ECE 5314: Power System Operation & Control

Lecture 9: Secure Power System Operation

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R1 A. J. Wood, B. F. Wollenberg, and G. B. Sheble, Power Generation, Operation, and Control, Wiley, 2014, Chapter 7.

R2 A. Gomez-Exposito, A. J. Conejo, C. Canizares, *Electric Energy Systems: Analysis and Operation*, Chapters 6.

Power system security

Power system components can experience outages

- generators go offline
- transmission lines trip
- transformers fail

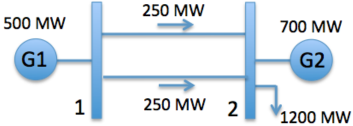
(N-1) reliability rule: system should be safely operating even if any single component fails; set by North American Electric Reliability Corp. (NERC)

Major power system security functions:

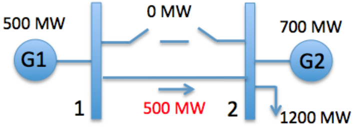
1. power system state estimation (see Lecture 11)
2. contingency analysis
3. security-constrained OPF (SC-OPF)

Normal dispatch

Assume double-circuit line with 400 MW limit



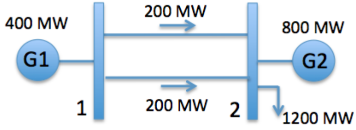
Normal dispatch: system is economically optimal, but not necessarily secure



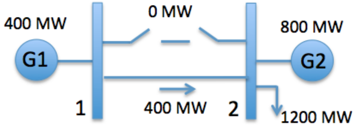
Post-contingency state: due to a single-component failure, operational constraints are now violated

Secure dispatch

Assume double-circuit line with 400 MW limit



Secure dispatch: security at the expense of increased cost



Secure post-contingency state: no line overloads

Contingency (*what-if*) analysis

Approximate but fast method for checking the effect of outages

Generator outage: if a generator fails, other generators take over

Line outage: *cascading failure example*

- single line trips (tree contact or insulation failure)
- flows automatically re-distributed over network
- other lines exceed their flow limits and trip
- phenomenon travels fast (few min to an hour for entire ISO footprint)

Linear sensitivity factors: model effect of above events to line flows

- Power Transfer Distribution Factors (PTDF)
- Line Outage Distribution Factors (LODF)

Review of DC power flow model

- Branch-bus incidence matrix (bus 1 as reference *and* slack bus)

$$\tilde{\mathbf{A}} = [\mathbf{a}_1 \;\; \mathbf{A}]$$

- If line $l : (i, j)$, then the l-th row of $\tilde{\mathbf{A}}$ is $\mathbf{a}_l = \mathbf{e}_i - \mathbf{e}_j$
- Bus reactance matrix $\tilde{\mathbf{B}} = \begin{bmatrix} B_{11} & \mathbf{b}_1^\top \\ \mathbf{b}_1 & \mathbf{B} \end{bmatrix}$
- Matrices $(\mathbf{B}, \mathbf{A})$ termed $(\mathbf{B}_r, \mathbf{A}_r)$ in Lecture 8; simpler notation here
- Flows relate linearly to injections $\mathbf{f} = \mathbf{S}\mathbf{p}$
- Matrix $\mathbf{S}$ describes how changes in $\mathbf{p}$ reflect on changes in $\mathbf{f}$

$$\Delta\mathbf{f} = \mathbf{S}\Delta\mathbf{p} \qquad \text{where} \qquad \mathbf{S} = [\mathbf{0} \;\; \mathbf{X}^{-1}\mathbf{A}\mathbf{B}^{-1}]_{L\times N}$$

Power transfer between two buses

- Assume GEN$_n$ goes offline and its power is picked up by GEN$_m$

$$\hat{p}_n = p_n + \Delta p_n \quad \text{and} \quad \hat{p}_m = p_m - \Delta p_n$$

  with $\Delta p_n = -p_n$

- Change in power injections ($\mathbf{e}_n$: n-th column of $\mathbf{I}$)

$$\Delta \mathbf{p} = \hat{\mathbf{p}} - \mathbf{p} = \Delta p_n(\mathbf{e}_n - \mathbf{e}_m)$$

- Change in all line flows

$$\Delta \mathbf{f} = \hat{\mathbf{f}} - \mathbf{f} = \mathbf{S}\Delta \mathbf{p} = \Delta p_n(\mathbf{S}\mathbf{e}_n - \mathbf{S}\mathbf{e}_m) = \Delta p_n(\mathbf{S}_{:,n} - \mathbf{S}_{:,m})$$

- Flow on line l changes by $\Delta f_l = \Delta p_n(S_{l,n} - S_{l,m})$

Finding entry $S_{l,n}$

- Interested in finding the (l,n)-th entry of

$$\mathbf{S} = \mathbf{X}^{-1}[\mathbf{0} \;\; \mathbf{A}\mathbf{B}^{-1}]$$

- Since l-th row of $\tilde{\mathbf{A}}$ is $\mathbf{a}_l = \mathbf{e}_i - \mathbf{e}_j$, then

$$\begin{aligned} S_{l,n} &= \frac{1}{x_l}(\mathbf{e}_i - \mathbf{e}_j)^\top [\mathbf{B}^{-1}]_{:,n} \\ &= \frac{[\mathbf{B}^{-1}]_{i,n} - [\mathbf{B}^{-1}]_{j,n}}{x_l} \end{aligned}$$

- Exceptions because $\mathbf{A}$ has the first column of $\tilde{\mathbf{A}}$ removed
  - if $n = 1$, then $S_{l,n} = 0$
  - if $j = 1$ and $n \neq 1$, then $S_{l,n} = \frac{[\mathbf{B}^{-1}]_{i,n}}{x_l}$
  - if $i = 1$ and $n \neq 1$, then $S_{l,n} = -\frac{[\mathbf{B}^{-1}]_{j,n}}{x_l}$

Power Transfer Distribution Factors

- How a power transfer from bus n to bus m affects flow on line l

$$\begin{aligned} \mathsf{PTDF}_{n,m,l} &= \frac{\text{flow change on line } l}{\text{power transfer from bus } n \text{ to } m} \\ &= \frac{\Delta f_l}{\Delta p_n} \\ &= S_{l,n} - S_{l,m} \end{aligned}$$

- It can be shown that $|\mathsf{PTDF}_{n,m,l}| \leq 1$

- Typically the outage of generator n is shared by multiple generators (AGC)

$$\Delta f_l = \sum_{m \neq n} \gamma_m \Delta p_n \mathsf{PTDF}_{n,m,l}$$

for participation factors $\gamma_m \geq 0$ and $\sum_{m \neq n} \gamma_m = 1$

Power transfers through reference bus

- Recall that $\text{PTDF}_{n,1,l} = S_{l,n}$ because $S_{l,1} = 0$
- Power transfer between two buses through the reference bus

$$\text{PTDF}_{n,m,l} = \text{PTDF}_{n,1,l} - \text{PTDF}_{m,1,l} = S_{l,n} - S_{l,m}$$

- Hence, no need to compute all $\text{PTDF}_{n,m,l}$
- Need only to compute $\text{PTDF}_{n,1,l}$; that is the entries of $\mathbf{S}$
- Power sharing after generator outage

$$\Delta f_l = \Delta p_n S_{l,n} - \sum_{m \neq n} \gamma_m \Delta p_n S_{l,m}$$

Line Outage Distribution Factors

- How flows change when a single line $k : (n \rightarrow m)$ is removed?

- How are flows redistributed under a transmission topology change?

- If line k was originally carrying f_k, the post-contingency flow on line l is

$$\hat{f}_l = f_l + \mathsf{LODF}_{l,k} f_k$$

- Two ways for finding $\mathsf{LODF}_{l,k}$
    1. compensation trick
    2. matrix inversion lemma

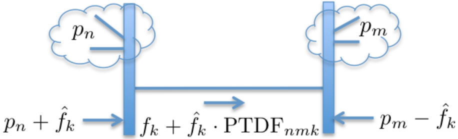
LODFs via the compensation trick

Pretend no line outage has happened

Modify (p_n, p_m) by $(\hat{f}_k, -\hat{f}_k)$ such that the new flow on line $k : (n, m)$ is $\hat{f}_k$

$$\hat{f}_k = f_k + \mathsf{PTDF}_{n,m,k}\hat{f}_k \quad \Rightarrow$$

$$\hat{f}_k = \frac{1}{1 - \mathsf{PTDF}_{n,m,k}} f_k$$



This implies that line k has no effect for the rest of the grid

Line outage has been interpreted as a power transfer $\tilde{f}_k$ from bus n to m

$$\Delta f_l = \mathsf{PTDF}_{n,m,l}\hat{f}_k = \frac{\mathsf{PTDF}_{n,m,l}}{1 - \mathsf{PTDF}_{n,m,k}} f_k$$

Result: $\mathsf{LODF}_{l,k} = \frac{\mathsf{PTDF}_{n,m,l}}{1 - \mathsf{PTDF}_{n,m,k}}$ for $l \neq k$; and $\mathsf{LODF}_{k,k} = -1$

LODFs via the matrix inversion lemma

- Applying the **matrix inversion lemma**

$$(\mathbf{A}+\mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}(\mathbf{C}^{-1}+\mathbf{DA}^{-1}\mathbf{B})^{-1}\mathbf{DA}^{-1}$$

- yields the inverse of the modified bus-reactance matrix

$$\hat{\mathbf{B}}^{-1} = \left(\mathbf{B} - \frac{1}{x_k}\mathbf{a}_k\mathbf{a}_k^\top\right)^{-1} = \mathbf{B}^{-1} + \frac{\mathbf{B}^{-1}\mathbf{a}_k\mathbf{a}_k^\top\mathbf{B}^{-1}}{x_k - \mathbf{a}_k^\top\mathbf{B}^{-1}\mathbf{a}_k}$$

- the post-outage flows are then

$$\hat{\mathbf{f}} = \mathbf{f} + \frac{\mathbf{X}^{-1}\mathbf{A}\mathbf{B}^{-1}\mathbf{a}_k\mathbf{a}_k^\top\mathbf{B}^{-1}\mathbf{p}_r}{x_k - \mathbf{a}_k^\top\mathbf{B}^{-1}\mathbf{a}_k}$$

- focusing on the l-th entry

$$\begin{aligned}\Delta f_l = \hat{f}_l - f_l &= \left(\frac{\mathbf{a}_l^\top\mathbf{B}^{-1}\mathbf{a}_k}{x_l}\right)\left(\frac{1}{1-\mathbf{a}_k^\top\mathbf{B}^{-1}\mathbf{a}_k/x_k}\right)\left(\frac{\mathbf{a}_k^\top\mathbf{B}^{-1}\mathbf{p}_r}{x_k}\right)\\ &= \frac{\mathsf{PTDF}_{n,m,l}}{1-\mathsf{PTDF}_{n,m,k}}f_k\end{aligned}$$

Combining sensitivity factors

Study simultaneous power transfer of δ_{ij} between $i \to j$ *and* outage of line k

- First, model the power transfer on line k and any other line l

$$\hat{f}_k = f_k + \delta_{ij} \cdot \mathsf{PTDF}_{i,j,k}$$

$$\hat{f}_l = f_l + \delta_{ij} \cdot \mathsf{PTDF}_{i,j,l}$$

- Secondly, capture the line outage

$$\begin{aligned}\hat{\hat{f}}_l &= \hat{f}_l + \mathsf{LODF}_{l,k}\hat{f}_k \\ &= (f_l + \mathsf{LODF}_{l,k} f_k) + (\mathsf{PTDF}_{i,j,l} + \mathsf{LODF}_{l,k}\mathsf{PTDF}_{i,j,k})\,\delta_{ij}\end{aligned}$$

- Why not implement the changes in reverse order?

Sensitivity factors can be used to simplify: *i)* monitoring security of current system state; and also *ii)* security-constrained OPF (SC-OPF)

Preventive SC-OPF

Recall network-constrained DC-OPF:

$$\min_{\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}} \sum_{m=1}^{N} C_m(p_m) \tag{P1}$$

$$\text{s.to } \mathbf{1}^\top \mathbf{p} = 0$$

$$-\overline{\mathbf{f}} \leq \mathbf{S}\mathbf{p} \leq \overline{\mathbf{f}}$$

To ensure flows remain safe even after any single-line outage, add constraints

for each line l : $\quad -\overline{\mathbf{f}}^l \leq \mathbf{S}^l\mathbf{p} \leq \overline{\mathbf{f}}^l \qquad$ where $\mathbf{S}^l$ relates to each line outage

How to compute $\mathbf{S}^l$ from LODFs?

Note that power injections remain unchanged after line outage

Preventive SC-OPF

In a power system with L lines, we need L sets of constraints $-\bar{\mathbf{f}}^l \leq \mathbf{S}^l\mathbf{p} \leq \bar{\mathbf{f}}^l$; hence total of $\mathcal{O}(L^2)$ linear inequality constraints

With L in the order of thousands, we end up with millions of constraints...

Successively adding credible contingencies

- solve **base** DC-OPF in (P1) to find $\mathbf{p}^*$ and $\mathbf{f}^*$
- apply LODFs on $\mathbf{f}^*$ to find flows under contingencies
- if contingency flows are safe, output dispatch $\mathbf{p}^*$
- else, rank outages or individual constraints based on severity of violation
- add only credible contingencies to (P1), resolve, and iterate ...

Reserves

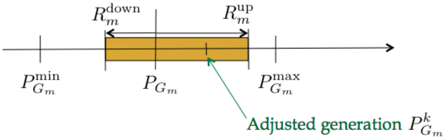
With preventive SC-OPF, we assumed there was no time to react

Reserves: generation available to be deployed under contingencies

Categorized as regulating (real-time); spinning (10'); and non-spinning (30');

Enforced system-wide and regionally.

Scheduling of reserves: up-spinning R_m^{up}; down-spinning R_m^{down}



Available reserves depend on generation; co-optimized with energy schedules

Corrective SC-OPF

Decides reserves and plans $\mathbf{p}^k$ for each contingency (AC-OPF versions exist!)

$$\min_{\mathbf{p},\mathbf{r}^{\text{up}},\mathbf{r}^{\text{down}},\{\mathbf{p}^k\}} \quad \sum_{m=1}^{N} C_m(p_m) + C_m^{\text{up}}(r_m^{\text{up}}) + C_m^{\text{down}}(r_m^{\text{down}}) \tag{1}$$

$$\text{s.to} \quad \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}; \mathbf{1}^\top \mathbf{p} = 0; -\overline{\mathbf{f}} \leq \mathbf{S}\mathbf{p} \leq \overline{\mathbf{f}} \tag{2}$$

$$\underline{\mathbf{p}} \leq \mathbf{p} - \mathbf{r}^{\text{down}}; \ \mathbf{p} + \mathbf{r}^{\text{up}} \leq \overline{\mathbf{p}} \tag{3}$$

$$\underline{\mathbf{p}}^k \leq \mathbf{p}^k \leq \overline{\mathbf{p}}^k; \mathbf{1}^\top \mathbf{p}^k = 0; -\overline{\mathbf{f}}^k \leq \mathbf{S}^k \mathbf{p}^k \leq \overline{\mathbf{f}}^k \tag{4}$$

$$\mathbf{p} - \mathbf{r}^{\text{down}} \leq \mathbf{p}^k \leq \mathbf{p} + \mathbf{r}^{\text{up}} \tag{5}$$

- (1): generation and reserve cost
- (2): network constraints for base case
- (3): reserve levels
- (4)–(5): network constraints and adjustments for credible contingency k