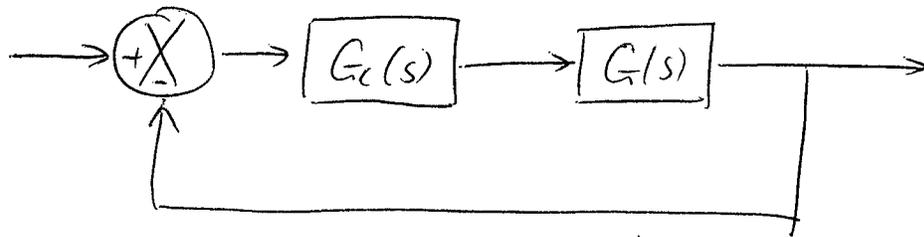


Lag Compensation

P429-438



$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \quad \alpha > 1$$

Assume that the root-locus already passes through the desired location s_1 . We can set both $\frac{1}{T}$ & $\frac{1}{\alpha T}$ to be very small such that

$$\angle G_c(s) = \left| \angle \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\alpha T}} \right|$$

to be less than 5° , i.e.,

$$-5^\circ \leq \angle \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\alpha T}} \leq 0^\circ$$

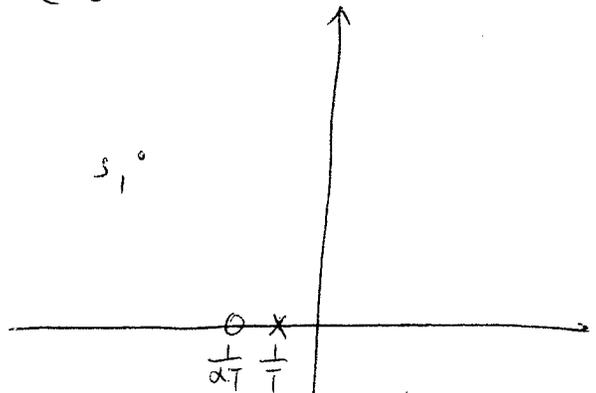
and we set $K_c = 1$.

$$\text{Hence, } \angle G(s_1) \approx \underbrace{\angle G_c(s_1)}_{180^\circ} \angle G(s_1)$$

$\Rightarrow s_1$ is close to the new root-locus

\Rightarrow Transient response does not change much
But produce a DC-gain of α

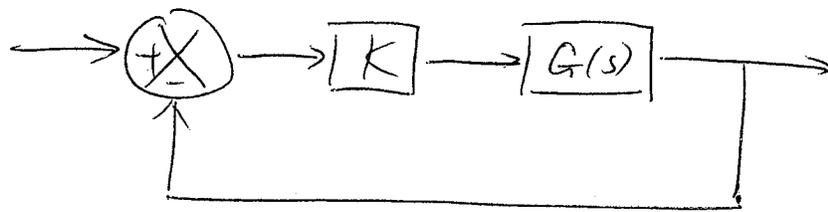
\Rightarrow Increase the static error-constants by α .



Steps for Lag-Compensator Design

- ① Draw the root-locus of the uncompensated system with open-loop transfer function $G(s)$.
Based on the transient-response spec, locate the dominant closed-loop poles on the root-locus.
- ② Evaluate the steady-state error constant of the uncompensated system.
Determine the amount of increase in the steady-state error constant necessary to satisfy the spec.
- ③ Determine the pole & zero of the lag compensator that produces the necessary increase in the steady-state error constant, without appreciably altering the original root-locus.
- ④ Draw the new root-locus, and adjust the gain K_c so that the dominant closed-loop pole is at the desired location.

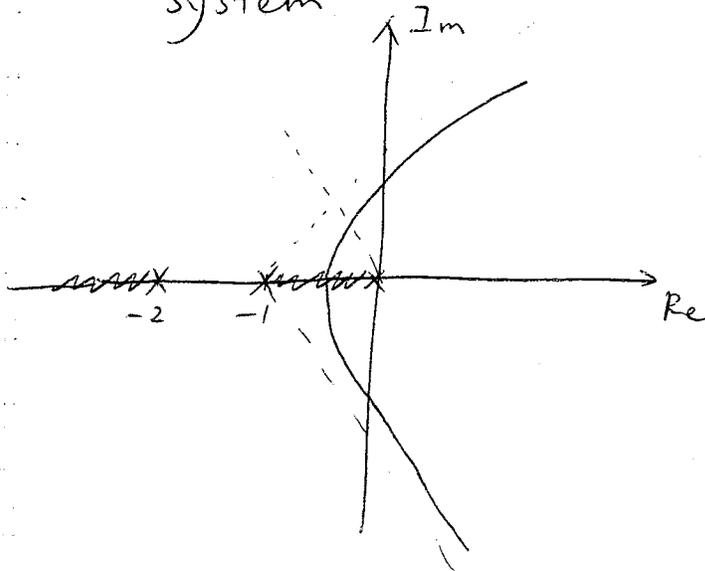
Ex) Similar to Example 7-2 in text p 432



$$G(s) = \frac{1}{s(s+1)(s+2)}$$

Desired spec: $\zeta = 0.5$, $K_v = 5$

Step 1: Draw the root-locus of the uncompensated system



With $\zeta = 0.5$, the dominant closed-loop is at $-0.333 \pm 0.574j$, with $K = 1.03$.

Step 2: Evaluate the steady-state error constant

$$\lim_{s \rightarrow 0} s \cdot K G(s) = \lim_{s \rightarrow 0} \frac{1.03}{(s+1)(s+2)} = 0.52$$

Since we need $K_v = 5$, we will need ten times more DC-gain.

Step 3: Choose $G_c(s) = K_c \frac{s+0.05}{s+0.005}$

The angle contribution to the dominant pole is about 3.5°

The new open-loop transfer function is

$$G_c(s)G(s) = K \cdot \frac{s+0.05}{s+0.005} \cdot \frac{1}{s(s+1)(s+2)}$$

Step 4. Draw the new root-locus.

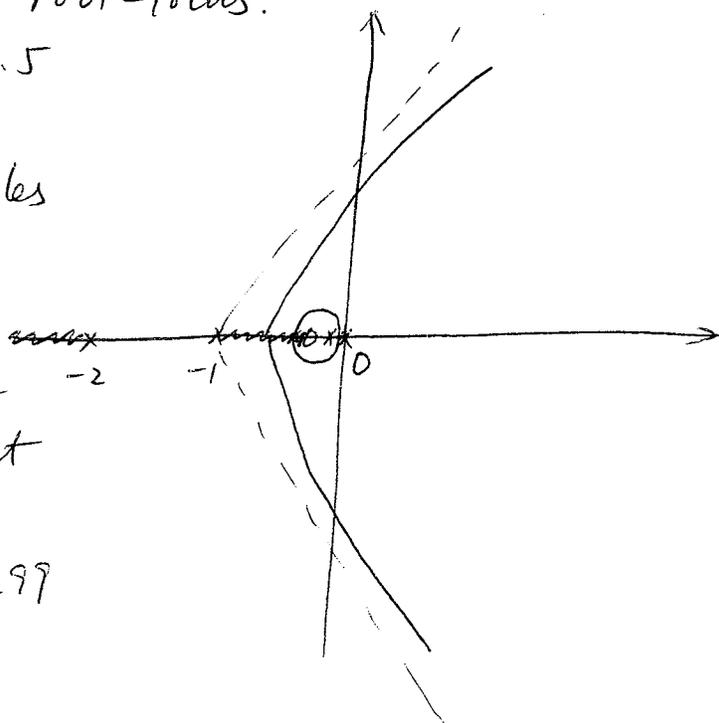
Since we want $\zeta = 0.5$

We have $K = 0.998$

The new closed-loop poles are $-0.315 \pm 0.539j$

The new steady-state velocity error constant is

$$K_v = \frac{0.998}{2} \times 10 = 4.99$$



We can use MATLAB to compare their time-response.

Original system:

$$\text{Root-locus: } 1 + \frac{K}{s(s+1)(s+2)} = 0$$

To get $\zeta = 0.5$, we need $K = 1.03$

The closed loop transfer function is

$$\frac{\frac{1.03}{s(s+1)(s+2)}}{1 + \frac{1.03}{s(s+1)(s+2)}} = \frac{1.03}{s^3 + 3s^2 + 2s + 1.03}$$

(42)

New system

Root-locus

$$1 + K \cdot \frac{s+0.05}{s+0.005} \cdot \frac{1}{s(s+1)(s+2)} = 0$$

To get $\zeta = 0.5$, we need $K = 0.998$
Closed-loop transfer function is

$$\frac{0.998 \cdot \frac{s+0.05}{s+0.005} \cdot \frac{1}{s(s+1)(s+2)}}{1 + 0.998 \cdot \frac{s+0.05}{s+0.005} \cdot \frac{1}{s(s+1)(s+2)}}$$
$$= \frac{0.998s + 0.998 \times 0.05}{s^4 + 3.005s^3 + 2.015s^2 + 1.008s + 0.998 \times 0.05}$$

With ramp-input, the new-system will have smaller steady-state error.

Additional example see A-7-7 in text.

Lag-Lead Compensator

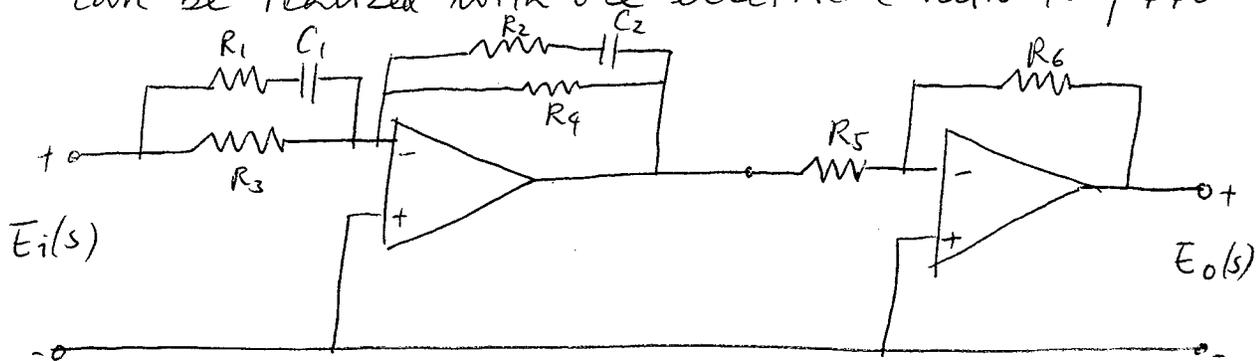
P439 - 450

(43)

Used if improvements in both transient response and steady-state response are required.

$$G_c(s) = K_c \underbrace{\frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}}}_{\text{lead-part}} \cdot \underbrace{\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}}_{\text{lag-part}}, \quad \beta > 1, \quad \rho > 1$$

- possess two poles & two zeros
- use the lead-part to improve transient response
- use the lag-part to improve steady-state response.
- can be realized with one electrical network p440



$$\frac{E_o(s)}{E_i(s)} = \frac{R_4 R_6}{R_3 R_5} \underbrace{\left(\frac{(R_1 + R_3) C_1 s + 1}{R_1 C_1 s + 1} \right)}_{\text{lead-part}} \cdot \underbrace{\left(\frac{R_2 (C_2 s + 1)}{(R_2 + R_4) (C_2 s + 1)} \right)}_{\text{lag-part}}$$

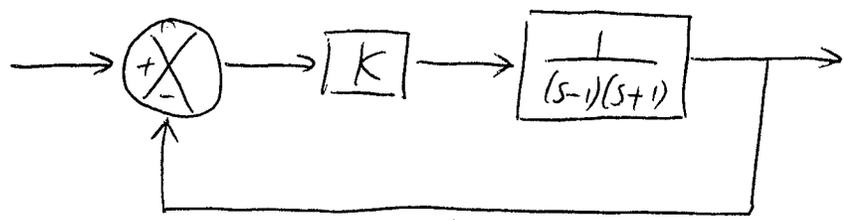
Steps for Lag-Lead compensator

(44)

- Lead-part: with the mind that $\frac{1}{T_2}$ will be small, and therefore will not affect the closed-loop poles much, design the lead part such that the desired closed-loop poles are achieved.
(Step 1-4 of lead-compensator design)
- Check the steady-state error, and determine the amount of increase in the steady-state error constant necessary to satisfy the spec.
- Determine the lag part that produces the necessary increase in the steady-state error constant without appreciably altering the root-locus
(Step 3-4 of lag-compensator design)

In textbook, there is another procedure that ensures $\sigma = \beta$ (p442), which is not required for our class.

Ex) Design a compensator for the following system



such that the closed-loop poles are $-1 \pm j2$ and $K_p = 50$

Note: The original system is UNSTABLE for any K!

Lead-part:

- ① The desired closed-loop poles are at $-1 \pm j2$
- ② Sketch the root-locus of the uncompensated system

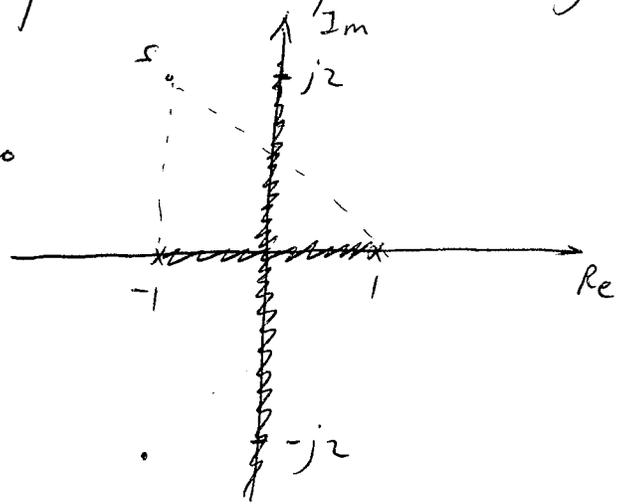
It does not pass through

$$s = -1 \pm j2.$$

$$\angle G(s) = -90^\circ - 135^\circ = -225^\circ = 135^\circ$$

Hence, the angle of compensation is

$$\phi = 180^\circ - 135^\circ = 45^\circ.$$

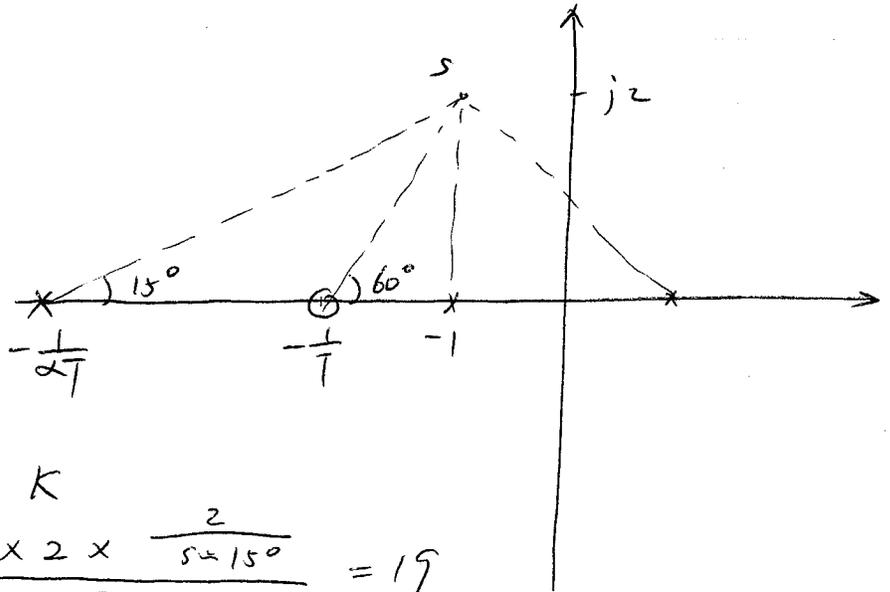


- ③ Design the lead-part.

$$K \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{2T}}$$

Pick the zero at $-1 - \frac{2}{\tan 60^\circ} = -2.15$

Pick the pole at $-1 - \frac{2}{\tan 15^\circ} = -8.46$



④ Determine K

$$K = \frac{2\sqrt{2} \times 2 \times \frac{2}{s+15^\circ}}{\frac{2}{s+60^\circ}} = 19$$

Lag-part:

① Check the steady-state error constant

$$\lim_{s \rightarrow 0} G_c(s)G(s) = 19 \cdot \frac{s+2.15}{s+8.46} \cdot \frac{1}{(s-1)(s+1)} = 4.8$$

Since we require $K_p = 50$, need 10 times increase.

② Design the Lag-part

$$\frac{s+0.01}{s+0.001}$$

The angle from the desired closed-loop pole is 2.1°

③ Can still use the gain $K=19$.

The closed-loop pole should be close to $-1 \pm j2$.

We can use MATLAB to compare the performance of the original system and the new system.