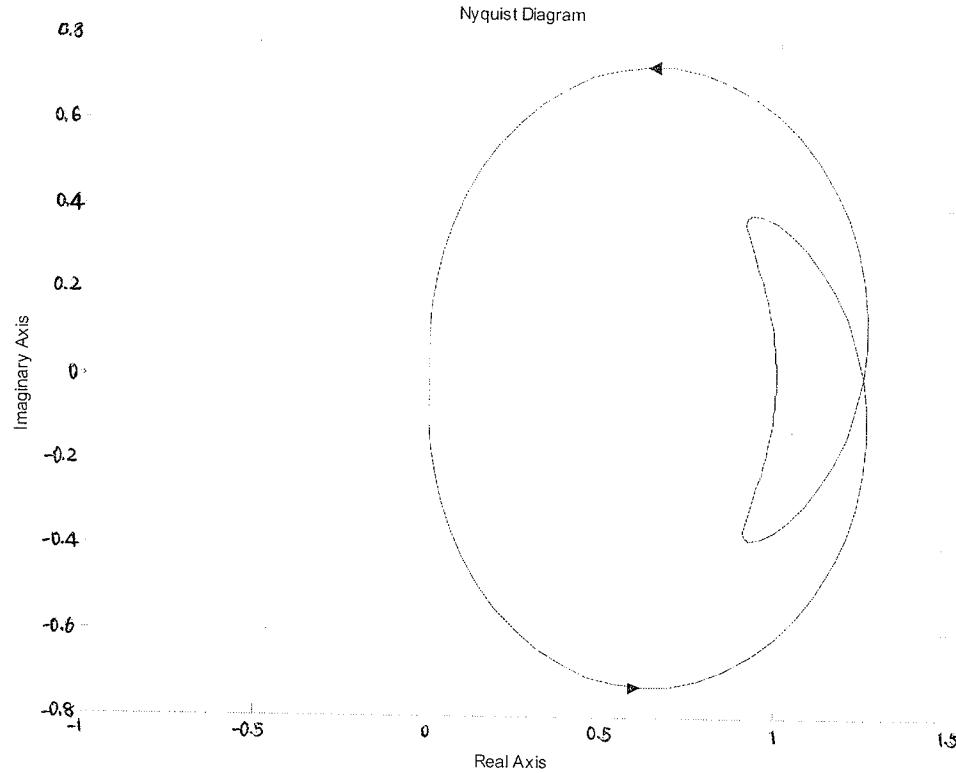


$$B-8-15 \quad G(s) = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

$G(s)$ has 2 open-loop poles in the right-half s plane

The following MATLAB program produces the Nyquist plot shown below.

```
num=[0 0 0 1];
den=[1 0.2 1 1];
nyquist(num, den)
grid
```



From the plot notice that the critical point $(-1+j0)$ is not encircled.

$$N = Z - P$$

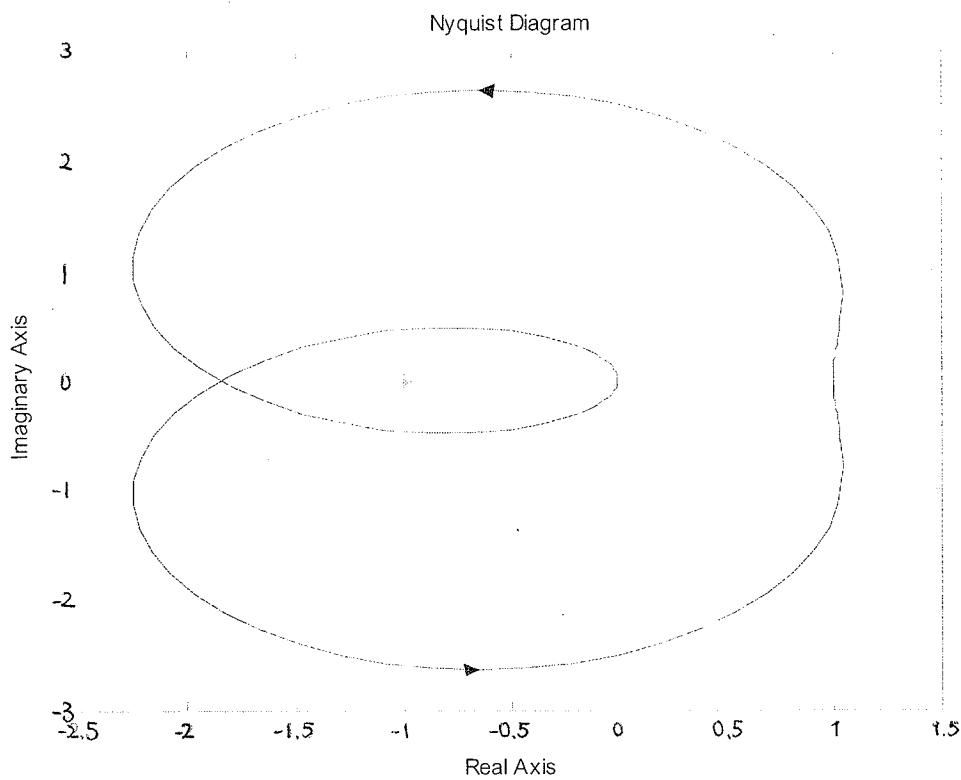
$$0 = Z - 2 \Rightarrow Z = 2$$

The system is unstable.

$$B-8-16 \quad G(s) = \frac{s^2 + 2s + 1}{s^3 + 0.2s^2 + s + 1}$$

The following MATLAB program produces the Nyquist plot shown below.

```
num=[0 1 2 1];
den=[1 0.2 1 1];
nyquist(num,den)
grid
```



Since $G(s)$ has 2 open-loop poles in the right-half s plane and the Nyquist plot encircles the critical point $(-1+0j)$ twice counter-clockwise,

$$N = Z - P$$

$$2 = Z - 2 \Rightarrow Z = 0$$

The system is stable.

B-8-17 The open-loop transfer function is

$$G(s) = \frac{1}{s(s-1)}$$

The points corresponding to $s=j\omega$ and $s=-j\omega$ on the locus of $G(s)$ in the $G(s)$ plane are $j\omega$ and $-j\omega$, respectively. On the semicircular path with radius Σ (where $\Sigma \ll 1$), the complex variable s can be written as

$$s = \Sigma e^{j\theta}$$

where θ varies from -90° to $+90^\circ$. Then $G(s)$ becomes

$$G(\Sigma e^{j\theta}) = -\frac{1}{\Sigma e^{j\theta}} = \frac{1}{\Sigma} e^{-j(\theta+180^\circ)}$$

The value K approaches infinity as Σ approaches zero, and $-\theta$ varies from -90° to -270° as a representative point s moves along the semicircle in the S plane. Thus the points $G(j\omega) = -j\omega$ and $G(-j\omega) = +j\omega$ are joined by a semicircle of infinite radius in the left-half G -plane. The infinitesimal semicircular detour around the origin in the S plane maps into the G plane as a semicircle of infinite radius. Figure(a) shows the $G(s)$ locus in the G plane.

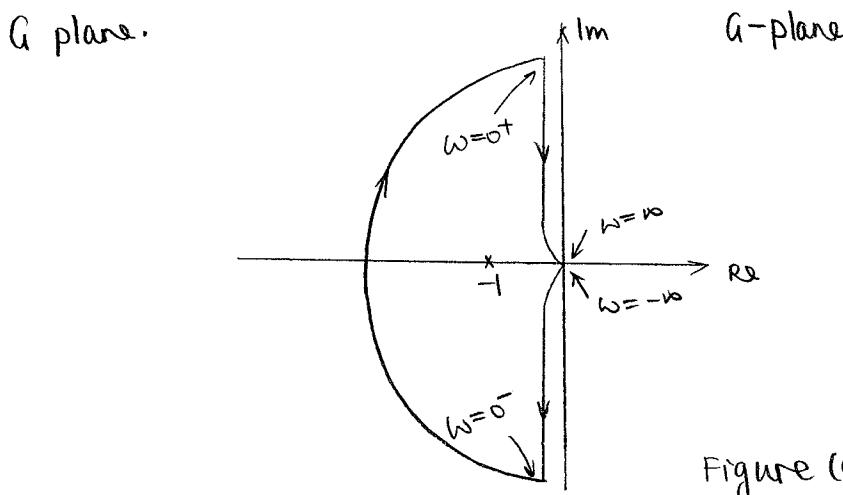


Figure (a)

Since $G(s)$ has one pole in the right-half s plane ($P=1$) and $G(s)$ locus encircles the $-1+j0$ point once clockwise ($N=1$), we have

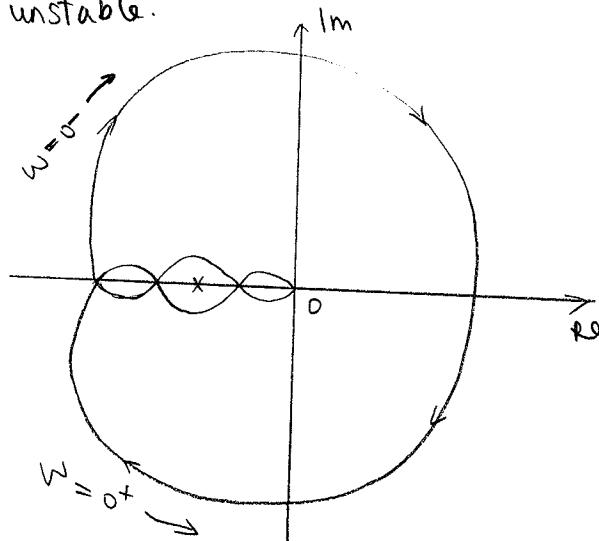
$$Z = N + P = 2$$

There are two zeros of $1+G(s)$ in the right-half s plane. Therefore, the system is unstable.

B-8-19 Consider the case where $G(s)$ has one pole in the right-half s plane. From the Nyquist plot of $G(j\omega)$ shown below, the $-1+j0$ point is encircled by the $G(j\omega)$ locus once clockwise and once counterclockwise. Hence $N=0$. Since $G(s)$ has one pole in the right-half s plane, we have $P=1$.

$$\text{Since } Z = N + P = 0 + 1 = 1$$

The system is unstable.



Next, consider the case where $G(s)$ has no pole in the right-half s plane, but has one zero in the right-half s -plane. The $-1+j0$ point is encircled by the $G(j\omega)$ locus once clockwise and once counterclockwise. Hence, $N=0$. Since $G(s)$ has no poles in the right-half s plane, we have $P=0$. Therefore,

$$Z = N + P = 0 + 0 = 0$$

The system is stable. (Note that the presence of a zero of $G(s)$ in the right-half s plane does not affect the stability of the system.)

Problem 2 (a) From the given figure, $|H(j\omega)| \neq \infty$. Hence there are no poles with zero real part. The # of unstable poles $p=1$. Also, the number of clockwise encirclements of 0 is 0, i.e., $N = z - p = 0$ for $H(j\omega)$

Therefore $z=1$.

Now, let N denote the # of clockwise encirclements of $-\frac{1}{K}$, by the Nyquist plot of $H(s)$; let p denote # of RHP open-loop poles, and z the # of RHP closed-loop poles.

(b) If $K=0.1$, $-\frac{1}{K}=-10$ and we have $N=0$

$$\therefore z = N+p = 0+1 = 1$$

(c) If $K=1$, $-\frac{1}{K}=-1$ and $N=1$

$$\therefore z = N+p = 2$$

(d) If $K=10$, $-\frac{1}{K}=-0.1$ and $N=0$

$$\therefore z = N+p = 1$$

(e) For stability, $z=0 \Rightarrow N=1$, i.e., one counterclockwise encirclement of $-\frac{1}{K}$.

As seen from the figure,

we need $-0.7 < -\frac{1}{K} < -0.6$

$$\therefore \frac{1}{0.7} < K < \frac{1}{0.6} \quad \text{i.e., } K \in (1.429, 1.667)$$

is the range of stability.