

$$2. f(t) = t$$

$$\begin{aligned}(a) F(s) &= \int_0^\infty t e^{-st} dt = -\frac{1}{s} \int_0^\infty t d e^{-st} \\&= -\frac{1}{s} (t e^{-st} \Big|_0^\infty - \int_0^\infty e^{-st} dt) \\&= 0 + \frac{1}{s} \left(-\frac{1}{s}\right) e^{-st} \Big|_0^\infty = \frac{1}{s^2}\end{aligned}$$

Since the integration converges for  $s > 0$ , the abscissa of convergence is  $\sigma_c = 0$ .

$$(b) \text{ Note that } f(t) = \int_0^t 1(\tau) d\tau$$

Since  $\mathcal{L}[1(t)] = \frac{1}{s}$ , using the integration property, we have

$$\mathcal{L}[f(t)] = \frac{1}{s} \mathcal{L}[1(t)] = \frac{1}{s^2}.$$

3. Note that

$$\cos \omega t = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

Since

$$\mathcal{L}[e^{j\omega t}] = \frac{1}{s-j\omega t}$$

$$\mathcal{L}[e^{-j\omega t}] = \frac{1}{s+j\omega t}$$

Hence

$$\begin{aligned}\mathcal{L}[\cos \omega t] &= \frac{1}{2} \left( \frac{1}{s-j\omega t} + \frac{1}{s+j\omega t} \right) \\&= \frac{1}{2} \cdot \frac{2s}{(s-j\omega t)(s+j\omega t)} \\&= \frac{s}{s^2 + \omega^2}\end{aligned}$$

B-2-2.

(a) \*  $f_1(t) = 3 \sin(5t + 45^\circ) = 2.121 \sin 5t + 2.121 \cos 5t$

$$F_1(s) = \frac{2.121 \times s}{s^2 + 5^2} + \frac{2.121 s}{s^2 + 5^2} = \frac{10.607 + 2.121 s}{s^2 + 25}$$

(b)  $f_2(t) = 0.03 - 0.03 \cos 2t$

$$\begin{aligned} F_2(s) &= \frac{0.03}{s} - \frac{0.03 s}{s^2 + 4} = \frac{0.03 s^2 + 0.12 - 0.03 s^2}{s(s^2 + 4)} \\ &= \frac{0.12}{s(s^2 + 4)} \end{aligned}$$

Remark: For Problem B-2-2(a), you should not use the time-shift property, for two reasons:

- a) time-shift property only works when you shift to the right.
- b) time-shift property only allows you to compute the L.T. of  $f(t-a)1(t-a)$ , but not  $f(t-a)1(t)$ .

B-2-4.

(a)  $f(t) = \sin wt \cdot \cos wt = \frac{1}{2} \sin 2wt$

Hence

$$\mathcal{L}[\sin wt \cdot \cos wt] = \mathcal{L}\left[\frac{1}{2} \sin 2wt\right] = \frac{\omega}{s^2 + 4\omega^2}$$

(b) Define

$$g(t) = e^{-t} \sin 5t$$

Then

$$\mathcal{L}[g(t)] = \mathcal{L}[e^{-t} \sin 5t] = \frac{5}{(s+1)^2 + 25} = G(s)$$

Using the complex-differentiation theorem, we have

$$\mathcal{L}[tg(t)] = -\frac{dG(s)}{ds}$$

Hence

$$\begin{aligned} \mathcal{L}[te^{-t} \sin 5t] &= \mathcal{L}[tg(t)] = -\frac{d}{ds}[G(s)] \\ &= -\frac{d}{ds}\left[\frac{5}{(s+1)^2 + 25}\right] = \frac{+10(s+1)}{[(s+1)^2 + 25]^2} \end{aligned}$$

Alternate solution to Problem B-2-4 (b)

Note that

$$\begin{aligned} te^{-t} \sin 5t &= \frac{te^{-t}}{2j} (e^{5jt} - e^{-5jt}) \\ &= \frac{t}{2j} [e^{(-1+5j)t} - e^{(-1-5j)t}] \end{aligned}$$

Since

$$\mathcal{L}[t] = \frac{1}{s^2},$$

using the exponential weighting property

$$\mathcal{L}\left[\frac{t}{2j} e^{(-1+5j)t}\right] = \frac{1}{2j} \frac{1}{[s - (-1+5j)]^2}$$

$$\mathcal{L}\left[\frac{t}{2j} e^{(-1-5j)t}\right] = \frac{1}{2j} \frac{1}{[s - (-1-5j)]^2}$$

Hence

$$\begin{aligned}\mathcal{L}[te^{-t}s^2] &= \\ &= \frac{1}{2j} \left\{ \frac{1}{[s - (-1+5j)]^2} - \frac{1}{[s - (-1-5j)]^2} \right\} \\ &= \frac{1}{2j} \frac{[s - (-1-5j)]^2 - [s - (-1+5j)]^2}{[s - (-1+5j)][s - (-1-5j)]^2} \\ &= \frac{1}{2j} \frac{20j(s+1)}{((s+1)^2 + 25)^2} \\ &= \frac{10(s+1)}{[(s+1)^2 + 25]^2}\end{aligned}$$

B-2-7.

$$f(t) = t \mathbf{1}(t) - (t-T) \mathbf{1}(t-T)$$

$$F(s) = \frac{1}{s^2} - \frac{e^{-Ts}}{s^2} = \frac{1 - e^{-Ts}}{s^2}$$