OFDM Downlink Scheduling for Delay-Optimality: Many-Channel Many-Source Asymptotics with General Arrival Processes

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***Abstract*—We consider the downlink of an OFDM system for supporting a large number of delay-sensitive users. The OFDM scheduling problem can be modeled as a discrete-time multi-source multi-server queuing system with time-varying connectivity. For such a system, the Max-Weight policy is known to be throughput-optimal and the Server-Side Greedy (SSG) policy has been recently shown to achieve small queue lengths for i.i.d. arrival processes. However, there is often significant difference between queue-length optimality and delay optimality, and there exist arrival patterns such that algorithms with small queue backlog can still lead to large delay. In this work, we propose a new OFDM scheduling algorithm that gives preference to packets with large delay. Assuming ON-OFF channels, we show that for a large class of arrival processes, the proposed policy is rate-function delay-optimal. We substantiate the result via both analysis and simulation.**

I. Introduction

Next generation OFDM-based wireless cellular systems (e.g. WiMax and LTE) are envisioned to provide much higher data rate and larger system capacity. It is conceivable that in the future, both voice, data, and video traffic can be carried on a single packet-based OFDM system, eliminating the need to maintain separate voice networks. An important problem in the realization of this goal is the design of scheduling algorithms that provide low-delay guarantees to delay sensitive voice/video users. In a typical OFDM system, the bandwidth available to the base-station is partitioned into hundreds of orthogonal carriers. A given user can be served by multiple frequency carriers simultaneously, and the allocation of carriers to users can change over every time-slot. How user transmissions should be scheduled over frequency and time will have a significant impact on the delay performance of the system, which needs to be carefully studied.

In this paper, we focus on the down-link OFDM scheduling problem in a single cell. Arriving packets get queued in the buffer at the base-station before they are transmitted to the users. The goal is to minimize the amount of time that any packet spends in the buffer of the base-station. In the literature, it is well-known that the Max-Weight algorithm is throughput optimal under such a setting, in the sense that it can stabilize the system under the largest set of offered loads. However, it has been observed in [1] [2] [3] that the Max-Weight algorithm can lead to large delays for users. Specifically, although a system can be stabilized by the Max-Weight algorithm, the queue length can be very large. [1] [2] [3] proposed a number of new scheduling algorithms that are efficient in maintaining low queue lengths for all users. They keep queue length small by serving the queues with higher weighted sum and at the same time balancing queues in each time-slot. The authors of [1] [2] [3] use large-deviation tools to study the asymptotic decay-rate of the probability that the queue-length of any user exceed a given threshold, as the number of users and the number of frequency carriers both increase. They show that for Bernoulli arrivals that are i.i.d. across time, the proposed algorithm is rate-function queue-length optimal i.e., they achieve the largest decay-rate for the above queue overflow probability. For more general arrival processes, the algorithms are shown to achieve strictly positive decay-rates for the queue overflow probability

However, simply maintaining low queue-lengths is insufficient in order to guarantee low waiting-time. When the number of arrival packets is constant over time, one may map the decay rate of the queue-overflow probability to that of the delay-violation probability [4]. However, for general arrival processes there may not exist such mappings. The discrepancy can be quite large especially when the arrivals are correlated over time. For example, a packet that is present in a queue with low queue-length may have to wait for a long time to get served if few packets are offered to this queue for several time-slots.

In this paper, we directly study the delay-optimality of OFDM down-link scheduling algorithms under an ON-OFF fading model. We use large-deviations tools and study the asymptotical decay-rate of the probability that the delay of any packet exceeds a threshold, as the number of users and the number of channels both increases. (The precise definition of the above delay-violation probability is given in Section II.)

We provide a new algorithm that is shown to achieve the largest decay-rate. Unlike [1] [2] [3], our optimality result holds for a large class of general arrival processes, which may be correlated across time. To the best of our knowledge this is the first work that deals directly with the design and analysis of rate-function delay-optimal scheduling policies in OFDM wireless cellular systems.

When a large number of users are served by a single-server queue with fixed capacity, it is easy to see that the delay-optimal policy should serve packets in a First-come First-serve manner. Previously, many-source large-deviations tools have been used to study the delay performance of First-come First-serve (FCFS) scheduling policy in such single server queues [5] [6]. Somewhat surprisingly, our analysis indicates that, when the number of users and the number of carriers are large, an OFDM system under ON-OFF fading behaves quite closely to a single-server queue with intermittent connectivity, provided that some conditions on the per-user transmission requirements are satisfied (see Lemma 6 in Section IV). However, the FCFS policy no longer satisfies these conditions. Specifically, due to the random connectivity between queues and servers, we may not always be able to serve a the set of packets with the highest delay in every time-slot. Hence, we must design a new policy, called DWM (delay-weighted matching) that respects the conditions on the per-user transmission requirement.

In summary, the main contributions of this work are,

- We develop a scheduling algorithm, called DWM (delay-weighted matching), that is rate-function delay-optimal under an ON-OFF channel model for a large class of arrival processes. The conditions on the arrival processes are very mild and the arrivals may be correlated across time. Specifically, we prove that both, arrivals that are i.i.d. across time and, arrivals driven by a two-state Markov chain, satisfy the assumptions required for optimality.
- The key insight that emerges from our work is that OFDM systems with a large number of users and channels may be approximately modeled as a single-server queue with intermittent connectivity. This however requires careful consideration of the restrictions imposed by the random nature of channel capacity in a wireless OFDM system. We provide the analytical techniques that successfully address these issues.

II. System Model

We model the down-link of a single cell in an OFDM system as a discrete-time multi-source multi-server system with stochastic connectivity. There are n users and the base-station maintains a queue for each user. There are n frequency sub-carriers each of which is represented by a server. The arrival process to each queue is assumed to be stationary and ergodic, and i.i.d. across queues. However, arrivals may correlate across time. We assume that time is slotted. Let $a_u(i)$ denote the number of packets that arrive to queue u at time i, and let $a_u(i,j) = \sum_{k=i}^{j} a_u(k)$ denote the total arrivals to queue u from time i to j. We use $\bar{a}$ to denote $\mathbb{E}[a_u(\cdot)]$. Let $A(i) = \sum_{u=1}^{n} a_u(i)$ denote the cumulative arrivals at time i and $A(i,j) = \sum_{k=i}^{j} A(k)$ denote the cumulative arrivals to all queues from time i to j. To model channel fading, the queue-server connectivity in time-slot i is given by the matrix $\mathbf{C}(i) = [c_{u,s}(i)]_{n\times n}$. We assume an ON-OFF model. When $c_{u,s}(i) = 1$ we say that the queue u is connected to the server s at time i. When $c_{u,s}(i) = 0$ we say that the queue u is disconnected from server s at time i. At every time-slot the resource manager or the scheduler at the base station allocates queues to servers. If a connected server s is allocated to a queue u in time-slot i, then one packet from u can be served by the end of the time-slot by server s. In a time-slot multiple servers may be assigned to a single queue, but each server can be assigned only to one queue. For concreteness we assume that all arrivals occur at the beginning of a time-slot followed by any possible service. We also assume that each queue has infinite buffer capacity so that no packets are ever dropped.

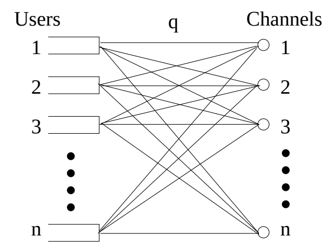

Fig. 1. System Model

Let $D_u(i)$ denote the maximum delay, starting from time i until the packet is served, of all packets that are present in queue u at time i. Note that if the packets of each queue u are served in a First-come First-serve (FCFS) manner and there is at least one packet that arrives to queue u at time i, then $D_u(i)$ is the maximum delay of all packets that arrive to queue u at time i. Further, this definition allows $D_u(i)$ to be well-defined even when there is no packet arriving to queue u at time i. Let $D(i) = \max_u\{D_u(i)\}$. Hence, $D(i) > d$ if and only if there exists a packet that arrived on or before time i and that has not been served till time $i+d$. Assume that the system is stationary and ergodic. We wish to design service rules that maximize:

$$I(d) = \lim_{n\to\infty} -\frac{1}{n} \log \mathbb{P}\Big(D(0) > d\Big).$$

Here, $I(d)$ is called the rate-function for delay d. One can imagine that a high value of rate-function would imply a low probability of packets getting delayed by d time-slots. In fact for large n we can estimate $\mathbb{P}\Big(D(0) > d\Big) \approx e^{-nI(d)}$, and the estimate becomes better for increasing values of n. It is then clear that our goal should be to design scheduling algorithms that achieve high values of delay rate-function. A policy is said to be rate-function optimal if it achieves the maximum value of $I(d)$ that any scheduling algorithm can achieve. Note that the large-n, fixed-d asymptotics are meaningful for the OFDM systems with a large number of users and carriers but requiring small delay.

We make the following assumptions about the arrival process:

Assumption 1: Arrivals are bounded. There exists $L < \infty$ such that $a_u(i) < L$ for any i and u.

Assumption 2: Given any $\epsilon > 0$ and $\delta > 0$, there exists $T > 0$, $N > 0$, and a positive function $I_B(\epsilon, \delta)$ independent of n and t such that

$$\mathbb{P}\Big(\frac{\sum_{i=1}^{t} \mathbf{1}_{\{|A(i)-\bar{a}n|>\epsilon n\}}}{t} > \delta\Big) < e^{-ntI_B(\epsilon,\delta)},$$

for all $t > T$ and $n > N$. For each $\epsilon > 0$ and $\delta > 0$, let $T_B(\epsilon,\delta)$ and $N_B(\epsilon,\delta)$ be one corresponding set of values for T and N, respectively.

Assumption 3: We assume that the channel process is *i.i.d.*, i.e.,

$$c_{u,s}(i) = \begin{cases} 1 & \text{with probability } q, \\ 0 & \text{with probability } 1-q, \end{cases}$$

independently across i, u, s.

A. Chernoff Bound and Cramer's Theorem

We use the following results from Probability Theory in our proofs. Let $X_i, 1 \le i \le n$ be a sequence of i.i.d. random variables. For any $x > \mathbb{E}[X_i]$, the Chernoff bound states that

$$\mathbb{P}\Big(\sum_{i=1}^{n} X_i \ge nx\Big) \le e^{-n[\theta x - \lambda_{X_1}(\theta)]}, \quad \text{for any real number } \theta > 0, \tag{1}$$

where $\lambda_{X_i}(\theta) = \log \mathbb{E}[e^{\theta X_i}]$ is the cumulant-generating function of X_i. The best bound is obtained by choosing the real number $\theta = \theta^*$ that maximizes $\theta x - \lambda_{X_1}(\theta)$, assuming that θ^* exists. Cramer's Theorem states that the upper bound of (1) is tight in the exponent [7, Chapter 2], i.e.

$$\lim_{n\to\infty} -\frac{1}{n}\mathbb{P}\Big(\sum_{i=1}^{n} X_i \ge nx\Big) = \theta^* - \lambda_{X_1}(\theta^*).$$

Note that the cumulative arrivals in our system in any time interval $-t+1$ to 0 , i.e. $A(-t+1,0) = \sum_{u=1}^{n} a_u(-t+1,0)$, is just the sum of n i.i.d. random variables. Hence, using Cramer's Theorem we have, for any $x \ge 0$,

$$\lim_{n\to\infty} -\frac{1}{n}\mathbb{P}\Big(A(-t+1,0) \ge n(t+x)\Big) = \sup_{\theta}[\theta(t+x) - \lambda_{a_u(-t+1,0)}(\theta)].$$

We define the quantity

$$I_A(t,x) := \sup_{\theta}[\theta(t+x) - \lambda_{a_u(-t+1,0)}(\theta)].$$

This quantity is the rate function for the probability that in t time-slots, the total number of arrivals is greater than $nx+nt$. The minimum of $I_A(t,x)$ taken over all positive integer values of t is defined as

$$I_A(x) := \inf_{t>0} I_A(t,x).$$

We now state a technical result that uses the Chernoff bound and is often referred to in this paper.

Lemma 1: Let X_i be a sequence of binary random variables

$$\mathbb{P}\Big(X_i = 1\Big) < c(n)e^{-nb}, \quad \text{for all } i$$

where $c(n)$ is a polynomial in n of finite degree. Let N_1 be such that $c(n) < e^{\frac{nb}{2}}$ for all $n > N_1$. Then, for any $0 < a < 1$,

$$\mathbb{P}\Big(\sum_{i=1}^{t} X_i > at\Big) < e^{-\frac{tnab}{3}}$$

for all $n > N := \max\{\frac{12}{ab}, N_1\}$.

Proof: Please refer to Appendix A of [8]. ∎

B. Discussion on Assumptions

Assumption 1 is mild and *Assumption 3* has been used in previous work [1] [2] [3] [9]. Assumption 1 states that the arrivals in every time-slot must be bounded above by a finite number L. While the ON-OFF channel model is a simplification, we believe that the insights will be useful for more general channel models, e.g., when the channel takes a finite set of values. *Assumption 2* is also very general. The intuition behind Assumption 2 is a statistical multiplexing effect when a large number of sources are multiplexed. The basic tenet of the assumption is that the arrivals to different queues are independent of each other. The arrivals to every queue may vary around the mean value $\bar{a}$. In some time-slots the arrivals to one queue may be higher or lower than $\bar{a}$. However, when considering a large number of such independent sources, one would expect that the sources with large arrivals would balance the sources with small arrivals so that the sum is close to $\bar{a}n$. Hence, the chance that the sum is far away from the mean $\bar{a}n$ is low, especially when n is large. Further, when the time correlation of arrivals is short-ranged, the chance that the sum arrival is far away from the mean in a large fraction of time-slots also diminishes as the time-interval increases. These statements can be mathematically proved for a large number of arrival processes. We provide here the proof for two special classes i.e., i.i.d. arrivals and arrivals driven by two-state Markov chains.

Lemma 2: Let $a(\cdot)$ be a packet arrival process such that in every time-slot,

$$a(i) = \begin{cases} r & \text{with probability } \ p, \\ 0 & \text{with probability } \ 1-p. \end{cases}.$$

Note that in this case $\bar{a} = pr$. Then, given any $\epsilon > 0$, and $\delta > 0$, there exist $T > 0$, $N > 0$, and a positive function $I_B(\epsilon,\delta)$ independent of n and t such that

$$\mathbb{P}\Big(\frac{\sum_{i=1}^{t} \mathbf{1}_{\{|A(i)-\bar{a}n|>\epsilon n\}}}{t} > \delta\Big) < e^{-ntI_B(\epsilon,\delta)}$$

for all $t > T$ and $n > N$.

Proof: Let $\bar{\epsilon} = \frac{\epsilon}{r}$. Then, it is clear that, if at any time i the fraction of queues that receive r arrivals belongs to the interval $(p-\bar{\epsilon}, p+\bar{\epsilon})$, then $|A(i) - \bar{a}n| < \epsilon n$. Moreover, the probability of this event is no smaller than $1 - 2e^{-n\min\{D_{KL}(p+\bar{\epsilon}||p), D_{KL}(p-\bar{\epsilon}||p)\}}$, where $D_{KL}(x||y) = x\log\frac{x}{y} + (1-x)\log\frac{1-x}{1-y}$, is the Kullback-Leibler divergence [7]. We define $S(i)$ to be a sequence of random variables such that $S(i) = 1$ if $|A(i) - \bar{a}n| > \epsilon n$ and $S(i) = 0$ otherwise.

Then, from Lemma 1 we know that there exists $N > 0$ such that

$$\mathbb{P}\Big(\frac{\sum_{i=1}^{t} S(i)}{t} > \delta\Big) < e^{-nt\frac{\delta \min\{D_{KL}(p+\bar{\epsilon}||p), D_{KL}(p-\bar{\epsilon}||p)\}}{3}}$$

for all $n > N$ and $t > 0$. The result then follows. ∎

The next lemma shows that Assumption 2 also holds for an arrival process driven by a two-state Markov chain.

Lemma 3: Let $a(\cdot)$ be a packet arrival process driven by a Markov chain with two states 1 and 2. Assume that whenever the Markov chain is in state i, r_i packets are generated in each time-slot. State transitions occur at the end of time-slots. Suppose the transition probability of the chain is given by the matrix, $\begin{bmatrix} 1-p_1 & p_1 \\ p_2 & 1-p_2 \end{bmatrix}$. Note that in this case $\bar{a} = \frac{p_2}{p_1+p_2} r_1 + \frac{p_1}{p_1+p_2} r_2$.
Then, given $\epsilon > 0$, and $\delta > 0$, there exists T, N and a positive function $I_B(\epsilon, \delta)$ independent of n and t such that

$$\mathbb{P}\Big(\frac{\sum_{i=1}^{t} \mathbf{1}_{\{|A(i)-\bar{a}n|>\epsilon n\}}}{t} > \delta\Big) < e^{-ntI_B(\epsilon,\delta)}$$

for all $t > T$ and $n > N$.

Proof: Please refer to Appendix B of [8]. ∎

III. An upper-bound on the rate function

In this section we derive an upper-bound on the rate function $I(d)$ of the delay asymptote for all scheduling algorithms.

Theorem 1: Given the system model as described in Section II, under any scheduling algorithm,

$$\limsup_{n\to\infty} -\frac{1}{n}\log \mathbb{P}\Big(D(0) > d\Big) \\ \leq \min\{(d+1)I_X, \min_{0\leq c\leq d}\{I_A(d-c) + cI_X\}$$

where $I_X = \log(\frac{1}{1-q})$.

Proof: We consider two events $\mathcal{E}_1$ and $\mathcal{E}_2$ that imply that $D(0) > d$.

Event $\mathcal{E}_1$: Suppose that there is a packet that arrives to queue 1 at time 0. Further, suppose that from time 0 to d queue 1 is disconnected from all servers. Then, at the end of time-slot d this packet is still in the buffer, hence $D(0) > d$.

$$\mathbb{P}\Big(\mathcal{E}_1\Big) = (1-q)^{n(d+1)} = e^{-n(d+1)I_X}. \tag{2}$$

Event $\mathcal{E}_2$: Consider the following sequence of events. Fix any $\epsilon > 0$. Choose t such that $I_A(t, d-c) < I_A(d-c) + \epsilon$. Suppose that from time $-t+1$ to 0 there are greater than $nt + n(d-c)$ arrivals to the system. Let the probability of this event be $p_{(d-c)}$. Then from Cramer's Theorem, we know that $\lim_{n\to\infty} -\frac{1}{n}\log p_{(d-c)} \leq I_A(t, d-c) \leq I_A(d-c) + \epsilon$. The total service at any time cannot exceed n. Hence, at the end of time 0, there are at least $n(d-c)+1$ packets in the buffer. Moreover, at the end of time $d-c$ the buffer must contain at least one packet that arrived before time 0. Without loss of generality, assume that this packet is present in queue 1. Now, assume that queue 1 remains disconnected from all servers in the next c time-slots. This occurs with probability $(1-q)^{cn} = e^{-nc\log\frac{1}{1-q}}$ independently of all past history. Hence, at the end of time d, there is still a packet that arrived before time 0 and that remains in queue 1. Hence, $D(0) > d$ in this case. In other words, the probability $\mathbb{P}\Big(D(0) > d\Big)$ is no smaller than $p_{(d-c)}e^{-ncI_X}$. Since this is true for any $0 \leq c \leq d$, by taking c that maximizes the above quantity, we have,

$$\mathbb{P}\Big(\mathcal{E}_2\Big) \geq \max_{0\leq c\leq d}\{p_{(d-c)}e^{-ncI_X}\}.$$

Thus,

$$\lim_{n\to\infty} -\frac{1}{n}\log \mathbb{P}\Big(\mathcal{E}_2\Big) \leq \min_{0\leq c\leq d}\{I_A(d-c) + \epsilon + cI_X\}.$$

Hence, by picking the more probable event from $\mathcal{E}_1$ and $\mathcal{E}_2$, we have

$$\lim_{n\to\infty} -\frac{1}{n}\log \mathbb{P}\Big(D(0) > d\Big) \\ \leq \min\{\min_{c\in\{0,1..d\}}\{I_A(d-c) + \epsilon + cI_X\}, (d+1)I_X\}.$$

As the above equation holds for all $\epsilon > 0$, we have

$$\lim_{n\to\infty} -\frac{1}{n}\log \mathbb{P}\Big(D(0) > d\Big) \\ \leq \min\{\min_{c\in\{0,1..d\}}\{I_A(d-c) + cI_X\}, (d+1)I_X\}.$$

∎

IV. Vector matching in bipartite graphs

In section V, we will propose a policy that attains the above optimal decay rate. Towards this end, we first study under what conditions a desired allocation of carriers to the users can be found. This problem can be viewed as a vector matching problem in bipartite graphs as we describe below.

We first introduce some notations that will be used in this section. We use $G[X \cup Y, E]$ to denote a general bipartite graph, where X and Y are two disjoint sets of vertices and E is the set of edges such that every edge $e \in E$ connects a vertex in X to a vertex in Y. Let $\partial_G(z)$ denote the set of neighbors of vertex z in G. Suppose that V is a set of vertices of G. We define $\partial_G(V) = \cup_{z\in V}\partial_G(z)$. $|\partial_G(V)|$ is called the degree of V and denotes the number of distinct neighbors of the vertices of V. If M is a subset of edges of G, i.e., $M \subset E$, then $G(M)$ is called the sub-graph induced by M and consists of all vertices of G and edges present in M.

The concept of matching is well known in Graph Theory [10, Chapter 16]. Let $G[X \cup Y, E]$ be a bipartite graph. Then a matching M is a subset of edges such that in the induced sub-graph $G(M)$ the degree of every vertex is at most one. A perfect matching is a matching such that in the induced sub-graph the degree of every vertex is exactly one.

In this section we generalize this idea of matching to vector matching. Let $G[X \cup Y, E]$ be a bipartite graph where the vertices of set X are indexed as $\{x_1, x_2, .., x_n\}$. Let v be a $|X|$-dimensional vector whose elements are non-negative integers. If V is a subset of X, then $v(V) = \sum_{\{i:x_i\in V\}} v_i$. Then, a v-matching M is a sub-set of edges M such that:

$$|\partial_{G(M)}(x_i)| \leq v_i, \text{ for all } 1 \leq i \leq |X|, \quad \text{and,} \\ |\partial_{G_{(M)}}(y)| \leq 1, \quad \text{for all } y \in Y.$$

In other words each vertex x_i in X is matched to at most v_i vertices in Y, but each vertex in Y is matched to at most one vertex in X. Note that a graph may have more than one v-matchings. A *perfect v-matching* is a v-matching (M) such that $|\partial_{G(M)}(x_i)| = v_i$ for all $x_i \in X$. If G admits a perfect v-matching then it is said to be perfectly v-matched.

Lemma 4: Let $G[X \cup Y, E]$ be a bipartite graph. Let v be a $|X|$ dimensional vector whose components are non-negative integers. Then G has a perfect v-matching if and only if for every $V \subset X$, $|\partial_G(V)| \geq v(V)$.

Remark: If $v_i = 1$ for all i, then the above result is equivalent to the well known Hall's Marriage Theorem in Graph Theory [10]. For a detailed proof of the result please refer to Appendix C of [8].

Lemma 5: Let $G[X \cup Y, E]$ be a random bipartite graph, in which for every pair of $x \in X$ and $y \in Y$ there is an edge between x and y with probability q, independently of other edges. Let v and w be vectors of length $|X|$ with non-negative integer components such that,

1) $w_1 \geq w_2 + 2$;
2) $v_1 = w_1 - 1; v_2 = w_2 + 1$;
3) $v_i = w_i \ \forall\ 3 \leq i \leq |X|$.

Then,

$$\mathbb{P}\Big(G \text{ has a perfect } v\text{-matching}\Big) \geq \mathbb{P}\Big(G \text{ has a perfect } w\text{-matching}\Big).$$

Remark: Note that v and w are the same everywhere except in the first two components. Moreover, v is more balanced than w. The above result then says that, if a vector is more balanced, then the probability of vector matching is higher. This basic result forms the basis of the next Corollary. For a detailed proof please refer to Appendix D of [8].

In the following results, H is a given positive integer independent of n.

Corollary 1: Let $G[X \cup Y, E]$ be a random bipartite graph in which for every pair of $x \in X$ and $y \in Y$ there is an edge between x and y with probability q, independently of other edges. Let v and w be two vectors of length $|X|$ with non-negative integer components such that,

1) $\max_i [v_i] \leq H$;
2) $\sum_{i=1}^{|X|} v_i = \sum_{i=1}^{|X|} w_i \leq n - H$;
3) $w_i = \begin{cases} n - (k+1)H, & \text{if } i = 1 \\ H, & \text{if } 2 \leq i \leq k+1; \\ 0, & \text{if } i > k+1, \end{cases}$

where $k = \lceil \frac{n}{H} \rceil - 2$. Then,

$$\mathbb{P}\Big(G \text{ has a perfect } v\text{-matching}\Big) \geq \mathbb{P}\Big(G \text{ has a perfect } w\text{-matching}\Big).$$

Proof: Let Ξ be the set of all $|X|$-dimensional vectors with non-negative integer components, such that for any vector ξ in Ξ, $\sum_{i=1}^{|X|} \xi_i = n - H$ and $\max_{\{1 \leq i \leq |X|\}} \xi_i \leq H$. Then we claim that out of all these vectors, the vector w defined in the assumption is the most unbalanced. Specifically, it can be shown that any vector v in the set Ξ can be constructed from w be a series of operations, such that in every step the new vector becomes more balanced (in the sense of Lemma 5). The result then follows from induction. Further, for any vector v with $\sum_{i=1}^{|X|} v_i < n - H$ and $\max_{\{1 \leq i \leq |X|\}} v_i \leq H$, it is easy to see that there exists a vector ξ' in Ξ, such that v is component-wise no greater than ξ'. Hence,

$$\mathbb{P}\Big(G \text{ has a perfect } v\text{-matching}\Big) \geq \mathbb{P}\Big(G \text{ has a perfect } \xi'\text{-matching}\Big).$$

The result then follows. For the details of the proof please refer to Appendix E of [8]. ∎

Lemma 6: Let $G[X \cup Y, E]$ be a random bipartite graph, in which for every pair of $x \in X$ and $y \in Y$ there is an edge between x and y with probability q, independently of all other edges. Let $|X| = |Y| = n$. Let w be a vector with non-negative integer components with $\sum_{i=1}^{n} w_i \leq n - H$ and $\max_{1 \leq i \leq n} w_i \leq H$, then for some finite value of N_X,

$$\mathbb{P}\Big(G \text{ has a perfect } w\text{-matching}\Big) \geq 1 - \Big(\frac{n}{1-q}\Big)^{7H} e^{-n \log \frac{1}{1-q}}$$

for all $n > N_X$.

Proof: Please refer to Appendix F of [8]. ∎

Remark: Let X denote the set of source queues, Y denote the servers, and use an edge between u and s if queue u is connected to server s. Then according to Lemma 6, with high probability an allocation of servers to queues can be found such that each user i will receive w_i service, provided that $\sum_{i=1}^{n} w_i \leq n - H$ and $\max_{1 \leq i \leq n} w_i \leq H$. Hence, this system is very similar to a single-server queue with service rate $n - H$ and intermittent connectivity, provided that the service requirement of each user is bounded by H. In the next section, we will use this insight to construct a delay-optimal policy.

V. Scheduling policy

In this section we describe two scheduling rules. We assume that in every time-slot the scheduler has perfect knowledge of queue-server connectivity which is represented by the matrix $\mathbf{C}(i)$. Also, it can use the past history of arrival and channel processes.

A. Intuition behind the proposed delay-based policies

Motivated by Lemma 6, we consider a single-server queue with intermittent connectivity. Specifically, in every time-slot the server is connected with probability $1 - \left(\frac{n}{1-q}\right)^{7H} e^{-nI_X}$, and disconnected otherwise. Whenever the server is connected it can serve $n_0 = n - H$ packets. However, it cannot serve any packets when disconnected. It is not difficult to see that, if we serve packets in such a single-server queue in a FCFS manner, then the delay rate–function is optimal, i.e., it is equal to the upper bound given in Theorem 1.

Now from Lemma 6, our OFDM system is in fact quite similar to the single-server queue in the sense that, under suitable restrictions, the probability that n_0 packets may be served in a time-slot is no less than $1 - \left(\frac{n}{1-q}\right)^{7H} e^{-nI_X}$. However, obviously we cannot use a FCFS policy directly, because it may violate the condition of Lemma 6, which translates to the restriction that in a frame every user can have no more than H packets to be served.

To circumvent the difficulty, we propose two policies FBS(h) and DWM. The policy FBS(h) approximates the FCFS policy, while respecting the restrictions mentioned above. However, the FBS(h) policy is conservative in nature, and may waste capacity. Hence, it may not be throughput-optimal. Therefore, we propose another policy DWM, which is more aggressive in serving packets and does not waste capacity. We further show that DWM always serves packets ahead of FBS(h) for every arrival process and every value of h, and hence the delay rate-function for DWM must be no smaller than the delay rate-function for FBS(h). Then, in Section VI we show that there exists a value of h such that FBS(h) is rate-function optimal, which implies that DWM is also rate-function optimal.

B. Policy FBS(h) (Frame Based Scheduling)

This policy serves packets in units of frames. Suppose that a positive integer h is given. Recall that no more than L packets arrive to any user in a time-slot. Let $n_0 = n - Lh$ be the capacity of each frame. In the policy FBS(h) each frame is composed of packets that satisfy the following two conditions:

1) The number of packets in the frame is no greater than n_0 (i.e., the capacity of a frame);
2) The difference of arrival times of any two packets in the frame must be no larger than h.

As packets arrive in each time-slot, the frames are constructed by filling in the packets sequentially. Specifically, packets belonging to queue 1 are filled before packets belonging to queue 2, and so on. Further, older packets are added before newer packets. We fill each frame until the above conditions cannot be maintained. Then we start a new frame. There might be a frame that is only partially filled at the end of a time-slot. In the next time-slot this frame is filled first, before starting a new frame.

A frame in general may be represented as a vector in $\mathbb{Z}^n$, where the j^{th} component of the vector represents the number of packets of user j in the frame. The policy FBS(h) serves the frames in the same order as they are constructed. Further, at most one frame is served in a time-slot. Specifically, let $v(i)$ denote the vector representing the head-of-line frame at time i. From the construction of frame described above and Assumption 1 on the boundedness of the arrival process, we have $\max_{1\le j\le n} v_j(i) < Lh$. Moreover, $\sum_j v_j(i) \le n_0 = n - Lh$ for all i. Note that a frame might contain less than n_0 packets if it is the only frame left or if it was full because of condition 2.

In each time-slot i the policy FBS(h) tries to schedule the head-of-line frame $v(i)$ for transmission. Let $H = hL$. We know from Section IV on vector matching that, with probability $1 - \left(\frac{n}{1-q}\right)^{7H} e^{-nI_X}$, the scheduler can transmit the whole frame in a given time-slot. If the policy FBS(h) cannot transfer the whole frame, then no packets are scheduled in this time-slot and the scheduler will try again in the next time-slot. Define the random variable $X_F(i) = 1$ if $v(i)$ is successfully transmitted at time i, and $X_F(i) = 0$, otherwise.

C. Policy DWM (Delay Weighted Matching)

The policy FBS(h) may not serve any packet in a time-slot, and may waste up to Lh packets in a time-slot when is serves a packet. The policy DWM, below, is meant to be efficient.

In every time-slot, define the waiting time of every packet as the time that the packet has spent in the buffer. We assign a weight to every packet as follows. If a packet has a waiting time of W and belongs to the queue with index u, then its weight is $W + \frac{u}{n+1}$. Next, construct a bipartite graph $G[X \cup Y, E]$ such that vertices in X correspond to the oldest n packets of every queue and Y is the set of servers. The edge set E is constructed as follows: if u is connected to s, then all vertices that correspond to packets of u are connected to s. The packets to transmit are then determined by a maximum-weight matching algorithm. In the following Lemma we compare policies DWM and FBS(h).

Lemma 7: For any given sample path and for any value of h, by the end of time-slot i, Policy DWM has served every packet that FBS(h) has served.

Proof: Please refer to Appendix G of [8]. ■

By Lemma 7, if we can show that the policy FBS(h) is rate-function optimal for some value of h, the the policy DWM must also be rate-function optimal without the need to know the exact value of h. The optimality of FBS(h) is studied next in section VI.

VI. Analysis of FBS(h)

We define $F(i)$ as the number of unserved frames in buffer at time i. Then, we can write a recursive equation for $F(i)$:

$$F(i) = \max\{F(i-1) + \left\lceil \frac{A(i) - R(i-1)}{n_0} \right\rceil - X_F(i), 0\}.$$

$$Z(i) = \begin{cases} [Z(i-1)+1]\mathrm{mod}(h), & \text{if } A(i) < R(i-1) \\ 1, & \text{if } A(i) > R(i-1) \\ 0, & \text{if } A(i) = R(i-1). \end{cases}$$

$$R(i) = \mathbf{1}_{\{F(i)>0\}}\mathbf{1}_{\{Z(i)>0\}}[R(i-1) - A(i)]\mathrm{mod}(n_0). \quad (3)$$

To explain this set of equations, recall that after each time-slot, the end-of-line frame may be only partially filled and thus can be filled with new arrivals in the next time-slot. We use $R(i)$ to represent the remaining available space in the end-of-line partially-filled frame at the end of time i. Hence, $\left\lceil \frac{A(i)-R(i-1)}{n_0} \right\rceil$ represents the number of new frames that are created at time i. Note that if $A(i) \le R(i-1)$, i.e., the number of arrivals at time i is less than the remaining available space in the end-of-line frame at the end of time $i-1$, then no new frame is added. $X_F(i)$ represents the number of frames served in time-slot i. Notice that a maximum of one frame and hence n_0 packets can be transmitted in a time-slot. The variable $Z(i)$ counts the number of time-slots for which the end-of-line frame has been open. It starts at 1 when a new frame is opened, i.e., when $A(i) > R(i-1)$. Then it is incremented by 1 every time when the number of arrivals $A(i)$ is less than $R(i-1)$. If it reaches h, then this frame is completed and a new frame is started, in which case $Z(i) = 0$ and $R(i) = 0$. Let v be an n-dimensional vector whose i^{th} component represents the number of packets of queue i in a frame. The construction of the frames ensures that for every frame $v_i \le hL = H$ and $\sum_{i=1}^{n} v_i \le n - H = n_0$. Hence, from Lemma 6 we have that $X_F(i) = 1$ with probability no smaller than $1 - \left(\frac{n}{1-q}\right)^{7H} e^{-nI_X}$ in every time-slot.

Let $R_0 = R(i-1)$ be the empty space in the end-of-line frame at the end of time $i-1$. Then $A_F^{R_0}(i,k)$ denotes the number of new frames created from time i to k, including any partially-filled frame at time k but excluding any partially-filled frame at time i. We use the notation $A_F(i,k)$ to denote $A_F^{R_0}(i,k)$, if $R_0 = 0$. Hence we can write,

$$F(k) = F(i-1) + A_F^{R_0}(i,k) - X_F(i,k),$$

where $X_F(i,k)$ denotes the total number of frames departing from the buffer in the time interval i to k. That is,

$$X_F(i,k) = \sum_{j=i}^{k} X_F(j) 1_{\{F(j)>0\}}.$$

In general the equations in (3) are complicated to analyze. However, if the arrival process satisfies some special conditions in a time interval (i,k), then we can derive some useful results as follows.

Lemma 8: Let $A(\cdot)$ be an arrival process. Let R_0 be the empty space in the end-of-line frame at the end of time $i-1$. Let the arrivals in the interval from i to k be such that

1) The buffer never becomes empty in the interval, i.e., $F(j) > 0$ for all $j \in \{i, i+1, ..., k\}$.
2) For any $h-1$ consecutive time-slots in the interval, the cumulative arrivals are greater than or equal to n_0, i.e., $\sum_{j=x}^{x+h-2} A(j) \ge n_0$, for any $x \in \{i, i+1, ..., k-h+2\}$.

Then the following holds for policy FBS(h),

$$A_F^{R_0}(i,k) = \left\lceil \frac{A(i,k) - R_0}{n_0} \right\rceil, \quad R(k) = [R_0 - A(i,k)]\text{mod}(n_0). \tag{4}$$

Remark: The condition of the Lemma implies that every frame has exactly n_0 packets. The result then follows. For details, please refer to Appendix H of [8]. ■

Corollary 2: Let $A(\cdot)$ be an arrival process such that $F(j) > 0$ for all $i \le j \le k$ and let $B = \{x_1, ..., x_{|B|}\}$ be a sequence of time-slots in increasing order, belonging to the interval from i to k, such that in every interval $(x_i + 1, x_{i+1})$, $i \in \{1, 2..., |B|-1\}$, the condition 2 of Lemma 8 is satisfied. Then,

$$A_F^{R_0}(x_1 + 1, x_{|B|}) \le \sum_{j=1}^{|B|-1} \left\lceil \frac{A(x_j + 1, x_{j+1})}{n_0} \right\rceil$$

Proof: From Lemma 8 we have that

$$A_F^{R_0}(x_1 + 1, x_{|B|}) = \sum_{j=1}^{|B|-1} \left\lceil \frac{A(x_j + 1, x_{j+1}) - R(x_j)}{n_0} \right\rceil.$$

Since $R(\cdot) \ge 0$, it follows that,

$$A_F^{R_0}(x_1 + 1, x_{|B|}) \le \sum_{j=1}^{|B|-1} \left\lceil \frac{A(x_j + 1, x_{j+1})}{n_0} \right\rceil.$$

■

Theorem 2: If the arrival process satisfies assumptions 1 and 2 and the channel process satisfies assumptions 3, then there exists a value of h for which the scheduling policy FBS(h) is rate-function optimal, that is

$$\lim_{n\to\infty} \inf \frac{-1}{n} \log \mathbb{P}\Big(D(0) > d\Big) \ge I_0 := \min\{ \min_{c\in\{0,1,...d\}} I_A(d-c) + cI_X, (d+1)I_X\}$$

Proof: We first choose the value of h based on the statistics of the arrival process. Let the mean of the arrival process be $\bar{a}$. We fix $\delta < \frac{2}{3}$ and $\epsilon < \frac{\bar{a}}{2}$. Then, from Assumption 2 on the arrival process, there exists a positive function $I_B(\epsilon, \delta)$, such that for all $n > N_B(\epsilon, \delta)$ and $t > T_B(\epsilon, \delta)$ we have,

$$\mathbb{P}\Big(\frac{\sum_{i=j+1}^{j+t} \mathbf{1}_{\{|A(i) - \bar{a}n| > \epsilon n\}}}{t} > \delta \Big) < e^{-ntI_B(\epsilon,\delta)},$$

for any integer j.

We then choose

$$h = \max\left\{ T_B(\epsilon, \delta), \left\lceil \frac{1}{(\bar{a}-\epsilon)(1-\frac{3\delta}{2})} \right\rceil, \left\lceil \frac{2I_0}{I_B(\epsilon,\delta)} \right\rceil \right\} + 1. \tag{5}$$

The reason for choosing such a value of h will become clear later on. Recall that L is the maximum number of packets that can arrive to a queue at any time-slot i and $H = hL$. Note that H is then the maximum number of packets that can arrive to a queue in h time-slots.

Let $L(0)$ be the last time, $-t$ before 0, when the buffer was empty, i.e., $D(-t) = 0$. Then given that $L(0) = -t$, the event $D(0) > d$ occurs if and only if the number of frames that arrive in the time interval from $-t+1$ to 0 is greater than the total number of frames that could be served in $-t+1$ to d. That is,

$$\Big\{D(0) > d, L(0) = -t\Big\} = \Big\{L(0) = -t, A_F(-t+1, 0) - X_F(-t+1, d) > 0\Big\}.$$

By taking the union over all possible values of $L(0)$ we get,

$$\mathbb{P}\Big(D(0) > d\Big) \le \sum_{t=1}^{\infty} \mathbb{P}\Big(L(0) = -t, A_F(-t+1, 0) - X_F(-t+1, d) > 0\Big).$$

We now fix any $1 > \hat{p} > \bar{a}$. Then define,

$$t^* := \max\left\{T_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)}), \left\lceil \frac{6}{1-\hat{p}} \right\rceil, \right.$$
$$\left. \left\lceil \frac{I_0}{\min\{I_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)}), (\frac{1-\hat{p}}{9})I_X\}} \right\rceil \right\} \qquad (6)$$

and split the summation as,

$$\begin{aligned}
&\mathbb{P}\Big(D(0) > d\Big) \\
&\leq \sum_{t=1}^{t^*} \mathbb{P}\Big(L(0) = -t, A_F(-t+1,0) - X_F(-t+1,d) > 0\Big) \\
&+ \sum_{t=t^*}^{\infty} \mathbb{P}\Big(L(0) = -t, A_F(-t+1,0) - X_F(-t+1,d) > 0\Big).
\end{aligned} \qquad (7)$$

We divide the proof into two parts. In Part 1 we prove that there exists $N_1 > 0$ such that for all $n > N_1$

$$\begin{aligned}
&\sum_{t=1}^{t^*} \mathbb{P}\Big(A_F(-t+1,0) - X_F(-t+1,d) > 0, L(0) = -t\Big) \\
&\qquad < c_1 t^* 2^{c_2 t^*} \left(\frac{n}{1-q}\right)^{7H} e^{-nI_0},
\end{aligned}$$

where c_1, c_2 are positive constants independent of t and n. And in Part 2 we prove that there exists $N_2 > 0$ such that for all $n > N_2$

$$\begin{aligned}
&\sum_{t=t^*}^{\infty} \mathbb{P}\Big(A_F(-t+1,0) - X_F(-t+1,d) > 0, L(0) = -t\Big) \\
&\qquad \leq 4e^{-nI_0}.
\end{aligned}$$

Finally, by substituting both parts into equation (7), we have that there exists $N := \max\{N_1, N_2\}$ such that for all $n > N$,

$$\begin{aligned}
&\sum_{t=1}^{\infty} \mathbb{P}\Big(D(0) > d, L(0) = -t\Big) \\
&\qquad \leq (c_1 t^* 2^{c_2 t^*} \left(\frac{n}{1-q}\right)^{7H} + 4) e^{-nI_0}.
\end{aligned}$$

By taking logarithm and limit as n tends to infinity, we get the desired result.

Part 1: Let us denote by $\mathcal{E}_t^\alpha$ the set of sample paths in which every $h-1$ time-slots in the interval $-t+1$ to 0 see at least n arrivals. Let $\mathcal{E}_t^\beta$ be the set of sample paths in which $\frac{A(-t+1,0)}{n_0} - \sum_{j=-t+1}^{d} X_F(j) > 0$. And let $\mathcal{E}_t$ be the sample paths such that $L(0) = -t$ and $D(0) > d$. Then, the following can be shown,

$$\mathcal{E}_t \subset (\mathcal{E}_t^\alpha)^c \cup \mathcal{E}_t^\beta. \qquad (8)$$

To see this, observe that $\mathcal{E}_t$ is the set of sample paths in which $L(0) = -t$ and $A_F(-t+1,0) - X_F(-t+1,d) > 0$. For all sample paths in the set $\mathcal{E}_t^\alpha \cap \mathcal{E}_t$, Lemma 8 holds and hence, $A_F(-t+1,0) = \left\lceil \frac{A(-t+1,0)}{n_0} \right\rceil$. Moreover, it is easy to observe that, for all sample paths in the set $\mathcal{E}_t^\alpha \cap \mathcal{E}_t$, $X_F(-t+1,d) = \sum_{j=-t+1}^{d} X_F(j)$. Hence, for a sample path belonging to $\mathcal{E}_t^\alpha \cap \mathcal{E}_t$, we must have $\frac{A(-t+1,0)}{n_0} - \sum_{j=-t+1}^{d} X_F(j) > 0$. This implies that, $\mathcal{E}_t \cap \mathcal{E}_t^\alpha \subset \mathcal{E}_t^\beta$. Thus we have, $\mathcal{E}_t = (\mathcal{E}_t \cap \mathcal{E}_t^\alpha) \cup (\mathcal{E}_t \cap (\mathcal{E}_t^\alpha)^c) \subset (\mathcal{E}_t^\beta \cup (\mathcal{E}_t^\alpha)^c)$. Hence, (8) holds. It then follows that,

$$\mathbb{P}(\mathcal{E}_t) L \leq \mathbb{P}((\mathcal{E}_t^\alpha)^c) + \mathbb{P}(\mathcal{E}_t^\beta). \qquad (9)$$

We now give the intuition behind the analysis of $\mathcal{E}_t^\alpha$ and $\mathcal{E}_t^\beta$. For a detailed proof, please refer to Appendix I of [8].

We note that the event $\mathcal{E}_t^\alpha$ implies that every frame formed in the interval from $-t+1$ to 0 will have n_0 packets, i.e. all frames served are completely full. It is then obvious that $\mathbb{P}(\mathcal{E}_t^\alpha)$ depends on h, i.e., it will be large if we increase the maximum time for which any frame can remain open. By choosing an h large enough we can ensure that the probability $\mathbb{P}((\mathcal{E}_t^\alpha)^c)$ is arbitrarily small. In particular, we can ensure that the rate-function of $\mathbb{P}((\mathcal{E}_t^\alpha)^c)$ is greater than the rate-function of $\mathbb{P}(\mathcal{E}_t^\beta)$. The cost that needs to be paid for having a large h is the loss in frame-size, which is $n_0 = n - Lh$. But this decrease in frame-size is independent of n and does not affect the performance of the system significantly for large n. Hence, it does not show up in the rate-function. Specifically, for the choice of h in (5), it can be shown that there exists $N_3, c_3 > 0$ such that we have

$$\mathbb{P}(\mathcal{E}_t^\alpha) > 1 - c_3 t e^{-nI_0}. \qquad (10)$$

for all $n > N_3$.

It can be seen that the event $\mathcal{E}_t^\beta$ is similar to the buffer overflow event in a single-server queues with intermittent connectivity as described earlier. Recall that as opposed to a single-server queue with constant rate, in every time-slot, with probability approximately $1 - e^{-nI_X}$ the service is equal to n_0 packets, i.e., one frame. So now there can be two factors responsible for $\mathcal{E}_t^\beta$. Firstly, if the arrival process is bursty, then $\mathcal{E}_t^\beta$ can be caused by a large burst of arrivals in a few time-slots. Secondly, if q is small $\mathcal{E}_t^\beta$ can be caused by a time interval of low service as frames get piled up in the buffer. For moderate values of q, one can expect that the most likely way in which $\mathcal{E}_t^\beta$ occurs is a mixture of bursty arrivals and sluggish service. From large deviations theory we know that the rate-function of $\mathcal{E}_t^\beta$ is determined by the probability of the most likely sample path leading to $\mathcal{E}_t^\beta$. More formally, it can be shown that there exists $c_2, c_4, N_4 > 0$ such that

$$\begin{aligned}
&\mathbb{P}\Big(\mathcal{E}_t^\beta\Big) \\
&= \mathbb{P}\Big(\frac{A(-t+1,0)}{n_0} - \sum_{j=-t+1}^{d} X_F(j) > 0\Big) \\
&= \sum_{a=0}^{t+d} \mathbb{P}\Big(\sum_{j=-t+1}^{d} X_F(j) = a\Big) \mathbb{P}\Big(A(-t+1,0) > an_0\Big) \\
&\leq (t+d+1) \max_{0 \leq a \leq t+d} \Big\{ \mathbb{P}\Big(\sum_{j=-t+1}^{d} X_F(j) = a\Big) \\
&\qquad \times \mathbb{P}\Big(A(-t+1,0) > an_0\Big) \Big\} \\
&\leq c_4 2^{c_2 t} \left(\frac{n}{1-q}\right)^{7H}
\end{aligned}$$

$$\times e^{-n\min\{(d+1)I_X, \min_{a\in\{0,1,2...d\}}\{I_A(a)+(d-a)I_X\}\}}$$
$$\leq c_4 2^{c_2 t}\Big(\frac{n}{1-q}\Big)^{7H} e^{-nI_0}, \tag{11}$$

for all $n > N_4$.

Let $c_1 = 2\max\{c_3, c_4\}$. Substituting (10) and (11) into (9) we then have

$$\mathbb{P}(\mathcal{E}_t) \leq c_3 t e^{-nI_0} + c_4 2^{c_2 t}\Big(\frac{n}{1-q}\Big)^{7H} e^{-nI_0}$$
$$\leq c_1 2^{c_2 t}\Big(\frac{n}{1-q}\Big)^{7H} e^{-nI_0}$$

for all $n > N_1 = \max\{N_3, N_4\}$. Finally, summing over $t = 1$ to t^* we have,

$$\sum_{t=1}^{t^*} \mathbb{P}\Big(D(0) > d, L(0) = -t\Big) = \sum_{t=1}^{t^*} \mathbb{P}\Big(\mathcal{E}_t\Big)$$
$$\leq c_1 t^* 2^{c_2 t^*}\Big(\frac{n}{1-q}\Big)^{7H} e^{-nI_0},$$

for all $n > N_1$.

Part 2: We would like to show that there exists $N_2 > 0$ such that for $n > N_2$

$$\sum_{t=t^*}^{\infty} \mathbb{P}\Big(A_F(-t, 0) - X_F(-t, d) > 0\Big) < 4e^{-nI_0}.$$

We noted earlier that the equations for evolution of $A_F(-t+1, 0)$ are in general complicated. But if an arrival process satisfies certain conditions then some simple results such as Lemma 8 and Corollary 2 can be obtained. Hence, to analyze $A_F(-t+1, 0)$ we first construct an arrival process $\hat{A}(\cdot)$ that satisfies the conditions of Lemma 8 and $\hat{A}_F(-t+1, 0) > A_F(-t+1, 0)$. We do this by adding some extra arrivals to the process $A(\cdot)$ in some strategic time-slots. The resulting arrival process $\hat{A}(\cdot)$ has the property that $\hat{A}(i) = \hat{p}n$ whenever $A(i) \leq \hat{p}n$ and $\hat{A}(i) = Ln$ whenever $A(i) > \hat{p}n$. (Please refer to Appendix I of [8] for the details of how to construct $\hat{A}(\cdot)$.) Hence, the resulting arrival process $\hat{A}(\cdot)$ is in fact very simple. We now get an upper bound on $\hat{A}_F(-t+1, 0)$, which, by construction, is also an upper bound on $A_F(-t+1, 0)$.

Let $B = \{b_1, b_2, , b_{|B|}\}$ be the set of time-slots in the interval $-t+1$ to 0 when $A(i) \geq \hat{p}n$. Then from Corollary 2 we have that, given $L(0) = -t$,

$$\hat{A}_F(-t+1, 0)$$
$$\leq \sum_{j=1}^{|B|-1}\left\lceil\frac{\hat{A}(b_j+1, b_{j+1}-1)}{n_0}\right\rceil + \sum_{j=1}^{|B|}\left\lceil\frac{\hat{A}(b_j, b_j)}{n_0}\right\rceil$$
$$+ \left\lceil\frac{\hat{A}(-t+1, b_1-1)}{n_0}\right\rceil + \left\lceil\frac{\hat{A}(b_{|B|}+1, 0)}{n_0}\right\rceil$$
$$\leq \frac{n}{n_0}[\hat{p}t + (L+2)|B| + 1].$$

From Assumption 2 on the arrival process we know that for large enough n and t, $|B|$ can be made less than an arbitrarily small fraction of t. Further, we can show that for $|B| < \frac{1-\hat{p}}{6(L+2)}t$, $n > \frac{H(2+\hat{p})}{1-\hat{p}}$ and $t > \frac{6}{1-\hat{p}}$, $A_F(-t+1, 0) \leq \hat{A}_F(-t+1, 0) \leq (\frac{2+\hat{p}}{3})t$. (Please refer to Appendix I of [8] for details.) Hence,

$$\mathbb{P}\left(A_F(-t+1, 0) \geq (\frac{2+\hat{p}}{3})t, L(0) = -t\right)$$
$$\leq \mathbb{P}\left(|B| > \frac{1-\hat{p}}{6(L+2)}t\right)$$
$$\leq e^{-ntI_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)})}, \tag{12}$$

for all $n > N_5 = \max\{N_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)}), \frac{H(2+\hat{p})}{1-\hat{p}}\}$ and $t > T_1 = \max\{T_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)}), \frac{6}{1-\hat{p}}\}$.

Moreover, we know that for each i, $X_F(i) = 1$ with probability greater than $1 - \left(\frac{n}{1-q}\right)^{7H} e^{-nI_X}$ for all $n > N_X$. Hence, using Lemma 1 we have that, there exists $N_6 > N_X$ such that,

$$\mathbb{P}\Big(X_F(-t+1, d) < (\frac{2+\hat{p}}{3})t, L(0) = -t\Big)$$
$$\leq \mathbb{P}\Big(X_F(-t+1, d) \leq (\frac{2+\hat{p}}{3})(t+d), L(0) = -t\Big)$$
$$\leq e^{-n(t+d)(\frac{1-\hat{p}}{9})I_X}$$
$$\leq e^{-nt\frac{(1-\hat{p})I_X}{9}}, \tag{13}$$

for all $n > N_6$ and $t > 0$. Combining the above two results, from (12) and (13) we have, for all $n > N_7 = \max\{N_5, N_6\}$ and $t > T_1$,

$$\mathbb{P}\Big(A_F(-t+1, 0) - X_F(-t+1, d) > 0, L(0) = -t\Big)$$
$$\leq 1 - (1 - e^{-nt\frac{(1+\hat{p})I_X}{9}})(1 - e^{-ntI_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)})})$$
$$\leq 2e^{-tnI_{BX}},$$

where I_{BX} is the minimum of $I_B(\hat{p}-\bar{a}, \frac{1-\hat{p}}{6(L+2)})$ and $\frac{(1-\hat{p})I_X}{9}$.

Recall that $t^* > \max\{T_1, \frac{I_0}{I_{BX}}\}$. Hence, summing over all $t > t^*$ we have, for all $n > N_2 = \max\{N_7, \left\lceil\frac{\log 2}{I_{BX}}\right\rceil\}$

$$\sum_{t=t^*}^{\infty} \mathbb{P}\Big(A_F(-t, 0) - X_F(-t, d) > 0, L(0) = -t\Big)$$
$$\leq \sum_{t=t^*}^{\infty} \mathbb{P}\Big(A_F(-t, 0) - X_F(-t, d) > 0, L(0) = -t\Big)$$
$$\leq \sum_{t=t^*}^{\infty} 2e^{-ntI_{BX}}$$
$$\leq \frac{2e^{-nt^*I_{BX}}}{1 - e^{-nI_{BX}}}$$
$$\leq 4e^{-nt^*I_{BX}} \qquad (\text{as } e^{-nI_{BX}} < \frac{1}{2})$$
$$\leq 4e^{-nI_0}.$$

The result of the theorem then follows. ■

VII. Simulation Results

In this section, we compare the performance of the proposed DWM algorithm with the standard Max Weight (MW) [11] and the recently-proposed Server Side Greedy (SSG) algorithm in [1] [2]. We simulate these algorithms and compare the empirical probabilities that the maximum delay at any given time exceeds a constant d. We consider two settings: (i) when the arrivals are i.i.d. across time-slots, and (ii) when arrivals are correlated across time-slots.

In the first setting, the arrivals to every queue are given by the following distribution:

$$a(i) = \begin{cases} 5 & \text{with probability } \ 0.167, \\ 0 & \text{with probability } \ 0.833, \end{cases}$$

independently for all time-slots i. We run the MW, SSG and DWM algorithms for a system with $n = 30$, i.e., a system with 30 users and 30 carriers/servers. The user-server connection probability is $q = 0.75$, so that the system is stable but heavily loaded, i.e. greater than 83.5 % of the maximum load. We run the simulation for 10^5 time-slots.

In the second setting, we consider arrivals that are driven by a Markov chain with two states. When the Markov chain is in state 1, 5 packets are generated in each-time slot, and when the chain is in state 2, no packets are generated. Further, state transitions occur at the end of time-slots. The transition probability of the chain is given by the matrix $\begin{bmatrix} .5 & .5 \\ .1 & .9 \end{bmatrix}$. The user-server connection probability is chosen as $q = 0.75$. We consider a system with $n = 30$ and run the simulation for 10^5 time-slots.

The results are summarized in figure 2.

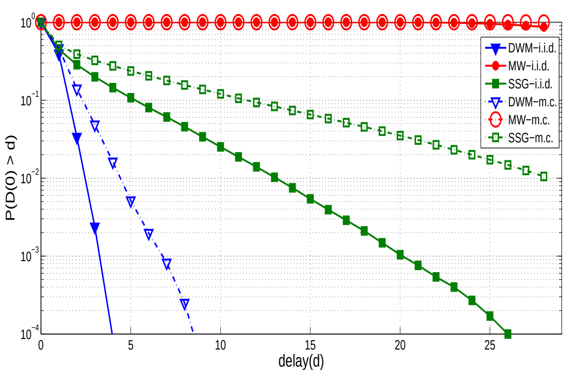

Fig. 2. Performance of DWM, MW and SSG for $n = 30$, $q = 0.75$, i.i.d. and 2-state Markov chain (m.c.) driven arrivals

As can be seen from the plots, the proposed DWM algorithm performs consistently better than the Max Weight algorithm and the SSG algorithm.

VIII. Conclusion

We consider the scheduling problem of the down-link of an OFDM system for supporting a large number of delay-sensitive users. Assuming an ON-OFF channel model, we show that when the scale of the system is large, the OFDM system can be approximated by a Single-Server Queue with intermittent connectivity. Inspired by this observation, we first construct the Frame Based Scheduling (FBS(h)) policy that emulates the single-serve queue by accounting for the restrictions placed by the wireless channel in an OFDM system. We then prove that, for a large class of arrival processes, there exists a value of h for which FBS(h) is rate-function delay-optimal. Since FBS(h) may waste capacity and the suitable value of h depends on the arrival process, we then design the Delay Weighted Matching (DWM) scheduling algorithm, which is both rate-function delay-optimal independently of the arrival process and achieves high throughput. Our simulations indicate that DWM can significantly improve the performance compared to the state-of-art algorithms in the literature.

There are many interesting directions for future work. First, we plan to use the insight gained from DWM to design scheduling algorithms for more general channel models. Second, although DWM achieves rate-function delay-optimality, it may have a high computational complexity. It would be worthwhile to consider scheduling algorithms that achieve good delay bounds and are of lower complexity. Third, in this work we consider the case when all users have similar arrival patterns, channel conditions, and delay requirements. It would be interesting to see how the DWM algorithm can be extended to users with different arrival patterns and channel conditions and with diverse delay requirements.

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