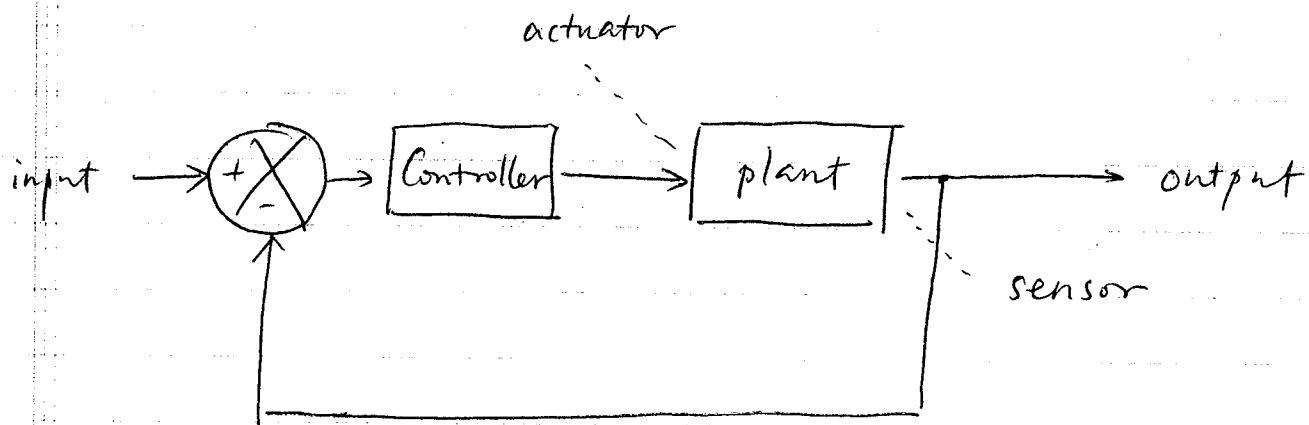
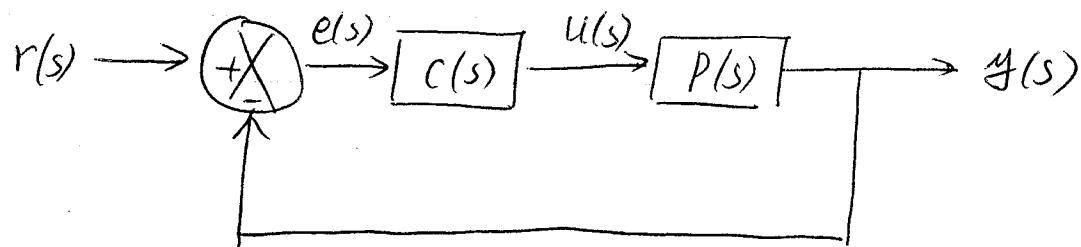


Common Types of Feedback Controllers p 281-287

Recall the basic control diagram



In typical control problems, we are given the transfer function of the target system $P(s)$. We are required to choose the right controller $C(s)$ to meet the design specifications.



Common types of controllers are

- ① Proportional controller $C(s) = K$
- ② Integral controller $C(s) = \frac{K_I}{s}$
- ③ Derivative controller $C(s) = K_D s$
- ④ PID controller
$$\begin{aligned} C(s) &= K_p + \frac{K_I}{s} + K_D s \\ &= K_p \left(1 + \frac{1}{T_2 s} + T_1 s \right) \end{aligned}$$

$\uparrow \quad \uparrow$
time-constants

It is said that "90% of controllers in industry are PID controllers".

We are interested in how these controllers (or their parameters K_p, K_i, K_d) affect the system performance.

- steady-state error
- stability : is the system stable? how far the parameters can be tuned to make the system unstable?
- oscillation in the transient response

Overview of results

	Steady-state error	Stability	Oscillation
Proportional feedback $K_p \uparrow$	↓	↓	↑
Integral feedback $K_i \uparrow$	↓	↓	↑
Derivative feedback $K_d \uparrow$	↑	↑	↓

Main idea

- Steady-state error: depends on the type of the open-loop transfer function $C(s)P(s)$, and the limit $\lim_{s \rightarrow 0} s^k C(s) P(s)$.
- Stability: Routh test
- Oscillation: depends on the damping ratio ξ

① Proportional controller $C(s) = K$

Controller action is proportional to error

$$U(s) = K E(s)$$

Ex 1) $P(s) = \frac{1}{1+Ts}$

ⓐ With step-input, the steady-state position error constant

$$K_p = \lim_{s \rightarrow 0} P(s) C(s) = K$$

Hence, the steady-state error for step input is

$$C_0 = \frac{1}{1+K_p} = \frac{1}{K+1}$$

The larger K is, the smaller the steady-state error.

ⓑ The end-to-end transfer function is

$$H(s) = \frac{Y(s)}{R(s)} = \frac{K}{(1+K)+Ts} = \frac{\frac{K}{1+K}}{1 + \frac{T}{1+K}s}$$

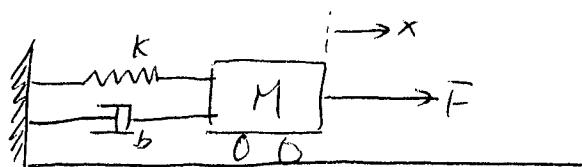
The system is always stable.

It has no oscillation.

↑ time-constant
of the closed-loop system.

The larger K is, the faster the response.

Ex 2) $P(s) = \frac{1}{s^2 + 2f\omega_n s + \omega_n^2} \quad 0 < f < 1$



$$F - kx - bx' = M\ddot{x}$$

$$\Rightarrow \frac{X(s)}{F(s)} = \frac{1}{MS^2 + bs + K}$$

(47)

- ④ With step-input, the steady-state position error constant

$$K_p = \frac{k}{\omega_n^2}$$

$$\Rightarrow C_0 = \frac{1}{1 + K_p} = \frac{1}{1 + \frac{k}{\omega_n^2}}$$

The larger K is, the smaller the steady-state errors.

- (b) The end-to-end transfer function is

$$H(s) = \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{k}{s^2 + 2\zeta\omega_n s + (\omega_n^2 + k)}$$

$$\text{Hence, } \omega_{n,\text{new}} = \sqrt{\omega_n^2 + K}$$

$$f_{\text{new}} = f \frac{\omega_n}{\sqrt{\omega_n^2 + K}}$$

Two poles at $-f \omega_n \pm j \sqrt{\omega_n^2(1-f^2) + K}$

The system is always stable.

The larger k is:

- $f_{new} V$, Overshoot $M_p \uparrow$

The system becomes more oscillatory (closer to ~~to~~ instability)

- W_d , new \uparrow , rise time \downarrow , peak time \downarrow
 - $T_{new} = T$, the settling time is not changed.

These observations are typical for higher order systems: Increase the gain K of proportional controller leads to faster response, but decreased damping \Rightarrow larger overshoot and even instability.

② Integral Controller $C(s) = \frac{K_I}{s}$

Control action is the integral of the error signal.

a) If $P(s)$ is a Type- N system, then $C(s)P(s)$ is a type- $(N+1)$ system.

\Rightarrow Improve steady-state tracking capability
steady-state error \checkmark

Intuitively, since the controller action $u(t)$ is the integral of the error signal $e(t)$, $e(t)$ must go to zero as $t \rightarrow +\infty$, otherwise $u(t)$ will be infinite!

b) Primary disadvantage: reduced stability

$$\text{Ex)} \quad P(s) = \frac{1}{s^2 + as + b} \quad C(s) = \frac{K}{s}$$

$$H(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{K}{s^3 + as^2 + bs + K}$$

Routh array:

$$s^3 \quad 1 \quad b$$

$$s^2 \quad a \quad K$$

$$s^1 \quad \frac{ab - K}{a}$$

$$s^0 \quad K$$

If K is too large, $b - \frac{K}{a}$ might be negative
 \Rightarrow system unstable.

(49)

③ Derivative controller $c(s) = K_s s$

Control action is the derivative of the error.

Opposite to integral controller:

- primary advantage: increase damping ratio and improve stability
- main disadvantage: lower system type, larger steady-state errors.

$$\text{Ex) } P(s) = \frac{1}{s^2 + as + b} \quad c(s) = Ks$$

(a) With step-input, the steady-state position error constant $K_p = \lim_{s \rightarrow 0} P(s) c(s) = 0$
 $\Rightarrow C_0 = 1$

④ The end-to-end transfer function

$$H(s) = \frac{Ks}{s^2 + (a+k)s + b}$$

$$\hat{\omega}_n = \sqrt{b} \quad \text{unchanged}$$

$$\hat{\zeta} = \frac{a+k}{2\sqrt{b}}$$

Improved stability: even if $a < 0$, $\hat{\zeta}$ can be positive with large k

Reduced oscillation: the larger k is, the larger the damping ration $\hat{\zeta}$.

(50)

④ PID Controller

A linear combination of Proportional, Integral and Derivative controllers.

$$c(s) = K_p + \frac{K_I}{s} + K_D s$$

$$= K_p \left(1 + \frac{1}{T_2 s} + T_D s \right)$$

$\nwarrow \uparrow$
time-constants

Choice of parameters K_p , K_I , K_D is called "tuning a PID controller". The general rule is a combination of the discussions earlier.

