ECE 382 Project Solution

April 20, 2007

1 Task 1

Substitute the values of M, m, l, I into equation (1), we can obtain the following transfer function,

$$P(s) = \frac{ml}{s^2[m^2l^2 - (M+m)(I+ml^2)] + (M+m)mgl}$$
$$= \frac{-63}{2835s^2 - 55566}.$$

Note that the numerator is negative, which is different from what we typically study in the class.

2 Task 2

Let us first draw the block-diagram of the system into a familiar form (Fig. 1). Note that this is a postive-feedback system. The characteristic equation is

$$1 - P(s)C(s) = 0.$$

Equivalently, we can rewrite the characteristic equation as

$$1 + (-P(s))C(s) = 0.$$

Now, $-P(s)$ has positive leading coefficients in both numerator and denominator, and the above characteristic equation is almost like that of negative feedback system. Assume $C(s) = K$. Using Matlab, we can obtain the root locus of $-P(s)$ as in Fig. 2. Note that there are two open-loop poles at ± 4.42.

Clearly, for any value of $K \geq 0$, there will exist a closed-loop pole that is either on the imaginary axis or with positive real part. Therefore, the system is stable for any value of $K \geq 0$.

We take $K = 1$ and obtain the unit impulse response of the system in Fig. 3. The system is clearly unstable.

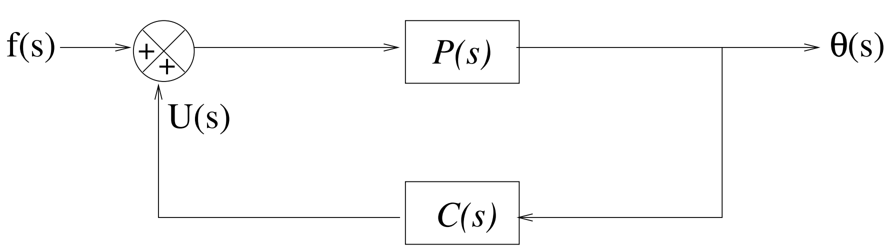

Figure 1: The equivalent model with external disturbance

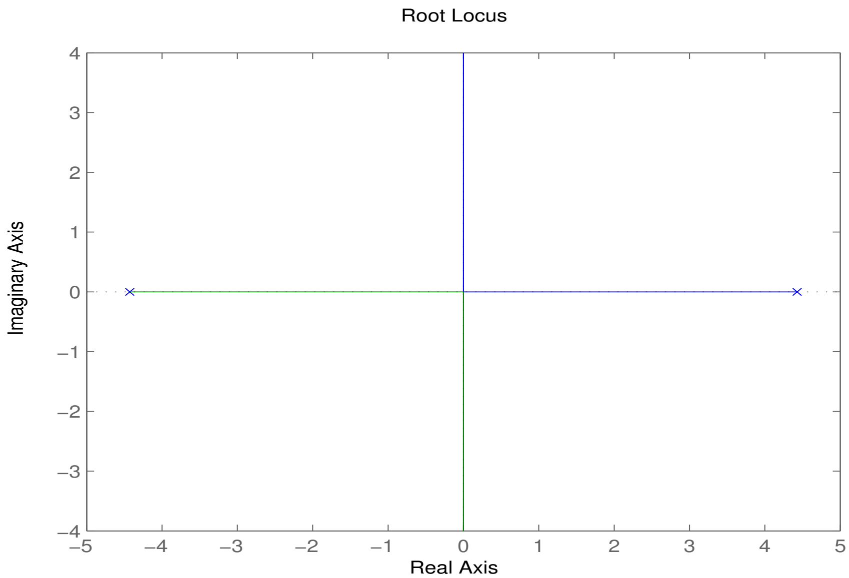

Figure 2: The root locus of the original system with a proportional controller

3 Task 3

According to the given Supplement Info, we can approximate the natual frequency and damping ratio with the following equations:

$$e^{\pi\xi/\sqrt{1-\xi^2}} = 5\%$$

$$\frac{4}{\xi\omega_n} = 2$$

Solving these two equations we obtain $\xi = 0.6901$ and $\omega_n = 2.8981$. Therefore, the desired closed-loop poles are

$$s_* = \omega_n\xi \pm \omega_n\sqrt{1-\xi^2} = -2 \pm 2.0974j.$$

4 Task 4

Assume the lead compensator is

$$C(s) = K\frac{s-z_c}{s-p_c}.$$

The angle caused by desired closed-loop pole $s_* = -2 + 2.0974j$ is

$$\begin{aligned}\alpha &= 0 - \angle(s_* - p_1) - \angle(s_* - p_2) \\ &= -\arctan\frac{2.0974}{-2+4.42} - (180^\circ - \arctan\frac{2.0974}{2+4.42}) + 360^\circ \\ &= 157.18^\circ\end{aligned}$$

The angle deficiency for this pole is: $\phi = 180^\circ - \alpha = 22.82^\circ$.

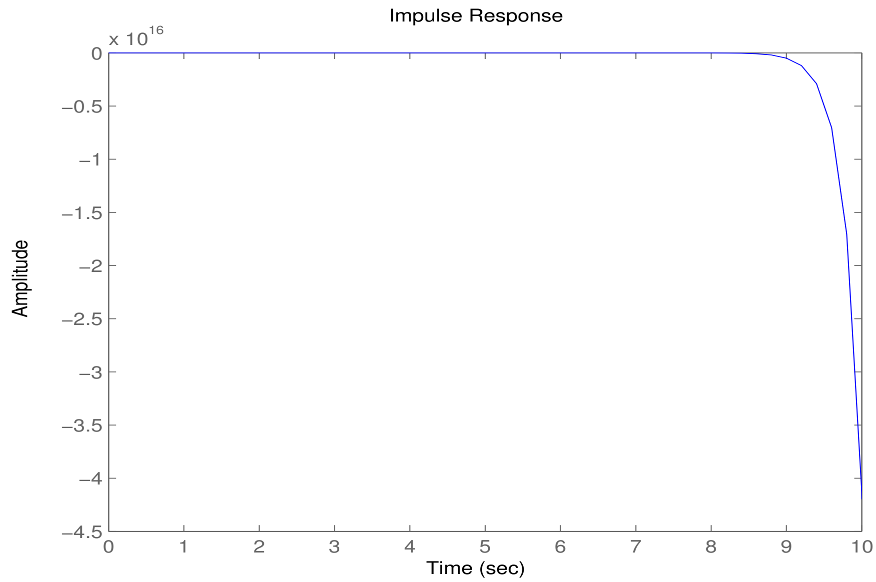

Figure 3: The unit impulse response of the original system with a proportional controller

We choose the open-loop zero of our lead comensator $C(s)$ such that it can provide $30°$; hence the the open-loop pole we choose will need to provide $\phi - 30° = 7.18°$. Hence, the open-loop zero is at $z_c = -5.63$ and the open-loop pole is $p_c = -18.66$.

The value of K is computed by,

$$K = \left| \frac{s - z_p}{s - z_c} \frac{2835s^2 - 55566}{-63} \right|_{s=-2+2.0974j} = 3906.$$

Therefore our lead compensator is

$$C(s) = \frac{3906s + 21991}{s + 18.66}.$$

The root locus for the new system is in Fig. 4.

5 Task 5

The unit impluse response of the new system is in Fig. 5. The largest initial value is -4.5^{-3}; the second largest initial value is 2.2×10^{-4}, which is 4.89% of the largest initial angle. At $t = 2$ second, the angle is 2.12×10^{-4}, which is 4.7% of the largest initial angle. This does not quite satisfy the design specification for settling time.

To achieve the design spec for settling time, we can increase the undamped natural frequency. We may chose $\xi = 0.6901$, and $\omega_n = 5$. The new closed-loop poles are $s_{**} = -3.4505 \pm 3.6186j$. Repeat the design process in Task 4, we can obtain the lead-compensator as

$$C(s) = \frac{2907s + 11890}{s + 9.797}.$$

The impluse reponse of this corrected system is in Fig. 6. We can see that the system now settles down to 2% in less than 2 seconds.

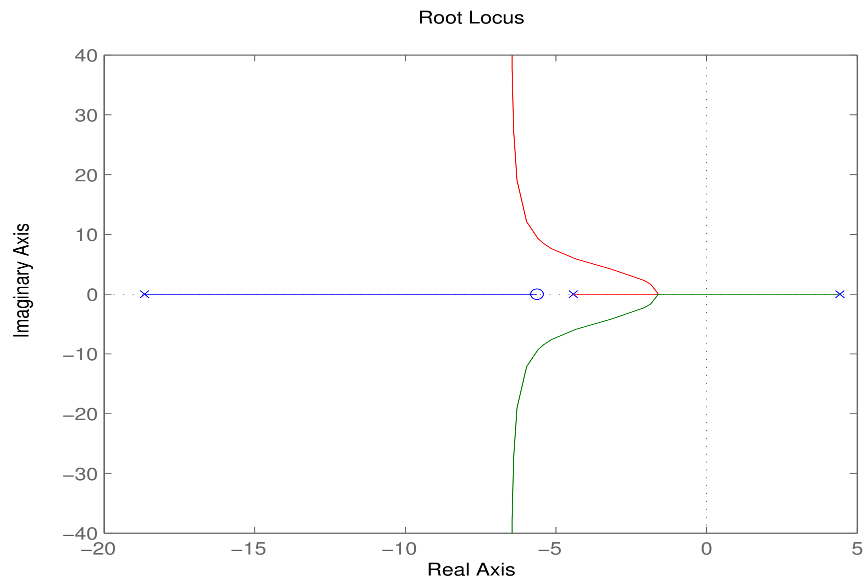

Figure 4: The root locus of the new system with a lead compensator

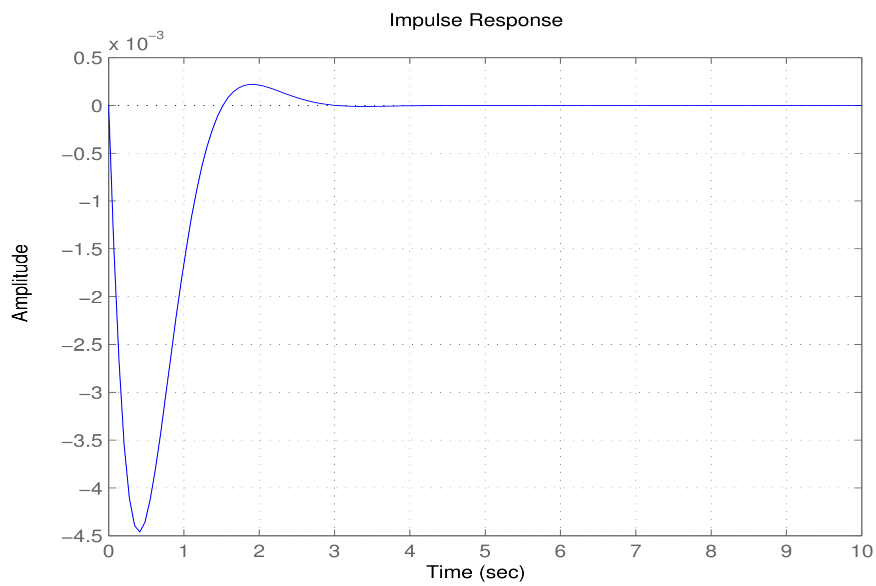

Figure 5: The unit impulse response with a lead compensator: First attempt.

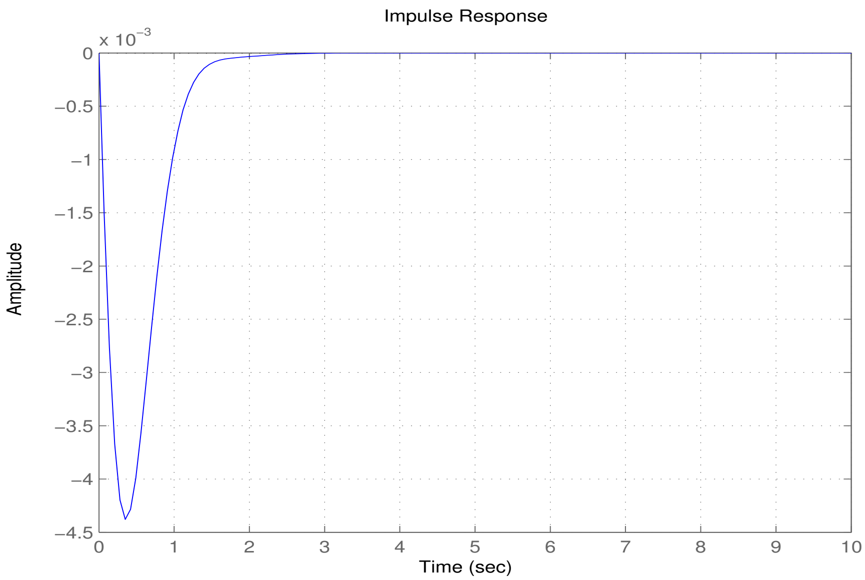

Figure 6: The unit impulse response of the corrected system: Final solution.

A MATLAB code for the first compensator

```
m = 70;
l = 0.9;
M = 20;
I = m * l * l / 3;
g = 9.8;
close all;

%Task 1: find P(s)
num = [m*l];
den = [(m*m*l*l - (M+m)*(I+m*l*l)) 0 (M+m)*m*g*l ];
disp('P(s) is');
sys = tf(num,den)

%Task 2: experiment with proportional controller
%Note that the system has positive feedback
%This is the root-locus of the uncompensated system
%Found two open-loop poles at +- 4.42
rlocus(-sys);

%This is the impulse response of the uncompensated system
disp('Close Loop:');
closed1=feedback(sys,1,1)
figure;
impulse(closed1,10);

%Task 3: choose the desired location of new closed-loop pole.
%From unit-step equations
```

```
xi=0.6901; wn=2.8981;
mag1=wn*xi; mag2=wn*sqrt(1-xi^2);
disp('Desired close-loop Pole')
s = -mag1+mag2*j

%Task 4: add compensator.
% Calculate the angle deficiency;
t1= 180 - atan(mag2/(mag1+4.42))*180/pi;
t2= atan(mag2/(-mag1+4.42))*180/pi;
angle = -t1-t2 + 360;

%Hence, the angle deficiency is
phi = 180-angle;

%Design the open-loop zero and pole for lead compensator
zero_phi=30;
zero=-mag1 - mag2/tan(zero_phi/180*pi);

pole_phi=zero_phi-phi;
pole=-mag1 - mag2/tan(pole_phi/180*pi);

%Calculate the gain
K = abs((2835*s*s-55570)*(s-pole)/(s-zero)/63);

% This is the compensator
num2=[1,-zero];
den2=[1,-pole];
disp('My C(s):');
sys2=tf(K*num2,den2)

% The following is the root-locus of new open loop
figure;
rlocus(-sys*sys2);

%Task 5: plot the impulse response of the compensated system
disp('New Closed-Loop Transfer Function');
closed2=feedback(sys,-sys2,-1)
figure;
impulse(closed2,10);
```

B MATLAB code for the final compensator

Note: the only changes is we now set $\mathrm{wn} = 5.0$ **in Task 3, and set** $\mathrm{zero\_phi} = 80$ **in Task 4.**

```
m = 70;
l = 0.9;
M = 20;
I = m * l * l / 3;
g = 9.8;
close all;

%Task 1: find P(s)
num = [m*l];
```

```
den = [(m*m*l*l - (M+m)*(I+m*l*l)) 0 (M+m)*m*g*l ];
disp('P(s) is');
sys = tf(num,den)

%Task 2: experiment with proportional controller
%Note that the system has positive feedback
%This is the root-locus of the uncompensated system
%Found two open-loop poles at +- 4.42
rlocus(-sys);

%This is the impulse response of the uncompensated system
disp('Close Loop:');
closed1=feedback(sys,1,1)
figure;
impulse(closed1,10);

%Task 3: choose the desired location of new closed-loop pole.
%From unit-step equations
xi=0.6901; wn=5; %increase the undamped natural frequency from 2.8981
mag1=wn*xi; mag2=wn*sqrt(1-xi^2);
disp('Desired close-loop Pole')
s = -mag1+mag2*j

%Task 4: add compensator.
% Calculate the angle deficiency;
t1= 180 - atan(mag2/(mag1+4.42))*180/pi;
t2= atan(mag2/(-mag1+4.42))*180/pi;
angle = -t1-t2 + 360;

%Hence, the angle deficiency is
phi = 180-angle;

%Design the open-loop zero and pole for lead compensator
zero_phi=80;
zero=-mag1 - mag2/tan(zero_phi/180*pi);

pole_phi=zero_phi-phi;
pole=-mag1 - mag2/tan(pole_phi/180*pi);

%Calculate the gain
K = abs((2835*s*s-55570)*(s-pole)/(s-zero)/63);

% This is the compensator
num2=[1,-zero];
den2=[1,-pole];
disp('My C(s):');
sys2=tf(K*num2,den2)

% The following is the root-locus of new open loop
figure;
rlocus(-sys*sys2);

%Task 5: plot the impulse response of the compensated system
```

```
disp('New Closed-Loop Transfer Function');
closed2=feedback(sys,-sys2,-1)
figure;
impulse(closed2,10);
```