ECE 5314: Power System Operation & Control

Lecture 11: Control of Power Generation

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R5 A. R. Bergen and V. Vittal, Power Systems Analysis, Prentice Hall, 2002, Chapter 11.

R2 A. Gomez-Exposito, A. J. Conejo, C. Canizares, *Electric Energy Systems: Analysis and Operation*, Chapter 9.

R1 A. J. Wood, B. F. Wollenberg, and G. B. Sheble, Power Generation, Operation, and Control, Wiley, 2014, Chapter 10.

Generation control hierarchy

Primary control: governor mechanism or droop control

response: fast (1-100 sec)

input: frequency

goals: *a)* rebalance power; *b)* stabilize/synchronize frequency

Secondary control: automatic generation control (AGC)

response: slower (1-2 min)

input: frequency and inter-area inter-changes

goal: *a)* restore nominal frequency; *b)* rebalance inter-area power exchanges

Tertiary control: economic dispatch, optimal power flow

response: 5-10 min (unit commitment over day)

input: demand and generation bids

goal: economical and secure dispatch of generation units

Laplace transform and basic properties

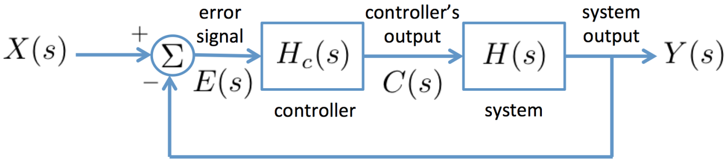
$$X(s) := \mathcal{L}[x(t)] = \int_0^\infty x(t)e^{-st}dt$$

- Unit step function: $\mathcal{L}[u(t)] = \frac{1}{s}$
- Differentiation: $\mathcal{L}[\dot{x}(t)] = sX(s) - x(0)$
- Integration: $\mathcal{L}[\int_0^t x(\tau)d\tau] = \frac{X(s)}{s}$
- Frequency shift: $\mathcal{L}[e^{at}x(t)] = X(s-a)$
- Final value theorem (FVT)

$$\lim_{t\to+\infty} x(t) = \lim_{s\to 0^+} sX(s)$$

[Proof: take $\lim_{s\to 0^+}$ on both sides of differentiation property]

Basic structure of feedback controller



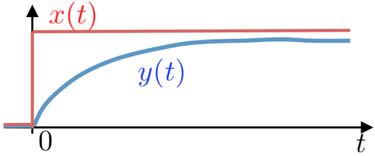
System output in Laplace domain:

$$\begin{aligned} Y(s) &= H(s)C(s) \\ &= H(s)H_c(s)E(s) \\ &= H(s)H_c(s)(X(s) - Y(s)) \end{aligned}$$

Input-output transfer function: $$\frac{Y(s)}{X(s)} = \frac{H_c(s)H(s)}{1 + H_c(s)H(s)}$$

Proportional (P-type) controller

- Special controller: $H_c(s) = G > 0$ (simple gain)
- What is the output $y(t)$ for a unit step input $x(t) = u(t)$?



- If $x(t) = u(t)$, then $X(s) = \frac{1}{s}$ and $Y(s) = \frac{1}{s}\frac{GH(s)}{1+GH(s)}$
- Output of controlled system in steady-state [FVT]

$$y(+\infty) = \lim_{t\to+\infty} y(t) = \lim_{s\to 0^+} sY(s) = \frac{1}{1+[GH(0)]^{-1}} < 1$$

Proportional-integral (PI-type) controller

- Special controller: gain plus integrator

$$c(t) = Ge(t) + A\int_0^t e(\tau)d\tau \quad \Longleftrightarrow \quad H_c(s) = G + \frac{A}{s}$$

- System output for unit step input: $Y(s) = \frac{1}{s}\frac{[sG+A]H(s)}{[s(G+1)+A]H(s)}$

- Output of controlled system in steady-state [FVT]

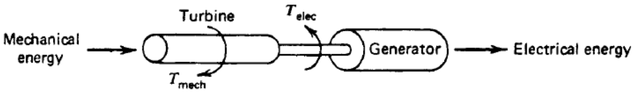
$$y(+\infty) = \lim_{t\rightarrow+\infty} y(t) = \lim_{s\rightarrow 0^+} sY(s) = \frac{AH(0)}{AH(0)} = 1$$

- Output of controlled system reaches desired value

Rotor dynamics

Control mechanical power output of prime mover (steam/gas/water turbine) to adjust the electrical power delivered by a generator

- T_m: net mechanical torque applied to shaft
- T_e: net electric torque applied to shaft (ignoring Ohmic losses)



- $\theta(t) = \omega_0 t + \delta(t)$: angular position [rad]

  ω_0: nominal rotor speed (60Hz for two poles); $\delta(t)$ instantaneous phase
- $\omega(t) = \dot{\theta}(t) = \omega_0 + \dot{\delta}(t)$: rotor speed [rad/sec]
- $\dot{\omega}(t) = \ddot{\theta}(t) = \ddot{\delta}(t)$: rotor acceleration [rad/sec^2]

Swing equation

- Second Newton's law [cf. $F = ma$] in rotational motion (ignoring friction)

$$I\ddot{\theta}(t) = T_m - T_e \tag{1}$$

where I **moment of inertia** of rotating masses

- Recall *power* = *force* × *speed*: $P = (T_m - T_e)\omega$

- Multiplying (1) by $\omega(t)$ yields the **swing equation**:

$$M\ddot{\theta}(t) = M\dot{\omega}(t) = P_m - P_e$$

where $M := I\omega$ is the **angular momentum** of rotating masses

Effect of power disturbances

$$M\dot{\omega}(t) = P_m - P_e$$

If no disturbances (steady state)

- $P_m = P_m^0$ and $P_e = P_e^0$ with $P_m^0 = P_e^0$
- $\dot{\omega}(t) = 0 \;\Rightarrow\; \omega(t) = \omega_0$
- **constant** rotor speed

Consider small disturbances $P_m = P_m^0 + \Delta P_m$ and $P_e = P_e^0 + \Delta P_e$

- if $\Delta P_m < \Delta P_e \quad \Rightarrow \quad \dot{\omega}(t) < 0$
- **frequency decreases**
- If $\omega(t) = \omega_0 + \Delta\omega(t)$, then $\dot{\omega}(t) = \Delta\dot{\omega}(t)$ and

$$M\Delta\dot{\omega}(t) = \Delta P_m(t) - \Delta P_e(t)$$

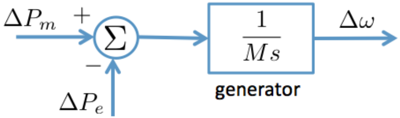
Generator modeling

- Time-domain behavior (differential equation)

$$M\Delta\dot{\omega}(t) = \Delta P_m(t) - \Delta P_e(t)$$

- Laplace-domain description

$$\Delta\omega(s) = \frac{1}{Ms}\left[\Delta P_m(s) - \Delta P_e(s)\right]$$



- Assume angle deviations in internal and terminal voltage of generator are approximately equal

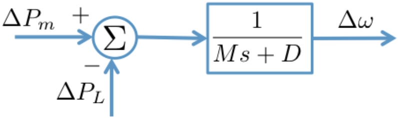
Generator plus load model

- Power consumed by load (frequency (in)dependent components)

$$\Delta P_e = \Delta P_L + \Delta P_{L,\omega} = \Delta P_L + D\Delta\omega \qquad (D > 0)$$

- Power consumption in motors increases with frequency due to friction
- Laplace-domain description

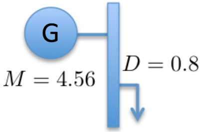
$$Ms\Delta\omega(s) = \Delta P_m(s) - \Delta P_L(s) - D\Delta\omega(s)$$

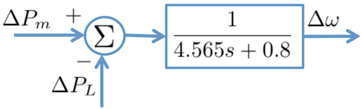


- Sensitivity factor D captures both motor loads and generation friction

Example

- Find the frequency change for a sudden load increase of 0.01 pu





- Output frequency in Laplace domain (partial fraction expansion)

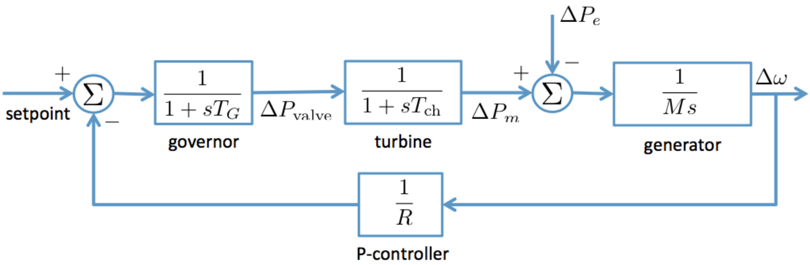
$$\Delta\omega(s) = \frac{\frac{1}{M}}{s + \frac{D}{M}}\left(0 - \frac{\Delta P_L}{s}\right) = -\frac{\Delta P_L}{D}\left[\frac{1}{s} - \frac{1}{s + \frac{D}{M}}\right]$$

- Output frequency in time domain

$$\Delta\omega(t) = -\frac{\Delta P_L}{D}\left[1 - e^{-\frac{D}{M}t}\right]u(t) = -0.0125\left[1 - e^{-0.1754t}\right]u(t)$$

- Frequency stabilized due to load: $\Delta\omega(+\infty) = -\frac{\Delta P_L}{D} = -0.0125$ Hz

Turbine-generator modeling



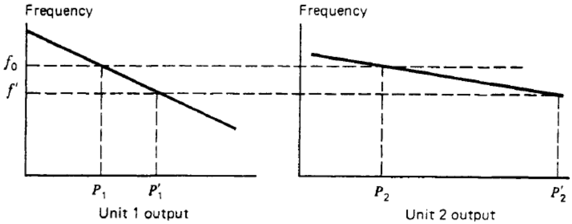
T_G (T_{ch}) time constant for governor (turbine); R is the **droop**

voltage magnitude control has been ignored

Input-output response:
$$\frac{\Delta\omega(s)}{\Delta P_e(s)} = -\frac{\frac{1}{Ms}}{1+\left(\frac{1}{Ms}\right)\left(\frac{1}{R}\right)\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{\text{ch}}}\right)}$$

If $\Delta P_e(t) = \Delta P_e \cdot u(t)$, then frequency changes to $\Delta\omega(+\infty) = -R\cdot\Delta P_e$

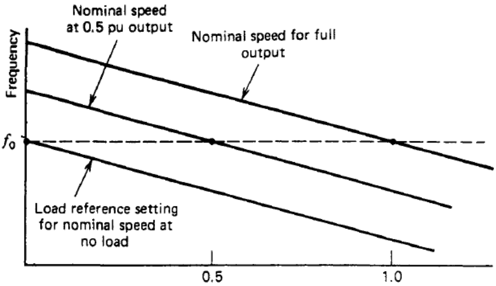
Droop characteristic



- Power output change in response to a frequency change $\Delta P_e^i = -\frac{1}{R_i}\Delta\omega$
- Related to participation factors from economic dispatch with quadratic costs!

Speed-changer settings

- Setpoint is the basic control variable to a generation unit
- Governor can provide f_0 for any desired unit output by changing setpoint

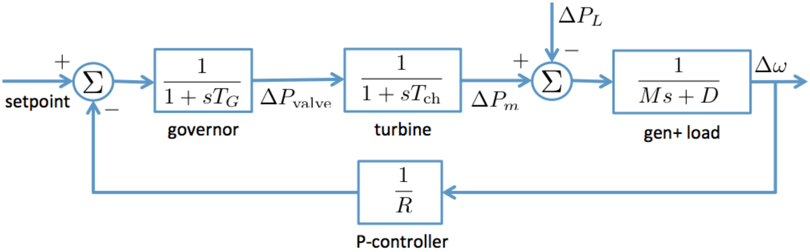


- Final frequency for setpoint $c(t) = C \cdot u(t)$ and $\Delta P_e(t) = \Delta P_e \cdot u(t)$

$$\Delta\omega(+\infty) = R(C - \Delta P_e)$$

Incorporating load

Unit output is connected to load $\Delta P_e = \Delta P_L + D\Delta\omega$



Input-output response:

$$\frac{\Delta\omega(s)}{\Delta P_L(s)} = -\frac{\frac{1}{Ms+D}}{1+\left(\frac{1}{Ms+D}\right)\left(\frac{1}{R}\right)\left(\frac{1}{1+sT_G}\right)\left(\frac{1}{1+sT_{\text{ch}}}\right)}$$

Final frequency for setpoint $c(t) = C \cdot u(t)$ and $\Delta P_L(t) = \Delta P_L \cdot u(t)$

$$\Delta\omega(+\infty) = \frac{C - \Delta P_L}{\frac{1}{R} + D}$$

Transmission line model

- Active power on purely inductive line between buses 1 and 2

$$P_{12} = \frac{V_1 V_2}{x_{12}} \sin(\theta_1 - \theta_2)$$

- Consider voltage angle deviations $\theta_i = \theta_i^0 + \Delta\theta_i$

- First-order approximation: $\sin\theta \simeq \sin\theta_0 + \cos\theta_0(\theta - \theta_0) \qquad \Rightarrow$

$$\sin(\theta_1 - \theta_2) \simeq \sin(\theta_1^0 - \theta_2^0) + \cos(\theta_1^0 - \theta_2^0)(\Delta\theta_1 - \Delta\theta_2) \quad [\Delta\theta_1 - \Delta\theta_2 \simeq 0]$$

- Deviation in line power flow

$$P_{12} = \frac{V_1 V_2}{x_{12}} \sin(\theta_1^0 - \theta_2^0) + T_{12}^0(\Delta\theta_1 - \Delta\theta_2) = P_{12}^0 + \Delta P_{12}$$

where $T_{12}^0 := \frac{V_1 V_2}{x_{12}} \cos(\theta_1^0 - \theta_2^0)$ is *line stiffness* at nominal voltages

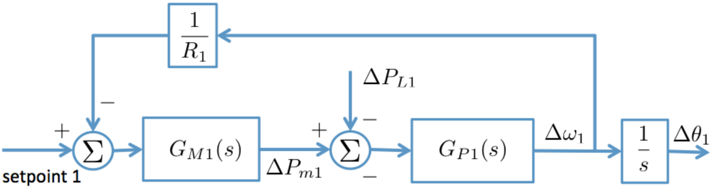
Deviation in line power flow

Time-domain description

$$\Delta P_{12}(t) = T_{12}^0(\Delta\theta_1(t) - \Delta\theta_2(t))$$

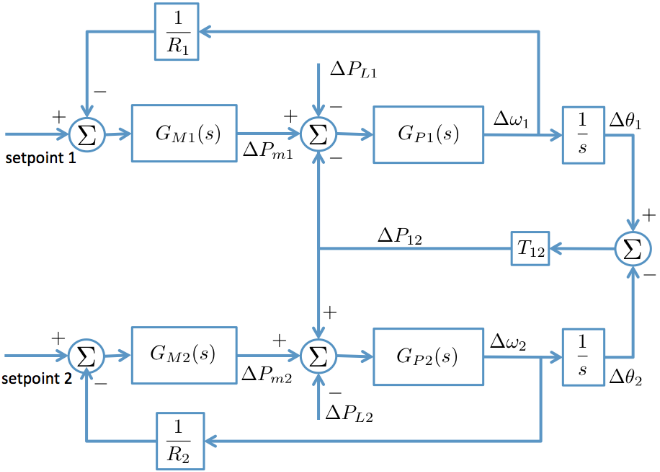
Laplace-domain description; due to $\omega(t) = \dot{\theta}(t) \;\Rightarrow\; \omega(s) = s\theta(s)$

$$\Delta P_{12}(s) = \frac{T_{12}^0}{s}(\Delta\omega_1(s) - \Delta\omega_2(s))$$



Define $G_{M1}(s) := \left(\frac{1}{1+sT_{G1}}\right) \cdot \left(\frac{1}{1+sT_{ch1}}\right)$ and $G_{P1}(s) := \frac{1}{M_1 s + D_1}$

Two-bus system



What happens when $\Delta P_{L1}(t) = \Delta P_{L1} \cdot u(t)$ and $\Delta P_{L2}(t) = \Delta P_{L2} \cdot u(t)$?

Frequency deviations in two-bus system

Define $H_i(s) := \frac{G_{Pi}(s)}{1+G_{Pi(s)}G_{Mi}(s)/R_i}$ for $i = 1, 2$ with $H_i(0) := \left(D_i + \frac{1}{R_i}\right)^{-1}$

After some algebra, we get the system of linear differential equations

$$\begin{bmatrix} 1 + \frac{H_1}{s}T_{12} & -\frac{H_1}{s}T_{12} \\ -\frac{H_2}{s}T_{12} & 1 + \frac{H_2}{s}T_{12} \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix} = \begin{bmatrix} -\frac{H_1}{s}\Delta P_{L1} \\ -\frac{H_2}{s}\Delta P_{L2} \end{bmatrix}$$

with solution:

$$\Delta\omega_1(s) = \frac{1}{s}\frac{H_1\Delta P_{L1}(s + H_2T_{12}) + H_2\Delta P_{L2}H_1T_{12}}{s + (H_1 + H_2)T_{12}}$$

Final **frequency deviation**: $\Delta\omega_1(+\infty) = \Delta\omega_2(+\infty) = -\frac{\Delta P_{L1}+\Delta P_{L2}}{D_1+\frac{1}{R_1}+D_2+\frac{1}{R_2}}$

- deviations converge to the same value
- smaller deviation than if buses were not connected $T_{12} = 0$ and there is load diversity (i.e., $\Delta P_{L1} > 0$ and $\Delta P_{L2} < 0$)

Line flow deviations in two-bus system

To find steady-state **power flow deviation**, exploit the fact

$$\Delta\omega_2 = G_{P2}\left(\Delta P_{12} - \frac{\Delta P_{L2}}{s} - \frac{G_{M2}\Delta\omega_2}{R_2}\right)$$

Solve for ΔP_{12} and apply FVT [$\Delta\omega_2(+\infty)$ found in previous slide]

$$\Delta P_{12}(+\infty) = \frac{\Delta P_{L2}(D_1 + \frac{1}{R_1}) - \Delta P_{L1}(D_2 + \frac{1}{R_2})}{D_1 + \frac{1}{R_1} + D_2 + \frac{1}{R_2}}$$

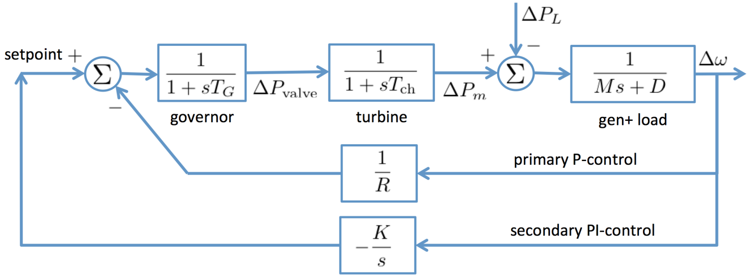
- If $\Delta P_{L1} > 0$ and $\Delta P_{L2} = 0$, then P_{12} decreases from the scheduled value
- No contradiction with slide 20:

$$\Delta P_{12}(+\infty) = T_{12}(\Delta\omega_1(0) - \Delta\omega_2(0))$$

where $\Delta\omega_i(0) \neq 0$ and can be found from initial value theorem

Secondary frequency control

How to maintain frequency at the nominal value?



Single-area system with primary and secondary frequency control

- We can show that $\Delta\omega(+\infty) = 0$
- Setpoint adjusted to $C(+\infty) = \Delta P_L$ (without knowing ΔP_L!)
- Larger K yields faster but more unstable response

Multi-area systems

Practically, power grids are partitioned in control areas

Each control area can be thought of as a bus in the previous analysis

Secondary frequency control should respect *pool operations*:

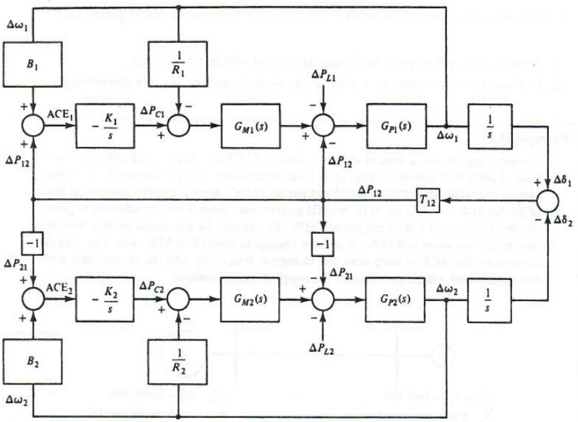
- each control area eventually balances its own load
- power flow schedules across areas remain unchanged
- frequency maintained at nominal value

Currently implemented using **Area Control Error (ACE)** signals:

$$\text{ACE}_i = \sum_{j:i\sim j} \Delta P_{ij} + B_i \Delta\omega_i$$

where $B_i > 0$ is the *frequency bias setting* for area i

Tie-line bias control



Power system with two control areas [*Power system analysis*, A. R. Bergen, V. Vittal]

It can be shown that $\Delta\omega_1(+\infty) = \Delta\omega_2(+\infty) = 0$

If $B_i = D_i + \frac{1}{R_i}$ for $i = 1, 2$, then $\Delta P_{12}(+\infty) = 0$