

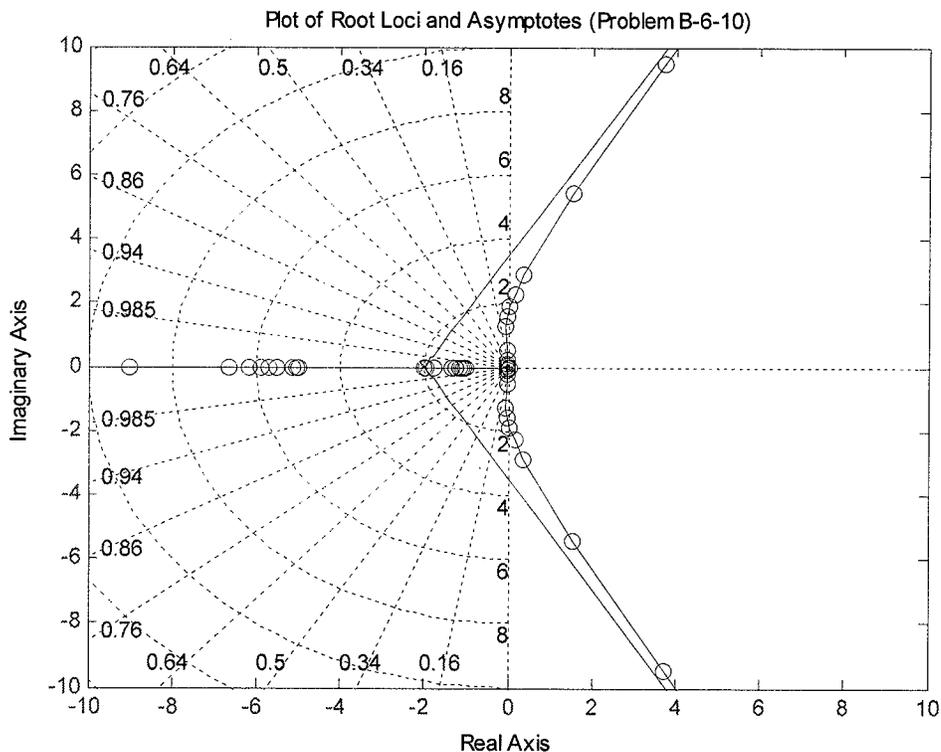
# ECE382 HW10 Solution

## B-6-10

MATLAB code to obtain the root locus:

```
num=[0 0 0 2 2];  
den=[1 7 10 0 0];  
numa=[0 0 0 1];  
dena=[0.5 3 6 4];  
r=rlocus(num,den);  
plot(r,'-');  
hold on  
plot(r,'o');  
rlocus(numa,dena);  
v=[-10 10 -10 10];  
axis(v);  
axis('square')  
grid;  
title('Plot of Root Loci and Asymptotes (Problem B-6-10)')
```

The resulting root-locus plot is shown below:



A root locus plot near the origin can be obtained by entering the following MATLAB program:

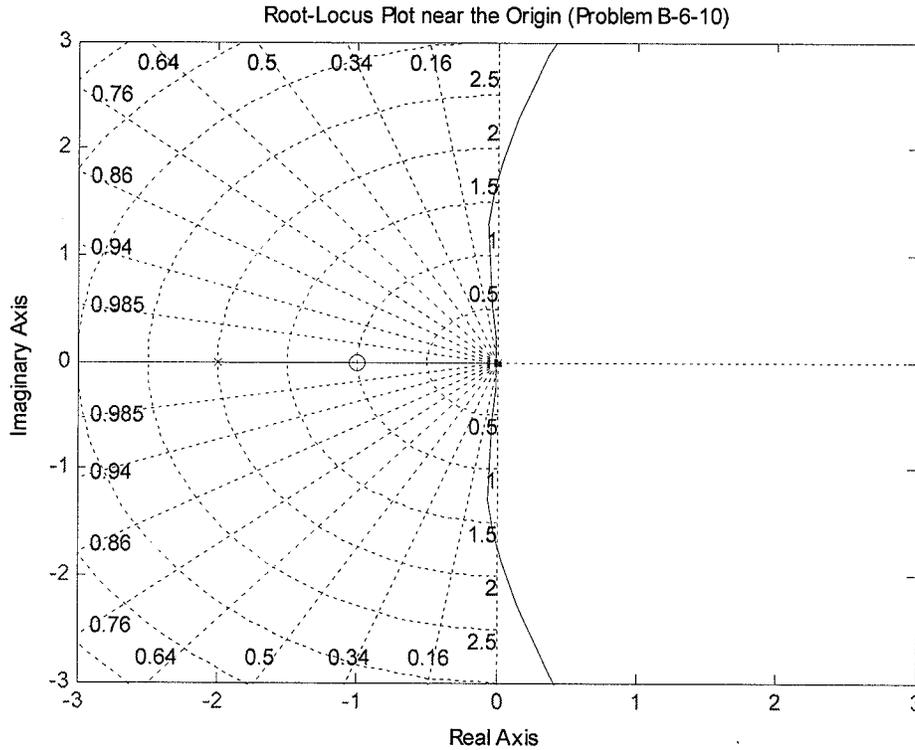
```
num=[0 0 0 2 2];  
den=[1 7 10 0 0];  
  
rlocus(num,den);
```

```

v=[-3 3 -3 3];
axis(v);
axis('square')
grid;
title('Root-Locus Plot near the Origin (Problem B-6-10)')

```

The resulting root-locus near the origin:



The range of  $K$  for stability can be determined by use of Routh stability criterion. Since the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{2K(s+1)}{s^4 + 7s^3 + 10s^2 + 2Ks + 2K}$$

The characteristic equation for the system is

$$s^4 + 7s^3 + 10s^2 + 2Ks + 2K = 0$$

Routh array:

$s^4$	1	10	2K	
$s^3$	7	2K		
$s^2$	$\frac{70-2K}{7}$	2K		
$s^1$	$\frac{(70-2K)2K}{7}$	-14K		0
	$\frac{70-2K}{7}$			
$s^0$	2K			

For stability, we require

$$\begin{aligned}
70 &> 2K \\
42 - 4K &> 0 \\
K &> 0
\end{aligned}$$

Thus, the range of  $K$  for stability is  $K \in (0, 10.5)$

B-6-13 The open-loop transfer function is given by

$$G(s)H(s) = \frac{K(s-0.6667)}{s^4+3.3401s^3+7.0325s^2} = \frac{K(s-0.6667)}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)}$$

Where  $p_i$ ,  $i=1, 2, 3, 4$  are poles and satisfy  $\sum_{i=1}^4 p_i = -3.3401$

Therefore the intersection point of the asymptotes is

$$\sigma_a = \frac{\sum p_i - \sum z_i}{n-m} = \frac{-3.3401 - 0.6667}{4-1} = -1.3356$$

And the equation for the asymptotes may be obtained as

$$G_a(s)H_a(s) = \frac{K}{(s-\sigma_a)^3} = \frac{K}{(s+1.3356)^3}$$

which also satisfies the angle condition that  $\phi_a = \frac{1}{3}(180^\circ + 360^\circ k)$   $k \in \mathbb{Z}$

Therefore

$$G_a(s)H_a(s) = \frac{K}{s^3+4.0068s^2+5.3525s+2.3825}$$

Hence, we enter the following numerators and denominators in the program.

For the system,

$$\text{num} = [0 \ 0 \ 0 \ 1 \ -0.6667]$$

$$\text{den} = [1 \ 3.3401 \ 7.0325 \ 0 \ 0]$$

For the asymptotes,

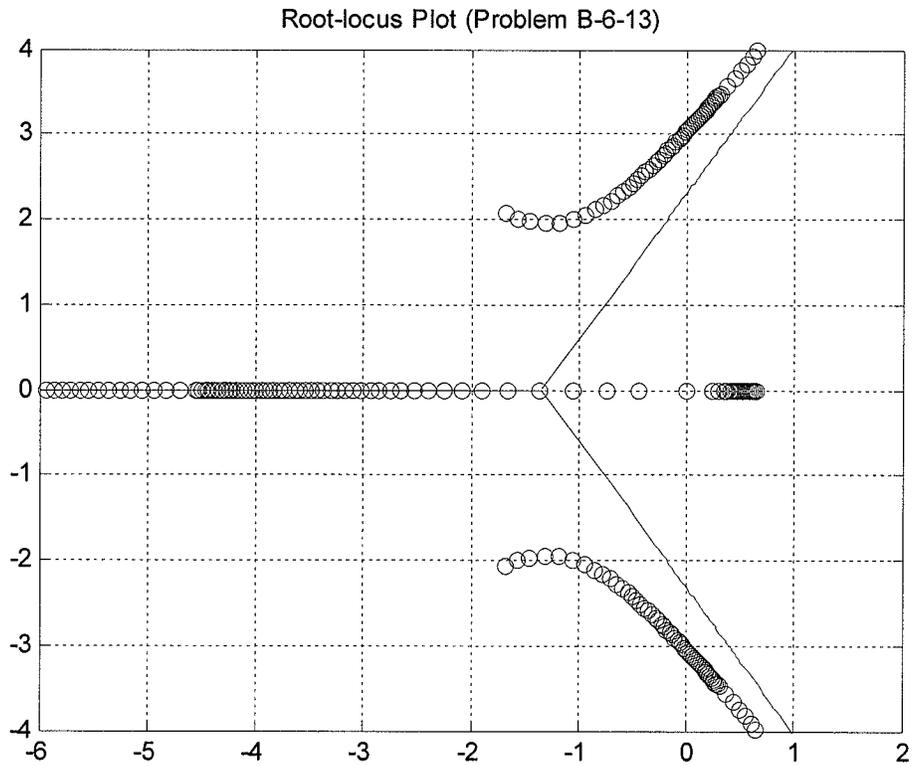
$$\text{num}_a = [0 \ 0 \ 0 \ 1]$$

$$\text{den}_a = [1 \ 4.0068 \ 5.3515 \ 2.3825]$$

```

num=[0 0 0 1 -0.6667];
den=[1 3.3401 7.0325 0 0];
numa=[0 0 0 1];
dena=[1 4.0068 5.3515 2.3825];
K1=0:1:50;
K2=50:5:200;
K=[K1 K2];
r=rlocus(num,den,K);
a=rlocus(numa,dena,K);
plot(r,'o');
v=[-6 2 -4 4];
axis(v);
hold on
plot(a,'-');
grid;
title('Root-locus Plot (Problem B-6-13)')

```



B-6-15

The term  $(s+1)$  in the feedforward transfer function and the term  $(s+1)$  in the feedback transfer function cancel each other.

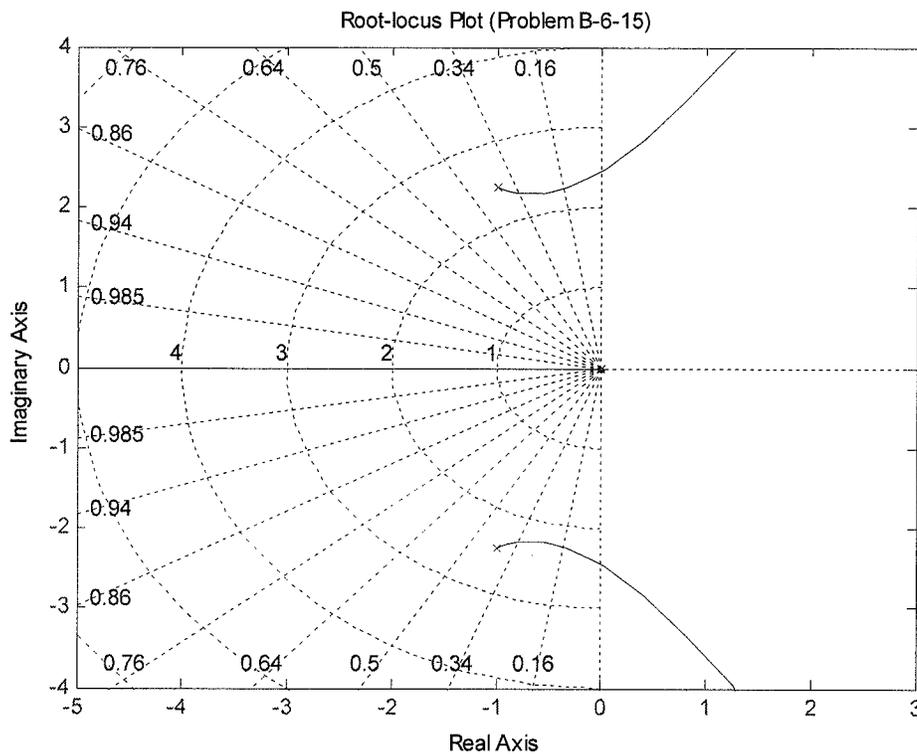
The reduced characteristic equation is

$$1 + G(s)H(s) = 1 + \frac{K(s+1)}{s(s^2+2s+6)} \cdot \frac{1}{s+1} = 1 + \frac{K}{s(s^2+2s+6)} = 0$$

∴ The open-loop poles of  $G(s)H(s)$  is at  $s=0$  and  $s=-1 \pm j\sqrt{5}$ .

MATLAB program and the resulting root-locus plot:

```
num=[0 0 0 1];  
den=[1 2 6 0];  
rlocus(num,den);  
v=[-5 3 -4 4];  
axis(v);  
grid;  
title('Root-locus Plot (Problem B-6-15)')
```



To find the closed-loop poles when the gain K is set equal to 2, we may enter the following MATLAB program into the computer:

```
p=[1 2 6 2];
roots(p)
```

we get:  
ans =

```
-0.8147 + 2.1754i
-0.8147 - 2.1754i
-0.3706
```

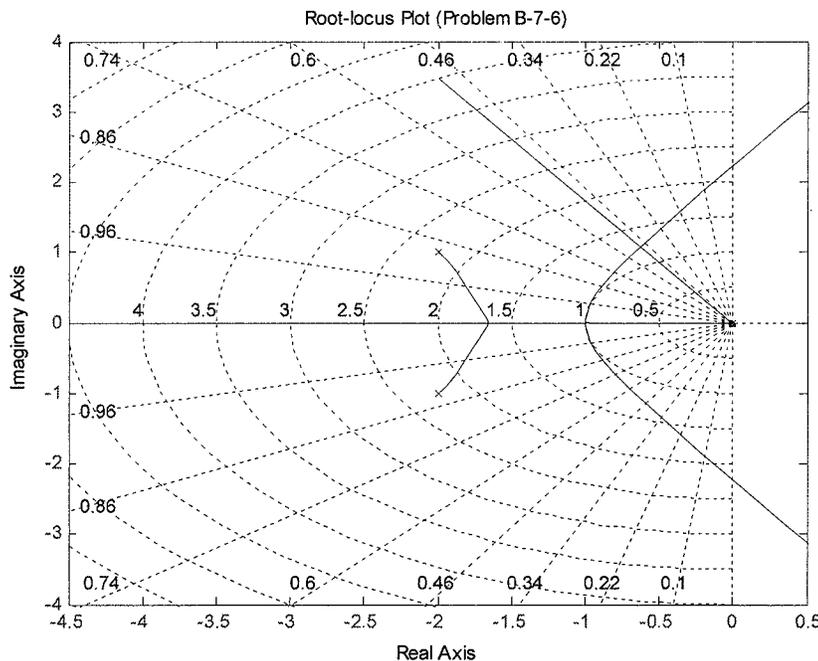
Thus, the closed-loop poles are located at

$$s = -0.8147 \pm j 2.1754, \quad s = -0.3706 \quad \text{and} \quad s = -1 \quad (\text{the cancelled pole})$$

### B-7-6

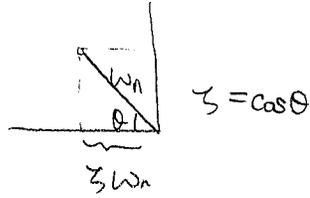
The following MATLAB program gives a root-locus plot for the system. The plot obtained is shown below.

```
num=[0 0 0 1];
den=[1 4 5 0];
rlocus(num,den);
hold;
x=[0 -2];
y=[0 3.464];
line(x,y);
axis('square');
grid;
title('Root-locus Plot (Problem B-7-6)')
```



Since the dominant closed-loop poles have the damping ratio  $\zeta$  of 0.5, we may write them as

$$s = -\alpha \pm j\beta x$$



The characteristic equation for the system is

$$s^3 + 4s^2 + 5s + K = 0$$

By substituting  $s = -\alpha + j\beta x$  into this equation, we obtain

$$(-\alpha + j\beta x)^3 + 4(-\alpha + j\beta x)^2 + 5(-\alpha + j\beta x) + K = 0$$

or

$$-8x^3 - 8x^2 + 5x + K + 2j\beta x(4x^2 + 2.5x) = 0$$

By equating the real part and imaginary part to zero, respectively, we get

$$-8x^3 - 8x^2 + 5x + K = 0 \quad (1)$$

$$4x^2 + 2.5x = 0 \quad (2)$$

Noting that  $x \neq 0$ , from equation (2), we obtain

$$4x + 2.5 = 0$$

$$\therefore x = -0.625$$

By substituting  $x = -0.625$  into equation (1), we get

$$K = 8x^3 + 8x^2 - 5x$$

$$= 8(-0.625)^3 + 8(-0.625)^2 - 5(-0.625)$$

$$= 4.296875$$

To determine all closed-loop poles, we may enter the following MATLAB program:

```
p=[1 4 5 4.296875];  
roots(p)  
and we get:  
ans =
```

```
-2.7500  
-0.6250 + 1.0825i  
-0.6250 - 1.0825i
```

Thus the closed-loop poles are located at

The unit-step response curve can be obtained by entering the following MATLAB program:

```
num=[0 0 0 4.2969];  
den=[1 4 5 4.2969];  
step(num,den)  
grid
```

The unit-step response curve is shown below:

