Pricing-based Energy Storage Sharing and Virtual Capacity Allocation

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***Abstract*—This paper develops a novel business model to enable virtual storage sharing among a group of users. Specifically, an aggregator owns a central physical storage unit and virtualizes the physical storage into separable virtual storage capacities that can be sold to users. Each user purchases the virtual storage capacity, and schedules the charge and discharge of the virtual storage to reduce his peak power consumption. We formulate the interaction between the aggregator and users in each operation horizon as a two-stage problem. At the beginning of the operation horizon, the aggregator first determines the unit price of virtual storage capacity to maximize her profit in Stage 1, and users decide the capacities to purchase and the storage scheduling during the operation horizon in Stage 2. Since the closed-form solution is not available and the decisions are coupled across the two stages, we characterize the solutions of the two-stage problem based on parametric linear programming. Simulation results show that compared to the case where each user acquires his own physical storage, storage virtualization reduces the overall physical capacity needed for all users by 34.9%, and the overall physical power rating by 45.1%.**

I. Introduction

Energy storage is becoming a crucial element to ensure the stable and high performance of the new generation of power system. While the benefits of the energy storage at the grid side have been well-recognized (e.g., for generation backup, transmission support, voltage control, and frequency regulation) [1], there is also an increasing interest to leverage energy storage at the end-user side to store energy from distributed generations, shave the peak load, and reduce the electrical bill [1]. Because of this, some recent storage products such as Tesla Powerwall have emerged targeting at residential customers [2], but often with a price tag that is quite high [3]. As such a consumer storage product can last for years, it can be challenging for a user to decide the optimal storage sizing due to the uncertainty of future energy demand. This further discourages the user to purchase such a product and enjoy the benefit. This motivates us to study the following key problem: *What would be an economical business model that promotes efficient utilization of the storage by end users?*

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The business model for end-user storage deployment is relatively under-studied in the literature. Most of the prior results considering storage-based demand management (e.g., [4]) assumed that users own the storage. They did not consider the impact of high storage cost on users' decisions. Studies in [5], [6] considered central storage platforms shared by a group of users. AlSkaif *et al.* in [5] designed a reputation-based policy to allocate energy in the storage to users. Mediwaththe *et al.* in [6] proposed a framework enabling users to trade energy with the central storage. Both [5] and [6] assumed that users need to have local renewable generations in order to share the energy in the storage. Furthermore, none of the above literature developed a clear business model for allocating central storage resources to users.

In this paper, we develop a business model that enables users to effectively share a central storage unit. We draw an analogy to the practice of cloud service providers, where users share the computing resources in a virtualized fashion [7]; in the power system, we can also envision that an aggregator owns and operates a central physical storage unit, who then virtualizes the storage into separable virtual storage capacities and sells them to different users. Specifically, we divide the time into many operation horizons. At the beginning of each operation horizon, the aggregator sets the unit price of the virtual capacity, and users decide their choices of the virtual capacities as well as storage charge and discharge schedules during the operation horizon. Users report their decisions to the aggregator, and the aggregator dispatches the central storage for them accordingly. Across different operation horizons, the aggregator and users can update their decisions. We formulate a two-stage problem for the interaction between the aggregator and users in each operation horizon, where the aggregator maximizes her profit and users minimize their costs. We will demonstrate that virtualization of storage will lead to more efficient use of the physical storage capacity, compared with the case where each user acquires his own physical storage.

The contributions of this paper are as follows:

- *Storage virtualization*: In Section II, we develop a virtual storage sharing framework. To the best of our knowledge, this is the first work that considers the virtualization of the central storage and allocates virtual storage capacities to serve users through a pricing mechanism.
- *Modelling and solution method*: In Section II, we model

the interaction between the aggregator and users as a two-stage problem. However, the decisions across the two stages are coupled, and it is also difficult to derive users' closed-form decisions in Stage 2. In Section III, we characterize the solutions of the two-stage problem based on the theory of parametric linear programming. We also propose a penalty term to resolve the multi-optima issue of the users' decision, without affecting the precision of the solution of the overall two-stage problem.

- *Simulation and benefits*: In Section IV, we show that storage virtualization can significantly reduce the cost of ownership. Specifically, compared to the case where each user acquires his own physical storage, storage virtualization reduces the overall physical capacity needed for all users by 34.9%, and the overall physical power rating[1] by 45.1%.

II. System Model

Figure 1 illustrates the system structure, where the main grid, a storage aggregator with a central storage unit, and a set of users are connected together. The set of users $\mathcal{I} = \{1, \ldots, I\}$ include both residential and commercial ones, whose load profiles can be different.[2] To satisfy his demand, a user purchases energy from the main grid, and purchases virtual storage capacity from the storage aggregator to perform demand management. Next, we introduce the economic interactions among various entities in the system.

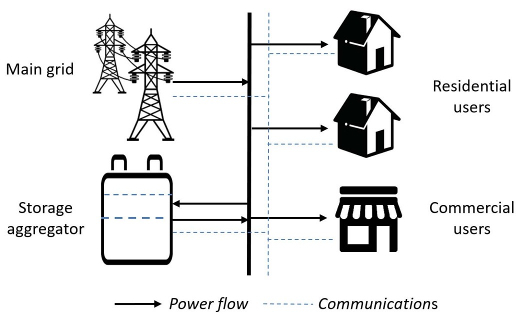

Fig. 1. System structure.

We assume that the main grid charges both commercial and residential users by a demand charge tariff [8]. Consider a billing cycle $\mathcal{T}^* = \{1, 2, ..., T^*\}$ of T^* time slots. If user i's electricity consumption from the main grid is $p_i^g[t]$ in time slot $t \in \mathcal{T}^*$, user i's electricity bill in $\mathcal{T}^*$ is [9]:

$$\pi_b \sum_{t \in \mathcal{T}^*} p_i^g[t] + \pi_p \max_{t \in \mathcal{T}^*} \; p_i^g[t], \tag{1}$$

where π_b is the unit price for total energy consumption in a billing cycle, and π_p is the unit price for the peak power consumption per slot in a billing cycle. To reduce the system peak, the utility company usually sets π_p much higher than π_b [8]. Based on the demand charge tariff in (1), there is a clear incentive for a user to shave the peak load.

[1]Power rating indicates the highest power that can flow in/out the storage.

[2]We leave the impact of renewable generation in our future work.

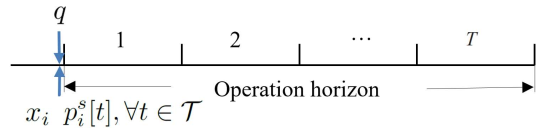

Fig. 2. Operation horizon.

We assume that an aggregator owns and operates the central storage. The aggregator virtualizes the physical storage into separable virtual capacities and sells them to users. Users purchase the virtual capacities to shave their peak load. Users can proactively charge their storage by purchasing more energy from the grid and discharge their storage to meet their peak load, thus reduce their peak consumption of the electricity bill. Since users can't control the central storage directly, they report their charge and discharge decisions to the aggregator, and the aggregator will dispatch the central storage on behalf of users accordingly. Note that the aggregator only cares about the net power flowing in and out the storage. As some users may choose charge while others choose discharge in the same time slot, part of the requests will cancel out at the aggregator's side. This suggests that even if all users are fully utilizing their virtual storage capacity, it is possible to support the needs of users by using a smaller central storage (comparing with the total virtual storage capacities). This is the key insight behind the benefit of storage virtualization. Furthermore, since users' storage is virtual, they can update their purchase flexibly over time based on their varying load profiles, which is difficult to realize if users own physical storage.

In the next two subsections, we show how the aggregator allocates the virtual storage to users through a pricing mechanism. We formulate the interaction between the aggregator and users as a two-stage problem.

A. Virtual storage allocation through pricing

We denote an operation horizon by $\mathcal{T} = \{1, 2, ..., T\}$ (e.g., it could represent one day with the time slot length of one hour). We assume that users can predict their load profiles for the entire operation horizon at the beginning of the horizon.[3] The aggregator and users will make their pricing, purchase, and virtual storage schedule decisions for the whole operation horizon at the beginning of the horizon. Figure 2 illustrates the decision timing in more details. At the beginning of each operation horizon, the aggregator determines the unit price q of virtual capacity for this horizon and announces it to users. Each user i decides the virtual storage capacity x_i and storage schedule decision $p_i^s[t]$ for each time slot $t \in \mathcal{T}$, and reports them to the aggregator. Here $p_i^s[t] > 0$ means that user i requests to charge his storage in time slot t, and $p_i^s[t] < 0$ means discharge. Since users' storage is virtual, we assume that the virtual storage has 100% charge or discharge efficiency and 0% energy leakage rate. The aggregator, however, will incur the extra energy loss during the charge and charge of the physical storage. To satisfy all users' requirement, the

[3]We leave the study on the impact of prediction error in the future work.

aggregator aggregates all users' charge and discharge decision $\sum_i p_i^s[t]$, which corresponds to the net charge and discharge operation of the central storage in time slot t. We assume that the central storage is close to users, hence the energy transmission loss is negligible [10].

In the next subsection, we formulate the interaction between the aggregator and users as a two-stage problem.

B. Two-stage formulation

We formulate a two-stage problem for the interaction between the aggregator and users as shown in Figure 3:

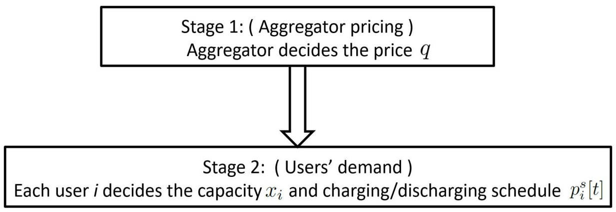

Fig. 3. Interaction between the aggregator and users.

1) Stage 2: User i's optimization problem: Given the unit price q determined by the aggregator, each user i decides the optimal virtual capacity x_i and storage schedule $p_i^s[t]$ in time slot t to minimize his cost, which includes the virtual storage payment qx_i and the electricity bill. If user i's original load profile is $P_i^l[t]$ in time slot t,[4] his total energy purchase from the grid in time slot t becomes $P_i^l[t] + p_i^s[t]$, therefore his overall electricity bill is given by

$$\pi_b \sum_{t\in\mathcal{T}} (P_i^l[t] + p_i^s[t]) + \pi_p \max_{t\in\mathcal{T}} \ (P_i^l[t] + p_i^s[t]). \tag{2}$$

We then discuss various constraints that the user needs to satisfy. We assume that the aggregator requires each user's total charge amount to be equal to the total discharge amount over the entire operation horizon. This ensures the independent operation of the virtual storage across different horizons [11].

$$\sum_{t\in\mathcal{T}} p_i^s[t] = 0. \tag{3}$$

Furthermore, user i's charge and discharge decision $p_i^s[t]$ should satisfy the constraint of the virtual capacity x_i:

$$E_i[t] = E_i[t-1] + p_i^s[t], \forall t \in \mathcal{T}, \tag{4}$$

$$0 \le E_i[t] \le x_i, \forall t \in \mathcal{T}', \tag{5}$$

where $E_i[t]$ denotes the energy level in user i's virtual storage in time slot t. We let $\mathcal{T}' = \{0\} \bigcup \mathcal{T} = \{0, 1, 2, ..., T\}$, and introduce the variable $E_i[0]$ for the initial energy level of user i. Since the user's storage is virtual, we allow user i to optimize $E_i[0]$ in each operation horizon. We assume that the aggregator can provision the initial energy level of the physical storage to satisfy users' requirement in each operation horizon.

We formulate the optimization problem for users:

[4] We assume the original load is fixed and there is no demand response.

Stage 2: User i's optimization problem $\mathbf{UP}_i$:

$$\begin{aligned} \min \quad & qx_i + \pi_b \sum_{t\in\mathcal{T}} (P_i^l[t] + p_i^s[t]) \\ & \qquad + \pi_p \max_{t\in\mathcal{T}} \ (P_i^l[t] + p_i^s[t]) \\ \text{subject to:} \quad & (3), (4)-(5), \\ \text{variables:} \quad & x_i; \ \{p_i^s[t], \forall t \in \mathcal{T}\}; \ \{E_i[t], \forall t \in \mathcal{T}'\}, \end{aligned}$$

where the unit price q is assumed to be fixed in Stage 2. We denote the optimal solutions to problem $\mathbf{UP}_i$ as $x_i^*(q)$, $p_i^s[t]^*(q)$ and $E_i[t]^*(q)$.

2) Stage 1: Aggregator's optimization problem: The aggregator determines the virtual storage unit price q to maximize her profit, which includes user's total payment $q\sum_i x_i^*(q)$ and storage operation cost. The charge and discharge operation $\sum_i p_i^s[t]^*(q)$ may cause the degradation of the storage. Furthermore, each charge or discharge will incur some energy loss, which the aggregator must compensate for. We adopt the linear operation cost model in the literature [11] [12]:

$$c \sum_{t\in\mathcal{T}} \mid \sum_{i\in\mathcal{I}} p_i^s[t]^*(q) \mid, \tag{6}$$

where c is the unit cost of charge and discharge amount.

We formulate aggregator's optimization problem as follows:

Stage 1: Aggregator's optimization problem AP:

$$\max_{q>0} \quad q \sum_{i\in\mathcal{I}} x_i^*(q) - c \sum_{t\in\mathcal{T}} \mid \sum_{i\in\mathcal{I}} p_i^s[t]^*(q) \mid,$$

where $x_i^*(q)$, $p_i^s[t]^*(q)$ are optimal decisions of user i by solving Problem $\mathbf{UP}_i$.

III. Solving Two-stage Problem

To solve the two-stage problem, we first characterize the optimal solution of each user i's problem $\mathbf{UP}_i$ under the price q, and then incorporate users' optimal solutions into the aggregator's problem to determine the optimal pricing.

A. Solution of Stage 2

For Stage 2, we prove that the optimal virtual capacity $x_i^*(q)$ that user i purchases is stepwise non-increasing in price q. However, for some values of q, the optimal capacity and charge and discharge profile may have multiple optimal solutions. To resolve this issue, we impose a small penalty term on the user i's optimization problem to ensure the unique solutions. We show that we can make the penalty term arbitrarily small such that it affects the user's choice of optimal capacity little.

We first analyze how the optimal capacity $x_i^*(q)$ changes with the price q, which leads to the following proposition:

Proposition 1 (Stepwise of virtual capacity): The optimal capacity $x_i^*(q)$ of Problem $\mathbf{UP}_i$ is a non-increasing and stepwise correspondence of the price q.

Specifically, we denote the set of threshold prices as $\mathcal{Q}_i = \{q_i^1, q_i^2, ..., q_i^{K_i}\}$ of K_i elements, then we have

$$x_i^*(q) = \begin{cases} x_i^0, q \in (0, q_i^1), \\ x_i^1, q \in (q_i^1, q_i^2), \\ ... \\ x_i^{K_i}, q \in (q_i^{K_i}, \infty), \end{cases}$$

where $x_i^0 > x_i^1 > \cdots > x_i^{K_i} = 0$. For any $q = q_i^k \in \mathcal{Q}_i$, $x_i^*(q)$ can be any value in the set $[x_i^{k_i-1}, x_i^k]$.

Figure 4(a) illustrates Proposition 1. We can see that as the price q increases, a user purchases a smaller virtual storage capacity. If the price is higher than the threshold $q_i^{K_i}$, user i will buy none. Between two adjacent threshold prices, the user's optimal choice of the virtual capacity remains the same. Furthermore, at a threshold price, the optimal capacity $x_i^*(q)$ is not unique. Even when we fix the value of $x_i^*(q)$, there may also be multiple corresponding $p_i^s[t]^*(q)$ and $E_i[t]^*(q)$ as the solutions of Problem $\mathbf{UP}_i$. The reason is that user can have multiple charge and discharge profiles as long as the profiles satisfy the capacity constraint. We prove Proposition 1 based on parametric linear programming [13].

The possibility of multiple optimal decisions in Stage 2 will make it difficult to solve the aggregator's problem in Stage 1, as the aggregator cannot accurately predict the users' behaviors under a given price. To address this issue without significantly changing the solution structure, we impose a penalty term on the user's Problem $\mathbf{UP}_i$ based on user i's charge and discharge amount as $\varepsilon \sum_t (p_i^s[t])^2$, where $\varepsilon > 0$ is a small value. Modifying the objective function of Problem $\mathbf{UP}_i$, we obtain Problem $\mathbf{UPP}_i$ as follows:

$$\min \quad \pi_b \sum_{t\in\mathcal{T}} (P_i^l[t] + p_i^s[t]) + \pi_p \max_{t\in\mathcal{T}} \, (P_i^l[t] + p_i^s[t]) + q x_i + \varepsilon \sum_{t\in\mathcal{T}} (p_i^s[t])^2$$

subject to: $(3), (4)-(5),$

variables: $x_i;\ \{p_i^s[t], \forall t \in \mathcal{T}\};\ \{E_i[t], \forall t \in \mathcal{T}'\}.$

We denote the optimal solutions to users' optimization Problem $\mathbf{UPP}_i$ as $x_i^\star(q,\varepsilon)$, $p_i^s[t]^\star(q,\varepsilon)$ and $E_i[t]^\star(q,\varepsilon)$ for any $\varepsilon > 0$. We show the uniqueness of each user's decision:

Proposition 2 (Uniqueness): For any $\varepsilon > 0$, and any q, $x_i^\star(q,\varepsilon)$, $p_i^s[t]^\star(q,\varepsilon)$, and $E_i[t]^\star(q,\varepsilon)$ are unique.

Next, we show that as ε is sufficiently small, the optimal capacity $x_i^\star(q,\varepsilon)$ of Problem $\mathbf{UPP}_i$ approaches $x_i^*(q)$ of Problem $\mathbf{UP}_i$ in Theorem 1:

Theorem 1 (Asymptotic optimality): For any fixed $q \notin \mathcal{Q}_i$ and $\delta > 0$, there exists an $\epsilon_0 > 0$, such that for any $\epsilon \in (0, \epsilon_0)$, we have $\mid x_i^\star(q,\varepsilon) - x_i^*(q) \mid < \delta$.

For the rest of discussion, we adopt the optimal solutions $x_i^\star(q,\varepsilon)$, $p_i^s[t]^\star(q,\varepsilon)$ and $E_i[t]^\star(q,\varepsilon)$ to Problem $\mathbf{UPP}_i$ as user i's best choice of virtual storage given the unit price q.

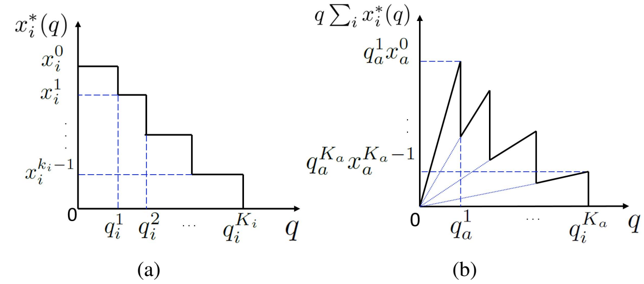

Fig. 4. (a) The optimal capacity $x_i^*(q)$ of Problem $\mathbf{UP}_i$; (b) Revenue $q\sum_i x_i^*(q)$.

B. Solution of Stage 1

At the aggregator's side, each price q will lead to users' best choice of $x_i^\star(q,\varepsilon)$, $p_i^s[t]^\star(q,\varepsilon)$ and $E_i[t]^\star(q,\varepsilon)$, and the aggregator can calculate her profit from user's payment and the operation cost accordingly. We will prove that with a sufficiently small penalty coefficient ε on the users' problems in Stage 2, the aggregator's profit has a piecewise linear property and the optimal price $q^\star$ of Problem **AP** occurs around one of the threshold prices.

First, we consider the optimal solutions to Problem $\mathbf{UP}_i$. We show how the aggregate optimal capacity $\sum_i x_i^*(q)$ changes with the price q. By aggregating all users' threshold price set $\mathcal{Q}_i$ and optimal capacity set $\{x_i^0, x_i^1, ..., x_i^{K_i}\}$ for each $i \in \mathcal{I}$, we can show that $\sum_i x_i^*(q)$ is also non-increasing stepwise correspondence of the price q. We denote the threshold price set by $\mathcal{Q}_a = \bigcup_i \mathcal{Q}_i = \{q_a^1, q_a^2, ..., q_a^{K_a}\}$, and the corresponding optimal capacity set $\{x_a^0, x_a^1, ..., x_a^{K_a}\}$. Similarly, $x_a^0 > x_a^1 > \cdots > x_a^{K_a} = 0$ and for any $q = q_a^k \in \mathcal{Q}_a$, $\sum_i x_i^*(q)$ can be any value in the set $[x_a^{k-1}, x_a^k]$. We then discuss the relationship between the aggregator' revenue $q\sum_i x_i^*(q)$ and the price q. Since the aggregate capacity $\sum_i x_i^*(q)$ is stepwise in price q, the corresponding revenue $q\sum_i x_i^*(q)$ is piecewise linear in price q as illustrated in Figure 4(b): the revenue increases linearly from one threshold price and then decreases vertically at the next threshold price. It repeats this process until the revenue is zero. The maximum revenue is achieved at one of the threshold prices.

Next, we consider the optimal solution of Problem $\mathbf{UPP}_i$. Similar to Theorem 1, we can show that when ε is very small, aggregator's revenue $q\sum_i x_i^\star(q,\varepsilon)$ approaches $q\sum_i x_i^*(q)$. Furthermore, when ε is sufficiently small, the operation cost $c\sum_t \mid \sum_i p_i^s[t]^\star(q,\varepsilon) \mid$ also has a stepwise property over the threshold price set $\mathcal{Q}_a$ as illustrated by $C^*(q)$ in Figure 5(a). We show the property in Proposition 3:

Proposition 3 (Operation cost): There exits $C^*(q)$,

$$C^*(q) = \begin{cases} C^0, q \in (0, q_a^1), \\ C^1, q \in (q_a^1, q_a^2), \\ ... \\ C^{K_a}, q \in (q_i^{K_a}, \infty), \end{cases}$$

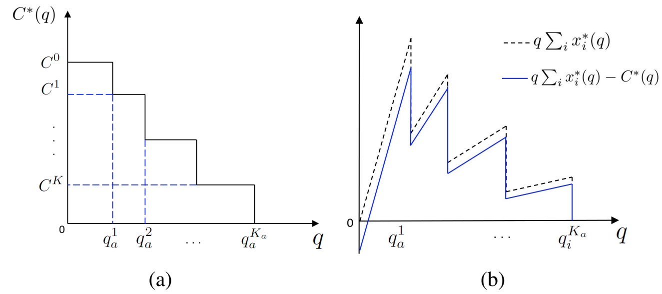

Fig. 5. (a) $C^*(q)$ in the price q; (b) $q\sum_i x_i^*(q) - C^*(q)$ in the price q.

where $C^0, C^1, ..C^{K_a-1}, C^{K_a} = 0$ are constant. For any fixed $q \notin \mathcal{Q}_a$ and $\delta > 0$, there exists $\epsilon_0 > 0$, such that for any $\epsilon \in (0, \epsilon_0)$, we have

$$| \, c \sum_{t\in\mathcal{T}} | \sum_{i\in\mathcal{I}} p_i^s[t]^\star(q,\varepsilon) \, | - C^*(q) \, | < \delta.$$

Further, combining the revenue and operation cost, we have that if ε is sufficiently small, the aggregator's profit approaches $q\sum_i x_i^*(q) - C^*(q)$ which has the piecewise linear property as illustrated in Figure 5(b).[5] We show the property in Proposition 4:

Proposition 4 (Profit) : For any fixed $q \notin \mathcal{Q}_a$ and $\delta > 0$, there exists $\epsilon_0 > 0$, such that for any $\epsilon \in (0, \epsilon_0)$, we have

$$\begin{aligned} | \, q \sum_{i\in\mathcal{I}} x_i^\star(q,\varepsilon) - c \sum_{t\in\mathcal{T}} | \sum_{i\in\mathcal{I}} p_i^s[t]^\star(q,\varepsilon) \, | \\ - (q \sum_{i\in\mathcal{I}} x_i^*(q) - C^*(q)) \, | < \delta. \end{aligned}$$

Due to the space limit, we present the detailed proofs of all theorems and propositions in the online technical report [14].

Utilizing the properties of the objective function of Problem **AP** that we have obtained so far, we can obtain the optimal price $q^\star$ through a search algorithm. Basically, the aggregator communicates with users in an iterative fashion. The aggregator increases q by a small increment in each iteration until no capacities can be sold out,[6] and accordingly, each user i solves Problem $\mathbf{UUP}_i$ under the price q to determine the purchased capacities and the storage schedule over the operation horizon. Finally, the aggregator chooses the optimal $q^\star$ that maximizes her profit. Note that the aggregator only observes users' decisions without knowing their load profiles, which protects the users' privacy. Due to the space limit, we present the algorithm in the online technical report [14].

IV. Simulation Result

In this section, we conduct simulation based on four types of typical load profiles. We compute aggregator's optimal pricing, and users' optimal choices of virtual storage. We will then show the benefits of the storage virtualization in terms of reducing the storage ownership cost.

[5] Note that if the price q is too low, the revenue approaches zero. Since the operation cost is positive, the aggregator's profit will be negative.

[6] Note that the choice of increment Δq should tradeoff between the accuracy and the computational burden.

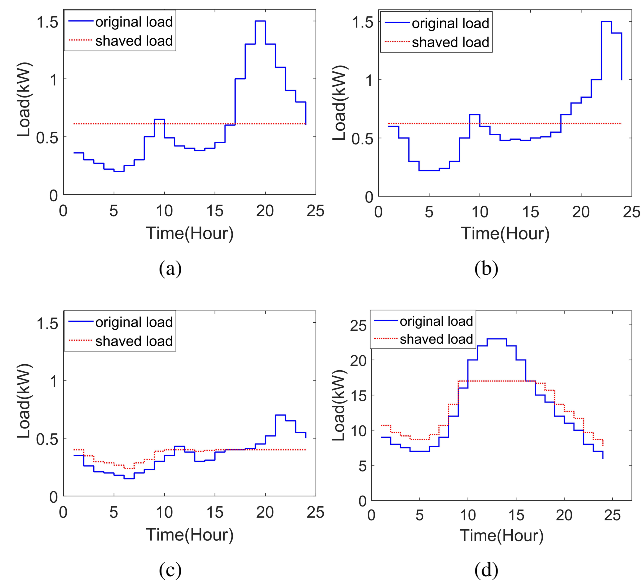

Fig. 6. Four types load: (a)(b)(c) residential load of Type 1, 2, 3; (d) commercial load of Type 4.

A. Parameter

In the simulation, we consider one day as the operation horizon, which is equally divided into $T = 24$ time slots. For residential users, we consider 3 types of load profiles on a typical weekday [15], and each type has 10 users. For commercial users, we consider 1 type of load profile [16], with only 1 user. These load profiles are the "original load" $P_i^l[t]$ during an operation horizon as shown in Figure 6. Among the 3 types of residential load, Type 1 has one peak in the early evening (around 19:00), Type 2 has one peak in the late evening (around 22:00), and both Type 1 and 2 have one small peak in the morning(around 9:00). Comparing with Type 1 and 2, Type 3 consumes less energy and has a smaller load variation in one day. The commercial user of Type 4 consumes much more energy than residential users, and his load profile has the peak around noon.

We choose the price of electricity charged to the users based on [10], where the peak demand charge per month is $\pi_p = 10.28$\$/kW, and the energy charge is $\pi_b = 0.034$\$/kWh. Since our simulation is for one day, in order to make the comparisons fair, we scale the demand charge to a day by setting $\pi_p = 10.28/30 = 0.34$\$ /kW. That way, π_b can remain unchanged. We choose a sufficiently small $\varepsilon = 0.003$ cents/(kWh)^2. We set the operation cost $c = 1$ cents/kWh [11].

B. Optimal solution of the two-stage problem

We search the price to find the optimal price $q^\star$=4.7 cents/kWh that maximizes the aggregator's profit. The optimal capacities $x_i^\star$ that users purchase are as follows: Type 1-3.61kWh; Type 2-2.88kWh; Type 3-0.98kWh; Type 4-28kWh.

We notice that a Type-3 residential user buys a much smaller capacity than users of other types as his load profile is lower. The commercial user buys a much larger capacity due to the high load. Each user's total energy purchase from the grid $P_i^l[t] + p_i^s[t]$ during an operation horizon is shown in Figure 6 as the "shaved load". Comparing with the original load in Figure 6, we can see that a user will discharge to shave the peak when the original load is high. Since we require that each user's total charging amount equals the discharging amount during an operation horizon, he has to charge his storage when the original load is relatively low.

C. Benefits of reducing ownership cost

We show that through virtualization, a physical storage unit can support users' need of a much larger virtual storage. Based on the central storage's schedule $\sum_i p_i^s[t]$, we define as follows the minimal effective physical capacity $X_{\min}$ and minimal power rating $P_{\min}$ to support this schedule:

Definition 1 ($X_{\min}$): The minimal effective physical capacity to support the given schedule $\sum_i p_i^s[t]$ is :

$$X_{\min} = \max_{t\in\mathcal{T}} \sum_{\tau=1}^{t} \sum_{i\in\mathcal{I}} p_i^s[\tau] - \min_{t\in\mathcal{T}} \sum_{\tau=1}^{t} \sum_{i\in\mathcal{I}} p_i^s[\tau]. \quad (7)$$

Definition 2 ($P_{\min}$): The minimal power rating to support the given schedule $\sum_i p_i^s[t]$ is:

$$P_{\min} = \max_{t\in\mathcal{T}} \mid \sum_{i\in\mathcal{I}} p_i^s[t] \mid . \quad (8)$$

In other words, the power rating $P_{\min}$ is the maximum value of the aggregate charging or discharging power across all time slots. The effective capacity $X_{\min}$ is obtained by the difference between the maximum energy level and the minimal energy level in the storage of all time slots. We present the detailed explanation in the online technical report for (7) and (8) [14].

We compare the minimal physical size (measured in $X_{\min}$ and $P_{\min}$) and total allocated virtual storage size at different prices in Figure 7. Intuitively, as the price increases, users will purchase less amount of virtual storage, which will also reduce the required physical storage size. At the optimal price $q^\star$=4.7 cents/kWh, the minimum physical capacity $X_{\min}$ is reduced by 34.9% comparing with the total virtual capacity $\sum_i x_i^\star$, and the minimum power rating $P_{\min}$ is reduced by 45.1% comparing with the total virtual power rating $\sum_i \max_t \mid p_i^s[t]^\star \mid$. This indicates that central storage virtualization will lead to more efficient use of the physical storage capacity, which will potentially benefit both users and aggregator.

V. Conclusion

In this paper, we proposed a pricing-based virtual storage sharing scheme among a group of users. An aggregator owns the central storage and virtualizes the storage into separable virtual capacities, which can be sold to serve different users. We formulated a two-stage problem for the interaction between the aggregator and users. Simulation results showed that compared to the case where each user acquires his own storage, storage virtualization reduces the overall physical capacity needed for all users by 34.9%, and the overall physical power rating by 45.1%. For future work, we aim to further explore the benefits of storage virtualization in terms of renewable energy generation and frequency regulation support.

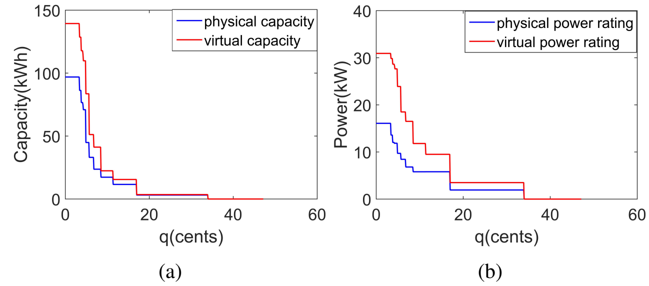

Fig. 7. (a) Required physical capacity/allocated virtual capacity; (b) Required physical power rating/allocated virtual power rating.

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