ECE-382: Midterm #2

March 28, 2007
8:30PM—9:30PM

- **Do not write your answers at the back of any page. Answers written at the back of page will NOT be graded.**
- This is a closed book exam. You are only allowed to bring a pen (or pencil), a one-sided crib sheet, and an eraser to the exam. No calculator is allowed.
- Some common sin/cos/tan function values and common square root values are provided at the end of the exam booklet.
- Write your name and PUID at the space provided below.
- **This exam has two parts.**
  Part I consists of three problems. Unless otherwise instructed, justify your answers to these problems completely. **Show all intermediate steps to get full credits for problems from Part I.**
  Part II consists of one question, with three sub-questions, for which no justification is required. Enter the answers to Part II in the spaces provided. **Partial credit will not be provided for problems from Part II.**
- The total points are 150. You have **ONE HOUR** to complete the exam.

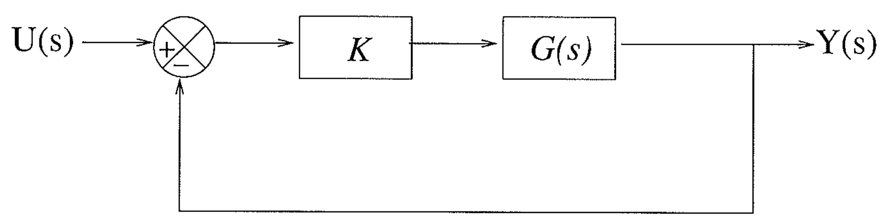
Solution
____
Your Name

____
10-digit PUID

Questions for Part I

Justify your answer completely. Show all intermediate steps to get full credits.

Problem 1 (60 points)



Consider the unit negative feedback system shown above with

$$G(s) = \frac{1}{(s+2)(s^2+2s+2)}.$$

Answer the following questions (use the blank figure on the next page for marking your answers on the complex plane).

(a) (5 points) Find the open-loop zeros and poles (if any), and plot them on the complex plane.

(b) (5 points) Find the segment (or segments) of the real axis that belong to the root locus of $G(s)$. Mark them on the complex plane.

(c) (10 points) For the root locus, find the asymptotes (if any) and the point (if any) that the asymptotes intersect the real axis. Mark them on the complex plane.

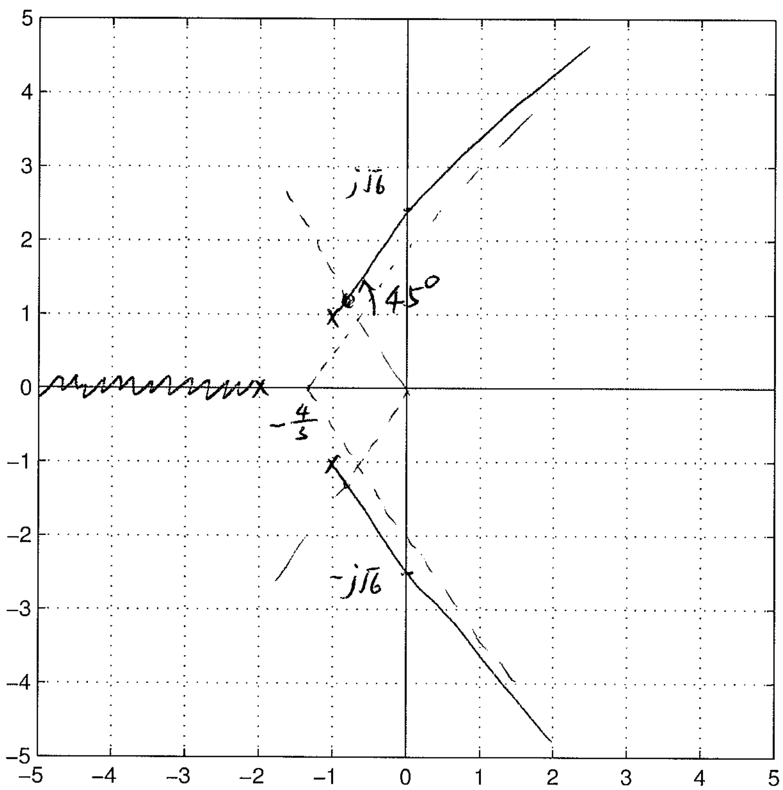
You are given the following information: there are NO break-in/break-away points on the root-locus.

(d) (10 points) Find the points (if any) where the root locus cross the imaginary axis, and the corresponding value of K.

(e) (10 points) For the root locus, find the angle of departure from the open loop poles (if any), and the angle of arrival at the open loop zeros (if any).

(f) (10 points) Sketch the complete root-locus on the complex plane.

(g) (10 points) Is it possible to choose $K > 0$ such that the closed-loop system has dominant poles with damping ratio $\zeta = 0.5$. Justify your answer. If the answer is yes, mark the corresponding location for the closed-loop poles on the complex plane, and write down the approximate values of the closed-loop poles.



Solution:

(a) The open-loop poles are: $-2,\ -1+j,\ -1-j$

There are no open-loop zeros

(b) The real-segment to the left of -2 is on the locus

(c) There will be three asymptotes

The angles are

$$\theta_a = \frac{180^\circ}{3} + l \cdot \frac{360^\circ}{3} = 60^\circ, 180^\circ, -60^\circ$$

The intersection point is

$$\sigma_a = \frac{\Sigma P_i}{3} = \frac{-2 + (-1+j) + (-1-j)}{3} = -\frac{4}{3}$$

(d) To compute the point the locus crosses the imaginary axis, rewrite the characteristic equation

$$(s+2)(s^2+2s+2) + K = 0$$

$$s^3 + 4s^2 + 6s + (K+4) = 0$$

The Routh array is

$$\begin{array}{lcc} s^3 & 1 & 6 \\ s^2 & 4 & K+4 \\ s^1 & \frac{24-(K+4)}{4} & \\ s^0 & K+4 & \end{array}$$

By $K=20$, the third row will be all zero.

The row above corresponds to

$$4s^2 + 24 = 0$$

$$s = \pm j\sqrt{6} \approx \pm j2.45$$

(e) The angle of departure from the real pole at -2 is $180°$.

Let $p_1 = -1+j$, $p_2 = -2$, $p_3 = -1-j$

To compute the angle of departure from p_1, use the angle condition

$$-\angle(s-p_1) - \angle(s-p_2) - \angle(s-p_3) = 180°$$

Let $s \to p_1$,

$$-\theta_{p_1} - \angle(p_1-p_2) - \angle(p_1-p_3) = 180°$$

$$\Rightarrow -\theta_{p_1} - 45° - 90° = 180°$$

$$\therefore \theta_{p_1} = -180° - 45° - 90° = -315°$$
$$= 45°$$

The angle of departure from p_2 is $-45°$ by symmetry

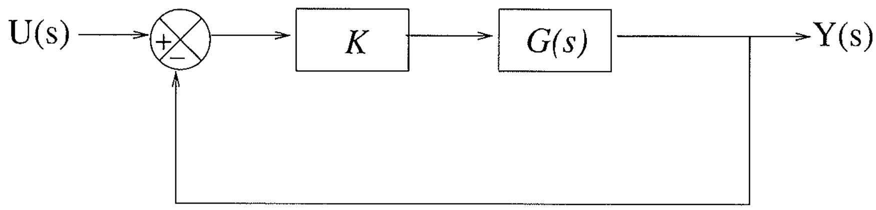
(f) see figure

(g) Draw a half line at angle $\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = 60°$ with the negative real axis.

Since the half-line intersects the locus, there is a value of K such that the damping ratio of the dominant closed-loop pole is 0.5.

The dominant closed-loop pole is approximately $-0.75 \pm j1.25$

Problem 2 (25 points)



Consider the unit negative feedback system shown above with

$$G(s) = \frac{s+2}{s(s+1)}.$$

Answer the following questions (use the blank figure on the next page for marking your answers on the complex plane).

(a) (15 points) Find the break-in/break-away points (if any) of the root locus, and the corresponding value of K. Mark these points on the complex plane.

You are given the following information: the root-locus does NOT intersect the imaginary axis.

(b) (10 points) Sketch the complete root-locus on the complex plane.

Solution:

(a) To compute the break-in/break-away points, write K as a function of s

$$1 + K \cdot \frac{s+2}{s(s+1)} = 0$$

$$\Rightarrow \quad K = -\frac{s(s+1)}{s+2} = -\left[(s-1) + \frac{2}{s+2}\right]$$

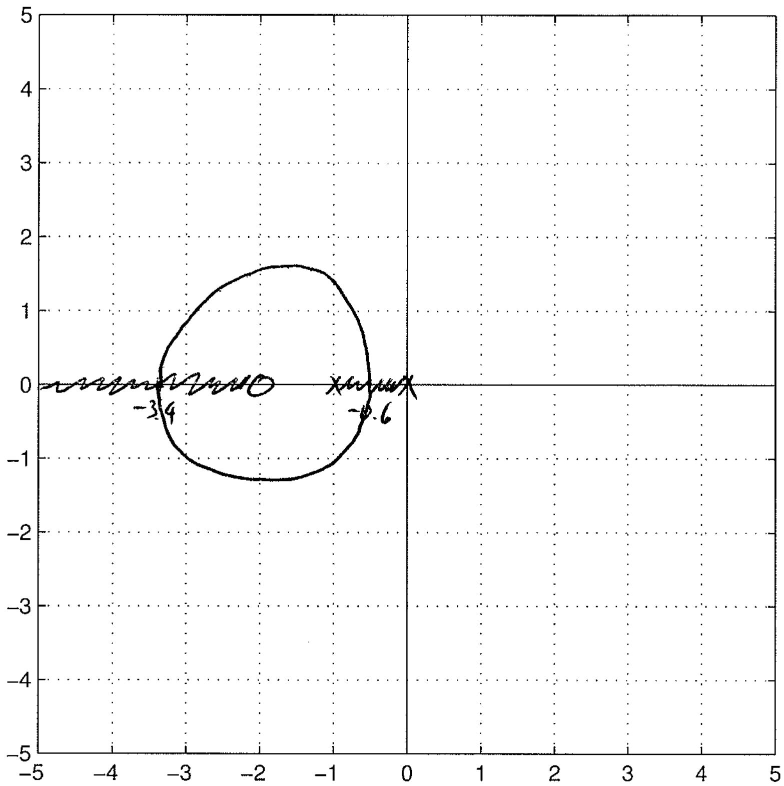
By setting $\frac{dK}{ds} = 0$, we have

$$-1 + \frac{2}{(s+2)^2} = 0$$

$$\Rightarrow s = -2 \pm \sqrt{2} = -0.6, \ -3.4$$

The corresponding value of K is

$$K = -\left[(-2 \pm \sqrt{2} - 1) + \frac{2}{-2 \pm \sqrt{2} + 2}\right] = 3 \mp 2\sqrt{2}$$



(b) see figure

Problem 3 (15 points)

Given the following equation

$$s^4 - s^2 + s + 1 = 0,$$

use the Routh criteria to find the number of roots with positive real parts, negative real parts, and zero real parts (i.e., purely imaginary roots), respectively. Summarize your final answers in the spaces below. (**Note: You are required to provide the details of the Routh array to get full credits!**.)

(a) Number of roots with positive real parts: 2

(b) Number of roots with negative real parts: 2

(c) Number of roots with zero real parts: 0

Solution: Construct the Routh array

$\varepsilon \to 0^+$	$\varepsilon \to 0^-$				
$+$	$+$	s^4	1	-1	1
$+$	$-$	s^3	$\cancel{0}\ \varepsilon$	1	
		s^2	$\cancel{\frac{-\varepsilon-1}{\varepsilon}}$	$\cancel{\frac{\varepsilon}{\varepsilon}}$	
$-$	$+$		$-1-\frac{1}{\varepsilon}$	1	
$+$	$+$	s^1	$\cancel{\frac{-1-\frac{1}{\varepsilon}-\varepsilon}{-1-\frac{1}{\varepsilon}}}$		
			$1+\frac{\varepsilon}{1+\frac{1}{\varepsilon}}$		
$+$	$+$	s^0	1		

$\therefore \quad N^{+} = 2, \quad N^{-} = 2$

$\max\{N^{+}, N^{-}\} = 2$ is the # of roots on RHP

$|N^{+} - N^{-}| = 0$ is the # of roots on the imaginary axis

$\Rightarrow$ 2 roots with positive real parts.

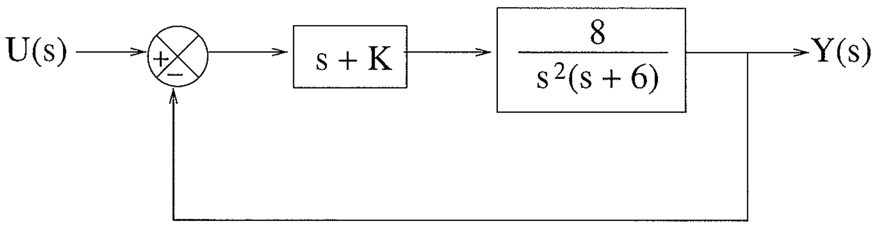
Since the polynomial has 4 roots $\Rightarrow$ 2 roots with negative real parts

Questions for Part II

Do not justify your answer
Partial credit is NOT available

Problem 4 (50 points)

- (10 points) Consider the feedback control system shown below, where $K > 0$ is a parameter that one can adjust freely.



Recall that the closed-loop poles are the roots of the characteristic equation. In order to carry out root-locus analysis, rearrange the characteristic equation such that it is in the standard form

$$1 + KG(s) = 0.$$

Find $G(s)$. Do not justify your answer.

Your answer:

$$G(s) = \frac{8}{s^3 + 6s^2 + 8s}$$

Detailed Solution: The characteristic solution is

$$1 + (s+k)\frac{8}{s^2(s+6)} = 0$$

$$\Rightarrow \quad s^3 + 6s^2 + 8s + 8k = 0$$

$$\Rightarrow \quad 1 + k \cdot \frac{8}{s^3 + 6s^2 + 8s} = 0$$

- (20 points) Assuming the system is of the following form.

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

Match the four **step-response** in Figure 1 (at the next page) with the four transfer functions $G(s)$ in the following table. Do not justify your answer.

Transfer function $G(s)$	$\frac{10}{(s+1)(s+2)(s+3)}$	$\frac{10}{(s+3)(s^2+2s+2)}$	$\frac{1}{(s-1)(s+1)}$	$\frac{10}{s^2+2s+2}$
Step-response #	3	2	1	4

Detailed solution: Let the four transfer functions be labeled as G_1, G_2, G_3, G_4, respectively.

G_1 is overdamped.
G_2 is underdamped
G_3 is unstable
G_4 is underdamped.

Now look at the step-responses.

Step-response 1 is unstable. It must be G_3
Step-response 2 & 4 has overshoots. They must be G_2 & G_4
Step-response 3 has no overshoot. It must be G_1.

To differentiate G_2 & G_4, we can check their final values of the step-response.

$\lim_{s\to 0} s\cdot\frac{1}{s} G_2(s) = \frac{5}{3}$. It must be step-response 2

$\lim_{s\to 0} s\cdot\frac{1}{s} G_4(s) = 5$. It must be step-response 4.

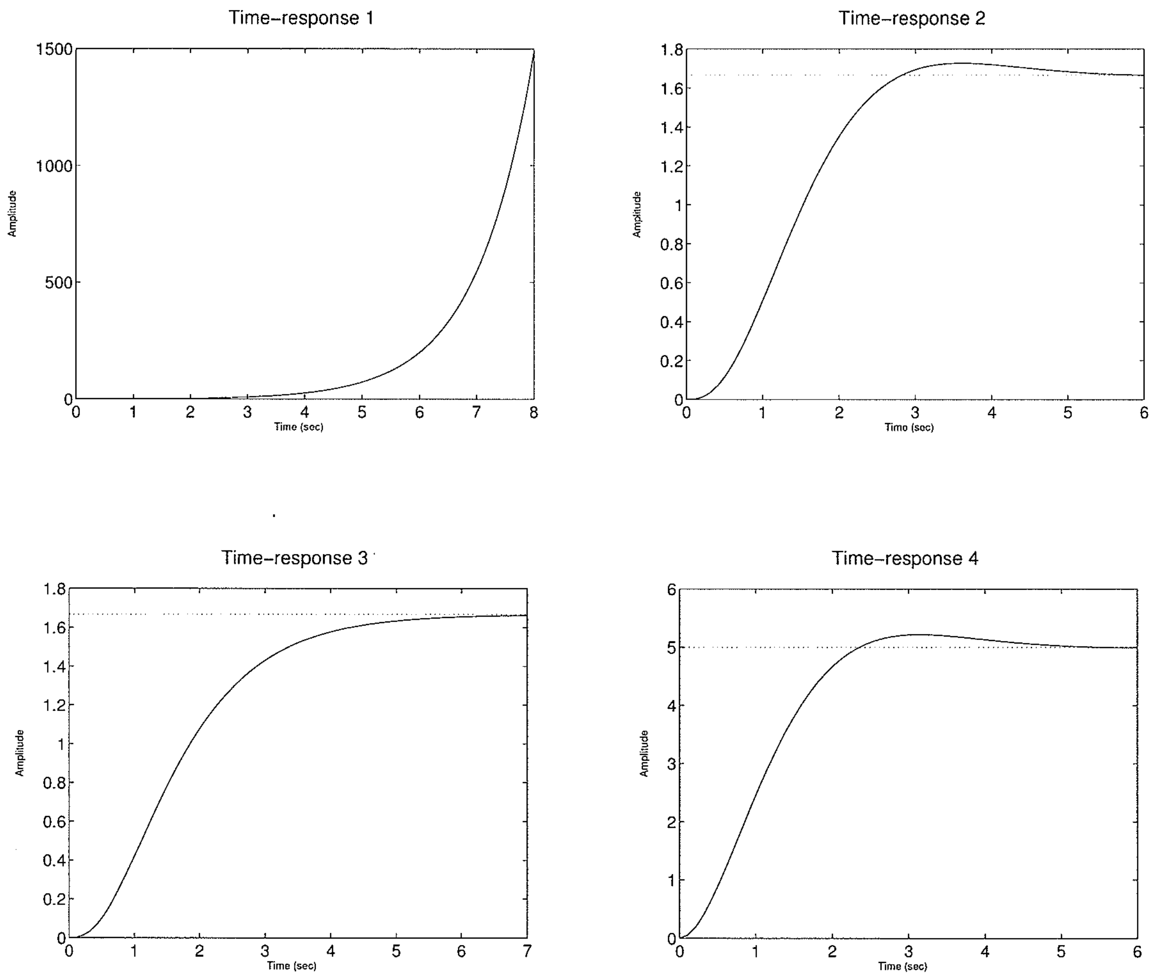
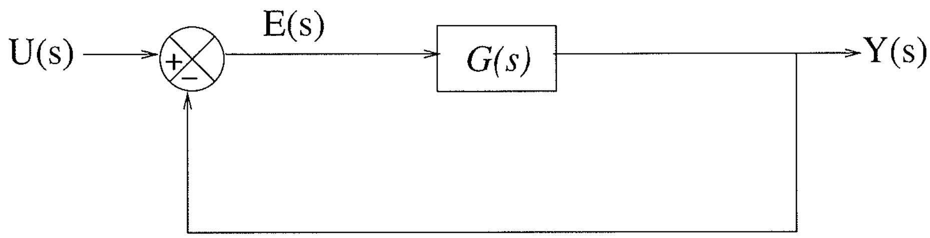


Figure 1: Step responses of four systems

- (20 points) Assume that a closed-loop system is of the following form.



In Figure 2 (at the next page), there are four time-response of the **error** $e(t) = u(t) - y(t)$ when the input signal $u(t)$ is a **ramp-function** (i.e., $u(t) = t\,1(t)$). Match these error response with the four transfer functions $G(s)$ in the following table. Do not justify your answer.

Transfer function $G(s)$	$\frac{s+3}{s^2(s+1)}$	$\frac{s+3}{(s+1)(s+2)(s+10)}$	$\frac{1}{s(s+1)}$	$\frac{1}{s(s+10)}$
Error-response #	4	2	3	1

Detailed solution: Let the four transfer functions be G_1, G_2, G_3, G_4, respectively.

G_1 is of type 2
G_2 is of type 0
G_3 & G_4 are of type 1.

Now look at the ramp-response of the error.

In ramp-response 1 & 3, the steady-state error approaches a non-zero constant. They must correspond to type-1 systems G_3 & G_4

In ramp-response 2, the steady-state error approaches infinity. It must be type-0 system G_2

In ramp-response 4, the steady-state error approaches zero. It must be type-2 system G_1.

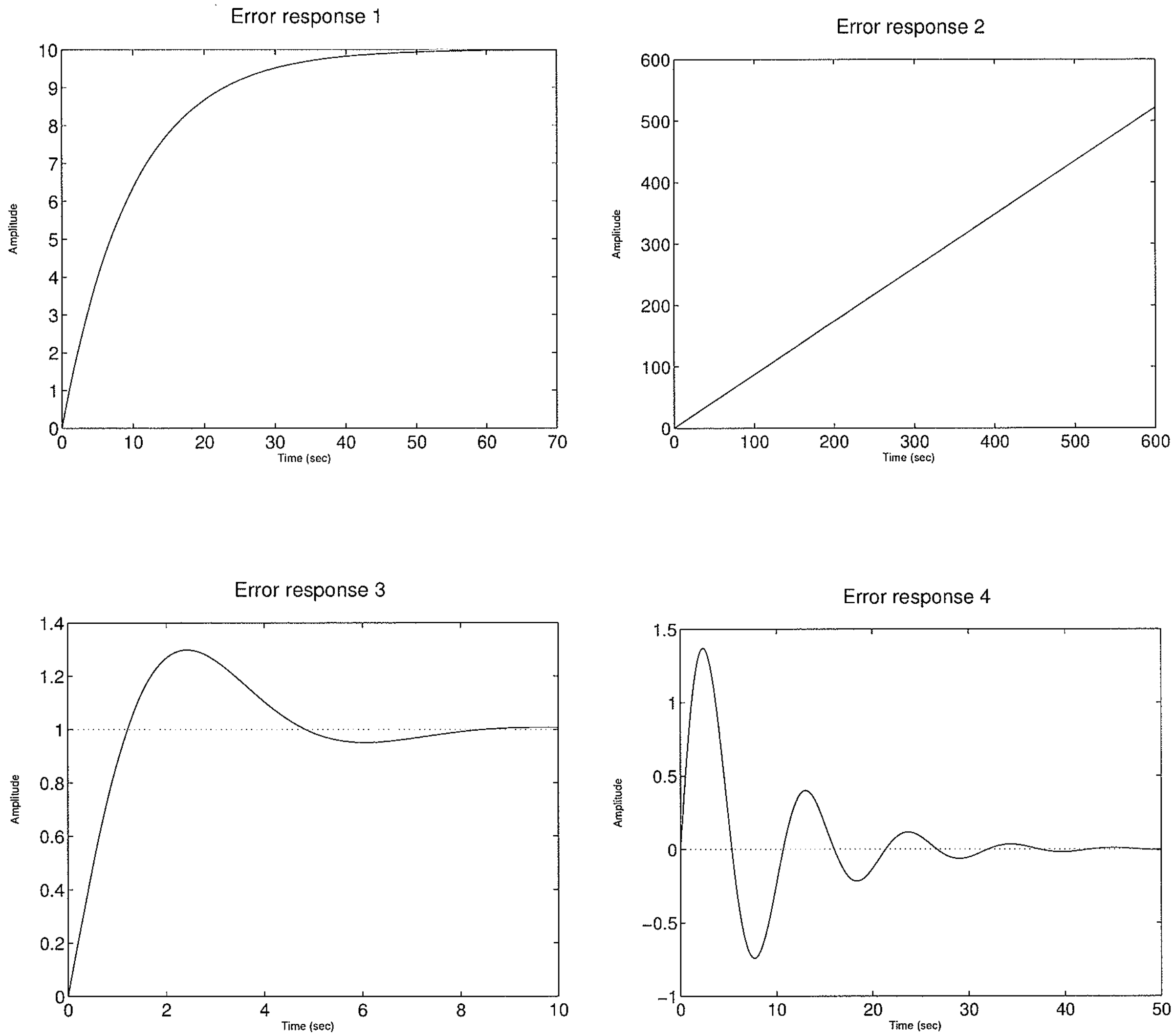


Figure 2: Error responses with ramp input

To differentiate G_3 & G_4 we can check the values of the steady-state error.

$$\frac{1}{\lim_{s\to 0} s \cdot G_3(s)} = \frac{1}{1} = 1 \quad . \text{ It must be response 3}$$

$$\frac{1}{\lim_{s\to 0} s \cdot G_4(s)} = \frac{1}{\frac{1}{10}} = 10 \quad . \text{ It must be response 1.}$$