ECE-382: Homework 9

Due: March 21, 2007

1. Problems B-6-1, B-6-3, B-6-6 and B-6-9 in textbook. For Problem B-6-9, just plot the root-locus, and you do not need to work on the second part of the problem, i.e., finding the gain K that corresponds to damping ratio 0.5. *Hint for Problem B-6-6*: Write the closed-loop poles as functions of K and show that they form a circle.

   Note: all root loci must be plotted by hand. In particular, you need to specify the following details of the root locus: open-loop poles/zeros, the segment on the real axis, the asymptotes, intersection with the imaginary axis, break-in/breakaway points, and departure/arrival angles at open-loop poles/zeros.

2. Recall the car suspension system that we introduced in Homework 5. (You may want to look at Homework 5 again for some of the details.) Recall that the transient response of the system was quite bad: when there is a bump, the body of the car will experience large oscillation, and the settling time is long. Now let us say we want to design an *active suspension* system to improve the transient response. The active suspension system produces a force u, which may depend on the value of x_1, x_2 and W (see Fig. 2). The relationship between the output $y = x_1 - x_2$, the input w, and the control u is given by:

$$Y(s) = G_1(s)U(s) + G_2(s)W(s),$$

where

$$G_1(s) = \frac{(M+m)s^2 + b_2 s + k_2}{\Delta}$$

$$G_2(s) = \frac{-Mb_2 s^3 - Mk_2 s^2}{\Delta}$$

and

$$\Delta = (Ms^2 + b_1 s + k_1)[ms^2 + (b_1 + b_2)s + (k_1 + k_2)] - (b_1 s + k_1)^2.$$

Now suppose we want to design a controller that computes the force u based on the output y. In other words,
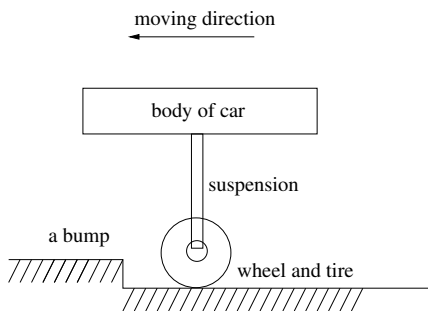
$$U(s) = -C(s)Y(s).$$

1

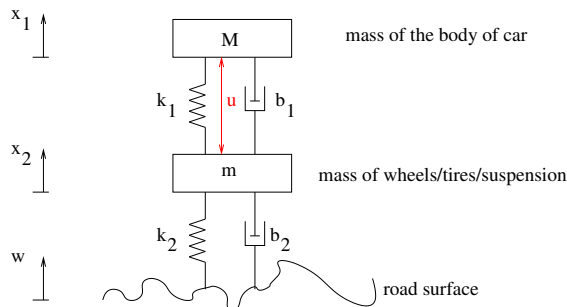Figure 1: A car suspension system.

Figure 2: One-dimensional mechanical model of an active suspension system.
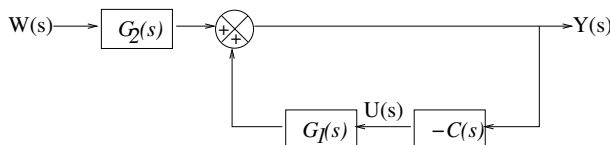

Figure 3: The block diagram of the active suspension system.

Note the sign of the feedback: when $y > 0$, the force u should be negative in order to reduce the displacement. The overall block diagram of the system is shown in Fig. 3.

Suppose we would like to use a PID controller:

$$C(s) = K_P + \frac{K_I}{s} + K_D s.$$

The goal of the this assignment is to give you some experience in tuning the parameters of the PID controller. You goal is to tune the PID controller such that, with a step input (i.e., the car goes through a bump), the maximum overshoot is less than 5%, and the settling time is less than 5 seconds. Thus, when the car runs onto a 10cm bump, the car body will oscillate within a range of $+/- 5$mm and will stop oscillating within 5 seconds.

The parameters of the system are given by: $M = 2500kg$, $m = 320kg$, $k_1 = 80000N/m$, $k_2 = 500000N/m$, $b_1 = 350Ns/m$, $b_2 = 15020Ns/m$.

- Derive the end-to-end transfer function of the closed-loop system $\frac{Y(s)}{W(s)}$.
- Write a MATLAB program that plots the step response of the close-loop system. You may start from the following parameters for the PID controller: $K_P = 832100$, $K_I = 624075$, $K_D = 208025$. Does the system performance meet your goals? (*Hint:* You will notice that, even with a step input, the response of the system converges to zero in the end. Hence, you can take the maximum overshoot as the

maximum magnitude of the response, and take the settling time as the time when the magnitude is very small, e.g., less than $5\% \times 2\% = 0.1\%$.)

- Which parameter should you tune to improve the system performance? Plot your new step response.

- Can you tune the PID controller so that the maximum overshoot is less than 5%, and the settling time is less than 1 second? (It is okay if you cannot achieve the design goal. In that case, submit the best controller you can find. Later in the class we will provide a more systematic design approach based on root-locus.)

(*Hint:* This problem is adapted from the website http://www.engin.umich.edu/group/ctm/. I would strongly encourage you to work on the problem independently before you look at the website.)