Thus, this implies that each node should schedule only one outgoing link.

We then show that the transmission power on the scheduled link should be either 0 or $\mathbb{P}_{i,\max}$. Suppose that node i schedules one of its outgoing link denoted by l_1 with transmission power x, i.e., $P_{l_1}=x$ ($0 \leq x \leq \mathbb{P}_{i,\max}$). Then, the objective function (3) under the linear SINR interference model can be expressed as follows,

$$\begin{aligned} & E\left[\sum_{l\in\mathcal{L}_i} r_l q_l \middle| P_{l_1}=x,\ P_{l'}=0\ \ \forall l'\in\mathcal{L}_{i,out}\setminus\{l_1\}\right] \\ = & \frac{B\,q_{l_1}}{K}\sum_{k=1}^{K}\frac{G_{l_1l_1}x}{I'_{l_1}[k]+\eta_{l_1}} \\ + & \sum_{l\in\mathcal{L}_i\setminus\mathcal{L}_{i,out}}\frac{B\,q_l}{K}\sum_{k=1}^{K}\frac{G_{ll}P_l[k]}{I'_l[k]+G_{l_1l}x+\eta_l} \end{aligned} \tag{18}$$

where $I'_l[k]$ is the total interference received by link l at the k-th past sample, excluding the interference from link l_1. Note that (18) is also strictly convex with respect to x. Thus, in order to maximize (18), x should be either 0 or $\mathbb{P}_{i,\max}$. This means that node i should schedule one of its outgoing links with full power or not schedule any link at all. The result of Proposition 1 then follows.

APPENDIX II
PROOF OF PROPOSITION 2

We use similar ideas as in Appendix I. We only need to show that, for any decision of node i that schedules more than one outgoing links with positive power, there exists another decision that schedules one less link with positive power that can achieve a larger value of the objective function. Let l_1 and l_2 be two of n outgoing links that node i schedules with positive power. Let P_0 denote the sum of the transmission power of links l_1 and l_2. We let $P_{l_1}=x$, then $P_{l_2}=P_0-x$, where $0 \leq x \leq P_0$ and $0 \leq P_0 \leq \mathbb{P}_{i,\max}$. Each node is assumed to know the empirical distribution of the other nodes' actions in the neighborhood. In other words, node i knows the power allocation $P_l[k]$ and the link capacity $r_l[k]$ of each neighboring link l ($l \in \mathcal{L}_i$) at the k-th past sample. Then, the objective function (5) under the logarithmic SINR interference model can be expressed as follows,

$$\begin{aligned} & E\left[\sum_{l\in\mathcal{L}_i} r_l q_l \middle| P_{l_1}=x, P_{l_2}=P_0-x\right] \\ = & \frac{B\,q_{l_1}}{K}\sum_{k=1}^{K}\log\left(1+\frac{G_{l_1l_1}x}{I_{l_1}[k]+G_{l_2l_1}(P_0-x)+\eta_{l_1}}\right) \\ + & \frac{B\,q_{l_2}}{K}\sum_{k=1}^{K}\log\left(1+\frac{G_{l_2l_2}(P_0-x)}{I_{l_2}[k]+G_{l_1l_2}x+\eta_{l_2}}\right) \\ + & \sum_{l\in\mathcal{L}_i\setminus\{l_1,l_2\}}\frac{B\,q_l}{K} \\ \times & \sum_{k=1}^{K}\log\left(1+\frac{G_{ll}P_l[k]}{I_l[k]+G_{l_1l}x+G_{l_2,l}(P_0-x)+\eta_l}\right), \end{aligned} \tag{19}$$

where $I_l[k]$ is the interference received by link l at the k-th past sample, excluding the interference from link l_1 and l_2.

Let $f_{l_1,k}(x) \triangleq \log\left(1+\frac{G_{l_1l_1}x}{I_{l_1}[k]+G_{l_2l_1}(P_0-x)+\eta_{l_1}}\right)$, $f_{l_2,k}(x) \triangleq \log\left(1+\frac{G_{l_2l_2}(P_0-x)}{I_{l_2}[k]+G_{l_1l_2}x+\eta_{l_2}}\right)$, and $f_{l,k}(x) \triangleq \log\left(1+\frac{G_{ll}P_l[k]}{I_l[k]+G_{l_1l}x+G_{l_2,l}(P_0-x)+\eta_l}\right)$. Then, (19) can be rewritten as follows,

$$\begin{aligned} & E\left[\sum_{l\in\mathcal{L}_i} r_l q_l \middle| P_{l_1}=x\right] \\ = & \frac{B}{K}\sum_{k=1}^{K}\Bigg(q_{l_1}f_{l_1,k}(x)+q_{l_2}f_{l_2,k}(x) \\ & \qquad + \sum_{l\in\mathcal{L}_i\setminus\{l_1,l_2\}} q_l f_{l,k}(x)\Bigg). \end{aligned} \tag{20}$$

From Assumption 1, $G_{l_1l_1}=G_{l_2l_1}$ and $G_{l_1l_2}=G_{l_2l_2}$ if $b(l_1)=b(l_2)$. Hence, $f_{l_1}(x)$ and $f_{l_2}(x)$ can be simplified to

$$f_{l_1,k}(x)=\log\left(\frac{a_1[k]}{a_1[k]-x}\right)$$

and

$$f_{l_2,k}(x) = \log\left(\frac{b_2[k]}{a_2[k]+x}\right),$$

where $a_1[k] = I_{l_1}[k]/G_{l_1l_1} + P_0 + \eta_{l_1}/G_{l_1l_1}$, $a_2[k] = I_{l_2}[k]/G_{l_2l_2} + \eta_{l_2}/G_{l_2l_2}$, and $b_2[k] = a_2[k] + P_0$. Note that $a_1[k] - x > 0$ and $a_2[k] + x > 0$ since $\eta_l > 0$ for $\forall l \in \mathcal{L}$ and $x \le P_0$. Both $f_{l_1,k}(x)$ and $f_{l_2,k}(x)$ are strictly convex on $[0, P_0]$. Similarly, since $G_{l_1l} = G_{l_2l}$, $f_{l,k}(x)$ is independent of x. Therefore, (20) is strictly convex with respect to a single variable x in $[0, P_0]$. Thus, in order to maximize (20), x should be either 0 or P_0. This means that for given P_{l_s} $(s \neq 1, 2)$, $P_{l_1} = 0$ and $P_{l_2} = P_0$, or $P_{l_1} = P_0$ and $P_{l_2} = 0$. Thus, node i should schedule either link l_1 or link l_2. This result can be iteratively applied to the remaining set of outgoing links. Thus, this implies that each node should schedule only one outgoing link. The result of Proposition 2 then follows.

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