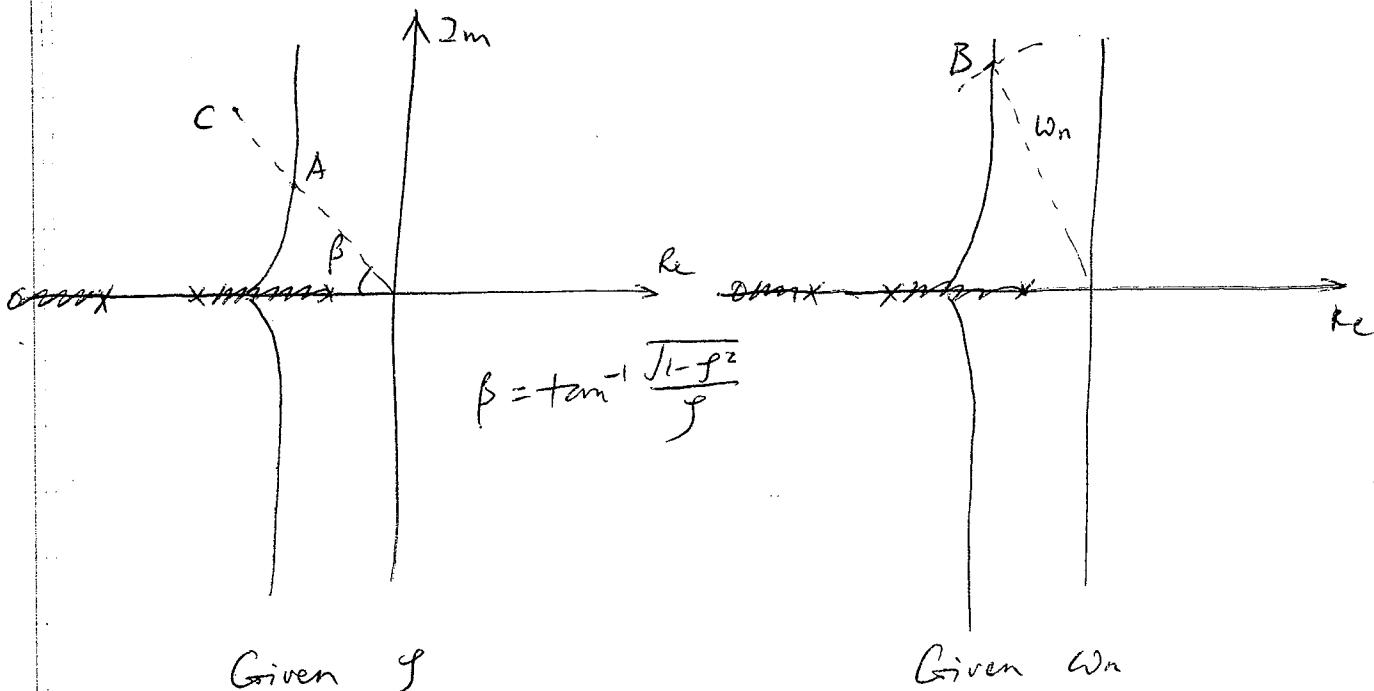


(26)

Root-locus Design P418

Recall that, from the root-locus, we can determine:

- ① What are the locations of closed-loop poles?
- ② What is the range of K for stability?
- ③ What is the value of K that corresponds to a given spec?

Given J Given w_n

- ④ Or, if the design specs can't be achieved by adjusting K alone, how to design a compensator to move the closed-loop poles to the desired location?

Example: Specifying both J & w_n

- ⇒ desired pole location C is not on the locus
- ⇒ Need compensation.

(27)

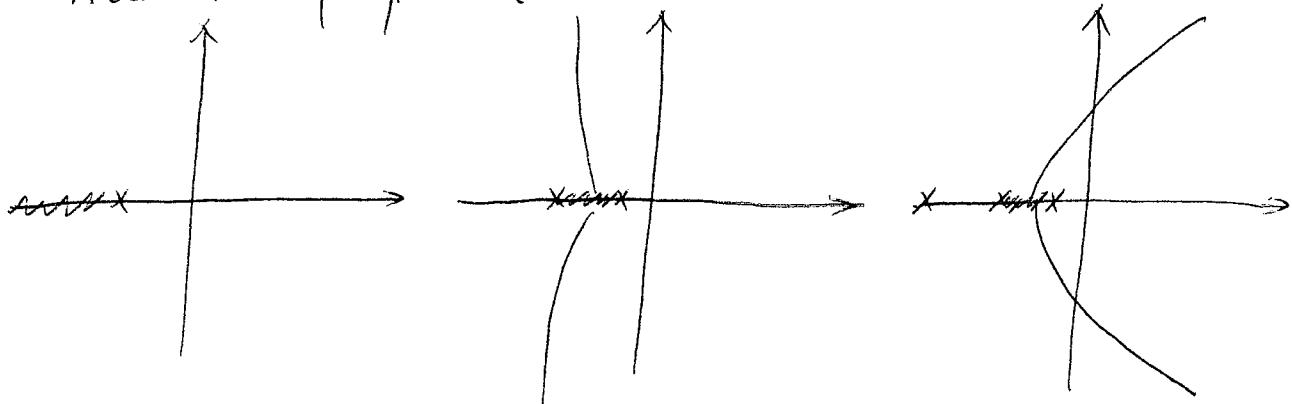
Compensation: Redesign the controller (typically by adding open-loop poles & zeros) such that the modified root-locus passes through the desired closed-loop pole location.

- ⇒ such a controller is called a compensator.
- ⇒ Basic compensators are
 - Lead-compensator
 - Lag-compensator
 - ~~Lead~~ Lag-lead compensator.

Basics:

If we add an open-loop pole/zero, What is the effect on the root-locus P420

① Addition of poles:



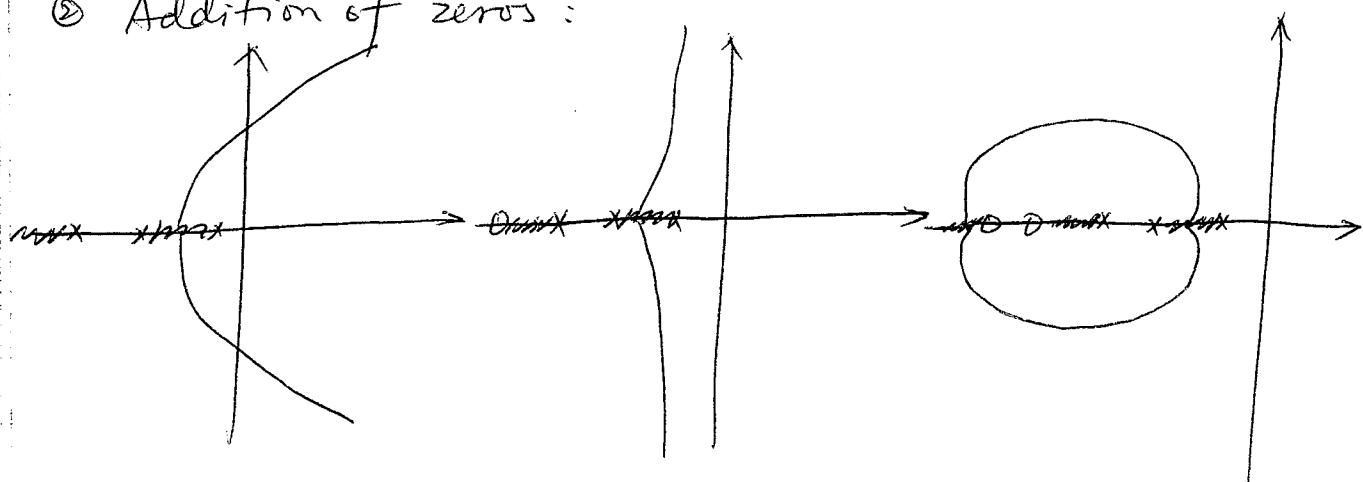
Note: # of Asymptotes = # of poles - # of zeros

- pull the root-locus to the right
- tend to reduce stability, and slow down the settling of the response.

(28)- (29)

Recall that the addition of integral control adds a pole at the origin, thus makes the system less stable.

④ Addition of zeros :



- pull the root-locus to the left
- tend to improve stability, and speed up the settling of the response .

Recall that the addition of derivative control adds a zero at the origin, and makes the system more stable.

In summary : zeros are the "good" guys
poles are the "bad" guys.

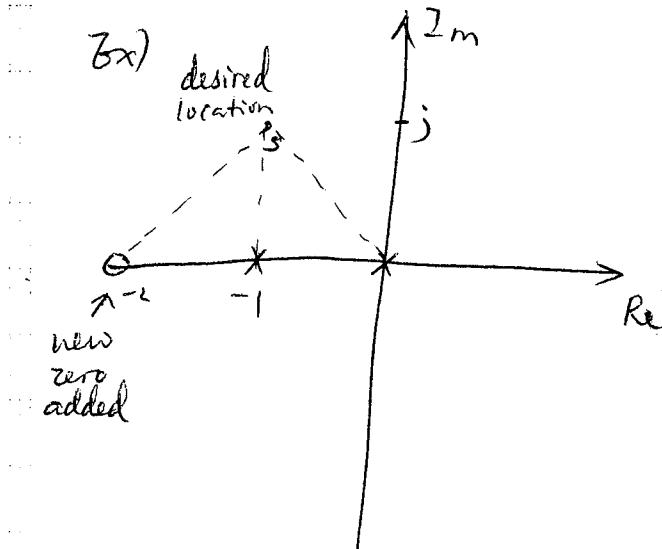
(30)

The idea of Compensation:

If a point s is not on the existing root-locus, the angle of $G(s)$ is not 180° :

$$\angle G(s) = \sum_{j=1}^m \angle(s - z_j) - \sum_{i=1}^n \angle(s - p_i) \neq 180^\circ$$

The difference $180^\circ - \angle G(s)$ is the amount of compensation required to adjust the root-locus so that it passes through s .



$$\begin{aligned}\angle G(s) &= -135^\circ - 90^\circ = -225^\circ \\ &= 135^\circ\end{aligned}$$

Compensation required:
add 45° .

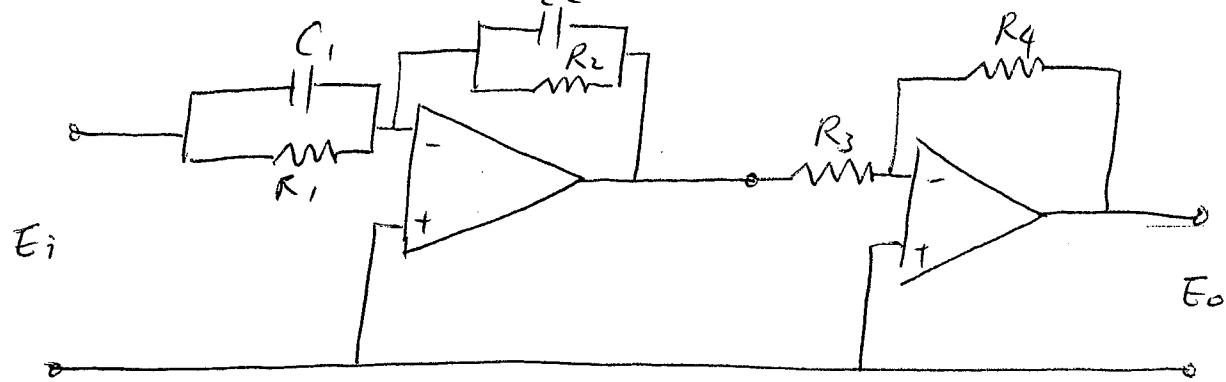
Possible solution: add a zero at -2 . Now $\angle G(s)$ is $45^\circ - 135^\circ - 90^\circ = 180^\circ$.

In summary, for points above the real axis, an additional zero adds angle to $\angle G(s)$, and an additional pole subtracts angle from $\angle G(s)$.

The example is typical: often we need to add an angle to move the root-locus to the left (i.e., make the system more stable and have better transient response). We then need to add ~~an~~ a zero.

(3)

However, in practice it is difficult to only add a zero (involving differentiation). Instead, it is easier to add a pair of pole & zero.

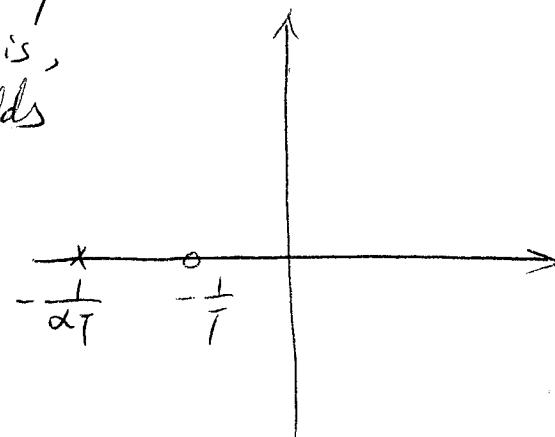


$$\begin{aligned}\frac{E_o(s)}{E_i(s)} &= \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}} \cdot \frac{R_4 C_1}{R_3 C_2} \\ &= K \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}\end{aligned}$$

where $K = \frac{R_4 C_1}{R_3 C_2}$ $T = R_1 C_1$, $\alpha T = R_2 C_2$

- ① If $\alpha < 1$, lead-compensator
For s above the real axis,
a lead compensator adds
an angle to $\angle G(s)$

"zero leads the pole"
like adding a zero.



Use of a lead-compensator

- Provide the amount of compensation required to move the root-locus to the left so that it passes through the desired location.
- $\frac{1}{T}, \alpha$ — determined by the angle of compensation
- K — determined by magnitude condition.

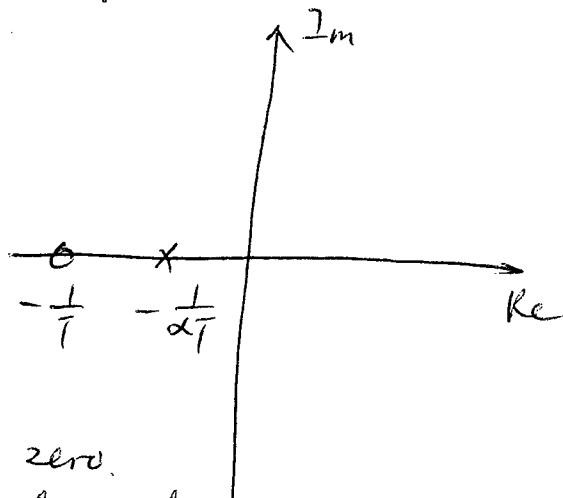
Potential drawback

- When α is small, the DC-gain decreases
 \Rightarrow larger steady-state error

② If $\alpha > 1$ Lag-compensator.

For s above the real-axis,

a lag-compensator subtracts an angle from $\angle G(s)$.



We often pick $\frac{1}{T}$ very small, so that this angle is close to zero.
 Thus the root-locus is not changed much.

Use of a lag-compensator

- Pull up the DC-gain by α
 \Rightarrow smaller steady-state error

Potential drawback

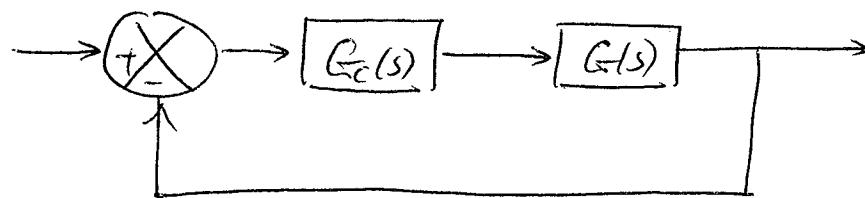
- Add a slowly-decaying transients $e^{-\frac{1}{\alpha T} t}$.

Design with Lead & Lag-compensators P 421- 450

Basic idea:

- use lead-compensator to move the closed-loop pole to the desired location
⇒ improve transient response
- use lag-compensator with small zeros & poles to improve the DC-gain
⇒ improve steady-state performance.

Lead-compensator P 421- 428



$$G_c(s) = K_c \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \quad 0 < \alpha < 1$$

Used to move the root-loans to the desired location.

Steps:

- ① Determine the desired location for the dominant closed-loop poles from the performance specifications.

- ② By sketching the root-locus of the uncompensated system, check whether adjusting the gain K alone can yield the desired closed-loop poles.

If not, calculate the angle deficiency
 $\phi = 180^\circ - \angle G(j\omega)$

- ③ Determine the value of $\alpha & T$ of the lead-compensator that compensates the angle deficiency ϕ .

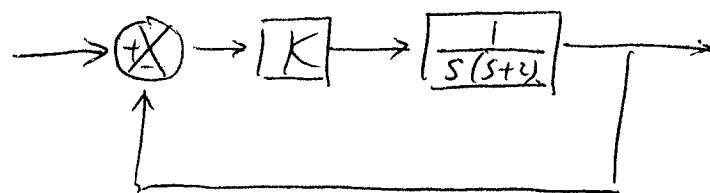
- ④ Determine the value of K_c through the magnitude condition.

- ⑤ Check steady-state error.

May add a lag-compensator if the steady-state error is too large.

Ex) P424 in text

Original System is



Design the controller such that
 $\omega_n = 4$ & $T = 0.5$.

Step 1: The desired location of the closed-loop pole is

$$s = (-\gamma \pm j\sqrt{1-\gamma^2})\omega_n = -2 \pm j2\sqrt{3}$$

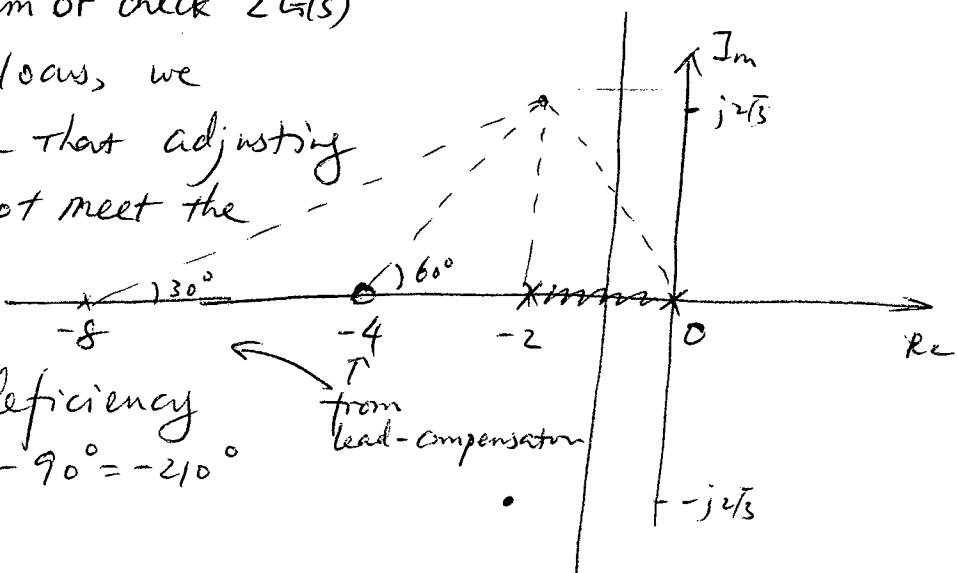
Step 2: Sketch the root-locus of the uncompensated system or check $\angle G(s)$

From the root-locus, we

can determine that adjusting K alone cannot meet the design spec.

Find angle deficiency
 $\angle G(s) = -120^\circ - 90^\circ = -210^\circ$
 $= 150^\circ$

$$\phi = 180^\circ - \angle G(s) = 30^\circ$$



Step 3: Determine the value of α, T in the lead-compensator $K \frac{s + \frac{T}{\alpha}}{s + \frac{1}{\alpha T}}$ that compensates the angle deficiency $\phi = 30^\circ$.

There are many choices. The textbook has a procedure for choosing (α, T) with the largest α (p425). We use a simpler procedure. Assume that you have been given the additional zero at -4 . Where should be the pole?

(38)

Since the additional zero at -1 produces an angle 60° , we need an additional pole to produce -30°

\Rightarrow the pole should be at

$$-2 - \frac{2\sqrt{3}}{\tan 30^\circ} = -8$$

The lead-compensator is then

$$K \frac{s+4}{s+8}$$

$$\text{where } T = \frac{1}{4}, \alpha = \frac{1}{2}$$

Step 4: Determine the value of K

$$K = \frac{\text{distances to the poles}}{\text{distances to the zeros}}$$

$$= \frac{2\sqrt{3} \times 4 \times 4\sqrt{3}}{4} = 24$$

The lead-compensator is then

$$24 \frac{s+4}{s+8}$$

Step 5: Check steady-state error constants

$$K_v = \lim_{s \rightarrow 0} s G_c(s) G(s)$$

$$= \lim_{s \rightarrow 0} 24 \frac{s+4}{s+8} \cdot \frac{1}{s+2}$$

$$= \frac{24 \times 4}{8} \cdot \frac{1}{2} = 6$$

We can use MATLAB to compare the root-locus & transient response of the new & old system

Original system :

Need $K=4$ for $f=0.5$. Root-locus $1 + \frac{K}{s(s+2)} = 0$
Closed-loop transfer function is

$$\frac{\frac{4}{s(s+2)}}{1 + \frac{4}{s(s+2)}} = \frac{4}{s^2 + 2s + 4}$$

New system :

Root-locus

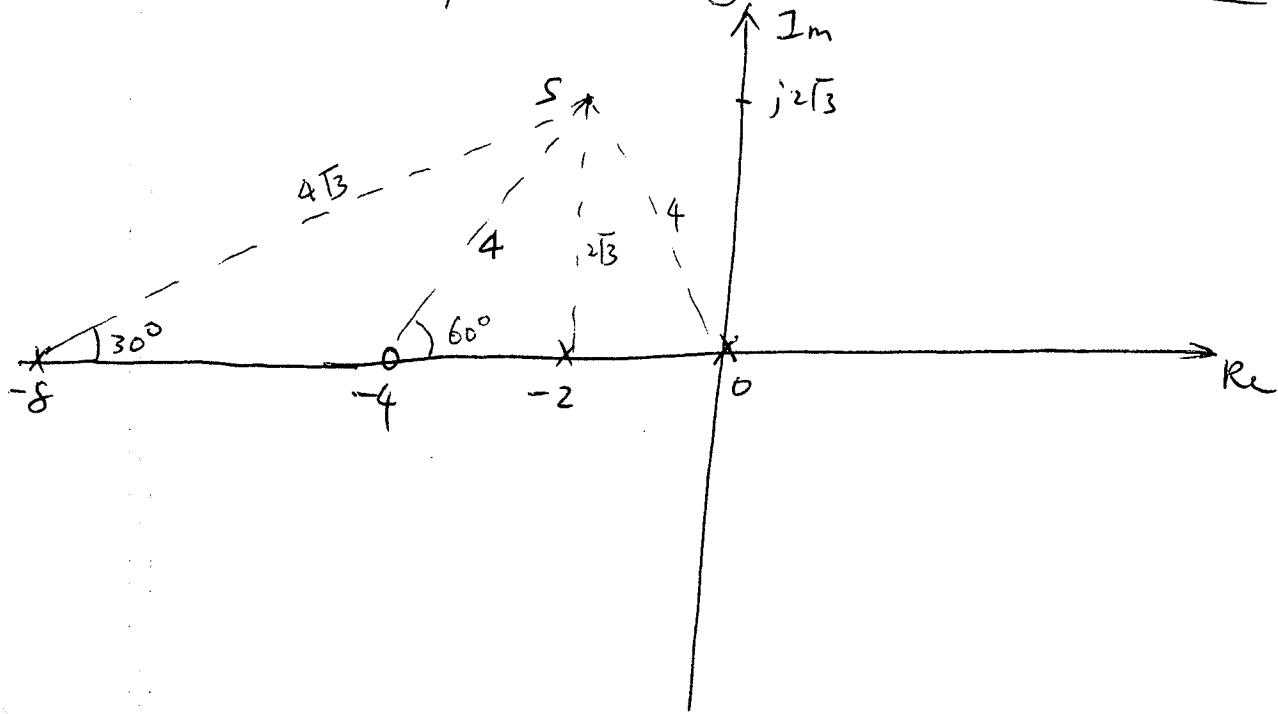
$$1 + K \cdot \frac{s+4}{s(s+8)(s+2)} = 0$$

Closed-loop transfer function ($K=24$)

$$\frac{\frac{24(s+4)}{s(s+8)(s+2)}}{1 + \frac{24(s+4)}{s(s+8)(s+2)}} = \frac{24(s+4)}{s^3 + 10s^2 + 40s + 96}$$

(37)-b

Worksheet for Designing Lead-Compensator



Desired closed-loop pole: $-2 \pm j2\sqrt{3}$

Open loop zeros	angle	distance
none	0°	1

new: -4

$$\text{sum: } \frac{+60^\circ}{0^\circ \quad 60^\circ} \quad 4$$

Open loop poles	angle	distance
0	120°	4

-2

90° $2\sqrt{3}$

new: -8

$$\text{sum: } \frac{+30^\circ}{210^\circ \quad 240^\circ} \quad 4\sqrt{3}$$

Total angle $\angle G(s) = -210^\circ + 180^\circ = 180^\circ$ ok.
 $\phi = 180^\circ - \angle G(s) = 390^\circ - 30^\circ = 30^\circ$

$$K = \frac{4 \times 2\sqrt{3} \times 4\sqrt{3}}{1 \times 4} = 24.$$