ECE 382 HW 8 Solution

\[ S^4 + 2S^3 + (4 + k)S^2 + 9S + 25 = 0 \] is the characteristic equation.

Routh array:

| \( S^4 \) | 1 | \( 4 + k \) | 25 |
| \( S^3 \) | 2 | 9 |
| \( S^2 \) | \( \frac{2k - 1}{2} \) | 25 |
| \( S' \) | \( \frac{18k - 90}{2k - 1} \) |
| \( S'' \) | 25 |

For stability, we require

\[ \frac{2k - 1}{2} > 0, \quad \frac{18k - 90}{2k - 1} > 0 \]

or \( k > 0.5, \quad 18k > 100 \)

Hence \( k > \frac{100}{18} = 5.56 \)

For stability, \( k \) must be greater than \( \frac{100}{18} \).

B-5-28 From the block diagram of Fig 5-90, we have

\[ \frac{C(s)}{R(s)} = \frac{20k}{S^3 + 5S^2 + (4 + 20kK)S + 20k} \]

The stability of this system is determined by the denominator polynomial (characteristic polynomial). The Routh array of the characteristic equation

\[ S^3 + 5S^2 + (4 + 20kK)S + 20k = 0 \]

is

| \( S^3 \) | 1 | 4 + 20kK |
| \( S^2 \) | 5 | 20k |
| \( S' \) | 4 + 20kK - 4k | 0 |
| \( S'' \) | 20k |

For stability, we require

\[ 4 + 20kK - 4k > 0, \quad 20k > 0 \]

or (given \( Kh > \)) \( 5K_k > k - 1, \quad k > 0 \)

The stable region in the \( K - Kh \) plane is the region that satisfies these two inequalities. Fig 1 shows the stable region in the \( K - Kh \) plane. If a point...
in the $K$-$K_h$ plane (that is, a combination of $K$ and $K_h$ values) lies in the shaded region, then the system is stable. Conversely, if a point in the $K$-$K_h$ plane lies in the nonshaded region, the system is unstable. The dividing curve is defined by $5K_h = K - 1$. Any point above this dividing curve corresponds to a stable combination of $K$ and $K_h$.

\[
\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b} \quad (\text{unity-feedback})
\]

Hence

\[(s^2+as+b)G(s) = (Ks+b)[1+G(s)]\]

or

\[G(s) = \frac{Ks+b}{s(s+a-K)}\]

The steady-state error in the unit-ramp response is

\[e_{ss} = \frac{1}{Kv} = \frac{s}{s+a} \quad sG(s) = \frac{a}{s} \quad s(s+a-k) = \frac{a-k}{b}\]
The characteristic equation is
\[ 2s^3 + s^2 - 3s + 1 = 0 \]

The Routh array becomes
\[
\begin{array}{ccc}
+ & s^3 & 2 & -3 \\
+ & s^2 & 1 & 10 \\
- & s^1 & -25 \\
+ & s^0 & 10 \\
\end{array}
\]

The system is unstable.
Two sign changes => The number of RHP poles is 2.

3. (b) \[ s^5 + s^3 + s^2 + 1 = 0 \]
\[ \Rightarrow s^5 + s^4 + s^3 + s^2 + 0.s + 1 = 0 \]
\[ 2 \rightarrow o^+ \quad 0^- \]
\[
\begin{array}{ccc}
+ & + & s^5 & 1 & 1 & 0 \\
+ & - & s^4 & \delta_s & 1 & 1 \\
- & + & s^3 & \frac{s-1}{2} & -2 & 1 \\
+ & + & s^2 & \frac{2s-1}{2s-1} & 1 & \delta^{-} \\
+ & - & s^1 & -\frac{2s-1}{2s-1} & 1 & \\
+ & + & s^0 & 1 & \\
\end{array}
\]

As \[ 2 \rightarrow o^+ \; , \; N^+ = 2 \; \quad + + - + + + \]
\[ 0^- \; , \; N^- = 4 \; \quad + - + - + \]

\# of RHP poles is \[ \max (N^+, N^-) = 4 \]
\# of RHP poles on imaginary axis is \[ |N^+ - N^-| = 2 \]

(b) \[ s^3 + 3s^2 + 2s + b = 0 \]
\[ S^3 \quad 1 \quad 2 \]
\[ S^2 \quad 3 \quad 4 \quad a_2(s) = 3s^2 + b \]
\[ S^1 \quad \not\exists \quad b \quad a_2'(s) = 6s \]
\[ S^0 \quad b \]

+++: No RHP pole from Routh Test

In addition, 2 roots from auxiliary equation
\[ 3s^2 + b = a_2' \Rightarrow s = \pm \alpha j \]

Therefore, the total number of RHP roots is 2.

4. From the pole-zero plots:
   - Plot I corresponds to an unstable system, because of the pole at \( s = -\alpha \). Let us call its transfer function \( G_1(s) \).
   - Plot II corresponds to a critically damped system. Let us call its transfer function \( G_2(s) \).
   - Plot III corresponds to an underdamped system, with \( \omega_d = 7.5 \text{ rad/sec} \) and \( \zeta = 1 \). Let us call its transfer function \( G_3(s) \).
   - Plot IV corresponds to an underdamped system, with \( \omega_d = 10 \text{ rad/sec} \) and \( \zeta = 1 \). Let us call its transfer function \( G_4(s) \).

With this information, we conclude the following from the step responses:
   - Plot A corresponds to the critically damped system with transfer function \( G_2(s) \).
   - Plot B has a greater frequency of oscillation than plot D, so it must correspond to the underdamped system with transfer function \( G_4(s) \).
   - Plot C corresponds to an unstable system with transfer function \( G_1(s) \), since the step response increases without bound.
   - Plot D corresponds to the other underdamped system with transfer function \( G_3(s) \).
Next, let us look at the steady-state error values. In order for \( K_v \) to be nonzero, the transfer function \( G(s)H(s) \) must have a pole at \( s=0 \), and the only candidate is \( G(s) \).

We also note the following:

\[
\begin{align*}
G_2(\infty) &= 1.5 \\
G_3(\infty) &= 1 \\
G_{4\infty} &= 0.5 \\
K_p &= 1.5 \\
K_p &= 1 \\
k_p &= 0.5 \\
C_0 &= 0.4 \\
C_0 &= 0.5 \\
C_0 &= \frac{2}{3}
\end{align*}
\]

\( (\because K_p = \lim_{s \to \infty} G(s) = G(\infty) \) \\
\( C_0 = \frac{1}{1+k_p} \) 

We can now easily fill the table:

<table>
<thead>
<tr>
<th>Steady-state error into</th>
<th>Pole-zero plot</th>
<th>Step response</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p = 1 )</td>
<td>III</td>
<td>D</td>
</tr>
<tr>
<td>( K_v = 1 )</td>
<td>I</td>
<td>C</td>
</tr>
<tr>
<td>( C_0 = 0.4 )</td>
<td>II</td>
<td>A</td>
</tr>
<tr>
<td>( C_0 = 2/3 )</td>
<td>IV</td>
<td>B</td>
</tr>
</tbody>
</table>