Randomized Pricing for the Optimal Coordination of Opportunistic Agents

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***Abstract*— In this work, we study the design of pricing mechanisms for the efficient and stable service of a large consumer base with deferrable demands, as envisioned in future smart electricity and data networks. When users with flexible demand are introduced into such large-scale dynamic markets with dynamic prices, they exhibit opportunistic behavior to minimize their cost per unit amount of service-received. This, in turn, generates highly fluctuating aggregate demand patterns that are destabilizing or costly to serve.**

To avoid this undesired outcome, we propose a randomized pricing mechanism that preserves the strengths of dynamic pricing in tracking a targeted aggregate demand while preventing the destabilizing effect of opportunistic users. Price randomization creates information asymmetry among consumers, but preserves fairness and also the economic benefits of the consumers. We show that the randomization operation can be captured by augmenting a related utility function to the cost minimization problem. This interesting connection enables the study of the steady-state behavior of our randomized pricing solution through the study of the associated optimization problem. Based on this connection, we investigate key metrics of supplier cost and profit and consumer payments.

I. INTRODUCTION

Many large scale systems, such as data networks, cloud computing services, and smart electricity grids, serve demands that inherently possess various types of flexibilities. These demand-side flexibilities can arise in various forms such as shifting or deferring the service time, allowing intermittent service, or adjusting the amount of service. For instance, smart electricity consumers can defer service times of home appliances like dishwashers, or customers of a cloud service can choose non-consecutive time blocks to execute chunks of a single task.

The unconventional consumer behavior generated by demand-side flexibilities brings both opportunities and challenges to system operation, and necessitates the design of novel management techniques. From a global perspective, demand-side flexibilities can be utilized to the advantage of the whole system. For example, in a smart electricity grid or a cloud computing system, consumer demand can be deferred to a later time to cut down peak load and reduce service and maintenance costs. On the other hand, self-interested and price-taking consumers will be willing to defer their service only if they can obtain economic benefits and reduce their payments. Therefore, there is a need to design real-time pricing schemes that incentivize economically-driven consumers to defer their flexible demand.

However, pricing-based dynamic control of self-interested users leads to its own challenges. Note that, in a real-time market, consumers can opportunistically exploit their flexibilities to obtain economic benefits by taking advantage of changing prices. In particular, users will likely defer their service to the periods of time with lower price to the extent that their flexibility allows. Indeed, previous works such as [1], [2] investigate consumer behavior under different flexibility and cost structures, and establish optimality of threshold-based policies in asymptotic and non-asymptotic regimes. However, the aggregate response of a large consumer base employing such threshold-based policies can potentially lead to highly and abruptly fluctuating total service and price (as will be demonstrated in Section III). In most systems, this volatile behavior is undesirable, if not acceptable, because it increases service costs, puts stress on the physical network and devices, and endangers the stability of the underlying infrastructure and the market [3].

Several works in the literature investigated this problem. In [4], auction strategies that require the knowledge of consumer demand and utility functions are designed. In [5], authors propose ex-post adjustments to the market price, but demands consumers to predict the effect of their aggregate behavior. Recent work in [6], [7], attempts to solve the volatility problem by assigning a price to the difference in consumption between consecutive time slots. The motivation in introducing a secondary price is to penalize large changes in individual consumer's service amounts. In this work, our goal is to design decentralized pricing mechanisms that are simple and computationally efficient, yet able to coordinate opportunistic users to mitigate volatility and instability problems while preserving fairness among users and their benefits.

In this work, we propose a novel approach to mitigate the volatility caused by opportunistic consumer behavior. The key idea is to communicate to each user a random perturbation of a common market price, in order to effectively guide their aggregate behavior. Price differentiation among users introduces heterogeneity to consumer decisions. As a result, the aggregate load is averaged out instead of having abrupt oscillations. Yet, this short-term heterogeneity is balanced in the long-run so as to sustain the fairness property of pricing. We capture the impact of price randomization as solving an augmented cost minimization problem where the

*The works of O. Dalkilic and A. Eryilmaz are supported by the NSF grant CAREER-CNS-0953515, the QNRF grants NPRP 7-923-2-344 and NPRP 09-1168-2-455, and the DTRA grant: HDTRA1-11-16-BRCWMD.

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augmented term is a function of the randomization procedure. Nonetheless, the randomized real-time pricing scheme does not require knowledge of consumers' utility functions. We then analyze the resulting system dynamics when the number of users is large, assuming that users are price-taking and that they implement threshold-policies that align with an individual consumer's economic interest. Interestingly, we show that both the system and the consumers economically benefit under the randomized pricing scheme. Further, information asymmetry created by price randomization yields significant global gains that symmetric information (i.e. a common market price) is unable to achieve.

We believe that the idea of randomized pricing presented in this paper applies to generic systems (including data networks, cloud servers, energy grids, etc.) that serve self-interested and price-taking smart consumers. However, to facilitate more concrete presentation, we specifically focus on the smart electricity market and present the system model in detail in Section II-A. In Section II-B, we formulate the optimization problem which embodies fundamental dynamics of various systems that have deferrable demand. We characterize the opportunistic consumer behavior in Section III-A, and demonstrate its effects on the system via two benchmark real-time pricing schemes in Section III-B. We present the randomized pricing algorithm in Section IV, and give theoretical results on its optimality and convergence characteristics. Another algorithm is proposed under the presence of a periodic inflexible load pattern in Section IV-B. We conclude our discussion in Section V, by presenting comprehensive numerical investigations on the performance of the randomized pricing algorithm compared to the benchmark schemes.

II. System Model and Problem Formulation

We consider a real-time market scenario where suppliers and consumers react to dynamically generated prices in short time scales possibly ranging from several seconds to minutes. The fundamental distinction that distinguishes our model from a traditional market and that motivates us to pursue novel pricing mechanisms is the flexibility of consumers to defer their demand. In the following, we present a real-time market model and introduce the market participants, with a focus on the electrical systems, and formulate the control and pricing problem under this setup. We note that the model and the proposed control and pricing mechanisms also apply to other types of systems where consumers have the flexibility to defer their consumption.

A. System Model

The real-time market comprises consumers, suppliers, and a non-profit entity called market manager. The market is operated over discrete time slots, $t = 0, 1, \ldots$, and at each time slot the market participants make their control decisions. The goal of the market manager is to ensure stable and efficient operation of the market. It sets the real-time market prices, maintains the infrastructure over which the transactions take place, and ensures that load and supply match each other at each time slot. Suppliers provide service to the market at predetermined costs per unit. The goal of each supplier is to maximize its profit. On the other hand, consumers seek to satisfy their demand by getting service with the aim of making the lowest payment for consumption. There are two types of consumers in the market. *Flexible* consumers have deferrable demand and they can delay their consumption to take advantage of low prices in the future. *Inflexible* consumers, however, cannot delay their consumption and must serve their demand at the time slot that the demand is realized.

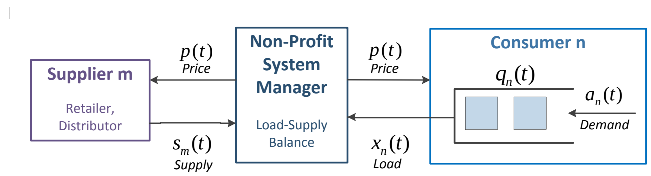

Fig. 1. Market model depicting the participants and their interactions.

The market type that we focus our investigation on is an electricity market depicted in Figure 1. This structure can be utilized to model a system in which consumers are households with small amount of demand and supplier is a single electricity retailer company. However, this market structure is generic and may also be used to model other different market types. Next, we present the market participants and the overall operation of the market in detail.

1) *Market Manager:* The manager institutes the electricity market and supervises its operation with the goal of providing a reliable market for supplier and consumer transactions. In a smaller setting, a utility company that procures electricity to its consumer base can also be considered a market manager.

The manager intends to coordinate the market participants by setting the real-time prices at each time slot, i.e. $p(t)$ for $t = 0, 1, \ldots$. The market price is generated *ex-ante*, meaning that the amount of supply and consumption are unknown at the time the price is set. Hence, suppliers and consumers are price-taking, and give their supply and consumption decisions at each slot based on the announced prices. Furthermore, the manager does not have the knowledge of supplier costs, consumer valuations, and their control strategies.

2) *Suppliers:* There are M electric suppliers, and the procurement of s watts of power incurs a cost of $C_m(s)$ to the supplier m. We assume that $C_m : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuously differentiable and increasing function of s for each m. We also assume that $\ddot{C}_m(s) > 0$, and hence C_m is strictly convex and $\dot{C}_m$ is invertible. At time slot t, supplier m decides on its supply offer $s_m(t)$ based on the announced market price, and receives payment $\omega_m(t) \triangleq p(t)s_m(t)$. Hence, supplier's goal is to maximize its profit $\omega_m(t) - C_m(s_m(t))$.

3) *Consumers:* There are N flexible and N_i inflexible consumers. At time slot t, consumer n generates demand $a_n(t)$. $a_n(t)$ is a random variable that is assumed to be

independent among consumers and i.i.d. over time slots for each consumer. The average demand arrival rate is λ_n, i.e. $\mathbb{E}\left[A_n(t)\right] = \lambda_n$ for all t. We further assume that demand arrivals are bounded such that $A_n(t) \in [0, a_{n,max}]$.

The amount of electric energy consumed, namely *load* [1] , by user n at slot t is denoted by $x_n(t) \in [0, x_n^m]$. For inflexible consumers, $x_n(t) = a_n(t)$ because the realized demand must be served immediately. We define $S_i(t) \triangleq \sum_{n=1}^{N_i} x_n(t)$, with mean $\lambda_S \triangleq \sum_{n=1}^{N_i} \lambda_n$, to be the total load of inflexible users. On the other hand, for flexible consumers, the amount of consumption is not necessarily equal to the amount of realized demand; Demand can be deferred and served later as a *load*.

The waiting queue (i.e. backlog) for flexible consumer n's deferrable demand at time t is $q_n(t)$ and its evolution is given by $q_n(t+1) = \left[q_n(t) + a_n(t) - x_n(t)\right]^+$. Queues are required to be stable, otherwise the delay experienced by the demand will approach infinity. We assume that the goal of flexible consumer n is to minimize its payment, $r_n(t) \triangleq p(t)x_n(t)$, under the queue stability constraint. Inflexible consumers do not have such objective since they do not have control on their load.

The market model described above is a stochastic dynamical system with closed-loop feedback . Suppliers and consumers react to the market prices generated by the operator, and the operator adjusts the prices based on the realizations of supply and load. Furthermore, the operator generates market prices without the knowledge of suppliers' cost structures and consumers' valuations. Under this model, we are interested in designing distributed control and pricing schemes that ensure the efficient and stable operation of the market and that are aligned with the market participant's individual objectives.

B. Problem Formulation

In the paper, we use boldface letters to denote vectors, e.g. $\mathbf{x} = (x_1, \ldots, x_N)$ is the N dimensional vector of the scalar quantities x_n for $n = 1, \ldots, N$. We use $\{.\}$ to denote a set of quantities whose cardinality should be understood from context, e.g. $\{y(t)\}$ for $t = 0, \ldots$.

The objective of the operator is to minimize the time-averaged expected cost of electricity procurement. The optimization problem is formally given by

$$\min_{\{\mathbf{x}(t)\}\{\mathbf{s}(t)\}} \quad \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m=1}^{M} \mathbb{E}\left[C_m\left(s_m(t)\right)\right] \tag{1}$$

$$\text{s.t.} \quad \sum_{n=1}^{N} x_n(t) + S_i(t) = \sum_{m=1}^{M} s_m(t),\ \forall t = 0, 1, \ldots \tag{2}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[x_n(t)\right] \geq \lambda_n,\ \forall n. \tag{3}$$

[1]To be precise, in this paper we make a distinction between "demand" and "load". Demand is externally generated according to a, but can be delayed. Load is the actual consumption at each time instant.

In problem (1), constraint (2) is necessary for keeping supply and load balanced in the electric grid at all time slots. Constraint (3) ensures that the consumers experience finite delay, i.e. consumer backlogs remain stable.

To simplify the design and analysis, instead of problem (1) we will consider the following static (one time-slot) problem:

$$\begin{aligned} \min_{\mathbf{x},\mathbf{s}} \quad & \sum_{m=1}^{M} C_m\left(s_m\right) \\ \text{s.t.} \quad & \sum_{n=1}^{N} x_n + \lambda_S \leq \sum_{m=1}^{M} s_m \\ & \lambda_n \leq x_n,\ \forall n. \end{aligned} \tag{4}$$

It can be shown that the optimum objective value in (4) is a lower bound for the objective value of problem (1). We will demonstrate with simulation that, under properly designed pricing mechanisms (which will become more clear later on), the optimum value of problem (1) will also be close to that of problem (4). Hence, by solving problem (4), whether exactly or approximately, we can obtain a solution for problem (1). Furthermore, problem (4) has a simpler structure, thus we can apply well-known techniques such as duality to derive iterative algorithms and tools.

However, Problem (4) may be solved by various iterative algorithms. Such algorithms may dictate undesirable control rules on the consumer side that do not align with flexible consumers' objective to minimize their payments. On the other hand, as we will demonstrate soon, allowing flexible consumers to fully exhibit their opportunistic behavior may cause instability and inefficiency by generating abrupt changes and fluctuations in power generation . Therefore, our goal is to design control and real-time market pricing schemes that will give flexible consumers the freedom to opportunistically consume electricity for their own interest, and that will also achieve close-to-minimum electricity procurement cost.

III. Flexible Consumer Behavior and Benchmark Real-time Pricing Schemes

In the following, we will first characterize the opportunistic behavior of flexible consumers, and then discuss its implications on the electricity market. To demonstrate the detrimental effects of consumer-side flexibility, we present two benchmark real-time pricing schemes which lead to undesirable oscillatory behavior even though they are simple and intuitive.

In the rest of the paper, for numerical investigations, we consider a market where a certain percentage of consumers have deferrable demand and the rest of the load, which we call inflexible load, has a daily predetermined pattern. Specifically, we use historical metered load data from PJM [8] as the inflexible load. Furthermore, for the simplicity of exposition and since our focus is on consumer behavior, we assume that there is a single supplier in the market, although the solution can be generalized to multiple suppliers.

A. Flexible Consumer Behavior

In our model, consumers are price-taking; At slot t, each consumer receives a price $p(t)$ for consuming a unit amount of power, and then decides on his load. Thus, the optimization problem faced by a flexible consumer can be formulated as

$$\begin{aligned} \min_{\{x_n(t)\}} \quad & \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[p(t)x_n(t)\right] \\ \text{s.t.} \quad & \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[x_n(t)\right] \geq \lambda_n, \ \forall n. \end{aligned} \tag{5}$$

From a single consumer's perspective, his load decisions have negligible effect on the future prices when the number of users is large. Hence, here when we are studying (5), we assume that $p(t)$ is exogenous; It is independent of $x_n(t)$. Under this assumption, the following policy asymptotically achieves the optimal value of (5) as the design parameter $\kappa_n^u > 0$ gets large:

$$x_n(t) = x_n^m \mathbb{1}\left\{p(t) \leq \frac{q_n(t)}{\kappa_n^u}\right\} \tag{6}$$

The policy in (6) results in an opportunistic behavior. Users consume electricity only when price is below a certain threshold, and when they consume they put their maximum available load to take full advantage of the low price. We note that this threshold policy and similar threshold-based policies have been shown to be asymptotically optimal when the prices are exogenous [1], [9], [10].

However, in reality the behavior of each individual user will eventually affect the price. Note that the above opportunistic behavior, when aggregated over a large consumer base, will cause very high (low) load when price is low (high). Thus, the resulting load pattern will not be flat and will be costly to supply. Furthermore, electric supply and market prices will be highly fluctuating since the prices are adjusted in real time by the operator as a response to the changes in load. Next, by comparing two benchmark pricing schemes, we demonstrate that this fluctuation is a real challenge when there are flexible consumers.

B. Benchmark Real-time Pricing Schemes

i) Scheme I (Real-time Pricing With Zero Penetration of Flexible Consumers): In this scheme, all consumers are inflexible so they do not have the ability to defer their loads; Arriving demand is served immediately, i.e. $x_n(t) = a_n(t)$ for all n, regardless of the market price. Supplier is obliged to procure supply equal to the realized load and receive a payment accordingly. On the other hand, the operator uses $\sum_n x_n(t)$ as the prediction of the load on the next time slot, and sets the market price to the total marginal procurement cost of the supplier, i.e. $p(t+1) = \dot{C}(s(t))$ subject to $s(t) = \sum_n x_n(t)$. This choice of price maximizes supplier's profit assuming that the load prediction is accurate. Scheme 1 is summarized as follows:

Scheme 1. *At time t:*

- *Consumer n sets $x_n(t) = a_n(t)$.*
- *The operator computes:*

$$p(t+1) = \dot{C}\left(s(t)\right), \ \textit{s.t.} \ s(t) = \sum_n x_n(t)$$

We note that, scheme 1 serves a base setup which will be useful in assessing both the advantages and disadvantages of consumer-side flexibility.

ii) Scheme II (Gradual Real-time Price Update Under the Presence of Flexible Consumers): We next introduce flexible consumers by setting a certain percentage of the users to have deferrable demand. We assume that flexible consumers implement the threshold policy (6). Under this policy, we expect the aggregate load to become either very large or too small, since the consumers use the maximum amount x_n^m or nothing based on the common market price. If the operator sets the price to marginal cost as it does in Scheme I, which implicitly assumes that the load is static, the price will fluctuate violently. Such closed-loop feedback between load and price degrades the system performance by increasing the procurement cost and putting more stress on the electrical grid.

One seemingly plausible direction to overcome this problem is to smoothen the price updates. We can consider an iterative solution by means of a primal-dual approach to problem (4), and gradually update the price at each time slot. Scheme 2, which is presented next, achieves the optimal solution of problem (4) asymptotically as $\kappa^s \to \infty$.

Scheme 2. *At time t:*

- *Consumer n computes:*

$$x_n(t) = x_n^m \mathbb{1}\left\{p(t) \leq \frac{q_n(t)}{\kappa_n^u}\right\} \tag{7}$$

$$q_n(t+1) = \left[q_n(t) + a_n(t) - x_n(t)\right]^+ \tag{8}$$

- *The supplier must meet the real load $\sum_n x_n(t)$. In addition it computes $s(t) = \dot{C}^{-1}\left(p(t)\right)$.*
- *The operator computes:*

$$p(t+1) = \left[p(t) + \kappa^s \left(\sum_n x_n(t) + S_i(t) - s(t)\right)\right]^+ \tag{9}$$

where $\kappa^s > 0$ is a positive step size.

Note that the supplier computes a fictitious amount of supply $s(t)$ based on the current price although the real supply should always be equal to the instantaneous load. On the other hand, this fictitious supply $s(t)$ is used by the operator to gradually adjust the price in (9).

Under Scheme 2, price exhibits relatively small oscillations due to the dampening effect of κ^s. However, the total load still abruptly fluctuates as seen in Figure 2. The threshold rule (7) allows flexible users to consume at their maximum rates x_n^m when the market price is low, and their aggregate load becomes too high since the same price is observed simultaneously by all users.

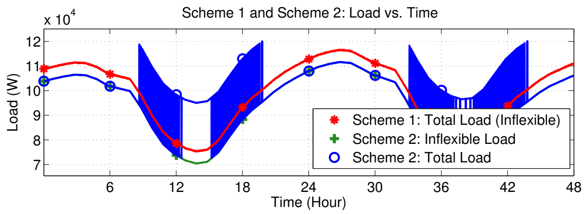

Fig. 2. Load evolution under Scheme 1 and 2. There are 1000 flexible consumers that receive Poisson distributed demand arrivals, and their load constitute 5% of the total load.

IV. Real-time Randomized Pricing Algorithm

In this section we present a real-time randomized pricing algorithm, and investigate its optimality and convergence characteristics via a fluid limit model.

Our algorithm aims to mitigate the volatility and instability problems that can arise due to the opportunistic behavior of flexible consumers in the real-time market. Towards this goal, the underlying motivation in the design of algorithm RP is two-fold. First, like Scheme 2, we consider updating the market price incrementally so that sudden changes in load do not directly translate into large fluctuations in price. Second, in order to prevent flexible consumers from aligning their load decisions together to create undesirable peaks and valleys in aggregate load, we differentiate the market price across the consumer base. In particular, each consumer is communicated an individual price that is randomly differentiated from the common price.

The real-time randomized pricing algorithm is given as follows:

Algorithm RP Randomized Pricing

At iteration t:

• Consumer n receives an individual price $p_n(t)$. Then, it computes its load and queue as

$$x_n(t) = x_n^m \mathbb{1}\left\{p_n(t) \leq \frac{q_n(t)}{\kappa}\right\} \tag{10}$$
$$q_n(t+1) = [q_n(t) + a_n(t) - x_n(t)]^+$$

• The supplier must meet the real load $\sum_n x_n(t)$. Further it computes $s(t) = \dot{C}^{-1}(p(t))$.

• The operator computes the common market price:

$$p(t+1) = \left[p(t) + \alpha\left(\sum_{n=1}^{N} x_n(t) + S_i(t) - s(t)\right)\right]^+$$

Then, the operator generates individual prices that are communicated to each consumer separately:

$$p_n(t+1) = p(t+1) + \epsilon_n(t+1)$$

where $\epsilon_n(t)$ are i.i.d. random variables over time and consumers with the CDF (Cumulative Distribution Function) F_ϵ.

In algorithm RP, individual prices are generated by adding i.i.d. random noise $\epsilon_n(t)$ to the common market price. The noise can have an arbitrary distribution as long as it satisfies the following assumption which is not restrictive.

Assumption 1. *F_ϵ is continuous on its domain and strictly increasing from 0 to 1 on an interval $[\epsilon_{min}, \epsilon_{max}]$.*

Users pay for their consumption at the individual price privately communicated to them by the operator. Since this price is generated by adding a random disturbance to the common market price, the revenue obtained by the supplier at each time slot will be different than the revenue anticipated at the common price. Hence, it is not surprising that RP does not achieve the optimal solution to problem (1). Instead, we will show that an approximate version of RP achieves the optimal solution to a welfare maximization problem that is closely related to the original problem. The basic idea is that communicating randomized prices to consumers induces a utility-function based decision at the consumer side. To demonstrate this, we present a continuous-time fluid approximation model of RP, which will also be instrumental in analyzing its behavior.

A. Continuous-time Fluid Approximation Model and Utility-Maximization-Based Formulation

In this section we derive a continuous-time fluid approximation for algorithm RP. Then, we relate the model to a utility maximization problem with modified consumer utility functions induced by price randomization.

The aggregate flexible consumer load is the sum of N binary variables, i.e. $X(t) \triangleq \sum_n x_n^m \mathbb{1}\left\{\epsilon_n(t) \leq \frac{q_n(t)}{\kappa} - p(t)\right\}$. For large number of users, the queue lengths of users, $q_n(t)$, will be roughly evenly distributed around $\kappa p(t)$ due to the diversity between internal states of users. In this case, $x_n(t)$ will be approximately i.i.d. Bernoulli random variables. Applying the Law of Large Numbers based on this assumption, we obtain the following expression for the aggregate load

$$X(t) \approx \sum_n x_n^m F_\epsilon\left(\frac{q_n(t)}{\kappa} - p(t)\right). \tag{11}$$

The above expression is the mean behavior for the aggregate load, and when the number of users is large it will well approximate the dynamics of the load.

We define $u_n(x) \triangleq x_n^m F_\epsilon(-x)$, and write

$$x_n(t) \approx u_n\left(p(t) - \frac{q_n(t)}{\kappa}\right),$$

which approximates the mean behavior of individual users. Next, we present below a continuous-time approximation to RP.

In algorithm (RP-C), consumer loads are computed via the smooth functions u_n. Since F_ϵ is continuous and strictly increasing on $[\epsilon_{min}, \epsilon_{max}]$, u_n is continuous and strictly decreasing, and it has an inverse u_n^{-1}. The domain of u_n^{-1} is the interval $[0, x_n^m]$, possibly with $u_n^{-1}(0) = \infty$ and $u_n^{-1}(x^m) = -\infty$, depending on the values of ϵ_{min} and ϵ_{max}. We define function U_n such that $\dot{U}_n(x) \triangleq u_n^{-1}(x)$ on $(0, x_n^m)$, which exists since $u_n^{-1}(x)$ is continuous, and

Algorithm RP-C Continuous-time Approximation of RP

The continuous-time fluid model of the pricing scheme is given by the following differential equations

$$x_n(t) = x_n^m F_\epsilon\left(\frac{q_n(t)}{\kappa} - p(t)\right) \quad (12)$$

$$s(t) = \dot{C}^{-1}(p(t)) \quad (13)$$

$$\dot{q}_n(t) = \begin{cases} \lambda_n(t) - x_n(t) & \text{if } {}^{q_n(t)>0, \text{ or}}_{\lambda_n(t)-x_n(t)\geq 0} \\ 0 & \text{otherwise} \end{cases}$$

$$\dot{p}(t) = \begin{cases} \gamma\left(\sum_{n=1}^N x_n(t) + \lambda_S - s(t)\right) & \text{if } {}^{p(t)>0, \text{ or}}_{\sum_{n=1}^N x_n(t)+S_i-s(t)\geq 0} \\ 0 & \text{otherwise} \end{cases}$$

hence integrable. If u_n^{-1} is bounded at the end points of its domain, i.e. 0 and x_n^m, we set $\dot{U}_n(x) = u_n^{-1}(x)$ at these points. Otherwise, we set $U_n(0) = -\infty$ if $u_n^{-1}(0) = \infty$, and $U_n(x_n^m) = -\infty$ if $u_n^{-1}(x_n^m) = -\infty$. Note that by this definition, U_n is strictly concave on $[0, x_n^m]$.

Having defined the functions U_n, we consider the following social welfare maximization problem

$$\max_{\mathbf{x},s} \quad \sum_{n=1}^N U_n(x_n) - C(s) \quad (14)$$

$$\text{s.t.} \quad \sum_{n=1}^N x_n + \lambda_S \leq s \quad (15)$$

$$\lambda_n \leq x_n, \ \forall n \quad (16)$$

In (14), U_n can be interpreted as a consumer utility function. U_n is strictly concave since u_n^{-1} is strictly decreasing and $\dot{U}_n(x) = u_n^{-1}(x)$. We do not restrict U_n to be a monotone function. Problem (14) is quite similar to problem (4) only with a change in the objective function, where the utility of consumption is amended.

Define p to be the dual variable corresponding to (15), and q_n to be the dual variables corresponding to (16). Let $(\hat{\mathbf{x}}, \hat{s}, \hat{p}, \hat{\mathbf{q}})$ be the optimal primal-dual solution to problem (14). The next theorem shows that RP-C converges to the optimal solution of (14)-(16).

Theorem 1. *The continuous-time approximation algorithm RP-C converges to the optimal solution* $(\hat{\mathbf{x}}, \hat{s}, \hat{p}, \hat{\mathbf{q}})$ *of Problem (14).*

Proof. First, we write the Lagrangian for Problem (14) as

$$L(\mathbf{x}, s, p, \mathbf{z}) = \sum_n U_n(x_n) - C(s) - p\left(\sum_n x_n + \lambda_S - s\right) - \sum_n \frac{q_n}{\kappa}(\lambda_n - x_n)$$

Applying KKT conditions, the optimal solution satisfies

$$\hat{s} = \sum_n \hat{x}_n + \lambda_S, \ \hat{p} = \dot{C}(\hat{s}), \ \hat{x}_n = \dot{U}_n^{-1}\left(\hat{p} - \frac{\hat{q}_n}{\kappa}\right), \forall n \quad (17)$$

$$\hat{x}_n \geq \lambda_n, \ \forall n \quad (18)$$

$$\hat{q}_n(\lambda_n - \hat{x}_n) = 0, \ \forall n \quad (19)$$

where (17)-(18) are optimality and feasibility conditions, and (19) is the complementary slackness conditions for the dual variables q_n and the corresponding inequality constraints.

Now, we consider RP-C, and treat the queue and the price values as the dual variables of Problem (14). In order to establish the convergence of RP-C to $(\hat{\mathbf{x}}, \hat{s}, \hat{p}, \hat{\mathbf{q}})$, we show that the following Lyapunov function is strictly decreasing.

$$V(t) = \frac{\kappa}{2\gamma}(p(t) - \hat{p})^2 + \frac{1}{2}\sum_n (q_n(t) - \hat{q}_n)^2. \quad (20)$$

Specifically, the drift is given by

$$\begin{aligned}\dot{V}(t) &= \frac{\kappa}{\gamma}(p(t) - \hat{p})\dot{p}(t) + \sum_n (q_n(t) - \hat{q}_n)\dot{q}_n(t) \\ &\leq \kappa(p(t) - \hat{p})\left(\sum_n x_n(t) + \lambda_S - s(t)\right) \\ &\quad + \sum_n (q_n(t) - \hat{q}_n)(\lambda_n - x_n(t)) \\ &= \kappa(p(t) - \hat{p})\left(\sum_n (x_n(t) - \hat{x}_n) + (\hat{s} - s(t))\right) \\ &\quad + \sum_n (q_n(t) - \hat{q}_n)((\lambda_n - \hat{x}) + (\hat{x} - x_n(t))) \quad (21)\end{aligned}$$

where (21) is obtained by adding and subtracting the optimum values $\hat{x}_n$ and $\hat{s}$, and noting from (17) that $\hat{s} = \sum_n \hat{x}_n$. Note that, from complementary slackness condition given in (19), $\hat{q}_n(\lambda_n - \hat{x}_n) = 0$ for all n. Also, $q_n(t)(\lambda_n - \hat{x}_n) \leq 0$ due to dual and primal feasibility. Therefore, $(q_n(t) - \hat{q}_n)(\lambda_n - \hat{x}_n) \leq 0$. Using this in (21), we obtain

$$\begin{aligned}\dot{V}(t) \leq\ & \kappa \sum_n \left(p(t) - \frac{q_n(t)}{\kappa} - \left(\hat{p} - \frac{\hat{q}_n}{\kappa}\right)\right)(x_n(t) - \hat{x}_n) \\ & - \kappa(p(t) - \hat{p})(s(t) - \hat{s})\end{aligned}$$

Recall that the function U_n is strictly concave from (17) and (12) we obtain

$$\begin{aligned}&\left(\left(p(t) - \frac{q_n(t)}{\kappa}\right) - \left(\hat{p} - \frac{\hat{q}_n}{\kappa}\right)\right)(x_n(t) - \hat{x}_n) \\ &= \left(\dot{U}_n(x_n(t)) - \dot{U}_n(\hat{x}_n)\right)(x_n(t) - \hat{x}_n) \leq 0,\end{aligned}$$

and the equality holds if and only if $x_n(t) = \hat{x}_n$. Similarly, due to strict convexity of C and from (13)

$$(p(t) - \hat{p})(s(t) - \hat{s}) \geq 0$$

where the equality holds if and only if $p(t) = \hat{p}$. Hence, we obtain $\dot{V}(t) < 0$, and the equality holds if and only if $p(t) = \hat{p}$ and $x_n(t) = \hat{x}_n$ for all n. Thus, $\dot{V}(t) < 0$ unless $(\mathbf{x}(t), s(t), p(t), \mathbf{q}) = (\hat{\mathbf{x}}, \hat{s}, \hat{p}, \hat{\mathbf{q}})$. As a result, the dual algorithm converges to the optimal point $(\hat{\mathbf{x}}, \hat{s}, \hat{p}, \hat{\mathbf{q}})$. □

B. The Algorithm and Consumer Behavior Under the Presence of Daily Inflexible Load

The problems in (1) and (4) disregard the fact that part of the grid load, which we model as inflexible load, has a daily pattern, . Load exhibits bottoms and peaks at approximately same hours each day, and its amount do not change significantly between the same hours of different days. Thus, inflexible load can be predicted day-ahead, and real-time control and pricing decisions can be made with this knowledge. Hence, a plausible direction is to formulate an optimization problem taking into account such daily patterns.

Let K be the number of time slots in a day over which supply and load matching should be ensured and control decisions are made. We assume that the inflexible load pattern is periodic with K, i.e. we expect to see the same load every day. The following problem aims to minimize the cost of electricity supply over K slots subject to the consumers' average consumption constraints.

$$\min_{\mathbf{x},\mathbf{s}} \quad \sum_{k=1}^{K} \sum_{m=1}^{M} C_m\left(s_m\right) \tag{22}$$

$$\text{s.t.} \quad \sum_{n=1}^{N} x_n^k + S_i^k \leq \sum_{m=1}^{M} s_m^k, \ \forall k = 1, \ldots, K \tag{23}$$

$$K\lambda_n \leq \sum_{k=1}^{K} x_n^k, \ \forall n. \tag{24}$$

Utilizing the line of reasoning used in the design of RP, we present algorithm RPD that aims to obtain an approximate solution to (22)-(24) by price randomization.

Algorithm RPD Randomized Daily Pricing

At iteration t:

- Consumer n computes its load and queue as

$$x_n(t) = x^m \mathbb{1}\left\{p_n(t) \leq \frac{q_n(t)}{\kappa}\right\}, \text{ for } t = 1, 2, \ldots,$$

$$q_n(t) = \left[q_n(lK) + \sum_{k=lK+1}^{(l+1)K} \left(a_n(k) - x_n(k)\right)\right]^+$$

$$\text{for} \quad t \in \left[(l+1)K+1, (l+2)K\right], \ l = 0, 1, \ldots$$

- The supplier must meet the real load $\sum_n x_n(t)$, and it computes $s(t) = \dot{C}^{-1}(p(t))$
- The operator computes the common market price and generates the individual prices for consumers:

$$p(t+K) = \left[p(t) + \alpha\left(\sum_{n=1}^{N} x_n(t) + S_i(t) - s(t)\right)\right]^+$$

$$p_n(t) = p(t) + \epsilon_n(t)$$

Algorithm RPD differs from RP in consumers' queue update rule. Since constraint (24) aims to satisfy average user demand over one optimization period, user queues are updated once at the end of each period and the same backlog value is used in the rest of the next period. Intuitively, consumers track their consumption on a daily basis.

In Figure 3, load evolutions obtained by running algorithms RP and RPD are plotted over two days. Historical metered load data from PJM, [8], is used as inflexible load, and flexible load is set to be 5% of the total load. We observe a *waterfilling* behavior for both algorithms; Flexible users consume electricity when the inflexible demand is low and fill the valleys in the daily pattern.

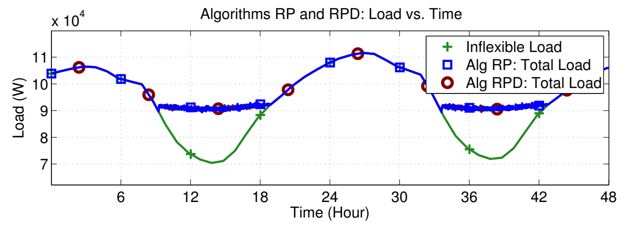

Fig. 3. Flexible load (5% of the total load) fill the valleys in the daily inflexible load pattern where the market price is lower.

V. Performance and Numerical Results

We next evaluate the proposed randomized pricing mechanisms using the following performance metrics: The payments made by the consumers, the payments received by the suppliers and the cost of generation. In terms of these metrics, we compare the performance of RP and RPD to the benchmark schemes Scheme 1 and 2.

The payment of consumer n under RP-C is given by

$$r_n(t) = p(t)x_n(t) + x_n^m \mathbb{E}\left[\epsilon_n \mathbb{1}\left\{\epsilon_n \leq -\dot{U}(x_n(t))\right\}\right]$$

which follows from (12). On the other hand, the supplier receives the payment $w(t) = p(t)s(t)$. Note that $w(t)$ is calculated based on the supplier's offer $s(t)$ instead of the actual generation $\sum_n x_n(t)$. At the equilibrium of RP-C, the supplier and total consumer payments are given by

$$\hat{r} = \hat{p}\sum_n \hat{x}_n + \sum_n x_n^m \mathbb{E}\left[\epsilon_n \mathbb{1}\left\{\epsilon_n \leq -\dot{U}(\hat{x}_n)\right\}\right] \tag{25}$$

$$\hat{\omega} = \hat{p}\hat{s} = \hat{p}\sum_n \hat{x}_n \tag{26}$$

where (26) follows from the KKT condition in (17).

Observe that, from (25) and (26), the amount of payment made to the supplier and the amount of payment received from flexible consumers do not necessarily match, and the difference is given by the second term in (25). We call this difference the *manager deficit*. Naturally, one wants to make this difference 0 so that the market actually clears in terms of payments. As an example, consider the case where $\lambda_n = \lambda$, $x_n^m = x^m$ for all n, and ϵ_n's have the identical uniform distribution over the interval $[\epsilon, \epsilon + a]$, i.e. $F_\epsilon(x) = \frac{x-\epsilon}{a}$. Then, setting $\epsilon = -\frac{a\lambda}{x^m}$ ensures that the deficit is 0. Indeed, the deficit is negligibly small even when $\epsilon(t)$ has a uniform distribution symmetric around 0.

Figure 4 plots on the horizontal axis the consumer payments that are obtained by running the benchmark schemes and the randomized algorithms. The simulations are run for different numbers of flexible consumers in the system. Particularly, load from flexible consumers constitutes 5%, 10%, and 20% of the total load in each simulation. The amount of total load in the system is kept constant for all levels of flexible consumer penetration. Furthermore, inflexible load is a sinusoid which approximates well the historical metered load data from PJM [8] (c.f. Figure 2&3). We observe that in Scheme 1, where demand has practically no flexibility, consumer payments are greater than all other pricing mechanisms. On the other hand, in Scheme 2 and in algorithms RP and RPD, consumers can take advantage of changing prices and reduce their payments. Figure 4 shows that consumer payments in randomized pricing algorithms and payments in Scheme 2 are quite close. We can conclude that randomized pricing preserves the economic benefits of flexible consumers, and hence consumers will be motivated to participate in a market where prices are differentiated. Furthermore, as seen in Figure 4, as the number of flexible consumers increases, payments tend to decrease due to lowered fluctuations in the common market price.

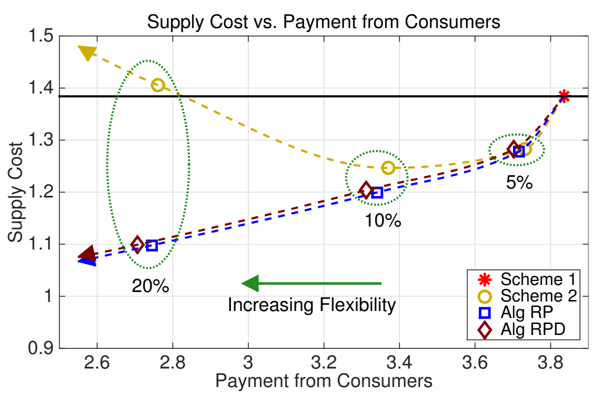

Fig. 4. Randomized pricing algorithms perform better with increasing flexible consumer penetration in the market.

Next, we study the impact of demand deferrability on supply cost and compare the costs achieved by the pricing mechanisms. Figure 4 plots on the vertical axis the supply costs that are achieved by running the benchmark schemes and the randomized algorithms. Since the cost function C is increasing and convex we expect it to be more costly to supply more variable load. However, in the figure, supply cost decreases as flexibility penetration increases from 0 to 10% under all pricing mechanisms except Scheme 1. In Scheme 1, the total load is the sum inflexible load and aggregation of random demand arrivals, thus it follows almost the same pattern in all flexibility levels. On the other hand, the decrease in cost under randomized mechanisms and Scheme 2 is due to the increased peak shaving effect with increasing flexibility penetration.

The benefit of price randomization becomes more dramatic when the percentage of flexible load increases. We observe in Figure 4 that supply cost starts increasing under Scheme 2 after a certain level of flexible load percentage whereas it continues to decrease under algorithms RP abd RPD. The increase under Scheme 2 is apparently due to large fluctuation in load and the convex cost structure. However, price randomization smoothens the flexible consumers' load and makes total load less costly to supply.

VI. Conclusion

We proposed a novel real-time pricing scheme that attempts to solve the volatility problem in a system where economically-driven consumers have the flexibility to defer their demand. We demonstrated the destabilizing effect of opportunistic consumer behavior on the load and market price, when conventional real-time pricing methods are employed.

We then proposed a new pricing scheme that is based on price differentiation among consumers; Individual consumers receive different prices that are randomly generated based on a common price. We numerically demonstrated that self-interested consumers economically benefit from deferring their demand while supply cost is kept low. The randomized pricing scheme is simple to implement since it does not require any knowledge on consumer strategies, and it can be employed in different systems where demand has time flexibilities.

References

[1] M. J. Neely, A. S. Tehrani, and A. G. Dimakis, "Efficient algorithms for renewable energy allocation to delay tolerant consumers," in *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*. IEEE, 2010, pp. 549–554.

[2] D. Materassi, M. Roozbehani, and M. A. Dahleh, "Equilibrium price distributions in energy markets with shiftable demand," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 3183–3188.

[3] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *Power Systems, IEEE Transactions on*, vol. 27, no. 4, pp. 1926–1940, 2012.

[4] A.-H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *Smart Grid, IEEE Transactions on*, vol. 1, no. 2, pp. 120–133, 2010.

[5] M. Roozbehani, M. Rinehart, M. Dahleh, S. Mitter, D. Obradovic, and H. Mangesius, "Analysis of competitive electricity markets under a new model of real-time retail pricing," in *Energy Market (EEM), 2011 8th International Conference on the European*. IEEE, 2011, pp. 250–255.

[6] O. Dalkilic, A. Eryilmaz, and X. Lin, "Stable real-time pricing and scheduling for serving opportunistic users with deferrable loads." in *Allerton*, 2013, pp. 1200–1207.

[7] A. Malekian, A. Ozdaglar, and E. Wei, "Competitive equilibrium in electricity markets with heterogeneous users and ramping constraints," in *Proc. IEEE Allerton Conference*, 2013.

[8] PJM. (2014) Historical metered load data. [Online]. Available: http://www.pjm.com/pub/operations/hist-meter-load/2014-hourly-loads.xls

[9] S. Chen, P. Sinha, and N. B. Shroff, "Scheduling heterogeneous delay tolerant tasks in smart grid with renewable energy," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 1130–1135.

[10] T. T. Kim and H. V. Poor, "Scheduling power consumption with price uncertainty," *Smart Grid, IEEE Transactions on*, vol. 2, no. 3, pp. 519–527, 2011.

[11] J. G. Dai, "On positive harris recurrence of multiclass queueing networks: a unified approach via fluid limit models," *The Annals of Applied Probability*, pp. 49–77, 1995.

[12] X. Lin and S. B. Rasool, "Constant-time distributed scheduling policies for ad hoc wireless networks," *Automatic Control, IEEE Transactions on*, vol. 54, no. 2, pp. 231–242, 2009.