ECE 5314: Power System Operation & Control

Lecture 8: Network-constrained Economic Dispatch

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R2 A. Gomez-Exposito, A. J. Conejo, C. Canizares, *Electric Energy Systems: Analysis and Operation*, Chapters 6.4 & 6.6.

R1 A. J. Wood, B. F. Wollenberg, and G. B. Sheble, Power Generation, Operation, and Control, Wiley, 2014, Chapter 8.

R4 J. Taylor, Convex Optimization of Power Systems, Cambridge University Press, 2015, Chapter 3.

Transmission congestion example

G1: 0-500 MW @ 5\$/MWh; **G2**: 0-500 MW @ 6 \$/MWh; **Load**: 400 MW

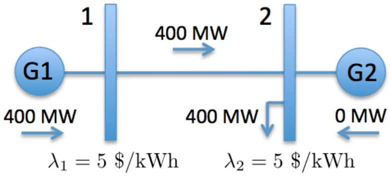


Figure: Transmission line with unlimited capacity

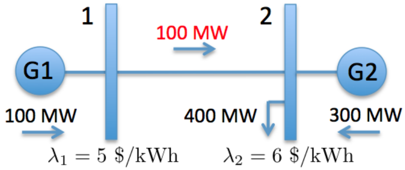
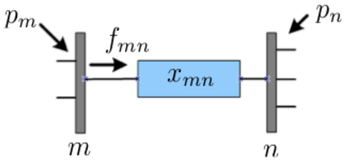


Figure: Line power flow limited to 100 MW in both directions (overheating, sagging)

Flows in DC model

- Grid with N buses and L lines $\mathcal{G} = (\mathcal{B}, \mathcal{L})$
- According to DC power flow model

$$f_\ell = f_{(m,n)} = \frac{\theta_m - \theta_n}{x_{mn}} \text{ for } \ell \in \mathcal{L}$$



- Collect all line flows in $\mathbf{f} = \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta} \in \mathbb{R}^L$
- Diagonal matrix with reactances $\mathbf{X} = \mathrm{diag}(\{x_{mn}\})$
- **Branch-bus incidence matrix**: captures network connectivity $(\mathbf{A1} = \mathbf{0})$

$$\begin{array}{cc}
 & \begin{array}{ccccccc} & & \text{source bus } m & & \text{dest. bus } n & & \\ & & \downarrow & & \downarrow & & \end{array} \\
\mathbf{A} = & \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & +1 & \cdots & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{L \times N} \leftarrow \text{line } (m,n)
\end{array}$$

Injections in DC model

- Conservation of power (lossless network)

$$p_m = \sum_{\ell:(m,n)} f_\ell - \sum_{\ell:(n,m)} f_\ell$$

- Collect active power injections in

$$\mathbf{p} = \mathbf{A}^\top \mathbf{f} = \mathbf{B}\boldsymbol{\theta}$$

where $\mathbf{B} = \mathbf{A}^\top \mathbf{X}^{-1} \mathbf{A}$ from DC power flow [recall $\mathbf{B} \succeq \mathbf{0}$ and $\mathbf{B1} = \mathbf{0}$]

- **Reduced bus reactance matrix**: $\mathbf{B}_r \in \mathbb{S}_{++}^{N-1}$ invertible for connected grids

$$\mathbf{B} = \begin{bmatrix} B_{11} & \mathbf{b}_1^\top \\ \mathbf{b}_1 & \mathbf{B}_r \end{bmatrix}$$

- **Reduced branch-bus incidence matrix**:

$$\mathbf{A} = [\mathbf{a}_1 \ \mathbf{A}_r] \quad \text{with} \quad \mathbf{B}_r = \mathbf{A}_r^\top \mathbf{X}^{-1} \mathbf{A}_r$$

Network-constrained economic dispatch or DC-OPF

Minimize generation cost subject to

$$\min_{\boldsymbol{\theta},\mathbf{p}} \quad \sum_{m=1}^{N} C_m(p_m) \tag{P1}$$

$$\text{s.to} \quad \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \tag{P1a}$$

$$\mathbf{p} = \mathbf{B}\boldsymbol{\theta} \tag{P1b}$$

$$-\overline{\mathbf{f}} \leq \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta} \leq \overline{\mathbf{f}} \tag{P1c}$$

- (P1a): generation constraints; it captures (in)elastic loads too
- (P1b): physical model (approx. DC power flow model)

  $\boldsymbol{\theta}$ in radians and $\mathbf{B}$ in pu
- (P1c): limits on line power flows [why two-sided?]

  line overheating; limits $\overline{\mathbf{f}}$ depend on weather conditions

Eliminating voltage angles

- Focus on constraints:

$$\mathbf{p} = \mathbf{B}\boldsymbol{\theta} \tag{P1b}$$

$$-\bar{\mathbf{f}} \leq \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta} \leq \bar{\mathbf{f}} \tag{P1c}$$

- Eliminate bus voltage angles by setting $\theta_1 = 0$ in (P1b)

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \mathbf{p}_r \end{bmatrix} = \begin{bmatrix} B_{11} & \mathbf{b}_1^\top \\ \mathbf{b}_1 & \mathbf{B}_r \end{bmatrix} \begin{bmatrix} \theta_1 \\ \boldsymbol{\theta}_r \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1^\top \boldsymbol{\theta}_r \\ \mathbf{B}_r \boldsymbol{\theta}_r \end{bmatrix}$$

- Constraint (P1b) equivalent to $\mathbf{1}^\top \mathbf{p} = 0$ and $\boldsymbol{\theta}_r = \mathbf{B}_r^{-1}\mathbf{p}_r$

- Flow vector in (P1c) can be expressed as $\mathbf{f} = \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta} = \mathbf{S}\mathbf{p}$

- **Power-transfer distribution factor (PTDF) matrix**:

$$\mathbf{S} := [\mathbf{0} \;\; \mathbf{X}^{-1}\mathbf{A}_r\mathbf{B}_r^{-1}]_{L\times N}$$

DC-OPF simplified

$$\min_{\mathbf{p}} \quad \sum_{m=1}^{N} C_m(p_m) \tag{P2}$$

$$\text{s.to} \quad \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \qquad \text{(kept implicit)} \tag{P2a}$$

$$\mathbf{1}^\top \mathbf{p} = 0 \qquad \longleftarrow \quad \lambda \tag{P2b}$$

$$-\overline{\mathbf{f}} \leq \mathbf{S}\mathbf{p} \leq \overline{\mathbf{f}} \qquad \longleftarrow \quad (\underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}) \tag{P2c}$$

How to solve this problem?

Lagrangian fun.: $L = \sum_{m=1}^{N} C_m(p_m) - \lambda \mathbf{1}^\top \mathbf{p} + \underline{\boldsymbol{\mu}}^\top (-\overline{\mathbf{f}} - \mathbf{S}\mathbf{p}) + \overline{\boldsymbol{\mu}}^\top (\mathbf{S}\mathbf{p} - \overline{\mathbf{f}})$

After re-arranging:

$$L = \sum_{m=1}^{N} C_m(p_m) - \left[\lambda \mathbf{1} - \mathbf{S}^\top (\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}})\right]^\top \mathbf{p} - (\overline{\boldsymbol{\mu}} + \underline{\boldsymbol{\mu}})^\top \overline{\mathbf{f}}$$

Optimality conditions

1. Primal feasibility: $\mathbf{p}^*$ satisfies (P2b) and (P2c)
2. Dual feasibility: $\overline{\boldsymbol{\mu}}^*, \underline{\boldsymbol{\mu}}^* \geq \mathbf{0}$
3. Lagrangian optimality:

$$\mathbf{p}^* \in \arg \min_{\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}} \sum_{m=1}^{N} C_m(p_m) - \left[\lambda^* \mathbf{1} - \mathbf{S}^\top (\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)\right]^\top \mathbf{p}$$

4. Complementary slackness: $(-\overline{\mathbf{f}} - \mathbf{S}\mathbf{p}^*) \odot \underline{\boldsymbol{\mu}}^* = \mathbf{0}$ and $(\mathbf{S}\mathbf{p}^* - \overline{\mathbf{f}}) \odot \overline{\boldsymbol{\mu}}^* = \mathbf{0}$

Economic interpretation: If $\boldsymbol{\pi} = \lambda^* \mathbf{1} - \mathbf{S}^\top (\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*) \in \mathbb{R}^N$ is used as a vector of prices, it maximizes the social welfare while adhering to network limits and power balance constraints!

$$p_m^* \in \arg \min_{\underline{p}_m \leq p_m \leq \overline{p}_m} C_m(p_m) - \pi_m p_m$$

Locational Marginal Prices (LMPs)

- **Locational marginal prices**: widely used in energy markets

$$\boldsymbol{\pi} := \lambda^* \mathbf{1} - \mathbf{S}^\top (\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)$$

- vector $\boldsymbol{\pi}$ shown to be equal to the vector of Lagrange multipliers for (P1b)

- **perturbation interpretation**: π_m is the cost of supplying the next demand increment at location m

- market practice: location can be a bus or a cluster of buses

- price at the reference bus is $\pi_1 = \lambda^*$ [the first column of $\mathbf{S}$ is zero]

LMP components

- **Locational marginal prices**: $\boldsymbol{\pi} := \lambda^* \mathbf{1} - \mathbf{S}^\top(\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)$
- If **no congestion** (line flows strictly within limits)

$$\underline{\boldsymbol{\mu}}^* = \overline{\boldsymbol{\mu}}^* = \mathbf{0} \quad \Longrightarrow \quad \boldsymbol{\pi} = \lambda^* \mathbf{1}$$

- **Marginal energy component (MEC)**: $\lambda^* \mathbf{1}$
- **Marginal congestion component (MCC)**: $-\mathbf{S}^\top(\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)$

  vector of **shadow prices** $(\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*) \in \mathbb{R}^L$

- **Marginal loss component (MLC)**: LMPs are heuristically adjusted by MLC to account for losses; e.g., MLC$= \lambda^* \nabla_{\mathbf{p}} P_{\text{loss}}(\mathbf{p})$ (typically small)
- In practice, LMP $=$ MEC $+$ MCC $+$ MLC

Reference and slack buses

- So far, reference and slack bus coincided; they don't have to
- *Reference bus* r: the bus for which we select $\theta_r = 0$
  - it determines which column of $\mathbf{B}$ is dropped to form $\mathbf{B}_r$
- *Slack bus* s: the bus s for which we select $p_s = -\sum_{n \neq s} p_n$
  - it determines which row of $\mathbf{B}$ is dropped to form $\mathbf{B}_r$
  - the s-th column of $\mathbf{S}_{s,r}$ is zero
- When changing reference and/or slack buses:
  - shadow prices $(\overline{\boldsymbol{\mu}}, \underline{\boldsymbol{\mu}})$ do not change
  - MEC $= \pi_s$ and MCC $= -\mathbf{S}_{s,r}^\top(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}})$ change
  - however, vector $\boldsymbol{\pi} =$ MEC $+$ MCC remains the same!
  - MLC components change ...

Transmission congestion surplus

How much money does the ISO collect?

$$
\begin{aligned}
s &= -\boldsymbol{\pi}^\top \mathbf{p}^* \\
&= -\lambda^* \mathbf{1}^\top \mathbf{p}^* + (\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)^\top \mathbf{S}\mathbf{p}^* \\
&= 0 + (\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*)^\top \mathbf{f}^*
\end{aligned}
$$

The optimal line flows $\mathbf{f}^* = \mathbf{S}\mathbf{p}^*$ satisfy:

$$
(-\overline{\mathbf{f}} - \mathbf{f}^*)^\top \underline{\boldsymbol{\mu}}^* = 0 \quad \text{and} \quad (\mathbf{f}^* - \overline{\mathbf{f}})^\top \overline{\boldsymbol{\mu}}^* = 0
$$

which yields a positive transmission congestion surplus

$$
s = (\overline{\boldsymbol{\mu}}^* + \underline{\boldsymbol{\mu}}^*)^\top \overline{\mathbf{f}} \geq 0
$$

Surplus is distributed via auctions of **financial transmission rights** (FTR)

AC optimal power flow (AC-OPF)

DC-OPF cannot handle constraints on reactive power and voltage magnitudes

$$\min_{\mathbf{p},\mathbf{q},\mathbf{v}} \quad \sum_{m=1}^{N} C_m(p_m) \tag{P3}$$

$$\text{s.to} \quad \mathbf{p} + j\mathbf{q} = \operatorname{diag}(\mathbf{v})(\mathbf{Y}\mathbf{v})^* \quad \text{(physical system)} \tag{P3a}$$

$$|P_{mn}| \leq P_{mn}^{\max} \quad \text{and/or} \quad |\mathcal{S}_{mn}| \leq S_{mn}^{\max} \tag{P3b}$$

$$V_m^{\min} \leq |\mathcal{V}_m| \leq V_m^{\max} \tag{P3c}$$

$$\mathbf{p}_{\min} \leq \mathbf{p} \leq \mathbf{p}_{\max}; \quad \mathbf{q}_{\min} \leq \mathbf{q} \leq \mathbf{q}_{\max} \tag{P3d}$$

- (P3): generation or any other meaningful cost
- (P3b): limits on line flows
- (P3c): limits on voltage magnitudes
- (P3d): generation and demand limits

AC-OPF

Fundamental tool in power system operations

- eliminate variables $(\mathbf{p}, \mathbf{q})$ using (P3a)

$$\min_{\mathbf{v}} \quad \sum_{m=1}^{N} C_m(\mathbf{v}) \tag{P4}$$

$$\text{s.to} \quad |P_{mn}(\mathbf{v})| \leq f_{mn}^{\max} \quad \text{and/or} \quad |\mathcal{S}_{mn}(\mathbf{v})| \leq S_{mn}^{\max} \tag{P4b}$$

$$V_m^{\min} \leq |\mathcal{V}_m(\mathbf{v})| \leq V_m^{\max} \tag{P4c}$$

$$\mathbf{p}_{\min} \leq \mathbf{p}(\mathbf{v}) \leq \mathbf{p}_{\max}; \quad \mathbf{q}_{\min} \leq \mathbf{q}(\mathbf{v}) \leq \mathbf{q}_{\max} \tag{P4d}$$

- voltages $\mathbf{v}$ in polar or rectangular coordinates
- comparison to power flow problem
- other variables: transformer tap ratios, phase shifters, shunt capacitors

Solving the OPF

- AC-OPF [Carpentier 1960] is a challenging problem (NP-hard)

- Several approaches have been developed:
  1. Augmented Langangian methods
  2. Primal-dual interior point methods
  3. Successive linear or quadratic approximations
  4. Semidefinite program (SDP) relaxation

- Under assumptions, convergence to optimum guaranteed by approach 4

H. Wang, Carlos E. Murillo-Sanchez, R. D. Zimmerman, and Robert J. Thomas, "On Computational Issues of Market-Based Optimal Power Flow," *IEEE Trans. on Power Systems*, Vol. 22, No. 3, Aug. 2007, pp. 1185–1193.

Successive linearization

1. Start from the current operating point $\mathbf{v}^0$

2. Linearize all functions involved around current $\mathbf{v}^t$

3. Solve linearized OPF with respect to constrained increment $\delta\mathbf{v}^{t+1}$

4. Evaluate system conditions at $\mathbf{v}^{t+1} = \mathbf{v}^t + \delta\mathbf{v}^{t+1}$ via power flow

5. Return to Step 2 until convergence

Convergence, feasibility, or optimality are not guaranteed

Electric quantities as quadratic functions of voltages

Similar to the power flow problem:

- (re)active power injections
- (re)active line power flows
- squared current magnitudes
- squared voltage magnitudes

are all quadratic functions of $\mathbf{v}$ in rectangular coordinates

Constraints can be expressed as non-convex quadratic inequalities

$$\underline{p}_m \leq p_m(\mathbf{v}) \leq \overline{p}_m \quad \Longleftrightarrow \quad \underline{p}_m \leq \mathbf{v}^H \mathbf{M}_{P_m} \mathbf{v} \leq \overline{p}_m$$

Semidefinite program relaxation

- Introduce $\mathbf{V} = \mathbf{v}\mathbf{v}^H$ and express optimal power flow as:

$$\min_{\mathbf{V}\succeq\mathbf{0},\ \textcolor{red}{\text{rank}(\mathbf{V})=1}} \quad \sum_{n=1}^{N} c_m \operatorname{Tr}(\mathbf{M}_{P_m}\mathbf{V})$$

$$\text{s.to} \quad \underline{s}_k \leq \operatorname{Tr}(\mathbf{M}_k\mathbf{V}) \leq \overline{s}_k, \quad \forall\ k$$

- Drop rank constraint to express OPF as an SDP
- Relaxation is exact under different operating assumptions
- Counterexamples with minimizers having rank higher than one do exist
- Problem simplifies to an SOCP for radial grids

X. Bai, H. Wei, K. Fujisawa, and Y. Wang, "Semidefinite programming for optimal power flow problems" *Int. J. Elect. Power Energy Syst.*, Vol. 30, No. 67, pp. 383-392, 2008.

J. Lavaei and S. Low, "Zero duality gap in optimal power flow problem," *IEEE Trans. on Power Systems*, Vol. 27, No. 1, Feb. 2012, pp. 92–107.

Generation planning

Operation vs. planning problems

Problem statement: where should we build new generation units?

$$\begin{aligned}
\min_{\mathbf{p},\mathbf{u}} \quad & \sum_{m=1}^{N} c_m u_m \\
\text{s.to} \quad & u_m \underline{p}_m \leq p_m \leq u_m \overline{p}_m \\
& \mathbf{1}^\top \mathbf{p} = 0; \quad -\overline{\mathbf{f}} \leq \mathbf{S}\mathbf{p} \leq \overline{\mathbf{f}} \\
& u_m \in \{0,1\}
\end{aligned}$$

- integer or continuous u_m for placement and sizing tasks
- solved as MILP for hundreds of variables
- $\mathbf{u}^* = \mathbf{0}$ if system already feasible

Transmission planning or switching

Braess' paradox: opening a new road may increase average traffic delays!

Problem statements: Where should we build new transmission lines? Which existing lines should be active?

$$\min_{\mathbf{p},\boldsymbol{\theta},\mathbf{f},\mathbf{u}} \quad \sum_{\ell=1}^{L} c_\ell u_\ell \quad \text{or} \quad \sum_{m=1}^{N} C_m(p_m)$$

$$\text{s.to} \quad \mathbf{p} = \mathbf{A}^\top \mathbf{f}; \quad \underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}$$

$$\left| f_\ell - \frac{\mathbf{a}_\ell^\top \boldsymbol{\theta}}{x_\ell} \right| \leq M(1-u_\ell)$$

$$|f_\ell| \leq u_\ell \overline{f}_\ell, \; u_\ell \in \{0,1\}$$

- **Big M trick**: satisfy one of two constraints: $f_1(\mathbf{p}) \leq Mu$ and $f_2(\mathbf{p}) \leq M(1-u)$ for $u \in \{0,1\}$ and for large M
- AC versions to incorporate reactive power and voltage stability issues