ECE-382: Midterm #1

February 26, 2007
8:30—9.30PM

- **Do not write your answers at the back of any page. Answers written at the back of page will NOT be graded.**
- This is a closed book exam. You are only allowed to bring a pen (or pencil), a one-sided crib sheet, and an eraser to the exam. No calculator is allowed.
- Common Laplace transform pairs are provided at the end of the exam booklet.
- Write your name and PUID at the space provided below.
- **This exam has two parts.**
  Part I consists of three problems. Unless otherwise instructed, justify your answers to these problems completely. **Show all intermediate steps to get full credits for problems from Part I.**
  Part II consists of one question, with three sub-questions, for which no justification is required. Enter the answers to Part II in the spaces provided. **Partial credit will not be provided for problems from Part II.**
- The total points are 150. You have one hour to complete the exam.

Solution

Your Name

10-digit PUID

Questions for Part I

Justify your answer completely. Show all intermediate steps to get full credits.

Problem 1 (30 points)

Consider the following system

$$U(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

where the transfer function is given by

$$H(s) = \frac{s}{s^2 + s + 1}.$$

(a) (5 points) Assume that the input is $u(t) = e^{-t}1(t)$. What is its Laplace transform $U(s)$?

(b) (5 points) Find the Laplace transform $Y(s)$ of the corresponding output signal.

(c) (10 points) Find the output time-response $y(t)$ when the input is $u(t) = e^{-t}1(t)$. You final answer should not contain imaginary parts.

(d) (10 points) Find the final value of $y(t)$ as $t \to +\infty$.

Solution:

(a) $U(s) = \frac{1}{s+1}$

(b) $Y(s) = H(s)\,U(s) = \frac{s}{(s+1)(s^2+s+1)}$

(c) Let $Y(s) = \frac{a}{s+1} + \frac{bs+c}{s^2+s+1}$

$$\Rightarrow \quad s = a(s^2+s+1) + (bs+c)(s+1)$$
$$= (a+b)s^2 + (a+b+c)s + (a+c)$$

$$\therefore \begin{cases} a+b = 0 \\ a+b+c = 1 \\ a+c = 0 \end{cases}$$

Solve for a, b, and c, we obtain

$$\begin{cases} a = -1 \\ b = 1 \\ c = 1 \end{cases}$$

$$\therefore Y(s) = \frac{-1}{s+1} + \frac{s+1}{s^2+s+1}$$

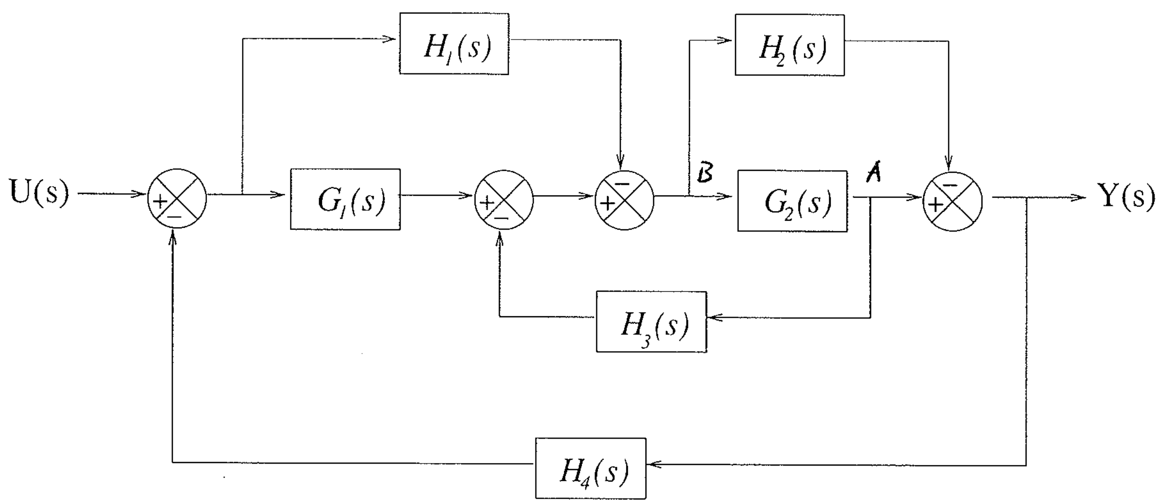
$$= \frac{-1}{s+1} + \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \frac{\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$\therefore y(t) = -e^{-t} + e^{-\frac{t}{2}}\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{-\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t$$

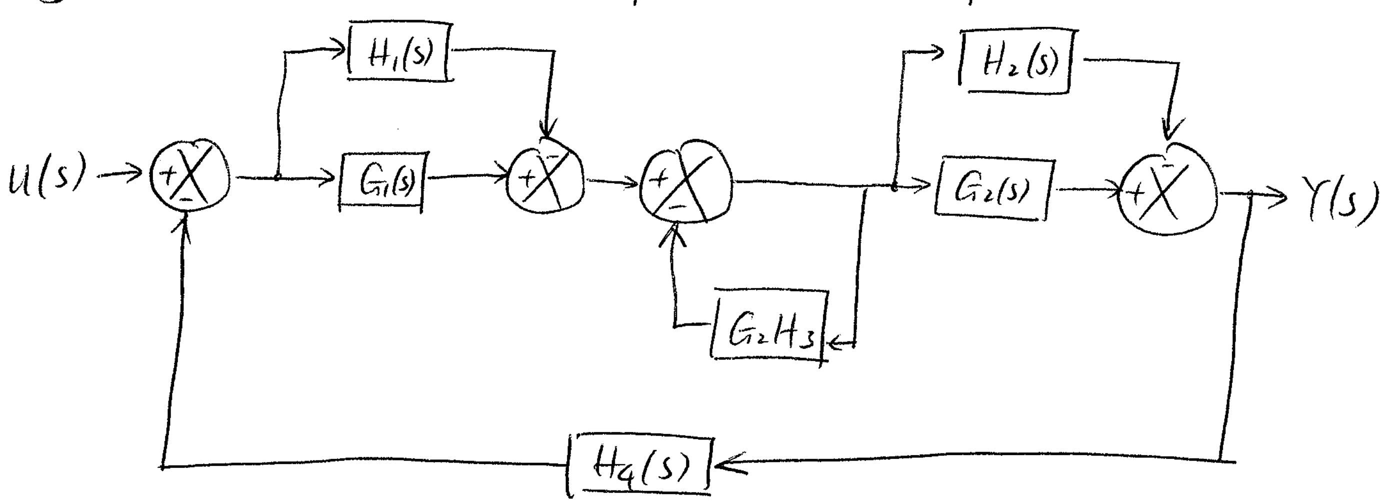
(d) As $t \to +\infty$, $y(t) \to 0$.

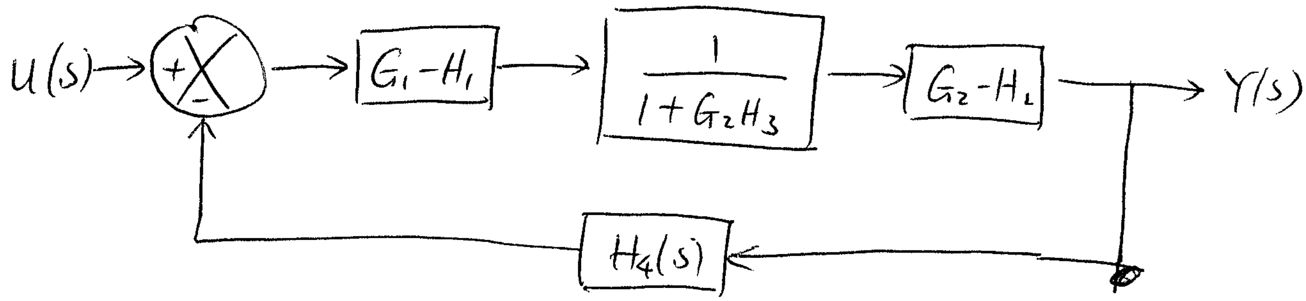
Problem 2 (40 points)

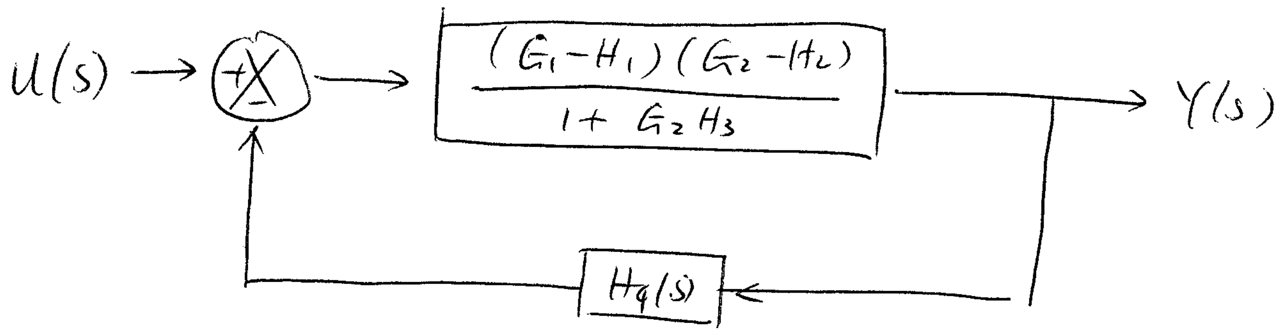
Using block-diagram reduction to simplify the following block diagram and derive the transfer function $\frac{Y(s)}{U(s)}$ from the input $U(s)$ to the output $Y(s)$. Clearly show all intermediate steps. Simplify your final transfer function so that it does NOT contain fraction of fractions. (Note: **you must use the block-diagram reduction method. Using other methods will result into zero credits.**)



Solution: Move the branch point A ahead of $G_2(s)$.





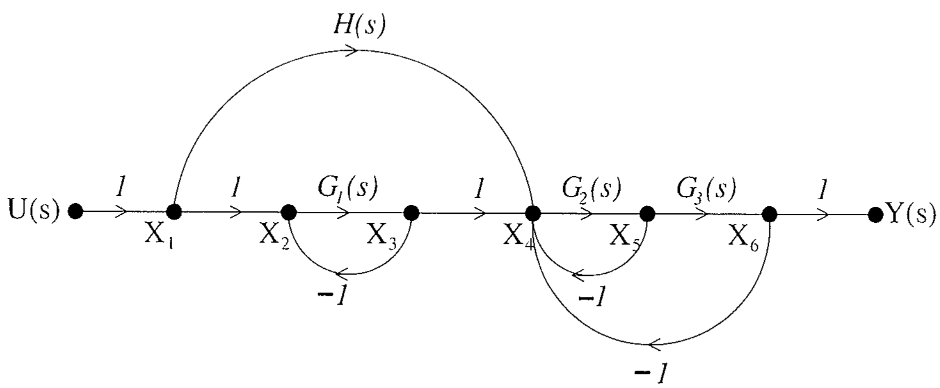


$$\therefore \quad \frac{Y(s)}{U(s)} = \frac{\dfrac{(G_1-H_1)(G_2-H_2)}{1+G_2H_3}}{1+\dfrac{(G_1-H_1)(G_2-H_2)}{1+G_2H_3}\cdot H_4}$$

$$= \frac{(G_1-H_1)(G_2-H_2)}{1+G_2H_3+\cancel{G_4}\,H_4(G_1-H_1)(G_2-H_2)}.$$

Problem 3 (50 points)

Follow the steps below and use Mason's Rule to derive the transfer function $\frac{Y(s)}{U(s)}$ for the following signal-flow-graph.



(a) (10 points) Identify all the loops and write down their individual loop gains.

(b) (10 points) Find the determinant of the graph.

(c) (10 points) Identify all the forward paths and write down their path gains.

(d) (10 points) Find the co-factors for each forward path.

(e) (10 points) Find the transfer function $\frac{Y(s)}{U(s)}$ using Mason's Rule.

Solution:

(a) The loops are

The loops are	Gains
$L1: X_2 X_3 X_2$	$-G_1$
$L2: X_4 X_5 X_4$	$-G_2$
$L3: X_4 X_5 X_6 X_4$	$-G_2G_3$

(b)

Two non-touching Loops	Gains
$L1$ & $L2$	$G_1 G_2$
$L1$ & $L3$	$G_1 G_2 G_3$

$\therefore \quad \Delta = 1 + G_1 + G_2 + G_2G_3 + G_1G_2 + G_1G_2G_3$

(c) Forward Paths | Gains

Forward Paths		Gains
$P1$:	$X_1 X_2 X_3 X_4 X_5 X_6$	$G_1 G_2 G_3$
$P2$:	$X_1 X_4 X_5 X_6$	$G_2 G_3 H$

(d) Cofactor for $P1$: 1

Cofactor for $P2$: $1 + G_1$

(e)
$$\frac{Y(s)}{U(s)} = \frac{G_1 G_2 G_3 + G_2 G_3 H (1 + G_1)}{1 + G_1 + G_2 + G_2 G_3 + G_1 G_2 + G_1 G_2 G_3}$$

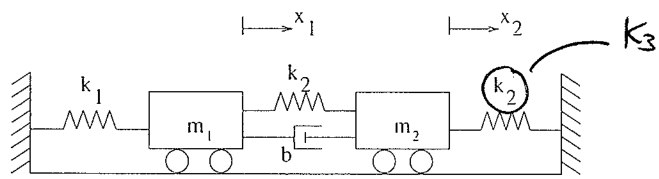
Questions for Part II

Do not justify your answer

Partial credit is NOT available

Problem 4 (30 points)

(a) (10 points) Consider the following mechanical system. Assume that each spring (with constants k_1, k_2 and k_3, respectively) produces a force that is proportional to the amount of extension/compression. Assume that the damper (with constant b) produces a force that is proportional to the velocity that it is extended/compressed. Assume that the springs are relaxed when $x_1 = x_2 = 0$. Use Newton's Second Law to write down the differential equations that characterize the dynamics of the objects m_1 and m_2. **(Note: You do NOT need to solve these equations!)**

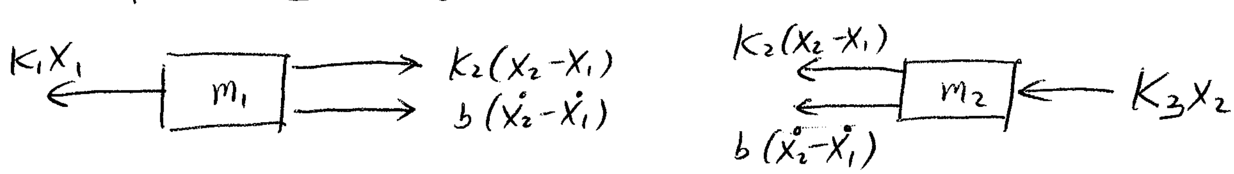


Your answer:

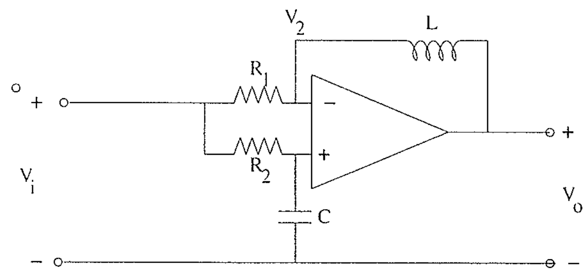
$$K_2(x_2-x_1)+b(\dot{x}_2-\dot{x}_1)-k_1x_1=m_1\ddot{x}_1$$

$$-K_3x_2-K_2(x_2-x_1)-b(\dot{x}_2-\dot{x}_1)=m_2\ddot{x}_2$$

The free-body diagrams:



(b) (10 points) Consider the following electrical system. Assume zero initial conditions for the inductor L and the capacitor C. Write down the two equations that governs the relationship between $V_i(s)$, $V_o(s)$ and $V_2(s)$. The unknowns in your equations should only contain $V_i(s)$, $V_o(s)$ and $V_2(s)$. **(Note: You do NOT need to solve these equations!)**



Your answer:

$$\frac{V_2(s) - V_i(s)}{R_2} + Sc \cdot V_2(s) = 0$$

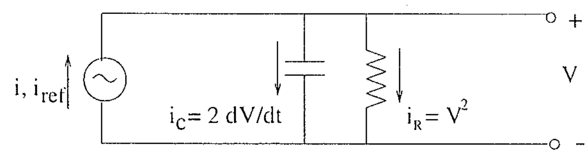
$$\frac{V_2(s) - V_i(s)}{R_1} + \frac{V_2(s) - V_o(s)}{SL} = 0$$

Note that the voltage at both inputs of the ideal op-amp is $V_2(s)$.

(c) (10 points) Consider the following non-linear circuit. Assume that the reference input from the current source i is $i_{\text{ref}} = 1$. The linear approximation of the system is of the following form:

$$(i - 1) = a(V - 1) + b\frac{d}{dt}(V - 1),$$

where a and b are two constants. Find the values of a and b.



Your answer:

$a =$ 2 , $b =$ 2

First write down the equation that govern the dynamics of the circuit:

$$i = 2\frac{dV}{dt} + V^2$$

When the reference input is $i_{ref} = 1$, set $\frac{dV}{dt} = 0$

$$1 = V_{eq}^2 \Rightarrow V_{eq} = 1$$

Note that the only non-lineal term is $f(V) = V^2$, then

$$f(V) \approx V_{eq}^2 + \left.\frac{\partial f}{\partial V}\right|_{V=V_{eq}} (V - V_{eq})$$

$$\left.\frac{\partial f}{\partial V}\right|_{V=V_{eq}} = \left.2V\right|_{V=1} = 2$$

$\therefore$ The linear approximation is

$$i = 2\frac{dV}{dt} + 1 + 2(V-1)$$

$$\Rightarrow (i-1) = 2\frac{d}{dt}(V-1) + 2(V-1).$$