

B-8-26

$$|G(j\omega)| = \frac{\sqrt{\alpha^2 \omega^2 + 1}}{\omega^2} \quad \angle G(j\omega) = \tan^{-1} \alpha \omega - 180^\circ$$

The phase margin of 45° at $\omega = \omega_1$ requires that

$$\frac{\sqrt{\alpha^2 \omega_1^2 + 1}}{\omega_1^2} = 1$$

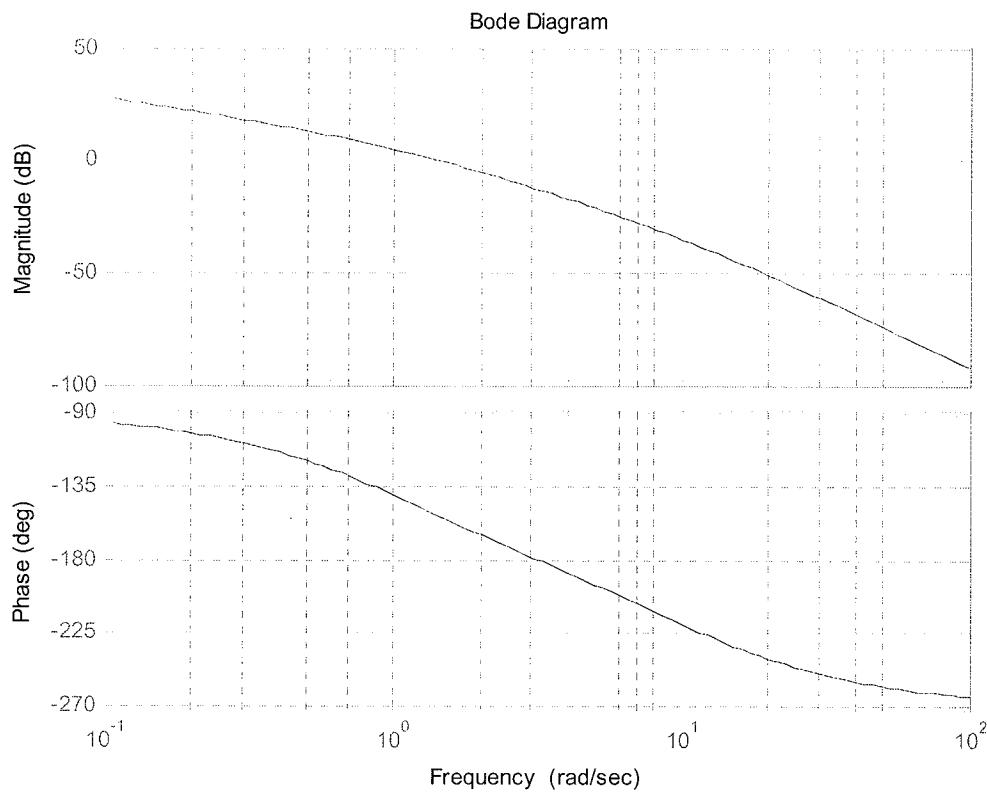
$$\tan^{-1} \alpha \omega_1 - 180^\circ = 45^\circ - 180^\circ$$

Thus, we have $\alpha^2 \omega_1^2 + 1 = \omega_1^4$, $\alpha \omega_1 = 1$

Solving for α , we obtain

$$\alpha = \left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} = 0.841$$

B-8-27 A Bode diagram of the system is shown below.



From this Bode diagram, we find the phase margin and gain margin to be 27° and 13 dB, respectively.

The phase margin, gain margin, phase crossover frequency, and gain crossover frequency can be obtained easily with MATLAB. Use the command,

$[Gm, pm, wcp, wcg] = \text{margin}(\text{sys});$

$GmdB = 20 * \log10(Gm);$

$[GmdB, pm, wcp, wcg]$

ans =

12.8691 26.9973 3.1623 1.4230

B-8-28

$$G(s) = \frac{2s(s+1)}{s(s^2 + 2s + 10)(s+5)}$$

The phase margin, gain margin, phase crossover frequency, and gain crossover frequency are obtained by use of the command

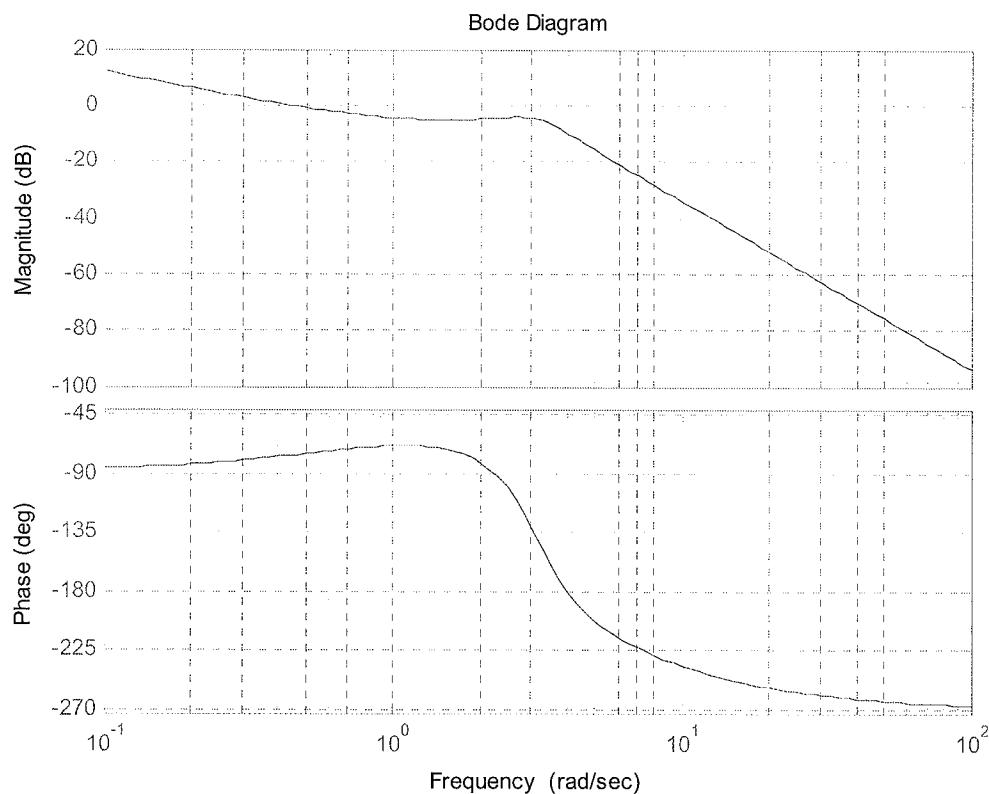
[Gm, pm, wcp, wcg] = margin(sys);

A MATLAB program to solve this problem is given below. The Bode diagram shown below verifies the phase margin, gain margin, phase crossover frequency, and gain crossover frequency obtained with MATLAB.

```
num=[0 0 20 20];
den=conv([1 2 10 0],[1 5]);
sys=tf(num,den);
w=logspace(-1,2,100);
bode(sys,w);
grid;
[Gm,pm,wcp,wcg]=margin(sys);
GmdB=20*log10(Gm);
[Gm,pm,wcp,wcg]
```

ans =

9.9301 103.6573 4.0132 0.4426



B-8-30 Note that

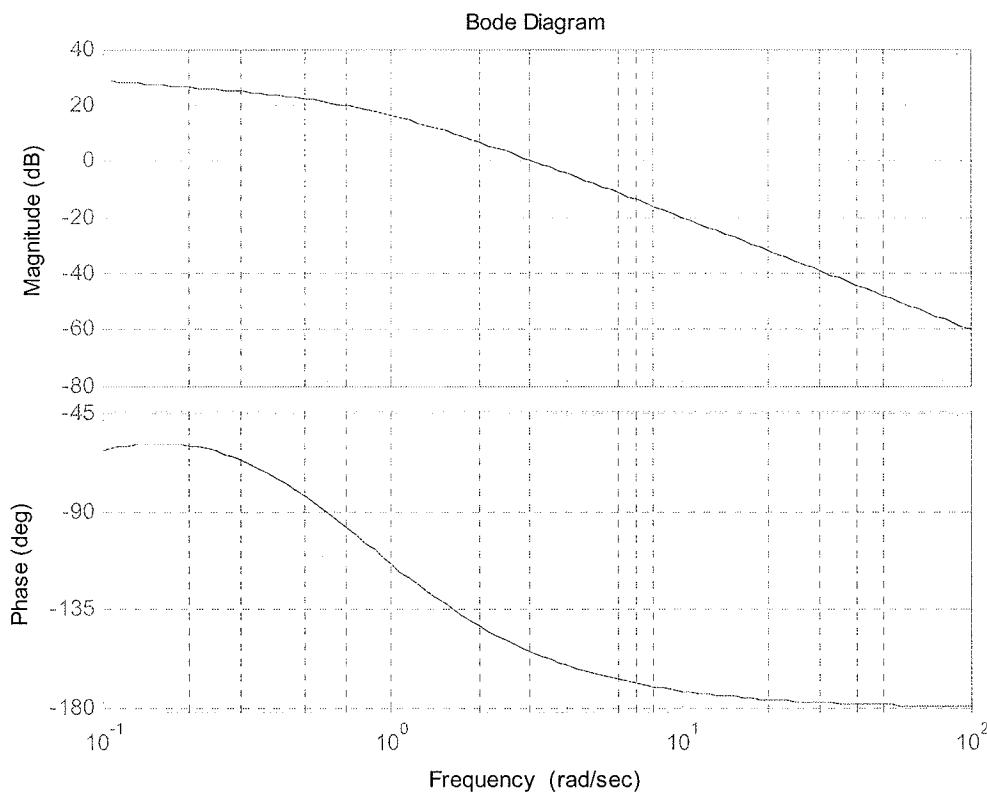
$$K \frac{j\omega+0.1}{j\omega+0.5} \frac{10}{j\omega(j\omega+1)} = \frac{2K(10j\omega+1)}{j\omega(2j\omega+1)(j\omega+1)}$$

We shall plot the Bode diagram when $2K=1$. That is, we plot the Bode diagram of

$$G(j\omega) = \frac{10j\omega+1}{j\omega(2j\omega+1)(j\omega+1)}$$

The diagram is shown below. The phase curve shows that the phase angle is -130° at $\omega=1.438$ rad/sec. Since we require the phase margin to be 50° , the magnitude of $G(1.438)$ must be equal to 1 or 0 dB. Since the Bode diagram indicates that $G(1.438)$ is 5.48 dB, we need to choose $2K=-5.48$ dB, or

$$K=0.266$$



Since the phase curve lies above the -180° line for all ω , the gain margin is $+\infty$ dB.