

## Steps for lead-compensator design p623

Recall the key equations of lead-compensators

- \* Maximum phase-lead

$$\phi_m = \sin^{-1} \frac{1-\alpha}{1+\alpha}$$

- \* Corresponding frequency

$$\omega_m = \frac{1}{T\alpha}$$

- \* Corresponding gain

$$\left| \frac{1+j\omega T}{1+j\alpha\omega T} \right|_{\omega_m} = \frac{1}{\sqrt{\alpha}}$$

### Steps for Lead compensator design

(given the desired steady-state error constant & phase margin)

Step 1 : Assume the lead-compensator

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

$$G_c(j\omega) = \frac{K_c \alpha}{j\omega} \frac{1 + j\omega T}{1 + j\alpha\omega T}$$

Determine gain  $K$  to satisfy the requirement on the given steady-state error constant.

Step 2: Draw Bode-plot of  $KG(s)$ , i.e., the gain-adjusted but uncompensated system.  
Evaluate the phase margin.

Step 3: Determine the necessary phase-lead angle to be added to the system. Add an additional  $5^\circ$  to  $12^\circ$  to the phase-lead angle required, because the addition of the lead compensator shifts the 0dB cross-over frequency to the right and will tend to reduce the phase margin a little bit.

Step 4: Use  $\phi_m = \sin^{-1} \frac{1-\alpha}{1+\alpha}$  to determine the value of  $\alpha$ .

(26)

Step 5: Find the frequency where the magnitude of  $KG(s)$  is  $\frac{1}{\alpha}$  (i.e.,  $20 \log_{10} \frac{1}{\alpha} \text{ dB}$ ).

We will make this frequency the new 0dB cross-over frequency. (Recall that the gain of the lead-compensator is  $\frac{1}{\alpha}$  when the maximum phase shift  $\phi_m$  occurs.)

Use  $\omega_m = \frac{1}{\alpha T}$  to determine the value of  $\frac{1}{\alpha}$ .

Step 6: Determine the parameters of the lead-compensator based on  $\alpha$  and  $T$ .

$$\text{zero: } -\frac{1}{T}$$

$$\text{pole: } -\frac{1}{\alpha T}$$

$$\text{gain } K_c = \frac{K}{\alpha}$$

Step 7: Check the gain margin of the compensated system to be sure that it is satisfactory. Otherwise, need to increase the phase-lead, or use lag-compensators.

(27)

Example: 9-1 - p624

$$G(s) = \frac{4}{s(s+2)}$$

Need static velocity error constant  $K_v = 20 \text{ sec}^{-1}$   
and phase margin  $> 50^\circ$ , gain margin  $> 10 \text{ dB}$ .

Step 1: Use lead-compensator

$$G_c(s) = K_c \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

$$G_c(j\omega) = \frac{K_c \alpha}{K} \frac{1 + j\omega T}{1 + j\alpha\omega T}$$

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} s \cdot G_c(s) G(s) \\ &= \lim_{s \rightarrow 0} s \cdot K_c \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \cdot \frac{4}{s(s+2)} \\ &= K_c \cdot \alpha \cdot 2 = 20 \\ \Rightarrow K &= K_c \alpha = 10 \end{aligned}$$

Step 2: Draw the Bode-plot for

$$KG(s) = \frac{40}{s(s+2)}$$

The phase margin is  $17^\circ$

Step 3: In order to increase PM to  $50^\circ$ ,  
we need a phase-lead of  $50^\circ - 17^\circ = 33^\circ$   
Add additional angle of  $5^\circ$

$$\phi_m = 33^\circ + 5^\circ = 38^\circ$$

This has to be provided by the lead-compensator.

Step 4: Use  $\phi_m = -1 - \frac{1-\alpha}{1+\alpha}$   
 $\Rightarrow \alpha = 0.24$

Step 5: Find the frequency where the magnitude of  $KG(s)$  is  $\sqrt{\alpha} = \sqrt{0.24} = -6.2 \text{ dB}$   
 $\Rightarrow \omega_m = 9 \text{ sec}^{-1}$

Select this as the new 0dB cross-over frequency, and it is also the frequency corresponding to the maximum phase-lead  $\phi_m$ .

Use  $\omega_m = \frac{1}{\sqrt{2T}} = 9$

$$\Rightarrow \frac{1}{T} = 9 \cdot \alpha = 4.41$$

Step 6: The parameters of the lead-compensator is then:

zero  $-\frac{1}{T} = -4.41$

pole  $-\frac{1}{\alpha T} = -18.4$

gain  $K_C = \frac{K}{\alpha} = \frac{10}{0.24} = 41.7$

$$\therefore G_C(s) = 41.7 \frac{s + 4.41}{s + 18.4} = 10 \cdot \frac{1 + 0.227s}{1 + 0.054s}$$

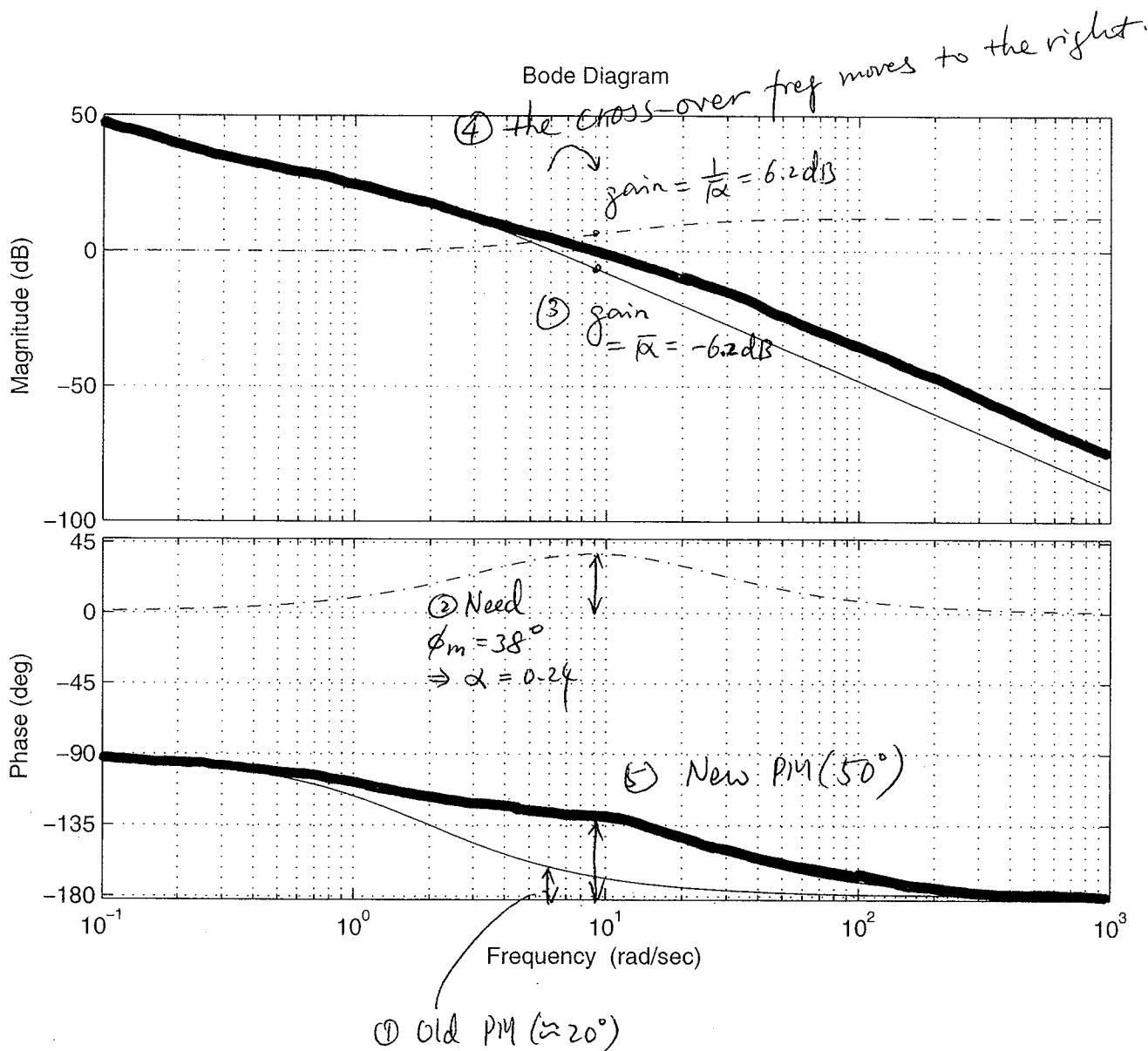
Step 7: Plot the Bode-plot of

$$G_C(s) G(s) = 41.7 \frac{s + 4.41}{s + 18.4} \cdot \frac{4}{s(s+2)}$$

We find  $PM = 50^\circ$ ,  $GM = +10$

(28)-B

# Curve-shaping with a lead-compensator



Remark : If the required phase-lead is too large ( $> 65^\circ$ ), it is difficult to achieve with lead-compensators. We should then use a lag-compensator, or lag-lead compensator.

## Steps for Lag-Compensator Design P63

Step 1: Assume the following lag-compensator

$$G_C(s) = K_C \frac{s + \frac{1}{T}}{s + \frac{\beta}{T}}$$

$$G_C(j\omega) = \frac{K_C \beta}{K} \frac{1 + j\omega T}{1 + j\beta\omega T}$$

Determine  $K$  such that  $KG(s)$  satisfy the requirement on the given steady-state error constant.

Step 2: Draw the Bode-plot of  $KG(s)$ , i.e., the gain adjusted but uncompensated system. Evaluate the phase margin.

Step 3: If original phase margin is too small, add  $5^\circ$  or  $12^\circ$  to the required PM. Then find the frequency where the phase of  $KG(s)$  satisfies the required PM.

Select this frequency as the new 0dB cross-over frequency.

Step 4: Choose the corner frequency  $\frac{1}{T}$  of the lag-compensator to be one decade below the new 0dB cross-over frequency.

Find  $T$ .

(31)

Step 5: Find the magnitude of  $KG(s)$  at the new cross-over frequency.

This magnitude must be equal to  $20 \log \beta$  because we will use the lag-compensator to attenuate the gain at this frequency to 0dB.

Find the value of  $\beta$ . The other corner frequency  $\frac{1}{\beta T}$  is also determined

Finally, find  $K_c = \frac{k}{\beta}$

Example 9-2 : p633

$$G(s) = \frac{1}{s(s+1)(0.5s+1)}$$

Need  $K_v = 5 \text{ sec}^{-1}$ ,  $\text{PM} > 40^\circ$ ,  $\text{GM} > 10 \text{ dB}$

Step 1: Assume a lag-compensator

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

$$G_c(j\omega) = \frac{K_c \beta}{K} \frac{1 + j\omega T}{1 + j\beta\omega T}$$

$$K_v = \lim_{s \rightarrow 0} s \cdot G_c(s) G(s) = \lim_{s \rightarrow 0} s \cdot K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \cdot \frac{1}{s(s+1)(0.5s+1)} \\ = K_c \beta = 5$$

$$\therefore K = 5$$

Step 2: Draw the Bode-plot of

$$KG(s) = \frac{5}{s(s+1)(0.5s+1)}$$

The phase-margin is  $-20^\circ$ . The system is unstable and it would be difficult to increase the PM to  $40^\circ$  using a lead-compensator.

Step 3: Since we need  $\text{PM} = 40^\circ$ ,

adding  $12^\circ$  ~~to get~~ makes the required  $\text{PM} = 52^\circ$

Find the frequency such that the angle of  $KG(s)$  is  $-180^\circ + 52^\circ = -128^\circ$   
 $\Rightarrow \omega = 0.5 \text{ sec}^{-1}$

This will be the new cross-over frequency.

Step 4: Choose the corner frequency of the lag-compensator at

$$\frac{1}{T} = 0.1 \text{ sec}^{-1}$$

(This is about one decade below  $\omega$ , although not quite accurate.)

Step 5: The magnitude of  $KG(s)$  at  $\omega = 0.5 \text{ sec}^{-1}$  is  $20 \text{ dB} = 10$ . This is to be attenuated to 0 dB. Hence, we need

$$\beta = 10$$

Hence, the other corner-freq of the lag-compensator is

$$\frac{1}{\beta T} = 0.01$$

$$\text{and } K_C = \frac{K}{\beta} = 0.5$$

$\therefore$  The lag-compensator is  
 $G_C(s) = 0.5 \frac{s+0.1}{s+0.01}$

When we draw the Bode-plot for

$$G_C(s) G(s) = 0.5 \frac{s+0.1}{s+0.01} \cdot \frac{1}{s(s+1)(0.5s+1)}$$

we have  $PM = 40^\circ$ ,  $GM = 11 \text{ dB}$ ,  $K_V = 5 \text{ sec}^{-1}$ .

# Curve - shaping with a lag - compensator

(34)

Bode Diagram

