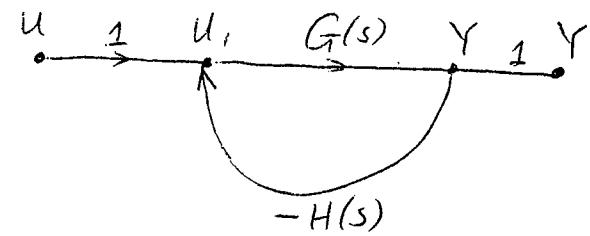
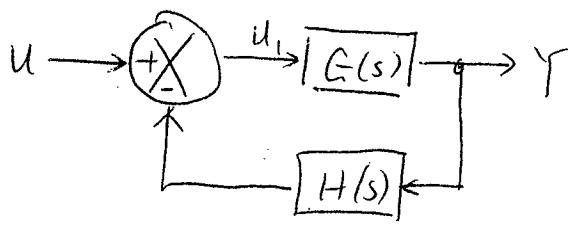


Signal Flow Graphs

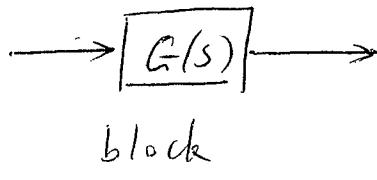
P104-P112

SFG provide an ~~alternate~~ alternate (and equivalent) way to represent block diagrams. Advantage is that there is a formula called "Mason's rule" that enables us to compute transfer functions between any input & output.

Example:

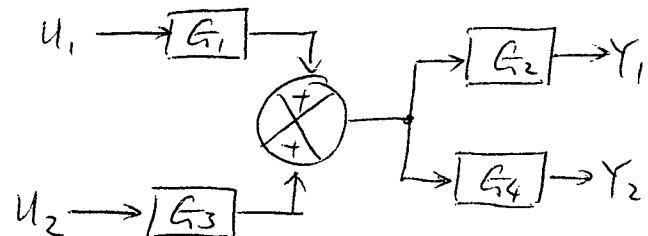
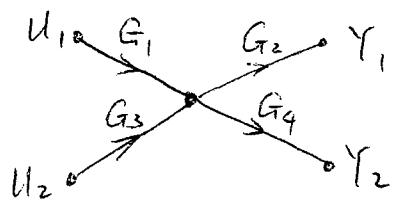


Basics: represent a block in a block diagram by a branch in SFG.



In SFGs:

- ① Node: represents a signal
- ② Transmittance: "gain" between two nodes (i.e., the transfer function).
- ③ Branch: a directed line segment connecting two nodes.
Gain between nodes is written below or above the arrow.
- ④ Input node: node with only outgoing branches
- ⑤ Output node: node with only incoming branches
- ⑥ Mixed node: has both incoming & outgoing branches.
A mixed node adds all incoming branches and transmits this to all outgoing branches.



- ⑦ A mixed node can be made into an output node by adding an outgoing branch of gain 1.

- ⑧ Path: a traversal of connected branches in the direction of the branch arrows.

$$U \rightarrow U_1 \rightarrow Y$$

$$U \rightarrow U_1 \rightarrow Y \rightarrow U_1$$

- ⑨ Loop or closed path: a path that ends at the beginning node and does not cross any other

node more than once

$$U_i \rightarrow Y \rightarrow U_j$$

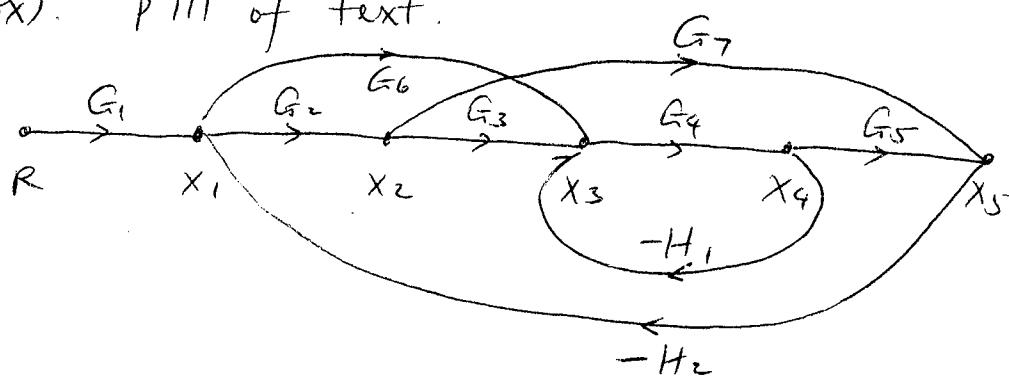
- (i) Loop gain: the product of the branch transmittance of a loop.
- (ii) Non-touching loops: multiple loops with no common nodes
- (iii) Forward path: path from the input node to output node with no node encountered more than once.

Mason's rule

Based on determinant Δ of a graph.

$$\begin{aligned} \Delta = & 1 - (\text{sum of all individual loop gains}) \\ & + (\text{sum of gain products of all combinations of two non-touching loops}) \\ & - (\text{sum of gain products of all combinations of three non-touching loops}). \end{aligned}$$

Ex). p 111 of text.



(JH)

<u>Loops</u>	<u>Gains</u>
$L_1 : X_1 X_2 X_3 X_4 X_5 X_1$	$- G_2 G_3 G_4 G_5 H_2$
$L_2 : X_3 X_4 X_3$	$- G_4 H_1$
$L_3 : X_1 X_2 X_5 X_1$	$- G_2 G_7 H_2$
$L_4 : X_1 X_3 X_4 X_5 X_1$	$- G_6 G_4 G_5 H_2$

(Q) is $X_1 X_3 X_4 X_3$ a loop?

Two non-touching loops

Gains

$L_2 \& L_3$

$G_2 G_4 G_7 H_1 H_2$

Hint: You can tell whether loops touch each other by looking at the node sequence

Hence, the determinant is

$$\Delta = 1 - \left[- G_2 G_3 G_4 G_5 H_2 - G_4 H_1 - G_2 G_7 H_2 - G_6 G_4 G_5 H_2 \right] + G_2 G_4 G_7 H_1 H_2$$

$\nwarrow_{L_2 \& L_3}$

Procedures of carrying out Mason's rule

Suppose we want to compute the end-to-end transfer function of a SFG. The steps are

- ① Compute Δ , the determinant of the SFG

- ② Find all forward paths and the associated forward path gains
 P_1, P_2, \dots, P_k (K is the number of forward paths)
- ③ For each forward path $P_i, i=1, \dots, k$ find the determinant Δ_i of the SFG gotten by removing ~~the~~ from Δ the terms that correspond to loops touching path P_i

Δ_i — cofactor of the forward path P_i

- ④ Then the required transfer function is

$$H = \frac{1}{\Delta} \sum_{i=1}^k P_i \Delta_i$$

Continuing the earlier example

	<u>Forward Path</u>	<u>Gains</u>	<u>Cofactor</u>
P_1	$R X_1 X_2 X_3 X_4 X_5$	$G_1 G_2 G_3 G_4 G_5$	1
P_2	$R X_1 X_2 X_5$	$G_1 G_2 G_7$	$1 - (-G_4 H_1)$
P_3	$R X_1 X_3 X_4 X_5$	$G_1 G_6 G_4 G_5$	1

Hint: To compute the cofactor of a path, check whether the path has a common node with each loop, and keep those terms of Δ that

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correspond to non-touching loops.

Hence

$$H(s) = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_7 (1 + G_4 H_1) + G_1 G_6 G_4 G_5}{1 + G_2 G_3 G_4 G_5 H_2 + G_4 H_1 + G_2 G_7 H_2} \\ + G_6 G_4 G_5 H_2 + G_2 G_4 G_7 H_1 H_2$$

Additional examples in A-3-24, p142 of text.
 You can also try A-3-25, but do not follow the reduction approach.