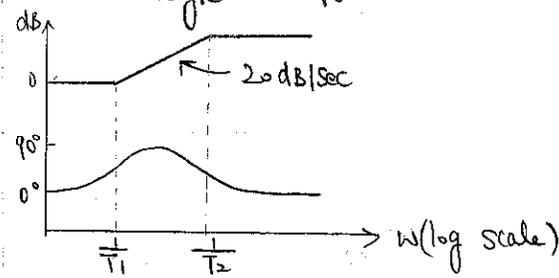


ECE 382 HW 12 Solutions

B-8-4

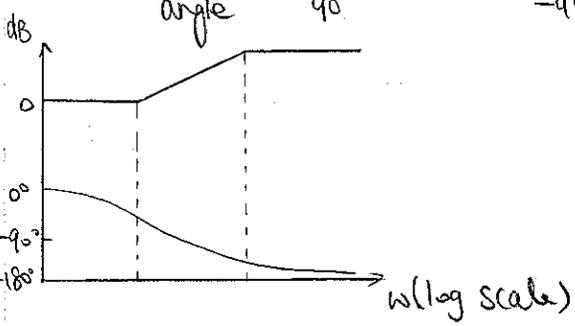
(a) $G(s) = \frac{T_1s+1}{T_2s+1}$

	$j\omega T_1+1$	$\frac{1}{j\omega T_2+1}$	Total
$\omega < \frac{1}{T_1}$	slope 0	0	0
	angle 180°	0°	180°
$\frac{1}{T_1} < \omega < \frac{1}{T_2}$	slope 20dB/dec	0	20dB/dec
	angle 90°	0°	90°
$\omega > \frac{1}{T_2}$	slope 20dB/dec	-20dB/dec	0
	angle 90°	-90°	0°



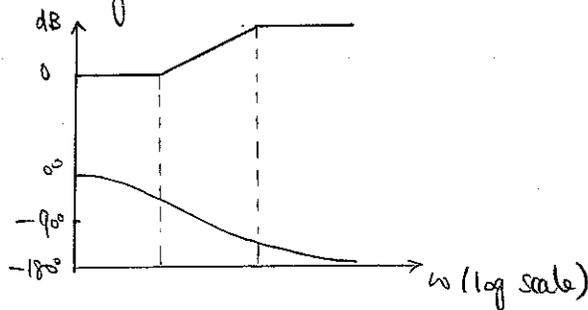
(b) $G(s) = \frac{T_1s-1}{T_2s+1}$

	$j\omega T_1-1$	$\frac{1}{j\omega T_2+1}$	Total
$\omega < \frac{1}{T_1}$	slope 0	0	0
	angle 180°	0°	180°
$\frac{1}{T_1} < \omega < \frac{1}{T_2}$	slope 20dB/dec	0	20dB/dec
	angle 90°	0°	90°
$\omega > \frac{1}{T_2}$	slope 20dB/dec	-20dB	0
	angle 90°	-90°	0°



(c) $G(s) = \frac{-T_1s+1}{T_2s+1}$

	$-j\omega T_1+1$	$\frac{1}{j\omega T_2+1}$	Total
$\omega < \frac{1}{T_1}$ slope	0	0	0
angle	0°	0°	0°
$\frac{1}{T_1} < \omega < \frac{1}{T_2}$ slope	20dB/dec	0	20dB/dec
angle	-90°	0°	-90°
$\omega > \frac{1}{T_2}$ slope	20dB/dec	-20dB/dec	0
angle	-90°	-90°	-180°



B-8-5

$$G(s) = \frac{10(s^2 + 0.4s + 1)}{s(s^2 + 0.8s + 9)}$$

$$G(j\omega) = \frac{10((j\omega)^2 + 0.4(j\omega) + 1)}{j\omega \left(\left(\frac{j\omega}{3}\right)^2 + 2 \cdot \frac{0.4}{3} \cdot \frac{j\omega}{3} + 1 \right)}$$

The function consists of the following factors

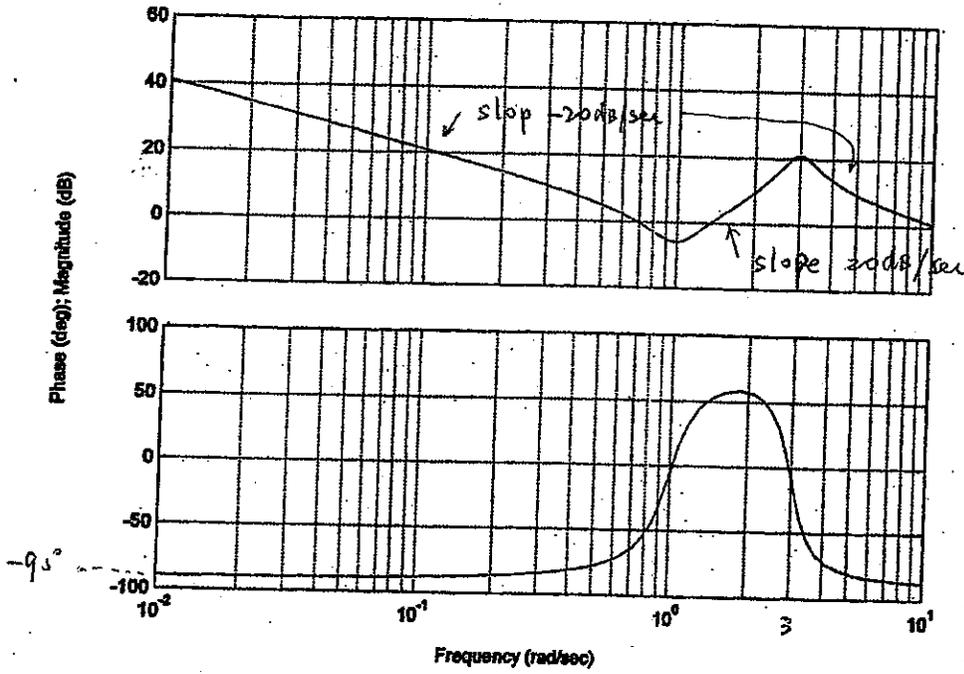
$$\frac{10}{9}, (j\omega)^{-1}, (j\omega)^2 + 0.4j\omega + 1, \left[\left(\frac{j\omega}{3}\right)^2 + 2 \cdot \frac{0.4}{3} \cdot \frac{j\omega}{3} + 1 \right]^{-1}$$

corner freq					Total	
$\omega < 1$	slope	0	-20dB/dec	0	0	-20dB/dec
	angle	0°	-90°	0°	0°	-90°
$1 < \omega < 3$	slope	0	-20dB/dec	40dB/dec	0	20dB/dec
	angle	0°	-90°	$< 180^\circ$ *	0°	$< 90^\circ$
$\omega > 3$	slope	0	-20dB/dec	40dB/dec	-40dB/dec	-20dB/dec
	angle	0°	-90°	180°	-180°	-90°

* Since $\omega_1=1$ and $\omega_2=3$ are close to each other, the angle can't reach 180° when $1 < \omega < 3$.

The Bode diagram is illustrated below:

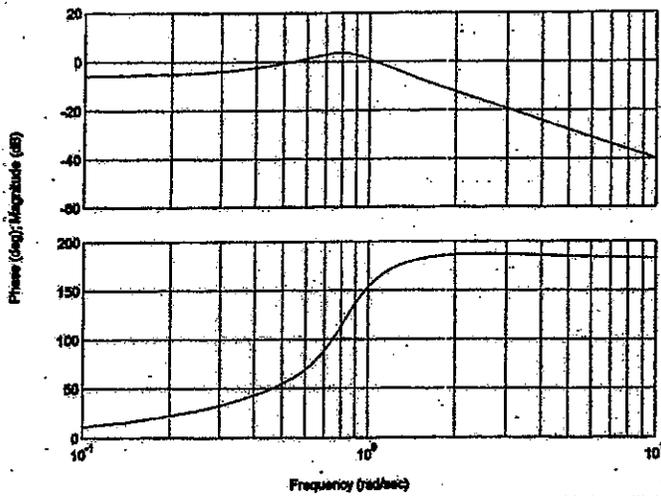
Bode Diagram of $G(s) = 10(s^2 + 0.4s + 1) / [s(s^2 + 0.8s + 8)]$



B-8-7 $G(s) = \frac{s+0.5}{s^3+s^2+1}$

```
% ***** Bode Diagram *****
num = [0 0 1 0.5];
den = [1 1 0 1];
bode(num,den)
title(Bode Diagram of G(s) = (s+0.5)/(s^3+s^2+1))
```

Bode Diagram of $G(s) = (s+0.5)/(s^3+s^2+1)$



To verify why the phase angle starts from 0° and ends at 180° , we may compute angles $\angle G(j\omega)$ and $\angle G(j\infty)$. Since

$$G(s) = \frac{s+0.5}{(s+1.4656)(s-0.2328-j0.7926)(s-0.2328+j0.7926)}$$

We have

$$\begin{aligned} \angle G(j\omega) &= \angle 0.5 - \angle 1.4656 - \angle -0.2328 - j0.7926 - \angle -0.2328 + j0.7926 \\ &= 0^\circ - 0^\circ - \tan^{-1} \frac{0.7926}{0.2328} + \tan^{-1} \frac{0.7926}{0.2328} = 0^\circ \end{aligned}$$

and

$$\begin{aligned} \angle G(j\infty) &= 90^\circ - 90^\circ - \tan^{-1} \frac{\infty}{-0.2328} - \tan^{-1} \frac{\infty}{-0.2328} \\ &= 90^\circ - 90^\circ + 90^\circ + 90^\circ = 180^\circ \end{aligned}$$

B-8-8 $G(s)H(s) = \frac{K(Ta s + 1)(Tb s + 1)}{s^2(Ts + 1)}$

As $s = j\omega$, $G(j\omega)H(j\omega) = \frac{K(Ta\omega j + 1)(Tb\omega j + 1)}{(j\omega)^2(T\omega j + 1)}$

If $\omega \rightarrow 0$ $G(j\omega)H(j\omega) \rightarrow \frac{K}{-\omega^2} = \frac{K}{\omega^2} \angle 180^\circ$

If $\omega \rightarrow \infty$ $G(j\omega)H(j\omega) \rightarrow \frac{-KTaTb\omega^2}{-T\omega^3 j} = \frac{KTaTb}{\omega T} \angle -90^\circ$

Hence, both curves start from around the negative axis, and approach the origin from the negative $j\omega$ axis.

(a) $Ta > T > 0, Tb > T > 0$

Assume $Ta = Tb \gg T$

As $\omega \approx \frac{1}{Ta}$, $G(j\omega)H(j\omega) = \frac{K(Hj)^2}{-\frac{1}{Ta^2}(\frac{T}{Ta}j + 1)} \approx \frac{K \cdot 2j}{-\frac{1}{Ta^2}} = 2KTa^2 \angle 90^\circ$

As $\omega \approx \frac{1}{T}$, $G(j\omega)H(j\omega) = \frac{K(H + \frac{Ta}{T})^2}{-\frac{1}{T^2}(Hj)} \approx \frac{-K(\frac{Ta}{T})^2}{-\frac{1}{T^2}(Hj)} = \frac{KTa^2}{\sqrt{2}} \angle -45^\circ$

Hence, the Nyquist diagram enters the fourth quadrant.

(b) $T > Ta > 0, T > Tb > 0$

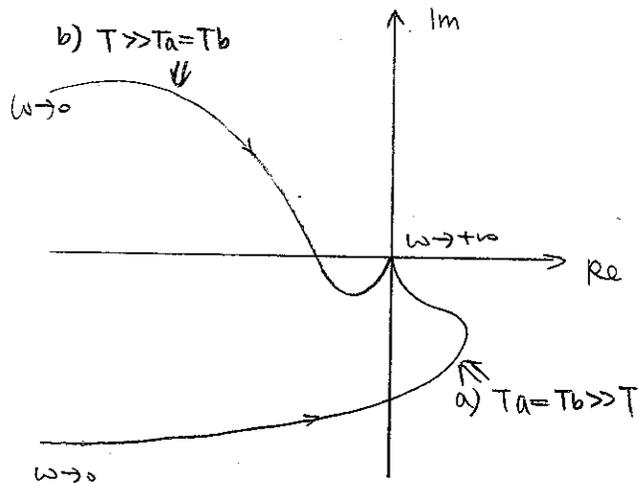
Assume $Ta = Tb < T$.

$$\text{As } \omega \approx \frac{1}{T}, G(j\omega)H(j\omega) = \frac{K(1 + \frac{T_a}{T}j)^2}{-\frac{1}{T}(1+j)} = \frac{KT^2}{\sqrt{2}} \angle 135^\circ$$

$$\text{As } \omega \approx \frac{1}{T_a}, G(j\omega)H(j\omega) = \frac{K(1+j)^2}{-\frac{1}{T_a}(\frac{T}{T_a}j+1)} = \frac{2KT_a^3}{T} \angle 180^\circ$$

Hence, the Nyquist diagram goes from the second quadrant to the third quadrant.

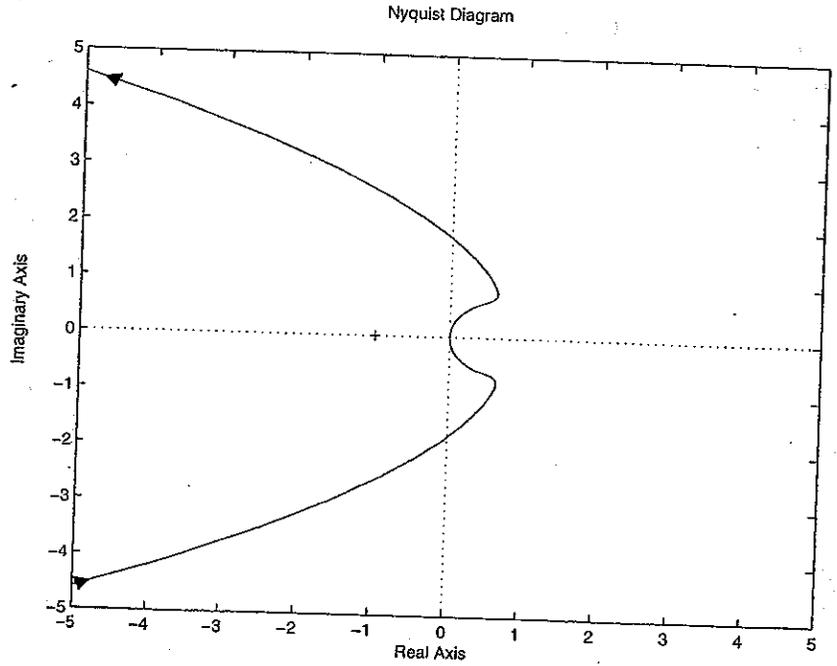
Typical Nyquist plots.



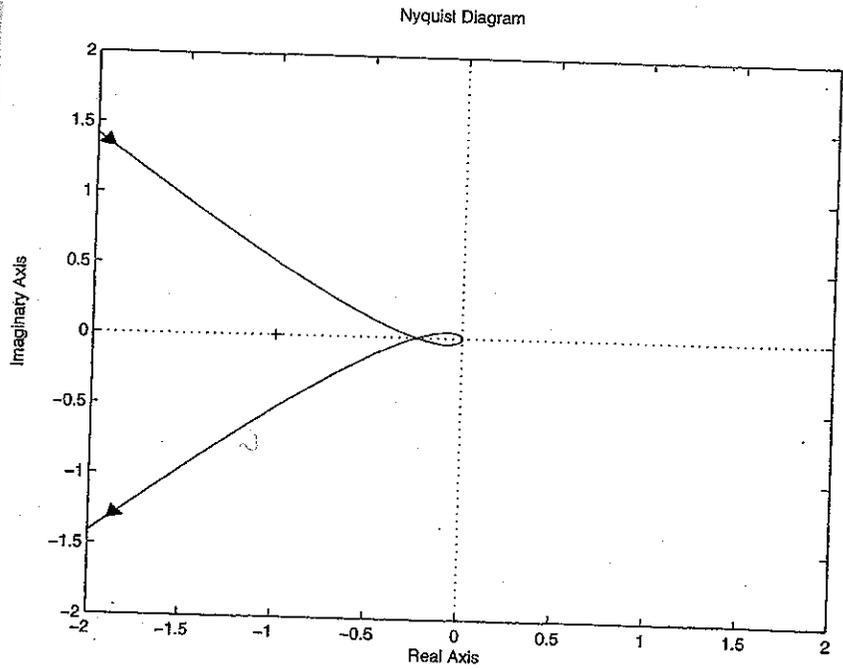
Note: It is possible that you take other combinations of T_a, T_b & T , and get slightly different curves.

Nyquist plots by matlab

a) $T_a = T_b = 1$
 $T = 0.1$



b) $T_a = T_b = 1$
 $T = 1$



B-8-14 $G(s) = \frac{1}{s(s^2 + 0.8s + 1)}$

$$G(j\omega) = \frac{1}{j\omega(0.8\omega j + 1 - \omega^2)}$$

As $\omega \rightarrow 0$ $G(j\omega) = \frac{1}{\omega} \angle -90^\circ$

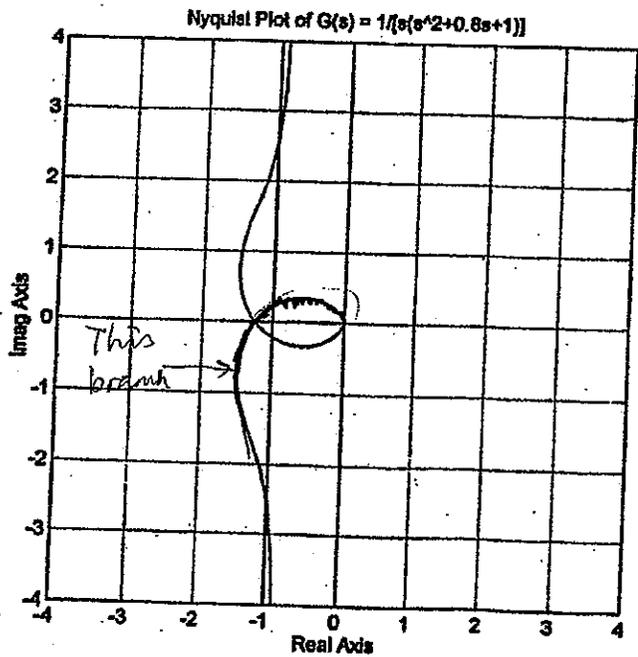
As $\omega \rightarrow \infty$ $G(j\omega) = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} \angle 90^\circ$

As $\omega \rightarrow 1$ $G(j\omega) = \frac{1}{0.8} \angle 180^\circ$

We can then sketch the Nyquist plot, which is consistent with what we get from Matlab.

The following MATLAB program will produce the Nyquist plot shown below.

```
% ***** Nyquist plot *****
num = [0 0 0 1];
den = [1 0.8 1 0];
nyquist(num,den)
v = [-4 4 -4 4]; axis(v); axis('square')
grid
title('Nyquist Plot of G(s) = 1/[s(s^2+0.8s+1)]')
```



Problem 3.

(a) The answer is no, since the phase angle of $G(s)$ evaluated at $pd = -1 + j\sqrt{3}$ is

$$\angle G(pd) = -\angle pd - \angle (pd+2) - \angle (pd+4) = -120^\circ - 60^\circ - 30^\circ = -210^\circ,$$

which is not of the form $180^\circ + k \cdot 360^\circ$. Thus pd is not on the root locus of $G(s)$, or equivalently, one can not achieve the pole at pd by merely using a proportional compensator K .

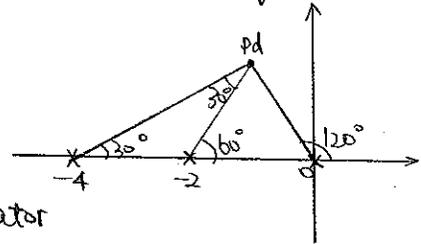


Fig 1

(b) From the answer to part (a), the lead compensator needs to contribute the deficiency angle

$$\phi = -180^\circ - (-210^\circ) = 30^\circ, \text{ i.e., } \angle (pd+2) - \angle (pd+\alpha) = 30^\circ.$$

From the geometric relation shown in Fig 1, we conclude that $\alpha = 4$.

The corresponding constant K can be obtained from the magnitude condition

$$|C(pd)G(pd)| = \left| \frac{K}{pd(pd+4)^2} \right| = 1 \Rightarrow K = |pd| |pd+4|^2 = 2 \cdot (2\sqrt{3})^2 = 24.$$

(c) Since $k_v = \lim_{s \rightarrow 0} sC(s)G(s) = 24/4^2 = 3/2$, the steady-state velocity error is

$$e_v = \frac{1}{k_v} = \frac{2}{3}.$$

(d) Since pd is the same as in part (c), the lead compensator designed previously can serve as the lead part of the lag-lead compensator. For the lag part, we need to increase k_v from $3/2$ to $1/0.01 = 100$ by a factor of $100/(3/2) = 200/3$. Thus β in the lag compensator should be $\beta = 200/3$. T_2 can be chosen to be a large number, say, $T_2 = 100$. Thus the lag-lead compensator is

$$C(s) = 24 \cdot \frac{s+2}{s+4} \cdot \frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}} = \frac{24(s+2)(s+0.01)}{(s+4)(s+0.00015)}$$

(e)
$$C(s)G(s) = \frac{24(s+0.01)}{s(s+4)^2(s+0.00015)}$$