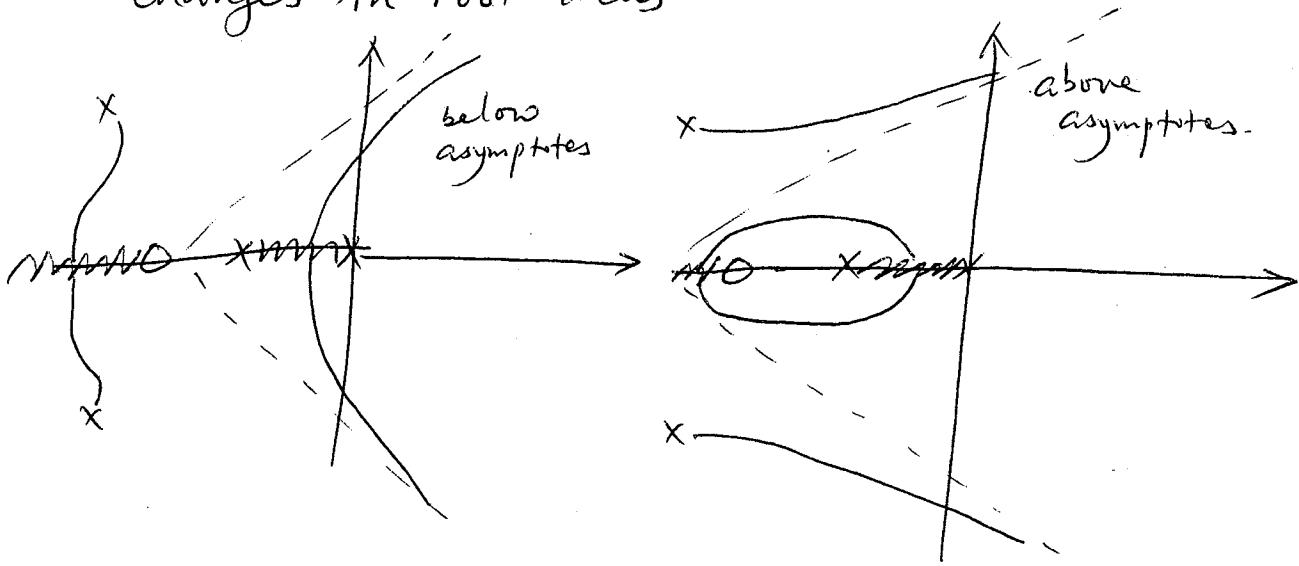


(2)

Note: ① Sometimes small change in pole-zero configuration may cause significant changes in root-locus

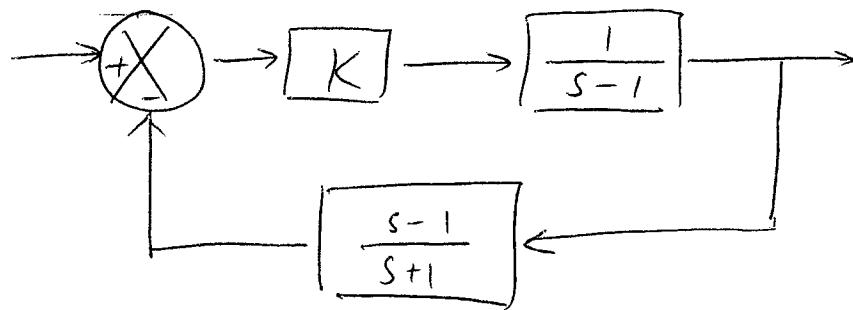


- ② If  $n \geq m+3$  (i.e., the number of open-loop poles exceeds the number of open-loop zeros by three or more), one of the asymptotes will enter the RHP. Hence, there exists a value of  $K$  beyond which the root-locus enters the RHP  
 $\Rightarrow$  the system will become unstable.

## More on root-locus

- ① Pole-zero cancellations P 356 in text

Sometimes a feedback loop may have pole-zero cancellations:



Thus the closed-loop poles are obtained by solving

$$1 + K \cdot \frac{1}{s-1} \cdot \frac{s-1}{s+1} = 0$$

It would be tempting to cancel the  $(s-1)$  term, and get

$$1 + K \frac{1}{s+1} = 0$$

$$\Rightarrow s = -1(K+1) \leq -1 \quad \text{for all } K$$

$\Rightarrow$  system is stable for all  $K$

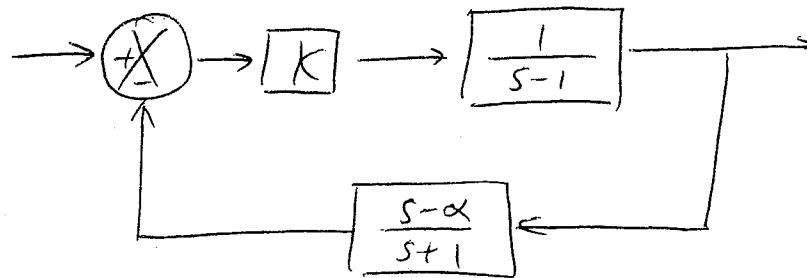
This is WRONG!

Correct procedure: Suppose there are pole-zero cancellations in  $G(s)$ . Then, the root-locus consists of the "reduced" root-locus, obtained

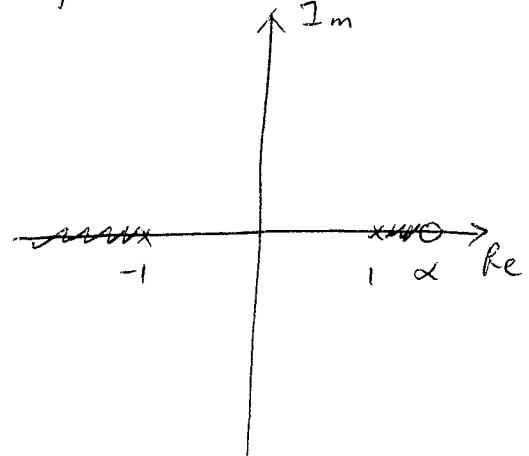
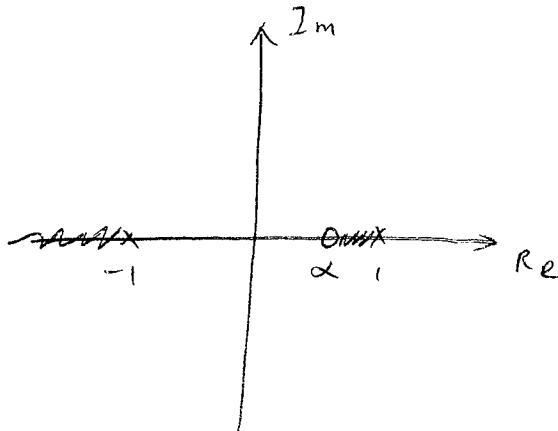
from  $G(s)$  in its reduced form }  
 U { cancelled poles }

In this example, the correct root-locus should include the pole/zero at 1. Hence, the system is always unstable.

Why? In real systems, due to uncontrollable variations in electrical/mechanical components, we will never be able to cancel a pole exactly



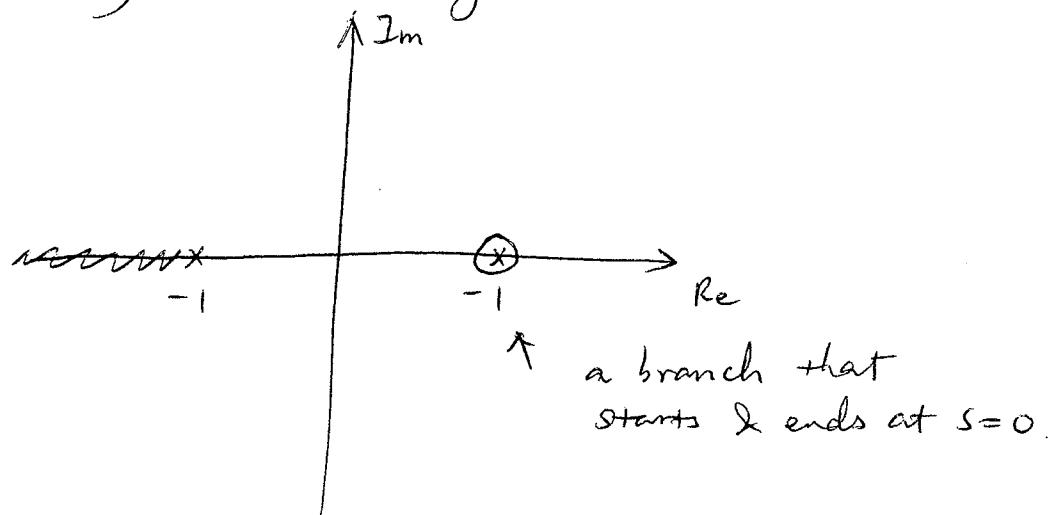
We can make  $\alpha$  close to 1, but never exactly 1.  
 Let us now plot the root-locus for  $\alpha$  near 1



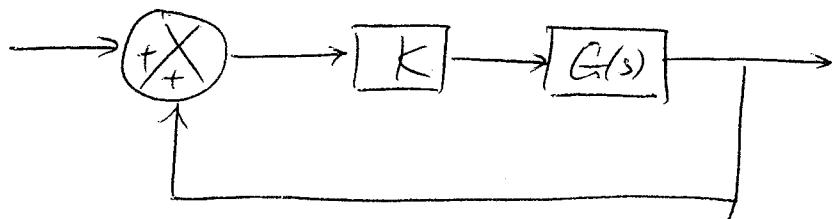
$$1 + K \cdot \frac{s-\alpha}{(s+1)(s-1)} = 0$$

(24)

There is always a branch near the pole 1  
 As  $\alpha \rightarrow 1$ , the branch shrinks to a point 1.  
 $\Rightarrow$  system is always unstable.



② Root-locus with positive feedback p 373-377



The characteristic equation is

$$1 - KG(s) = 0$$

Changes:

## ① Basic Relationships

- Angle condition:

$$\angle G(s) = 360^\circ \cdot l$$

- Magnitude condition: unchanged

- Start/end of branches: unchanged

## ② Steps

Step 1: Standard form

$$1 - K \frac{(s-z_1)(s-z_2) \cdots (s-z_m)}{(s-p_1)(s-p_2) \cdots (s-p_n)} = 0$$

Step 2: Locate poles & zeros

Step 3: Find real segments of the locus.

A point on the real axis lies on the locus  
iff there is an even number of real poles  
& zeros to its right.

Step 4: Find asymptotes

$$\text{Angle of asymptotes } \phi_a = \frac{360^\circ \cdot l}{n-m},$$

Intersection of asymptotes  $\sigma_a$  unchanged.

Step 5: Find imaginary closed-loop poles

Use Routh array (unchanged).

Step 6: Find break-in/break-away points

$$\text{Use } \frac{dK}{ds} = 0 \quad (\text{unchanged})$$

Step 7: Find angles of arrival and departures

Angles must add up to  $360^\circ$ !