Low-Complexity and Distributed Energy Minimization in Multi-hop Wireless Networks

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Abstract—In this work, we study the problem of minimizing the total power consumption in a multi-hop wireless network subject to a given offered load. It is well-known that the total power consumption of multi-hop wireless networks can be substantially reduced by jointly optimizing power control, link scheduling, and routing. However, the known optimal cross-layer solution to this problem is centralized, and with high computational complexity. In this paper, we develop a low-complexity and distributed algorithm that is provably power-efficient. In particular, under the node exclusive interference model and with suitable assumptions on the power-rate function, we can show that the total power consumption of our algorithm is at most \((2 + \epsilon)\)-times as large as the power consumption of the optimal (but centralized and complex) algorithm, where \(\epsilon\) is an arbitrarily small positive constant. Our algorithm is not only the first distributed algorithm with provable performance bound, but its power-efficiency ratio is also tighter than that of another sub-optimal centralized algorithm in the literature.

Index Terms—Energy Aware Routing, Duality, Mathematical Programming/Optimization, Cross-Layer Optimization, Simulations

I. INTRODUCTION

There has been significant recent interest in developing control protocols for multi-hop wireless networks. Many applications can benefit from the deployment of these networks. For instance, sensors can form multi-hop wireless sensor networks [2] for a variety of applications, such as habitat monitoring and the management of sewer overflow events [3]. Vehicles can form multi-hop wireless networks to exchange safety messages and traffic information [4]. Wireless LAN devices can form multi-hop mesh networks to provide wireless broadband access [5].

A key issue in developing control protocols for multi-hop wireless networks is to reduce the energy or power consumption. This is obviously an important issue for battery-powered networks since the power consumption often limits the lifetime of the network. Even for networks with access to power sources, the transmission power of the communication links may still need to be properly controlled, e.g., due to health or regulatory concerns.

In this work, we are interested in the problem of minimizing the total power consumption of a multi-hop wireless network, subject to a given offered load. It is well-known that the total power consumption of multi-hop wireless networks can be substantially reduced by jointly optimizing power control, link scheduling, and routing. However, known optimal solutions require centralized computation and high computational complexity. In this paper, we propose a new low-complexity and distributed solution to this problem under a widely-used interference model, called the node-exclusive interference model. Using this model, the work in [6] developed a centralized solution that yielded a 3-approximation ratio (i.e., the resultant power consumption is within a factor of 3 from the optimal power consumption). In contrast, in this paper, we will obtain a \((2 + \epsilon)\)-approximation algorithm that is fully distributed, where \(\epsilon > 0\) is an arbitrarily small constant. To the best of our knowledge, our proposed algorithm is the first distributed solution in the literature with a provable performance bound.

Our solution approach is inspired by the recent progress in using imperfect scheduling algorithms to develop distributed cross-layer congestion control and scheduling algorithms in multi-hop wireless networks [7]–[9]. We first formulate the energy minimization problem into a special form that naturally leads to a distributed solution. We then map the solution to corresponding components of the cross-layer control protocol, and rigorously establish the stability and power-efficiency of the protocol.

Our work is also related to the study of energy-aware routing protocols for minimizing energy consumption and extending network lifetime [10]–[14]. These works assume that the system capacity is battery-limited instead of interference-limited and therefore do not consider scheduling constraints. In contrast, our work explicitly considers scheduling, jointly with power control and routing.

The intellectual contribution of this work is summarized as follows:

- We develop a low-complexity and distributed joint routing, power control, and scheduling algorithm for multi-hop wireless networks with provable power-efficiency ratio. Further, our algorithm can guarantee a better power-efficiency level than some existing centralized algorithms.
- To the best of our knowledge, our solution cannot be obtained by extending the known optimal solution in the literature [15], [16]. Instead, we develop an optimization approach to the energy minimization problem that naturally leads to distributed solutions. We also develop
rigorous techniques for proving the stability and power-efficiency of the resulted control protocol.

The rest of this paper is organized as follows: In Section II, we present the system model and formulate the energy minimization problem. We present the solution in Section III, and discuss how to map the algorithm to different network protocol components in Section IV. In Section V, we present our main analytical results on the stability and power-efficiency of the proposed protocol. Numerical results are provided in Section VI. Then, we conclude in Section VII.

II. PROBLEM FORMULATION

We model a wireless multi-hop network by a directed graph $G(V, E)$, where $V$ is the set of vertices representing the nodes, and $E$ is the set of edges representing the communication links. We use $N_o(v)$ and $N_i(v)$ to denote the sets of outgoing and incoming links of node $v$, respectively. Their union $N(v)$ forms the set of all links incident on node $v$.

The system is time-slotted. We adopt the following node-exclusive interference model that is used to characterize FH-CDMA and UWB system with perfect orthogonal spreading codes and low power-spectrum density [8], [17]–[19]. Under this model, a node can only receive from or transmit to at most one node at any time-slot $m$. Further, each link is power-controlled. That is, if the node-exclusive interference constraint is satisfied, we assume that the possible data rate $R_e$ of link $e$ is a function of its own power assignment $p_e$. We use $p_e = h_e(R_e)$ to denote the power consumption for supporting data rate of $R_e$ on link $e$. For every link $e$, it is assumed that $h_e(\cdot)$ is a non-decreasing and convex function on $[0, a_e]$ satisfying $h_e(0) = 0$, where $a_e$ is the maximum rate supported on link $e$. An example of $h_e(\cdot)$ is the power-rate relationship in an Additive White Gaussian Noise (AWGN) channel.

Each packet may take multiple hops to be delivered from source to destination. Let $T_{vd}$ denote the long-term average data rate of the flow that needs to be supported from source node $v$ to destination node $d$. We use $D$ to denote the set of destinations.

The joint energy minimization problem is now formulated as follows:

\[ (\ast) \min_{f, R} \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} \sum_{e \in E} h(R_e(m)), \]  

subject to

\[ R_e(m) \leq a_e \quad \text{for all} \ e \ \text{and} \ m, \]  

\[ \hat{R}(m) \]  

satisfies the node-exclusive constraint for all time-slots $m$,  

\[ \sum_{e \in D} f_e^d = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} R_e(m), \]  

for all links $e$,  

\[ \sum_{e \in N_o(v)} f_e^d - \sum_{e \in N_i(v)} f_e^d - T_{vd} \geq 0, \]  

for all $d \in D$ and nodes $v \neq d$,  

where $R_e(m), m = 1, 2, \ldots$ is the rate assigned to link $e$ at time slot $m$, $\hat{R}(m) = [R_e(m)]$, $f = [f_e^d]$ and the quantity $f_e^d$ can be interpreted as the average data rate on link $e$ allocated for destination $d$. The objective function in (1) corresponds to the long-term average energy consumed by all links. The constraints in (3) require that the long-term average data rate, determined by the power allocation, should be able to support the total average data rate ($\sum_{e \in D} f_e^d$) on each link. The constraints in (4) require that the total outgoing flow of a node should be able to support the total incoming flow plus the locally-generated flow, for all destinations. In the rest of the paper, we will refer to the above problem as Problem ($\ast$).

III. SOLUTION METHODOLOGY

A. Approximating the Energy Minimization Problem

The optimal solutions developed in [15], [16] could be used to solve Problem ($\ast$). However, their solutions contain a scheduling component with high computational complexity. In order to compute at which power and at what time each link $e$ should be activated, these solutions need to solve a complex global optimization problem in each time-slot.

In this paper, in order to obtain a low-complexity and distributed solution, we take a different approach. We first approximate ($\ast$) by another optimization problem that is easier to solve. The following Lemma [6] provides the first step in this direction.

Lemma 1: There exists a power-optimal solution that solves Problem ($\ast$) such that for all time-slots $m$ when link $e$ is activated, the instantaneous data rate $R_e(m)$ is independent of $m$.

Lemma 1 follows from the convexity of the function $h_e(\cdot)$ [6]. According to this lemma, we only need to consider those solutions for which there exists a single value $R_e$, such that $R_e(m) = R_e$ holds for all time-slots $m$ when link $e$ is activated.

As a result, $\lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} R_e(m)$ is equal to the product of $R_e$ and the fraction of time that link $e$ is activated. Therefore, by (3), the objective function of Problem ($\ast$) can now be written as

\[ \sum_{e \in E} \frac{\sum_{d \in D} f_e^d}{R_e} h_e(R_e), \]

where $\frac{\sum_{d \in D} f_e^d}{R_e}$ is the fraction of time-slots that link $e$ is activated.

Further, using the results from low-complexity scheduling [7]–[9], we have

- **Fact 1:** In the optimal solution to ($\ast$), we must have
  \[ \sum_{e \in N(v)} \frac{\sum_{d \in D} f_e^d}{R_e} \leq 1, \]  
  for all nodes $v \in V$.

- **Fact 2:** Under the node-exclusive interference model, if
  \[ \sum_{e \in N(v)} \frac{\sum_{d \in D} f_e^d}{R_e} \leq \frac{1}{2} - \eta, \]  
  for all nodes $v \in V$.
where $\eta > 0$ is an arbitrarily small positive constant, then a maximal schedule [7]-[9] can be computed such that each link is activated for $\frac{\sum_{d \in D} f_e^d}{R_e}$ fraction of time-slots. We will discuss more about the role of maximal scheduling in our solution in Section IV-B.

Based on these two facts, in the rest of the paper, we will replace the scheduling constraints (2) by

$$\sum_{e \in N(v)} \sum_{d \in D} \frac{f_e^d}{R_e} \leq \beta, \text{ for all } v \in V.$$  \hspace{1cm} (5)

Problem (*) can then be approximated by the following problem:

$$\begin{align*}
\min_{f, t} & \quad \sum_{e \in E} \sum_{d \in D} \frac{f_e^d}{R_e} n_e(R_e), \\
\text{subject to} & \quad (4) \text{ and } (5), \\
& \quad (f, t) \in X,
\end{align*}$$

where $X = \{(f, t) : 0 \leq R_e \leq a_e, f_e^d \geq 0, \text{ for all links } e \text{ and destinations } d\}$. The formulation in (A) is not only easier to solve, but it also produces natural bounds for proving the power efficiency ratio of our solution. Indeed, solving (A) with $\beta = 1$ provides a lower bound on the minimum power of Problem (*), while $\beta = \frac{1}{2} - \eta, \eta > 0$ provides an upper bound.

In the rest of the paper, we assume that Problem (A) is strictly feasible for some $\beta = 1/2 - \eta$, i.e., there exists $(f, t) \in X$ such that the constraints (4) and (5) are satisfied with strict inequality. Note that in practice this assumption can easily be satisfied by picking the maximum data rate $a_e$ to be sufficiently large.

Remark: Problem formulation (A) also appears in [6]. However, our solution is different from this point on. As we mentioned earlier, their solution is a centralized one with an approximation ratio of 3, while our solution is a distributed one with a better approximation ratio.

B. Handling the Non-Convexity

In Problem (A), the objective function and the constraint (5) are non-convex. Problems of this type are considered to be difficult in general. To overcome this difficulty, the following change of variable is performed:

$$t_e = \sum_{d \in D} \frac{f_e^d}{R_e}, \text{ for all links } e \in E.$$ 

The parameter $t_e$ can be interpreted as the fraction of time-slots for which link $e$ is activated. Due to the constraint (5), there is no loss of optimality by assuming that $t_e \leq 1$ for each link $e$.

Later in Section IV-A, we will interpret $t_e$ also as the offered load on link $e$. The latter interpretation will become more appropriate in Section IV-A when we deal with $t_e(m)$ for each time-slot in the dual solution. With this change of variable, we can denote the long-term average power consumption of link $e$ as a function of $t_e$ and $f_e^d = [f_e^d, d \in D]$, i.e., $\Theta_e(f_e^d, t_e) = t_e h_e\left(\sum_{d \in D} \frac{f_e^d}{R_e}\right)$, for $t_e > 0$. To define the value of the function $\Theta_e$ for $t_e = 0$, note that $\sum_{d \in D} f_e^d/t_e = R_e \leq a_e$. Hence, the only feasible point when $t_e = 0$ is when $\sum_{d \in D} f_e^d$ is also equal to 0. Let

$$Y_e = \{(f_e^d, t_e) : f_e^d \geq 0, \text{ for all } d \in D; 0 \leq t_e \leq 1; \sum_{d \in D} f_e^d \leq a_e t_e\}.$$  \hspace{1cm} (7)

We can then define the function $\Theta_e(f_e^d, t_e)$ on the domain $Y_e$ as

$$\Theta_e(f_e^d, t_e) = \begin{cases} 0, & t_e = 0, \\
\sum_{d \in D} f_e^d/t_e, & 0 < t_e \leq 1. \end{cases}$$  \hspace{1cm} (8)

Note that this definition ensures that the function $\Theta_e$ is continuous on its domain.

Let $\tilde{t} = [t_e, e \in E]$, and let $Y$ denote the Cartesian product of $Y_e$ for all $e$, i.e.,

$$Y = \{(f_e^d, t_e) : f_e^d \geq 0, \text{ for all edges } e \text{ and destinations } d; 0 \leq t_e \leq 1, \sum_{d \in D} f_e^d \leq a_e t_e, \text{ for all edges } e\}.$$ 

Using the above notation, Problem (A) can be transformed into

$$\begin{align*}
\min_{\tilde{f}, \tilde{t}} & \quad \sum_{e \in E} \Theta_e(\tilde{f}_e, \tilde{t}_e), \\
\text{subject to} & \quad \sum_{e \in N(v)} \tilde{t}_e \leq \beta, \text{ for all nodes } v \in V, \\
& \quad \sum_{e \in N_e(v)} f_e^d - \sum_{e \in N_e(v)} f_e^d - T_{vd} \geq 0, \\
& \quad \text{for all } d \in D \text{ and nodes } v \neq d, \\
& \quad (\tilde{f}, \tilde{t}) \in Y.
\end{align*}$$

We emphasize that Problem (A) and Problem (B) are equivalent. We now show that Problem (B) is a convex program. We need the following lemma.

Lemma 2: Assume that $h(r)$ is a convex function on $r \geq 0$. Let

$$\theta(f, t) = \begin{cases} 0, & t = 0, \\
\frac{f}{a_t}, & t > 0. \end{cases}$$  \hspace{1cm} (12)

Then $\theta(f, t)$ is also convex on the domain $C = \{(f, t) : 0 \leq f \leq a_t, 0 \leq t \leq 1\}$.

Proof: First, note that a function of the form $g(f, t) = \theta(f, t)$, where $t > 0$, is known as the perspective of function $h(r)$. As shown in [20, Chapter 3.2.6, p89], the perspective is one of the transformations that preserve convexity. Hence, given that the function $h(r)$ is convex on $r \geq 0$, the function $\theta(f, t)$ must be convex on $C' = \{(f, t) : f + T_{vd} \leq a_t, 0 < t \leq 1\}$.

To deal with the case when $t = 0$, note that the only point of the set $C$ that is not in $C'$ is $(f, t) = (0, 0)$. By setting $\theta(0, 0) = 0$, the function $\theta(f, t)$ is continuous on the domain $C$ (including the point $(0, 0)$). Hence, the function $\theta(f, t)$ must be convex on the domain $C$.

From Lemma 2, each term $\Theta_f(f_e^d, t_e)$ in the objective function of Problem (B) is convex, and hence the entire problem is a convex program. We can then use the following duality approach to solve the problem.
C. Distributed Algorithm Based on Lagrange Duality

To use the duality approach to solve Problem (B), we first form the Lagrangian:
\[
L(\tilde{f}, \tilde{t}, \bar{\mu}, \bar{q}) = \sum_{e \in E} \Theta_e(\tilde{f}_e, t_e) + \sum_{v \in V} \mu_v \left( \sum_{e \in \mathcal{N}(v)} t_e - \beta \right) + \sum_{e \in V} \sum_{d \in D} q^d_e \left( \sum_{e \in \mathcal{N}_v(d)} f^d_e + T_{vd} - \sum_{e \in \mathcal{N}_o(v)} f^d_e \right),
\]
where \(\bar{\mu} = [\mu_v, v \in V] \geq 0\) and \(\bar{q} = [q^d_v, v \in V, d \in D] \geq 0\) are the Lagrange multipliers for the constraints (10) and (11), respectively. For ease of notation, we define \(q^d_v = 0\), for all \(d\). Rearranging the order of the summations, the above equation can be transformed into the following:
\[
L(\tilde{f}, \tilde{t}, \bar{\mu}, \bar{q}) = \sum_{e \in E} c_e(\tilde{f}_e, t_e) - \beta \sum_{v \in V} \mu_v + \sum_{v \in V} \sum_{d \in D} (q^d_v T_{vd}),
\]
where
\[
c_e(\tilde{f}_e, t_e) = \Theta_e(\tilde{f}_e, t_e) + (\mu_x(e) + \mu_r(e)) t_e - \sum_{d \in D} (q^d_x(e) - q^d_r(e)) f^d_e.
\]
and \(x(e)\) and \(r(e)\) are the transmitting node and receiving node, respectively, of link \(e\).

The dual objective function is then
\[
D(\bar{\mu}, \bar{q}) = \min_{(\tilde{f}, \tilde{t}) \in Y_e} L(\tilde{f}, \tilde{t}, \bar{\mu}, \bar{q}),
\]
\[
= \sum_{e \in E} \left[ \min_{(\tilde{f}_e, t_e) \in Y_e} c_e(\tilde{f}_e, t_e) - \beta \sum_{e \in V} \mu_v \right] + \sum_{v \in V} \sum_{d \in D} (q^d_v T_{vd}),
\]
where \(Y_e\) is given by (7). In other words, the minimization of the Lagrangian can now be decomposed into minimization subproblems for each link. Note that all the information needed in minimizing \(c_e(\tilde{f}_e, t_e)\) is local to link \(e\).

The dual optimization problem is
\[
(C) \max_{\bar{\mu} \geq 0, \bar{q} \geq 0} D(\bar{\mu}, \bar{q}).
\]
Let \(\Theta^*\) denote the optimal solution to Problem (B). Assuming that the primal problem (B) is strictly feasible, the Slater condition can be verified. We can then conclude that there is no duality gap as in standard results [20, Chapter 5.2.3, p226].

**Theorem 3: (Strong Duality)**

Assume that Problem (B) is strictly feasible (and hence its optimal value \(\Theta^*\) is finite.) Then there is no duality gap, i.e., \(\Theta^* = \max_{\bar{\mu} \geq 0, \bar{q} \geq 0} D(\bar{\mu}, \bar{q})\).

The next step is to solve the dual problem in a distributed fashion. We can show that \(D(\bar{\mu}, \bar{q})\) is convex and its subgra-
dient is given by
\[
\frac{\partial D}{\partial \mu_v} = \sum_{e \in \mathcal{N}(v)} t_e - \beta,
\]
\[
\frac{\partial D}{\partial q^d_v} = \sum_{e \in \mathcal{N}_v(d)} f^d_e + T_{vd} - \sum_{e \in \mathcal{N}_o(v)} f^d_e.
\]
We can then use the following subgradient-ascent method to solve the dual problem.

**Distributed Energy Minimization Algorithm**

At each iteration \(m\),

1) At link \(e\), the data rate \(\tilde{f}_e\) and the link assignment \(t_e\) are determined by:
\[
(\tilde{f}_e(m), t_e(m)) = \arg\min_{(\tilde{f}_e, t_e) \in Y_e} c_e(\tilde{f}_e, t_e, m) + (\mu_x(e) + \mu_r(e)) t_e - \sum_{d \in D} (q^d_x(e) - q^d_r(e)) f^d_e.
\]

2) At node \(v\), the dual variables are updated by:
\[
\mu_v(m+1) = \mu_v(m) + \alpha_m \left( \sum_{e \in \mathcal{N}(v)} t_e(m) - \beta \right)
\]
\[
q^d_v(m+1) = q^d_v(m) + \alpha_m \left( \sum_{e \in \mathcal{N}_v(d)} f^d_e(m) + T_{vd} - \sum_{e \in \mathcal{N}_o(v)} f^d_e(m) \right).
\]

Remark: In the above algorithm, we use the same stepsize for updating \(\bar{\mu}\) and \(\tilde{f}\) at each iteration. This is simply for ease of notation: the stepsizes can be different for each dual variable.

The above exercise of using Lagrange duality is standard. However, there are a number of questions that are not answered by the above algorithm alone. First, how does the algorithm translate to practical network protocol components? Second, note that the primal problem is not strictly convex. Specifically, the objective function of Problem (B) contains a linear term (see (8)). Hence, the dual objective function is not always differentiable. In practice, in order to dynamically track the changes of the network condition (e.g., as the offered load \(T_{vd}\) changes), it is typical that a constant stepsize is used. As a consequence of the lack of differentiability of the dual objective function, the algorithm will not always be able to converge to a single operating point. Rather, the dual variables \(\mu_v(m)\) and \(q^d_v(m)\) could oscillate around their corresponding optimal values. Further, even if the dual variables are close to the optimal values, the primal variables \(\tilde{f}(m)\) and \(\tilde{t}(m)\)
may not. Hence, it is not immediately clear what level of performance this algorithm will be able to achieve. In the next two sections, we will carefully address these questions and quantify the performance levels of the resulting protocol.

IV. MAPPING TO NETWORK PROTOCOL COMPONENTS

In this section, we will map the Distributed Energy Minimization Algorithm to various protocol components. Recall that in each iteration \( m \) the Distributed Energy Minimization Algorithm updates the variables \( f(m), \hat{t}(m), \hat{\mu}(m) \) and \( \hat{q}(m) \) according to Equations (16)-(18). We will basically identify iteration \( m \) with time-slot \( m \), and use the values of these variables as the control decision at time-slot \( m \).

A. Routing, Power Control and Link Assignment

In Step 1 of the Distributed Energy Minimization Algorithm, each link solves (16) by minimizing \( c_e(\hat{f}_e, t_e, m) \). We now investigate the structure of this minimization problem, and we will show that this step corresponds to the routing, power control and link assignment protocol components. We first introduce the following transformation:

\[
f^d_e(m) = R^d_e(m) t_e(m).
\]

Recall that \( f^d_e(m) \) is an estimate (at iteration \( m \)) of the average data rate on link \( e \) allocated for destination \( d \), and \( t_e(m) \) is an estimate (at iteration \( m \)) of the fraction of time-slots that link \( e \) is activated. Thus, \( R^d_e(m) \) can be viewed as an estimate of the instantaneous data rate allocated on link \( e \) for destination \( d \). Substituting the above equation into \( c_e(\hat{f}_e, t_e, m) \), we have (dropping the time index \( m \) when there is no source of confusion):

\[
c_e(\hat{f}_e, t_e) = t_e l_e(\hat{R}_e),
\]

where \( \hat{R}_e = \{ R^d_e, d \in D \} \), and \( l_e(\hat{R}_e) \) is defined as

\[
l_e(\hat{R}_e) = h_e \left( \sum_{d \in D} R^d_e \right) + (\mu_x(e) + \mu_r(e))
- \sum_{d \in D} \left[ q^d_{x(e)}(e) - q^d_{r(e)}(e) R^d_e \right].
\]

Since \( t_e \geq 0 \), to minimize (20), we should first minimize \( l_e(\hat{R}_e) \) as a function of \( \hat{R}_e \), over \( 0 \leq \sum_{d \in D} R^d_e \leq a_e \). Note that function \( h_e(\cdot) \) takes as input parameter the sum of the data rates allocated over all the destinations on this link. In other words, from the viewpoint of power consumption, it is indifferent to which destination the data rate is allocated for, as long as the total data rate is the same. Further, we can interpret \( q^d_{x(e)}(e) \) in (18) as the backlog at node \( v \) for destination \( d \) (since it captures the cumulative difference between the input rate and output rate at node \( v \) for destination \( d \)). Then, the minimum of \( l_e(\hat{R}_e) \) is attained when all the data rates are allocated to the destination with the maximum positive backlog difference. In other words, if we let

\[
d = \arg \max_{d \in D} (q^d_{x(e)} - q^d_{r(e)}),
\]

then to find the optimal value of \( \hat{R}_e \), we should let \( R^d_e = 0 \) for all \( d \neq d \). With this observation, the minimization of \( l_e(\hat{R}_e) \) can be reduced to a minimization problem of a single-variable function, i.e.,

\[
\min_{0 \leq R_e \leq a_e} l_e(R_e) = h_e(R_e) + (\mu_x(e) + \mu_r(e))
- \left[ \max_{d \in D} (q^d_{x(e)}(e) - q^d_{r(e)}(e)) \right] R_e.
\]

Let \( R_e(m) \) denote the optimal solution to (23). We can then set \( R^d_e(m) = R_e(m) \) if \( d = d \), and \( R^d_e(m) = 0 \), otherwise. For example, if \( h_e(x) = \exp(x) - 1 \), and \( \max_{d \in D} (q^d_{x(e)}(e) - q^d_{r(e)}(e)) \) is positive, then \( R^d_e(m) = \log(q^d_{x(e)}(e) - q^d_{r(e)}(e)) \), and \( R^d_e(m) = 0 \) for all other destinations \( d \neq d \).

Now that \( \hat{R}_e \) has been chosen to minimize \( c_e(\hat{f}_e, t_e) = t_e l_e(\hat{R}_e) \). Clearly, the optimal \( t_e \) value is

\[
t_e(m) = \begin{cases} 1, & \text{if } l_e(R_e(m)) \leq 0, \\ 0, & \text{if } l_e(R_e(m)) > 0. \end{cases}
\]

Substituting into (19), the values of \( \hat{f}(m) \) can then be set as

\[
f^d_e(m) = R^d_e(m) \hat{t}_e(m) = \hat{t}_e \hat{R}_e(m)
\]

So far we have derived the values of \( \hat{f}(m) \) and \( \hat{t}(m) \) at iteration \( m \) according to (16). We now use the values of these variables as the control of the various protocol components in time-slot \( m \). The minimization of \( c_e(\hat{f}_e, t_e, m) \) on each link then naturally translates into the following protocol components. At each time-slot \( m \),

1) Routing: Choose only the flow \( \hat{d}(m) \) with maximum positive backlog difference (cf. (22)). This is the flow that should receive service.

2) Power control: Choose \( R_e(m) \) to minimize \( l_e(R_e(m)) \) in (23). Then \( h_e(R_e(m)) \) is the power assignment that link \( e \) should use, and \( R_e(m) \) is the corresponding data rate assigned to link \( e \) when it is turned on.

3) Link assignment: Choose \( t_e(m) \) to minimize

\[
t_e(m) l_e(R_e(m))
\]

in such a way that \( t_e(m) \) takes its maximum value 1 if the optimal \( l_e(R_e(m)) \) is less than or equal to 0; and 0 otherwise. This determines the amount of time that link \( e \) should be on.

Therefore, when link \( e \) is turned on (by the scheduling component to be discussed in Section IV-B), it will then use the power level \( h_e(R_e(m)) \) (and the corresponding data rate \( R_e(m) \)) to transfer packets for destination \( d(m) \) from its transmitting node to its receiving node in the corresponding time-slot. (Note that each link will carry packets for at most one destination at each time-slot.) Once the decisions of the above protocol components are determined based on the dual variables \( \mu_x(m) \) and \( q^d_{x(e)}(m) \), these dual variables are then updated according to (17) and (18). Note that to carry out the above control protocol, each link only needs to know the dual variables at its end-points. Further, to update the dual variables, each node only needs to know the control decisions at the links incident to it. Hence, the communication overhead incurred by the algorithm is small.

In the above control protocol, we have effectively re-interpreted \( t_e(m) \) as the offered load to link \( e \) at time-slot...
$m$ (i.e., $t_e(m) = 1$ implies that link $e$ must be activated for 
one additional time-slot). We now provide an economic inter-
pretation of the decision rule of $t_e(m)$ based on Equations (21) 
and (24). Recall that there are three terms in $c_e(\vec{f}_e, t_e, m)$ (see 
(16):

- The term $\Theta_e(\vec{f}_e, t_e)$ can be viewed as the power cost, i.e., 
  the power consumption of supporting an average data rate of 
  $\vec{f}_e$ while link $e$ being activated for $t_e$ fraction of time.
- The term $(\mu_{x(e)}(m) + \mu_{r(e)}(m))t_e$ can be viewed as the 
  scheduling cost. If we interpret $t_e(m)$ as the offered-load 
  to link $e$, then according to (17) the scheduling cost $\mu_e$ 
  will increase when $\sum_{e \in N(v)} t_e(m) > \beta$, and 
  it will decrease otherwise. In other words, an idle link 
  (i.e., with $t_e(m) = 0$) gives the system more flexibility in 
  scheduling other links, which leads to a possible decrease 
  in the scheduling costs at either the transmitting node or 
  the receiving nodes.
- The term $\sum_{d \in D} (q^d_{x(e)}(m) - q^d_{r(e)}(m))f^d_e$ can be viewed as 
  the utility of supporting the total data rate on link $e$. 
  Note that according to (18), a positive value of $f^d_e(m) > 0$ 
  will help to reduce the backlog difference $(q^d_{x(e)}(m) - 
  q^d_{r(e)}(m))$.

Hence, we will assign $t_e(m) = 1$ only when the utility of 
transporting data to next hop is no smaller than the power 
plus the scheduling cost. Note that there are two possible 
scenarios where $t_e(m) = 0$ in (24) (i.e., no offered-load is 
assigned to link $e$ in this time-slot):

- All backlog differences $(q^d_{x(e)} - q^d_{r(e)})$ are negative, which 
  means that the utility does not increase by transporting 
data to next hop on this link for any destination.
- Although some backlog differences $(q^d_{x(e)} - q^d_{r(e)})$ are 
  positive, as in (21), the utility of transporting data to next 
  hop is not large enough to outweigh the importance of 
saving more energy (the $h(\sum_{t} P^e_t)$ term) and/or the need 
to gain more flexibility in scheduling (the $(\mu_{x(e)} + \mu_{r(e)})$ term). 
  As a result, optimal $t_e(\vec{R}_e)$ is non-negative, and 
  no load is assigned to this link.

Before we proceed to the next subsection, we note that if 
$\alpha_m = \alpha > 0$ for all time-slots $m$, and if all links $e$ with 
$t_e(m) = 1$ are immediately allowed to transmit packets in 
time-slot $m$, then according to (18) the value of $q^d_{x(e)}(m)$ can 
be viewed as $\alpha$-multiple of the queue length at node $e$ for packets 
destined to $d$. However, as we will discuss soon in Section IV-
B, not all links with $t_e(m)$ can be activated immediately in 
time-slot $e$. Hence, a scheduling component will be required, 
which leads to a discrepancy between the value of the real 
queue and $q^d_{x(e)}(m)$. This discrepancy needs to be carefully 
studied in Section V-B.

Remark: We also note that some of the quantities pro-
duced/monitored by this algorithm can actually help network 
engineering. For instance, the Lagrange multiplier $q^d_e$ can 
also be interpreted as the shadow price of the corresponding 
constraint. If a small change occurs in the amount of supported 
traffic from node $v$ to node $d$, $q^d_e$ measures the sensitivity of 
the optimal power consumption with respect to this perturbation.

In a different networking setting where the network has some 
control over the traffic matrix $T$, information such as $\vec{q}$ can be 
used as guidelines to optimize power consumption.

B. The Maximal-Matching Scheduling Component

As we have seen thus far, the duality approach exploits 
the problem structure and decomposes the primal problem into 
sub-problems that immediately translate to protocol compo-
nents. However, in the above discussions, although we have 
considered the scheduling constraint in the form of (10), we 
have not studied the actual schedule of activating the links. 
In particular, the links that are assigned $t_e(m) = 1$ may 
in fact interfere with each other, and hence cannot all be 
activated immediately in time-slot $m$. A scheduling algorithm 
is then needed to schedule (at least some of) these links for 
activation at a later time. As a consequence, the evolution of 
$q^d_e$ in (18) does not correspond to that of the real queues. 
In particular, for each link $e$ and each time-slot $m$ such that 
$t_e(m) = 1$, define $\delta_e(m) \geq m$ as the time-slot when the 
scheduling algorithm can actually activate the link $e$ for this 
particular link-assignment instance. For obvious reasons, we 
require a one-to-one mapping from each time-slot $m$ with 
t$e(m) = 1$ to $\delta_e(m)$, and a natural ordering of $\delta_e(m)$ such 
that $\delta_e(m_1) < \delta_e(m_2)$ for every $m_1 < m_2$ and $t_e(m_1) = 
t_e(m_2) = 1$. Hence, the inverse mapping of $\delta_e(m)$ is well-
deﬁned, and we denote it by $\delta_e^{-1}(m)$. For those time-slots $m$ 
when link $e$ is not activated, they do not correspond to any 
time-slot $m'$ with $t_e(m') = 1$ and $\delta_e(m') = m$. In this case, 
we deﬁne $\delta_e^{-1}(m) = -1$.

In this paper, we are interested in using a simple scheduling 
algorithm called maximal-matching. Note that under the node-
exclusive interference model, at any time slot the feasible 
schedule must be a matching. (A matching of a graph is a sub-
set of the links such that no two links share a common node.) 
A maximal matching is a matching such that no more links 
can be added without violating the node-exclusive interference 
constraint. It is well-known that, under the node-exclusive 
interference model, as long as (10) is satisfied with $\beta = 1/2 - \eta$ 
for some $\eta > 0$, maximal-matching can guarantee to produce 
a schedule such that each link is activated for $\sum_{d \in D} f^d_e/R_e$ 
fraction of time on average [7]-[9]. Hence, we are interested 
in using maximal-matching to dynamically assign the mapping 
from $m$ to $\delta_e(m)$. Speciﬁcally, the maximal-matching 
scheduling policy maintains a queue $\nu_e(m)$ at each link $e$, 
which corresponds to the difference between the number of 
time-slots that link $e$ should have been activated before time-
slot $m$, and the actual number of time-slots that the link has 
been activated before time-slot $m$, i.e.,

$$\nu_e(m) = \sum_{s=1}^{m-1} 1_{t_e(s)=1} - \sum_{s=1}^{m-1} 1_{\delta_e^{-1}(s)=-1}.$$ 

Clearly, $\nu_e(m) \geq 0$ for all $m$. Let $\bar{\nu}(m) = [\nu_e(m)]$. 
At each time-slot $m$, from the set of links with $\nu_e(m) \geq 1$, 
the maximal-matching policy activates any maximal subset of 
links that do not interfere with each other. More precisely, 
denote $M(m)$ as the actual set of links that are activated at 
time-slot $m$. Then for the maximal-matching scheduling policy 
one of the following statement must be true. For any link $e$:

- either $\nu_e(m) = 0$;
or \( l \in \mathcal{M}(m) \) for some link \( l \in N(x(e)) \);  
• or \( l \in \mathcal{M}(m) \) for some link \( l \in N(r(e)) \);
where \( x(e) \) and \( r(e) \) are the transmitting node and receiving node, respectively, of link \( e \). The evolution of \( \nu_e(m) \) is then given by

\[
\nu_e(m + 1) = \nu_e(m) + t_e(m) - 1_{\{e \in \mathcal{M}(m)\}}. 
\]

(25)

When a link \( e \) is activated at time slot \( m \), we relate it to the earliest time-slot \( m' \) that has \( t_e(m') = 1 \) but has not been assigned an activation time-slot yet, and we let \( \delta_e(m') = m \). Further, we use the value of \( f_d^e(m') \) to serve packets on link \( e \) at this time-slot \( m \).

Consider the following equation,

\[
Q_e^d(m + 1) = \left\{ Q_e^d(m) + \alpha_m \left[ T_v^d \right. \right. \\
+ \sum_{e \in N(v)} f_d^e(\delta_e^{-1}(m)) - \sum_{e \in N(v)} f_d^e(\delta_e^{-1}(m)) \left. \right] \right\}^+ 
\]

(26)

It is easy to see that when \( \alpha_m = \alpha \) and \( Q_0^d = 0 \), \( Q_e^d(m)/\alpha \) provides an upper bound on the real queue maintained at node \( v \) for packets destined to node \( d \). (It is an upper bound because the actual number of incoming packets to node \( v \) may be strictly less than \( \sum_{e \in N(v)} f_d^e(\delta_e^{-1}(m)) \).) Note that here we have used the convention that \( f_d^e(-1) = 0 \) for all \( d \) and \( e \), which is consistent with the value of \( f_d^e(m) \) for those time-slots when \( t_e(m) = 0 \).

Remark: The maximal-matching scheduling policy can be implemented in a distributed fashion. We refer readers to [21], [22] for more details on the distributed implementation of maximal schedules.

In the next section, we will carefully quantify the stability and power-efficiency of the cross-layer protocol proposed above.

V. PERFORMANCE ANALYSIS

In this section, we will answer the following two questions. First, can the protocol developed in Section IV support the offered load given by \( [T_v^d] \)? Second, what is the power-efficiency of the cross-layer protocol control protocol? We note that these questions cannot be answered by standard results in convex optimization and duality theory alone. The reason is because the introduction of the maximal-matching scheduling component in Section IV-B leads to some complication in analyzing the dynamics of the protocol. In particular, a link with \( t_e(m) = 1 \) may need to be activated at a later time \( \delta_e(m) \). This delayed activation not only leads to a discrepancy between the value of \( q_d^e(m) \) and that of the real queue, but also leads to difficulties in accounting for the actual energy consumption. For ease of exposition, in this section we will first ignore the maximal-matching scheduling component, and study the properties of the control variables \( f_d^e(m) \) and \( t_e(m) \) computed by (16)-(18) under the assumption that all links with \( t_e(m) = 1 \) can be activated immediately in time-slot \( m \). We then remove this unrealistic assumption, and relate the properties of \( f_d^e(m) \) and \( t_e(m) \) to the actual performance of the protocol when the maximal-matching scheduling component is used.

A. Properties of \( f_d^e(m) \) and \( t_e(m) \) with Constant Stepsizes

Since the algorithm in (16)-(18) is a standard subgradient-ascent algorithm for the dual problem, we would expect that the dual variables will converge to a neighborhood of some optimal value. However, the primal variables \( (\tilde{f}(m), \tilde{t}(m)) \) will likely oscillate. For example, \( t_e(m) \) is either 0 or 1 according to Equation (24). A natural question to ask then is the following: In what sense is the primal variables \( f_d^e(m) \) and \( t_e(m) \) optimal?

The following theorem answers this question. Recall that \( \Theta^* \) is the minimum value of Problem (B).

Theorem 4: Let the stepsizes in the Distributed Energy Minimization Algorithm be equal to a constant, i.e., \( \alpha_m = \alpha \), for all time-slots \( m \). Let \( \Phi \) be the set of \((\mu, \bar{q})\) that maximizes \( D(\bar{\mu}, \bar{q}) \), and define the distance metric \( d((\mu, \bar{q}), \Phi) \triangleq \min(\mu, \bar{q}) \in \Phi \| (\mu, \bar{q}) - (\bar{\mu}^*, \bar{q}^*)\| \). Given any \( \varepsilon > 0 \), there exists some \( \alpha_0 > 0 \) such that, for any \( \alpha \leq \alpha_0 \) and any initial implicit costs \((\bar{\mu}_0, \bar{q}_0)\), there exists a time \( M_0 \) such that for all \( m > M_0 \),

\[
d((\bar{\mu}(m), \bar{q}(m)), \Phi) < \varepsilon, \tag{27}
\]

and further,

\[
\lim_{m \to \infty} \frac{1}{m} \sum_{\tau = 1}^{m} \Theta_e(\tilde{f}_e(\tau), t_e(\tau)) < \Theta^* + \varepsilon. \tag{28}
\]

Remark: The above theorem shows that, when stepsizes are small, the dual variables eventually converge to within a small neighborhood of the optimal dual solution. Note that under the assumption that all links with \( t_e(m) = 1 \) can be activated immediately in time-slot \( m \), the boundness of \( q_d^e(m) \) immediately implies that the offered load \([T_v^d] \) are supported by the protocol. In other words, according to (18),

\[
\frac{1}{m+1} \sum_{s=m}^{m+t_1-1} \sum_{e \in N(v)} f_d^e(s) + T_v^d - \sum_{e \in N(v)} f_d^e(s) \]

must be bounded for all time-slots \( m \) and for all \( t_1 > 0 \). Hence, the constraint in (4) is satisfied when we take \( f_d^e \) in (4) as the long-term average of \( f_d^e(m) \). Further, Theorem 4 shows that the power consumption determined by the primal variables \( f_d^e(m) \) and \( t_e(m) \) is close to the minimal value \( \Theta^* \) of Problem (B). In other words, even though the primal variables \((\tilde{f}(m), \tilde{t}(m)) \) may not converge, by using \((\tilde{f}(m), \tilde{t}(m)) \) for each time-slot \( m \), the long-term average of the resultant power consumption is arbitrarily close to \( \Theta^* \).

Proof: (of Theorem 4) The proof technique here is similar to [16]. Note that the subgradient of the dual objective function is bounded because \( 0 \leq t_e(m) \leq 1 \) and \( f_d^e(m) \leq a_{e} t_e(m) \leq a_e \). Using the results of [23], we can then conclude that, given any \( \varepsilon > 0 \), there exists some \( \alpha_1 > 0 \) such that, for any \( \alpha \leq \alpha_1 \) and any initial implicit costs \((\bar{\mu}_0, \bar{q}_0)\), there exists a time \( M_0 \) such that (27) holds for all \( m > M_0 \). Therefore, as time index \( m \) increases, \((\bar{\mu}(m), \bar{q}(m)) \) converges to within a small neighborhood of the set of the maximizer of \( D(\bar{\mu}, \bar{q}) \) if the constant stepsizes is chosen to be small enough. This implies that the sequence \( \{(\bar{\mu}(m), \bar{q}(m))\}_m \) is bounded.
To show (28), we consider the following Lyapunov function:
\[
V(\bar{\mu}(m), \bar{q}(m)) = \frac{1}{2} \sum_{v \in V} \sum_{d \in D} \alpha \mu_v^2(m) + \frac{1}{2} \sum_{v \in V} \mu_v^2(m).
\]
The subgradient of \(D(\bar{\mu}, \bar{q})\) at time-slot \(m\) can be written as
\[
\Delta \mu_v(m) = \sum_{e \in N(v)} t_e(m) - \beta,
\]
\[
\Delta q_v^d(m) = \sum_{e \in N(v)} f_v^d(m) + T_v d - \sum_{e \in N(v)} f_v^d(m).
\]
Using the above notation, the one-step drift of the Lyapunov function can be calculated as follows:
\[
V(\bar{\mu}(m + 1), \bar{q}(m + 1)) - V(\bar{\mu}(m), \bar{q}(m)) = \frac{1}{2} \sum_{v \in V} \left\{ \left( \mu_v(m) + \alpha \Delta \mu_v(m) \right)^+ - \mu_v^2(m) \right\} + \frac{1}{2} \sum_{v \in V, d \in D} \left\{ \left( q_v^d(m) + \alpha \Delta q_v^d(m) \right)^+ - q_v^d(m)^2 \right\} \]
\[
\leq \alpha \sum_{v \in V} \mu_v(m) \Delta \mu_v(m) + \alpha \sum_{v \in V} \sum_{d \in D} q_v^d(m) \Delta q_v^d(m) + \alpha^2 W,
\]
where \(W\) is a sufficiently large constant. It is possible to choose such a constant since the subgradient of \(D(\bar{\mu}, \bar{q})\) is bounded. Adding \(\alpha \sum_{e \in E} \Theta_e(\bar{f}_e(m), t_e(m))\) to both sides of the above formula, we have
\[
V(\bar{\mu}(m + 1), \bar{q}(m + 1)) - V(\bar{\mu}(m), \bar{q}(m)) + \alpha \sum_{e \in E} \Theta_e(\bar{f}_e(m), t_e(m)) \leq \alpha \Theta^* + \alpha^2 W.
\]

For any given \(\varepsilon > 0\), there exist some \(\hat{\alpha}_2 > 0\) such that \(\alpha W < \varepsilon/2\) for all \(\alpha < \hat{\alpha}_2\). Further, note that for all \(\alpha < \hat{\alpha}_1\), all \(V()\) is bounded since the sequence \((\bar{\mu}(m), \bar{q}(m))\) is bounded. For any given \(\varepsilon > 0\) and \(\alpha\), there exists a number \(M_1(\alpha)\) such that for all \(M \geq M_1(\alpha)\), the first term on the LHS of (29) can be bounded from below by \(-\alpha^2/2\). Letting \(\hat{\alpha}_0 = \min(\hat{\alpha}_1, \hat{\alpha}_2)\), we then have, for all \(\alpha < \hat{\alpha}_0\) and \(M \geq M_1(\alpha)\),
\[
\frac{1}{M} \sum_{m=1}^M \sum_{e \in E} \Theta_e(\bar{f}_e(m), t_e(m)) \leq \Theta^* + \varepsilon.
\]
The result of Theorem 4 then follows.

B. Stability and Power-Efficiency with the Maximal-Matching Scheduling Component

Theorem 4 establishes the stability and optimality of the primal variables \(f_v^d(m)\) and \(t_e(m)\). However, as we discussed at the beginning of this section, due to the delayed-activation of the links, there is a discrepancy between the real queue and the value of \(q_v^d(m)\). Hence, in order to ensure that the offered load \(T_v^d\) is supported, we must prove that the real-queue is stable. Further, we must show that the delayed-activation of the links does not change the average energy consumption in the system. Towards this end, we first show that the delay in activating the links is bounded.

**Lemma 5:** Assume that the positive stepsizes \(\alpha_m\) are fixed, i.e., \(\alpha_m = \alpha\) for all time-slots \(m\), where \(0 < \alpha < \hat{\alpha}_0\) and \(\hat{\alpha}_0\) is given in Theorem 4. There exists a constant \(T_0\) such that for any \(m\) with \(t_e(m) = 1\), the delay in activating the link \(e\) is no greater than \(T_0\), i.e., \(\delta_e(m) - m \leq T_0\).

**Proof:** We first show that \(\nu_e(m)\) in (25) is bounded for all \(e\) and \(m\). From Theorem 4, we know that \(\mu_v(m)\) is bounded for all nodes \(v\) and all time-slots \(m\). Let this bound be \(Q_1\). Using (17), we then have, for any \(t_1\),
\[
\frac{Q_1}{\alpha} \geq \frac{\mu_v(m + t_1) - \mu_v(m)}{\alpha} \geq \sum_{e \in E} \sum_{s=m}^{m+t_1-1} t_e(s) - \beta.
\]
Hence,
\[
\frac{Q_1}{\alpha \hat{\alpha}_1} \geq \frac{1}{t_1} \sum_{s=m}^{m+t_1-1} \sum_{e \in E} t_e(s) \leq \beta + \frac{Q_1}{\alpha \hat{\alpha}_1}.
\]
Let \(\epsilon = \frac{1}{2} - \beta\). Since \(\beta < 1/2\), by choosing \(t_1\) large enough, we can have
\[
\frac{1}{t_1} \sum_{s=m}^{m+t_1-1} \sum_{e \in E} t_e(s) \leq \frac{1}{2}(1 - \epsilon)
\]
(30) for all time-slot \(m\).

Next, define the Lyapunov function
\[
U(\bar{\nu}(m)) = \frac{1}{2} \sum_{e \in V} \left( \sum_{e \in E} \nu_e(m) \right)^2.
\]
Note that this Lyapunov function is standard in proving the stability of maximal-matching [7]. According to Equation (25) that governs the evolution of \( \bar{v} \), we can compute the drift of \( U(\cdot) \) as

\[
U(\bar{v}(m + t_1)) - U(\bar{v}(m)) \\
\leq \sum_{e \in E} \nu_e(m) \sum_{s = m}^{m + t_1 - 1} \left[ \sum_{l \in N(x(e))} t_l(s) + \sum_{l \in N(r(e))} t_l(s) \right] \\
- \sum_{l \in N(x(e))} 1_{\{l \in M(s)\}} - \sum_{l \in N(r(e))} 1_{\{l \in M(s)\}} + M_1,
\]

where \( M_1 \) is a positive constant that may depend on \( t_1 \). Note that for any link \( e \) with \( \nu_e(m) \geq t_1 + 1 \), we must have \( \nu_e(s) \geq 1 \) for \( s = m, m + 1, \ldots, m + t_1 \). Hence, according to the definition of maximal-matching,

\[
\sum_{l \in N(x(e))} 1_{\{l \in M(s)\}} + \sum_{l \in N(r(e))} 1_{\{l \in M(s)\}} \geq 1,
\]

for \( s = m, m + 1, \ldots, m + t_1 \). Therefore, for a sufficiently large positive constant \( M_2 \), we have

\[
U(\bar{v}(m + t_1)) - U(\bar{v}(m)) \\
\leq \sum_{e \in E} \nu_e(m) \sum_{s = m}^{m + t_1 - 1} \left[ \sum_{l \in N(x(e))} t_l(s) + \sum_{l \in N(r(e))} t_l(s) - 1 \right] \\
+ M_2, \\
\leq -e \sum_{e \in E} \nu_e(m) + M_2,
\]

where in the last step we have used (30). In other words, whenever \( \sum_{e \in E} \nu_e(m) \) is greater than \( 2M_2/e \), the value of \( U(\bar{v}(m)) \) must decrease in \( t_1 \) steps. This implies that \( U(\bar{v}(m)) \) must be bounded for all \( m \), and hence all \( \nu_e(m) \) must also be bounded for all links \( e \) and all time-slots \( m \). Thus, we can find a constant \( Q_2 \) such that

\[
\sum_{l \in N(x(e))} \nu_l(m) + \sum_{l \in N(r(e))} \nu_l(m) \leq Q_2
\]

for all \( e \) and \( m \).

We are now ready to show that the delay \( \delta_e(m) - m \) must also be bounded. Let \( K \) be the smallest integer that is greater than or equal to \( Q_2/(et_1) \). Suppose that for some \( e \) and \( m \), \( \delta_e(m) - m > Kt_1 \). We will show that this will result in \( \nu_e(m + Kt_1) = 0 \), which leads to a contradiction. To see this, note that since \( \delta_e(m) - m > Kt_1 \), we must have

\[
\nu_e(s) \geq 1, \text{ for } s = m, m + 1, \ldots, m + Kt_1.
\]

According to the definition of maximal-matching, we then have

\[
\sum_{l \in N(x(e))} 1_{\{l \in M(s)\}} + \sum_{l \in N(r(e))} 1_{\{l \in M(s)\}} \geq 1,
\]

for all \( m \leq s \leq m + Kt_1 \). Hence,

\[
\sum_{l \in N(x(e))} \nu_l(m + Kt_1) + \sum_{l \in N(r(e))} \nu_l(m + Kt_1) \leq Q_2 + \sum_{s = m}^{m + Kt_1 - 1} \left\{ \sum_{l \in N(x(e))} t_l(s) + \sum_{l \in N(r(e))} t_l(s) - \sum_{l \in N(x(e))} 1_{\{l \in M(s)\}} - \sum_{l \in N(r(e))} 1_{\{l \in M(s)\}} \right\}
\leq Q_2 - Kt_1 e 
\leq 0
\]

This implies that \( \nu_e(m + Kt_1) = 0 \). However, since \( \delta_e(m) - m > Kt_1 \), we must have \( \nu_e(m + Kt_1) \geq 1 \). This leads to a contradiction. Hence, the delay \( \delta_e(m) - m \) must be bounded by \( Kt_1 \) for all \( e \) and \( m \). Letting \( T_0 = Kt_1 \), the result of the lemma then follows.

We are now ready to state the main result of this section.

**Theorem 6:** Assume that the positive stepsizes \( \alpha_m \) are fixed, i.e., \( \alpha_m = \alpha \) for all time-slots \( m \), where \( 0 < \alpha < \tilde{\alpha}_0 \) and \( \tilde{\alpha}_0 \) is given in Theorem 4. With the maximal-matching scheduling policy stated earlier, the real queue at all nodes must be bounded at all time-slots. Hence, the offered-load [\( T_v^d \)] is supported by the cross-layer control protocol. Further, the long-term average energy consumption is no greater than \( \Theta^* + \epsilon \), where \( \Theta^* \) is the minimal value of Problem (B).

**Proof:** We first show that the real queues are bounded. By Lemma 5, the delay in activating the links is bounded by a number \( T_0 \). Consider Equation (26). Recall that \( Q_v^d(m) \) provides an upper bound on the real queue maintained at node \( v \) for packets destined to node \( d \). Hence, it suffices to show that \( Q_v^d(m) \) is bounded for all nodes \( v \), destinations \( d \) and time-slots \( m \). For any time-slot \( m \), assuming \( Q_v^d(0) = 0 \) for all \( v \) and \( d \), we then have

\[
\frac{Q_v^d(m)}{\alpha} \leq \sup_{0 \leq t_2 \leq m} \sum_{s = m - t_2}^{m - 1} \left[ T_v^d + \sum_{e \in N(s)} f_e^d(\delta_e^{-1}(s)) \right] \\
- \sum_{e \in N(v)} f_e^d(\delta_e^{-1}(s)).
\]

This is (31).

Since \( \delta_e(m) - m \leq T_0 \), we have, for all \( 0 \leq t_2 \leq m \),

\[
\sum_{s = m - t_2}^{m - 1} \sum_{e \in N(s)} f_e^d(\delta_e^{-1}(s)) \\
\leq \sum_{s = m - T_0}^{m - 1} \sum_{e \in N(s)} f_e^d(s),
\]

for all \( m \leq s \leq m + Kt_1 \). Hence,
where we have adopted the convention that $f^d_e(m) = 0$ for all $m < 0$. Similarly, we have, for all $0 \leq t_2 \leq m$,

$$
\sum_{s=m-t_2}^{m-1} \sum_{e \in \mathcal{E}_e(v)} f^d_e(\delta_e^{-1}(s)) - \sum_{e \in \mathcal{E}_e(v)} f^d_e(\delta_e^{-1}(s)) \geq \sum_{s=m-t_2}^{m-1} \sum_{e \in \mathcal{E}_e(v)} f^d_e(s) \geq \sum_{s=m-t_2}^{m-1} \sum_{e \in \mathcal{E}_e(v)} f^d_e(s) - M_3
$$

for some positive constant $M_3$. (Note that in the last step we have used the fact that $f^d_e(s) \leq a_e$.) We then have, for all $0 \leq t_2 \leq m$,

$$
\sum_{s=m-t_2}^{m-1} \left( T^d_v + \sum_{e \in \mathcal{E}_e(v)} f^d_e(\delta_e^{-1}(s)) - \sum_{e \in \mathcal{E}_e(v)} f^d_e(\delta_e^{-1}(s)) \right)
$$

Hence, the second part of Theorem 6 follows from the second part of Theorem 4.

**C. Power-Efficiency Ratio**

Setting $\beta = 1$ or $\beta = \frac{1}{2} - \eta$ in (10) gives necessary or sufficient conditions, respectively, for schedulability (i.e., in terms of stabilizing the overall queueing system [7]–[9]). It is then evident that, by choosing $\beta = \frac{1}{2} - \eta$, the maximum loss in throughput under the node-exclusive interference model is $\frac{1}{2} - \eta$. This is the throughput loss ratio in approximating Problem (2) by Problem (B) with $\beta = \frac{1}{2} - \eta$.

To derive the approximation ratio of our cross-layer control protocol, the throughput loss needs to be translated into power loss. We apply the same second-order approximation of the power-rate function $h_e(\cdot)$ as in [6]. More specifically, assume that the data rate $R_e$ in an AWGN channel is given by

$$
R_e = W \log_2 \left[ 1 + \frac{e^{-\eta} p_e}{N_0 W} \right],
$$

where $W$ is the available bandwidth, $\sigma_e$ is the channel gain of link $e$, $N_0$ is the noise spectral density, and $p_e$ is the transmission power. The power-rate function $h_e(R_e)$ is then given by

$$
p_e = h_e(R_e) = \frac{N_0 \log_2 \left( 1 + \frac{e^{-\eta} p_e}{N_0 W} \right)}{2 W \sigma_e}.
$$

Using a second-order approximation,

$$
2R_e/W \approx 1 + \frac{R_e \ln 2}{W} - \frac{1}{2} \frac{R_e^2 \ln^2 2}{W^2},
$$

the function $\Theta_e(f_e, t_e)$ can then be approximated by

$$
\Theta_e(f_e, t_e) = t_e h_e \left( \sum_{d \in D} f^d_e \right) t_e 
$$

$$
= \frac{N_0 \ln 2}{2 \sigma_e} \sum_{d \in D} f^d_e = \sum_{d \in D} f^d_e t_e 2W \sigma_e.
$$

(33)

Let $(\tilde{f}, \tilde{t})$ be the optimal solution to Problem (B) with $\beta = 1$. It is evident that $(\tilde{f}, \tilde{t})$, where $\varepsilon = 1/\left(1/2 - \eta\right) - 2$, is a feasible solution to Problem (B) with $\beta = 1/2 - \eta$. According to (33), this feasible solution results in a power consumption that is (approximately) at most $2 + \varepsilon$ of the optimal value of Problem (B) with $\beta = 1$. Since the optimal value of Problem (B) with $\beta = 1$ is a lower bound on the minimum power from Problem (2), we conclude that the power-efficiency ratio of our algorithm is upper-bounded by $(2 + \varepsilon)$. In contrast, under the same approximation of the power-rate function, the authors of [6] develop a centralized algorithm and show that its power consumption is at most 3-times as large as that of the optimum. Thus, our distributed algorithm is able to guarantee a better power-efficiency ratio.

**D. Performance with Diminishing Stepsizes**

We conclude this section with a characterization of the performance of the Distributed Energy Minimization Algorithm with diminishing stepsizes $\alpha_m$. Not only is this result of theoretical interest, it also provides additional insight into the optimality of the primal variables $f^d_e(m)$ and $t_e(m)$.
Theorem 7: (Primal Optimality with Diminishing Stepsizes)
(a) Let the stepsizes \( \{\alpha_m\} \) in the Distributed Energy Minimization Algorithm satisfy the following conditions:
\[
\sum_{m=1}^{\infty} \alpha_m^2 < +\infty, \quad \sum_{m=1}^{\infty} \alpha_m = +\infty. \tag{34}
\]
Then for any nonnegative starting point \((\bar{\mu}_0, \bar{q}_0)\), the dynamics of the Distributed Energy Minimization Algorithm converges to the minimum value of Problem (B), i.e.,
\[
\lim_{m \to \infty} (\bar{\mu}(m), \bar{q}(m)) = (\bar{\mu}^*, \bar{q}^*), \tag{35}
\]
\[
\lim_{m \to \infty} D(\bar{\mu}(m), \bar{q}(m)) = \Theta^*, \tag{36}
\]
where \((\bar{\mu}^*, \bar{q}^*)\) is a maximum point of \(D(\bar{\mu}, \bar{q})\).
(b) Let the stepsizes \(\{\alpha_m\}\) be chosen as
\[
\alpha_m = \frac{\kappa}{m + \rho}, \tag{37}
\]
where \(\kappa\) and \(\rho\) are some positive scalars (note that the above definition of \(\{\alpha_m\}\) satisfies the conditions in (34)). If the long-term average of the vector \((\bar{f}(m), \bar{t}(m))\) converges, i.e.,
\[
\lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} \bar{f}(m) = \bar{f}^*, \quad \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} \bar{t}(m) = \bar{t}^*, \tag{38}
\]
then the time-average version of \((\bar{f}(m), \bar{t}(m))\) is the optimal solution to Problem (B), i.e.,
\[
\sum_{e \in E} \Theta_e(\bar{f}_e^*, \bar{t}_e^*) = \Theta^*. \tag{39}
\]
The proof of Theorem 7 is included in our online technical report [24].

Remark: In the Distributed Energy Minimization Algorithm, the primal variables \(f_e^*(m)\) and \(t_e^*(m)\) may oscillate for ever (for example, \(t_e^*(m)\) can only be 0 or 1). From this perspective, our solution carries the same flavor as some of the related work [7], [15], namely, in the optimal solution, the resource, be it power or fraction of up-time, is used to the maximum extent, if the link is activated for the current time-slot. However, one would expect the optimal solution of \(t_e\) to Problem (B) to be a number anywhere from 0 to \(\beta\) for most of the links in a typical setting (cf. (10)). Theorem 7 reconciles the difference between these two viewpoints. Although \((\bar{f}(m), \bar{t}(m))\) is not a continuous mapping from the underlying and converging dual variables \((\bar{\mu}, \bar{q})\), as long as the long-term average of the primal variables \((\bar{f}(m), \bar{t}(m))\) converges, the limit is an optimal solution to the primal problem (B).

VI. NUMERICAL RESULTS

In this illustrative example, we consider a 7-node network, whose topology is depicted in Figure 1. The rate-power function is of the following form:
\[
R_e = W \log_2 \left[1 + \frac{\sigma_e p_e}{N_0 W}\right],
\]
where \(W = 1.0\) MHz is the available bandwidth, \(\sigma_e = 1.6 \times 10^{-13}\) is the channel gain of link \(e\), \(N_0 = 1.6 \times 10^{-18}\) mW/Hz is the noise spectral density, \(p_e\) is the transmission power, and \(R_e\) is the resultant instantaneous data rate of link \(e\). The power-rate function \(h_e(\cdot)\) is then given by (32). This network supports two flows, as shown in Table 1.

The node-exclusive interference model is considered, and \(\beta = (0.5 - 10^{-4})\) in Problem (B). The length of each time-slot is 1 second. The results reported in this section are the average over a moving time-window of length 120 seconds.
To show that our proposed solution can adapt to variations in the input parameters, we apply the following changes in the system setting. At time \(t = 4000s\), the channel gain \(\sigma_{(1,7)}\) of the direct link between node 1 and node 7 is decreased from \(1.6 \times 10^{-13}\) to \(0.4 \times 10^{-13}\). At time \(t = 8000s\), the data rate of flow 2 (from node 3 to node 6) is reduced from 500 kbps to 250 kbps.

<table>
<thead>
<tr>
<th>Flow 1</th>
<th>Flow 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>Destination</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. 1. Network topology.

Fig. 2. Power consumption from distributed algorithm and offline computation.

For each setting, offline computation is carried out to find the minimum value \(\Theta^*\) of Problem (B), which is given by the dashed line in Figure 2. The power consumption from the proposed distributed algorithm is shown as the solid line in the same figure. This simulation result shows that our proposed solution is capable of achieving the near-optimal power-
consumption $\Theta^*$ in a distributed manner, and automatically tracking the near-optimal operating point once the system parameters change.

Figure 3 shows the average data rates $f_l^{d}$ for different flows on four links. We now take a closer look at the routing of the flows.

- In the initial state, flow 1 concentrates on the minimum energy path, namely, link (1, 7).
- At $t = 4000s$, the channel gain $\sigma_{(1,7)}$ reduces by 75%, and part of flow 1 is shifted to path 1 – 2 – 7. Since the scheduling capacity of node 2 is saturated, a larger percentage of flow 2 is then routed through path 3 – 4 – 5 – 6 in the optimal solution.
- At $t = 8000s$, the traffic that the network has to support between node 3 and node 6 reduces (flow 2 is reduced to 250 kbps). As a consequence, part of the scheduling capacity of node 2 is freed, and more of flow 1 takes path 1 – 2 – 7 to reduce the overall power consumption.

The above example shows that the interaction between routing, scheduling, and power control is relatively complex even in a wireless network of small size. The correct way to deal with such interaction is difficult to summarize into general approaches such as minimum energy approach or load balancing approach. In our proposed solution, as shown in Figure 2 and Figure 3, low-complexity and distributed operation on each link accomplishes this joint optimization even in networks with non-stationarity.

VII. CONCLUSION

In this paper, we propose a joint power control, link scheduling, and routing algorithm to minimize the power consumption in multi-hop wireless networks. The known cross-layer solution to this problem is centralized, and with high computational complexity. In contrast, our algorithm is distributed, and with low computational complexity. We establish the power-efficiency ratio of our solution, and show that the performance bound of our solution, achieved in a distributed manner, is provably tighter than a centralized solution in the literature.

As in related works on cross-layer control and optimization of wireless networks [9], our solution borrows extensively the techniques from convex optimization and duality theory. However, we often observe that straightforward applications of optimization theory may not produce a control protocol that is directly usable in real systems. For example, for the problem that we studied in this paper, duality theory leads to a solution in (16)-(18) where the interference constraints could in fact be violated. Hence, additional modification of the solution is needed. One of the main contributions of the paper is to design an easy-to-implement scheduling component that accounts for the interference constraints, and to carefully quantify the stability and power-efficiency of the resulting protocol. Our simulation results verify that the proposed distributed solution can compute and track the near-optimal operating point whenever the system parameters change.

In this work, we have focused on the well-studied node-exclusive interference model. It is possible to extend the results to more general interference models. For example, we can define $\Gamma_e$ to be the set of links that interfere with link $e$. (The node-exclusive model can be viewed as a special case with $\Gamma_e = N(x(e)) \cup N(y(e))$.) Let $\omega_e$ denote the maximum number of links that can be scheduled simultaneously in $\Gamma_e$, and $\omega_{\text{max}} = \max_e \omega_e$. It can be shown that a distributed maximal schedule can stabilize the system while the throughput is reduced by at most a factor of $1/\omega_{\text{max}}$ [25]–[28]. In this case, the scheduling constraint (10) in our problem formulation (B) need to be modified to $\sum_{l \in \Gamma_e} t_l \leq \beta$, where setting $\beta = 1 - \eta$, $\eta > 0$, will provide a necessary condition for schedulability, and setting $\beta = \omega_{\text{max}}$ will provide a sufficient condition. The general methodology presented in this paper can then be carried through to this type of interference models as well.

REFERENCES


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