

ECE 382 HW 8 Solution

B-5-25

$s^4 + 2s^3 + (4+k)s^2 + 9s + 25 = 0$ is the characteristic equation

Routh array:

s^4	1	$4+k$	25
s^3	2	9	
s^2	$\frac{2k-1}{2}$	25	
s^1	$\frac{18k-109}{2k-1}$		
s^0	25		

For stability, we require

$$\frac{2k-1}{2} > 0, \quad \frac{18k-109}{2k-1} > 0$$

or $k > 0.5, \quad 18k > 109$

Hence $k > \frac{109}{18} = 6.056$

For stability, k must be greater than $109/18$.

B-5-28

From the block diagram of Fig 5-90, we have

$$\frac{C(s)}{R(s)} = \frac{20K}{s^3 + 5s^2 + (4 + 20KKh)s + 20K}$$

The stability of this system is determined by the denominator polynomial (characteristic polynomial). The Routh array of the characteristic equation

$$s^3 + 5s^2 + (4 + 20KKh)s + 20K = 0$$

is	s^3	1	$4 + 20KKh$
	s^2	5	$20K$
	s^1	$4 + 20KKh - 4K$	0
	s^0	$20K$	

For stability, we require

$$4 + 20KKh - 4K > 0, \quad 20K > 0$$

or (given $Kh > 0$) $5Kh > K - 1, \quad K > 0$

The stable region in the $K-Kh$ plane is the region that satisfies these two inequalities. Fig 1 shows the stable region in the $K-Kh$ plane. If a point

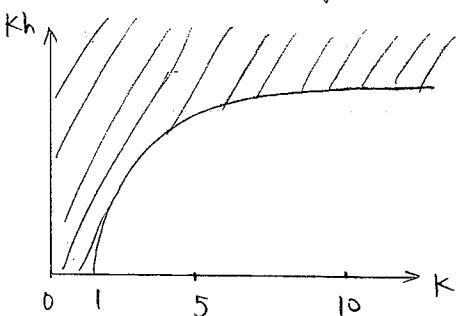


Fig. 1

in the $K-Kh$ plane (that is, a combination of K and Kh values) lies in the shaded region, then the system is stable. Conversely, if a point in the $K-Kh$ plane lies in the nonshaded region, the system is unstable. The dividing curve is defined by $5KKh = K - 1$. (Any point above this dividing curve corresponds to a stable combination of K and Kh .)

$$B-5-30 \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b} \quad (\text{unity-feedback})$$

Hence

$$(s^2+as+b)G(s) = (Ks+b)[1+G(s)]$$

or

$$G(s) = \frac{Ks+b}{s(s+a-k)}$$

The steady-state error in the unit-ramp response is

$$e_{ss} = \frac{1}{KV} = \lim_{s \rightarrow 0} \frac{1}{sG(s)} = \lim_{s \rightarrow 0} \frac{s(s+a-k)}{s(Ks+b)} = \frac{a-k}{b}$$

$$2. B-5-24. \frac{C(s)}{R(s)} = \frac{10}{s(s-1)(2s+3)+10}$$

The characteristic equation is

$$2s^3 + s^2 - 3s + 10 = 0$$

The Routh array becomes

$$\begin{array}{cccc} + & s^3 & 2 & -3 \\ + & s^2 & 1 & 10 \\ - & s^1 & -23 \\ + & s^0 & 10 \end{array}$$

The system is unstable.

Two sign changes \Rightarrow The number of RHP poles is 2.

$$3. (b) s^5 + s^3 + s^2 + 1 = 0$$

$$\Rightarrow s^5 + 0 \cdot s^4 + s^3 + s^2 + 0 \cdot s + 1 = 0$$

$$\Sigma \rightarrow 0^+ \quad 0^-$$

$$\begin{array}{cccccc} + & + & s^5 & 1 & 1 & 0 \\ + & - & s^4 & \cancel{s_2} & 1 & 1 \\ - & + & s^3 & \frac{\Sigma-1}{\Sigma} & \frac{-1}{\Sigma} \\ + & + & s^2 & \frac{2\Sigma-1}{\Sigma-1} & 1 \\ + & - & s^1 & -\frac{\Sigma}{2\Sigma-1} \\ + & + & s^0 & 1 \end{array}$$

$$\text{As } \Sigma \rightarrow 0^+, N_+ = 2 \quad ++ - + + +$$

$$\Sigma \rightarrow 0^-, N_- = 4 \quad + - + + - +$$

\therefore # of RHP poles is $\max(N_+, N_-) = 4$

of RHP poles on imaginary axis is $|N^+ - N^-| = 2$

$$(b) s^3 + 3s^2 + 2s + 6 = 0$$

s^3	1	2	
s^2	3	6	$a_2(s) = 3s^2 + b$
s^1	8	6	$a_2'(s) = 6s$
s^0	b		

+++ \therefore No RHP pole from Routh Test

In addition, 2 roots from auxiliary equation

$$3s^2 + b = 0 \Rightarrow s = \pm \sqrt{-\frac{b}{3}}$$

Therefore, the total number of RHP roots is 2.

4. From the pole-zero plots:

- Plot I corresponds to an unstable system, because of the pole at $s=0$. Let us call its transfer function $G_1(s)$.
- Plot II corresponds to a critically damped system. Let us call its transfer function $G_2(s)$.
- Plot III corresponds to an underdamped system, with $w_d = 7.5$ rad/sec, and $\sigma = 1$. Let us call its transfer fn. $G_3(s)$.
- Plot IV corresponds to an underdamped system, with $w_d = 10$ rad/sec, and $\sigma = 1$. Let us call its transfer function $G_4(s)$.

With this information, we conclude the following from the step responses:

- Plot A corresponds to the critically damped system with transfer function $G_2(s)$.
- Plot B has a greater frequency of oscillation than plot D, so it must correspond to the underdamped system with transfer function $G_4(s)$.
- Plot C corresponds to an unstable system with transfer function $G_1(s)$, since the step response increases without bound.
- Plot D corresponds to the other underdamped system with transfer function $G_3(s)$.

Next, let us look at the steady-state error values. In order for k_v to be nonzero, the transfer function $G_i(s)H(s)$ must have a pole at $s=0$, and the only candidate is $G_1(s)$.

We also note the following:

$$\begin{array}{lll} G_2(0) = 1.5 & G_3(0) = 1 & G_4(0) = 0.5 \\ K_p = 1.5 & K_p = 1 & K_p = 0.5 \\ \zeta = 0.4 & \zeta = 0.5 & \zeta = \frac{2}{3} \end{array}$$

$$(\because K_p = \lim_{s \rightarrow 0} G(s) = G(0))$$

$$\zeta = \frac{1}{1+K_p}$$

∴ We can now easily fill the table

Steady-state error info	Pole-zero plot	Step response
$K_p = 1$	III	D
$K_v = 1$	I	C
$\zeta = 0.4$	II	A
$\zeta = 2/3$	IV	B