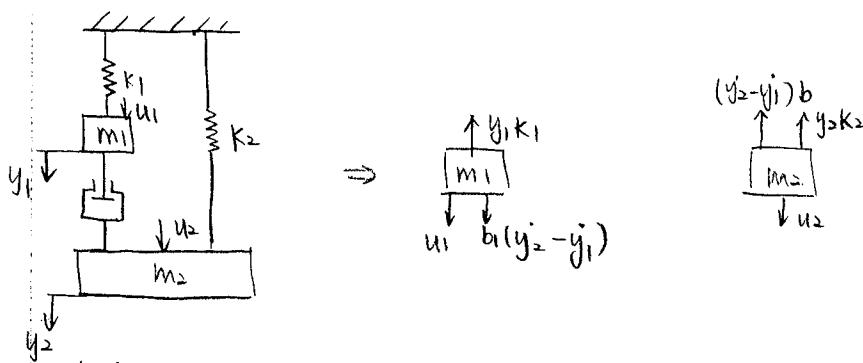


Fig 3-80



Using  $F = ma$

we have

$$m_1 \ddot{y}_1 = u_1 + (y_2 - y_1) b_1 - y_1 k_1 \quad (1)$$

$$m_2 \ddot{y}_2 = u_2 - (y_2 - y_1) b_1 - y_2 k_2 \quad (2)$$

Taking Laplace transform on (1) & (2), we have

$$(m_1 s^2 + b_1 s + k_1) Y_1(s) - b_1 s Y_2(s) = U_1(s) \quad (3)$$

$$(m_2 s^2 + b_1 s + k_2) Y_2(s) - b_1 s Y_1(s) = U_2(s) \quad (4)$$

$$\text{From (4)} \quad Y_2(s) = \frac{b_1 s}{m_2 s^2 + b_1 s + k_2} Y_1(s) + \frac{1}{m_2 s^2 + b_1 s + k_2} U_2(s) \quad (5)$$

Substituting (5) into (3) we get

$$(m_1 s^2 + b_1 s + k_1 - \frac{b_1^2 s^2}{m_2 s^2 + b_1 s + k_2}) Y_1(s) = U_1(s) + \frac{b_1 s}{m_2 s^2 + b_1 s + k_2} U_2(s)$$

Taking  $U_2(s) = 0$ , we get the transfer function for  $Y_1(s)$  w.r.t.  $U_1(s)$

$$\text{as } H_{11}(s) = \frac{m_1 s^2 + b_1 s + k_1}{m_2 s^2 + b_1 s + k_2} - b_1^2 s^2 ;$$

Taking  $U_1(s) = 0$ , we get the transfer function for  $Y_1(s)$  w.r.t.  $U_2(s)$

$$\text{as } H_{12}(s) = \frac{b_1 s}{(m_1 s^2 + b_1 s + k_1)(m_2 s^2 + b_1 s + k_2) - b_1^2 s^2} ;$$

Similarly, we can obtain

$$H_{21}(s) = \frac{b_1 s}{(m_1 s^2 + b_1 s + k_1)(m_2 s^2 + b_1 s + k_2) - b_1^2 s^2} \quad (Y_2(s) \text{ w.r.t. } U_1(s))$$

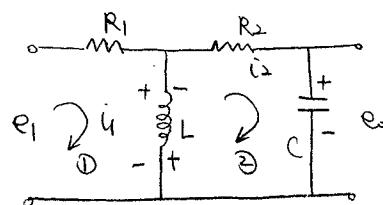
$$H_{22}(s) = \frac{m_1 s^2 + b_1 s + k_1}{(m_1 s^2 + b_1 s + k_1)(m_2 s^2 + b_1 s + k_2) - b_1^2 s^2} \quad (Y_2(s) \text{ w.r.t. } U_2(s))$$

The equations for the given circuit are as follows:

$$\text{Using KVL} \quad ① R_1 i_1 + L \left( \frac{di_1}{dt} - \frac{di_2}{dt} \right) = e_1$$

$$② R_2 i_2 + \frac{1}{C} \int i_2 dt + L \left( \frac{di_2}{dt} - \frac{di_1}{dt} \right) = 0$$

$$\frac{1}{C} \int i_2 dt = e_2$$



Taking the Laplace transforms of these three equations, assuming zero initial conditions, gives

$$R_1 I_1(S) + L [S I_1(S) - S I_2(S)] = E_i(S) \quad (1)$$

$$R_2 I_2(S) + \frac{1}{C} I_2(S) + L [S I_2(S) - S I_1(S)] = 0 \quad (2)$$

$$\frac{1}{C} I_2(S) = E_o(S) \quad (3)$$

From equation (2) we obtain

$$(R_2 + \frac{1}{C} + Ls) I_2(S) = Ls I_1(S)$$

$$\text{or } I_2(S) = \frac{Ls^2}{Ls^2 + R_2 s + 1} I_1(S) \quad (4)$$

Substituting equation (4) into equation (1), we get

$$(R_1 + Ls - Ls \frac{Ls^2}{Ls^2 + R_2 s + 1}) I_1(S) = E_i(S)$$

$$\text{or } \frac{LC(R_1 + R_2)s^2 + (R_1 R_2 C + L)s + R_1}{LCs^2 + R_2 Cs + 1} I_1(S) = E_i(S) \quad (5)$$

From eqns (3) & (4), we have

$$\frac{Ls}{LCs^2 + R_2 Cs + 1} I_1(S) = E_o(S) \quad (6)$$

From eqns (5) & (6), we obtain

$$\frac{E_o(S)}{E_i(S)} = \frac{Ls}{LC(R_1 + R_2)s^2 + (R_1 R_2 C + L)s + R_1}$$

B-3-24. For the op-amp circuit shown to the right, we have

$$E_A - E_B = Z_4 I_2$$

$$E_B - 0 = Z_3 I_1$$

$$E_A = E_B$$

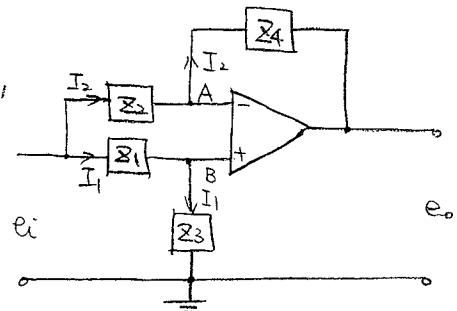
Hence

$$Z_4 I_2 + E_B = Z_3 I_1$$

$$\text{or } I_2 = \frac{1}{Z_4} (Z_3 I_1 - E_B) \quad (1)$$

$$\text{Also, } E_i - E_B = (Z_1 + Z_4) I_2 \quad (2)$$

$$E_i = (Z_1 + Z_3) I_1 \quad (3)$$



By substituting Eqn(11) into Eqn(12), we obtain

$$E_i - E_o = (\frac{Z_1 + Z_3}{Z_4}) \frac{1}{Z_4} (Z_3 I_1 - E_o)$$

By substituting eqn(13) into this last eqn., we get

$$(Z_1 + Z_3) I_1 - E_o = (\frac{Z_1}{Z_4} + 1) Z_3 I_1 - (\frac{Z_1}{Z_4} + 1) E_o$$

$$\text{or } (1 - \frac{Z_1}{Z_4} - 1) E_o = (Z_1 + Z_3 - \frac{Z_1 Z_3}{Z_4} - Z_3) I_1$$

$$\text{Hence } -Z_3 E_o = (Z_1 Z_4 - Z_1 Z_3) I_1$$

From eqns 13 & 14, we have

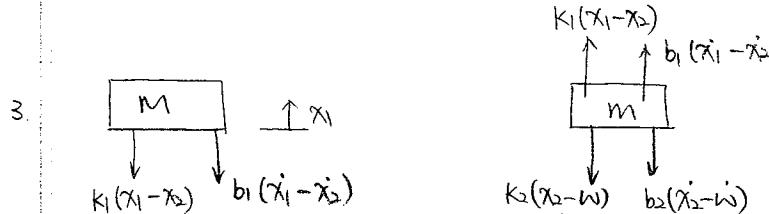
$$\frac{E_o}{E_i} = \frac{\frac{Z_2 Z_3 - Z_1 Z_4}{Z_2}}{Z_1 + Z_3} = \frac{Z_2 Z_3 - Z_1 Z_4}{Z_1 Z_2 + Z_2 Z_3}$$

For the current op-amp circuit, we have

$$Z_1 = \frac{1}{C_s}, Z_2 = R_1, Z_3 = R_2, Z_4 = R_1$$

Hence

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{R_1 R_2 - \frac{1}{C_s} R_1}{C_s R_1 + R_1 R_2}}{1} = \frac{R_2 - \frac{1}{C_s}}{\frac{1}{C_s} + R_2} = \frac{R_2 C_s - 1}{R_2 C_s + 1}$$



Using  $F = ma$

$$\text{we have } M \ddot{x}_1 = 0 - K_1(x_1 - x_2) - b_1(\dot{x}_1 - \dot{x}_2)$$

$$\text{Take LT for both sides: } m \ddot{x}_2 = K_1(x_1 - x_2) + b_1(\dot{x}_1 - \dot{x}_2) - K_2(x_2 - w) - b_2(\dot{x}_2 - \dot{w})$$

$$\Rightarrow M s^2 X_1(s) = -K_1 Y(s) - b_1 s Y(s)$$

$$m s^2 X_2(s) = K_1 Y(s) + b_1 s Y(s) - K_2(X_2(s) - W(s)) - b_2 s (X_2(s) - W(s))$$

$$\Rightarrow M s^2 X_1 = (-K_1 - b_1 s) Y$$

$$(M s^2 + b_1 s + K_1) X_1 = (K_1 + b_1 s) Y + (K_2 + b_2 s) W$$

$$\Rightarrow X_1 = -\frac{K_1 + b_1 s}{M s^2} Y \quad (1)$$

$$X_2 = \frac{K_1 + b_1 s}{M s^2 + b_2 s + K_2} Y + \frac{K_2 + b_2 s}{M s^2 + b_2 s + K_2} W \quad (2)$$

(1)-(2), we get

$$X_1 - X_2 = - \frac{K_1 + b_1 S}{M S^2} Y - \frac{K_1 + b_1 S}{m S^2 + b_2 S + K_2} Y - \frac{K_2 + b_2 S}{m S^2 + b_2 S + K_2} W$$

$$\therefore Y = - \left( \frac{K_1 + b_1 S}{M S^2} + \frac{K_1 + b_1 S}{m S^2 + b_2 S + K_2} \right) Y - \frac{K_2 + b_2 S}{m S^2 + b_2 S + K_2} W$$

$$\left( 1 + \frac{K_1 + b_1 S}{M S^2} + \frac{K_1 + b_1 S}{m S^2 + b_2 S + K_2} \right) Y = - \frac{K_2 + b_2 S}{m S^2 + b_2 S + K_2} W$$

Multiply by  $(m S^2 + b_2 S + K_2)$  on both sides:

$$\left( m S^2 + b_2 S + K_2 + \frac{(K_1 + b_1 S)(m S^2 + b_2 S + K_2)}{m S^2} + K_1 + b_1 S \right) Y = (K_2 + b_2 S) W$$

$$\therefore \frac{Y}{W} = - \frac{(K_2 + b_2 S) M S^2}{(K_2 + b_2 S) M S^2}$$

$$= - \frac{M m S^4 + M b_2 S^3 + K_2 M S^2 + m b_1 S^3 + b_1 b_2 S^2 + K_2 b_1 S + K_1 m S^2 + b_2 K_1 S + K_1 K_2 + \cancel{M K_1 S^2} + \cancel{M b_1 S^3}}{(K_2 + b_2 S) M S^2}$$

$$= - \frac{M m S^4 + (M b_2 + m b_1 + M b_1) S^3 + (K_2 M + b_1 b_2 + K_1 m) S^2 + (K_2 b_1 + K_1 b_2) S + K_1 K_2}{(K_2 + b_2 S) M S^2}$$

```
M=2500;
m=320;
k1=80000;
k2=500000;
b1=350;
b2=15020;
num=[b2*M k2*M 0 0];
den=[M*m M*b2+m*b1+M*b1 k2*M+b1*b2+k1*m+k1*M k2*b1+k1*b2 k1*k2];
sys=tf(num, den);
step(0.05*num, den);
```

Step Response

