ECE 5314: Power System Operation & Control

Lecture 10: Power System State Estimation

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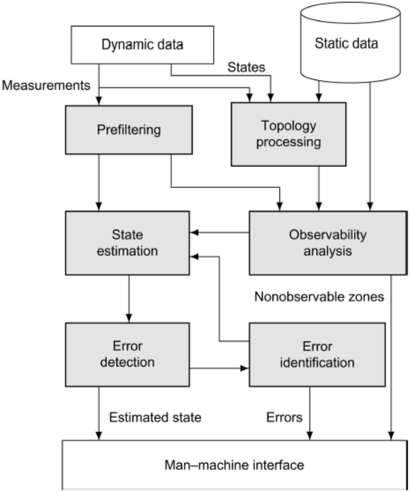
R2 A. Gomez-Exposito, A. J. Conejo, C. Canizares, *Electric Energy Systems: Analysis and Operation*, Chapter 4.

R1 A. J. Wood, B. F. Wollenberg, and G. B. Sheble, Power Generation, Operation, and Control, Wiley, 2014, Chapter 9.

Power system monitoring



Critical for

- situational awareness
- contingency analysis
- load forecasting
- economic operations
- billing

Power system state estimation (PSSE)

Problem: given meter readings and grid parameters, find actual state $\mathbf{v}$

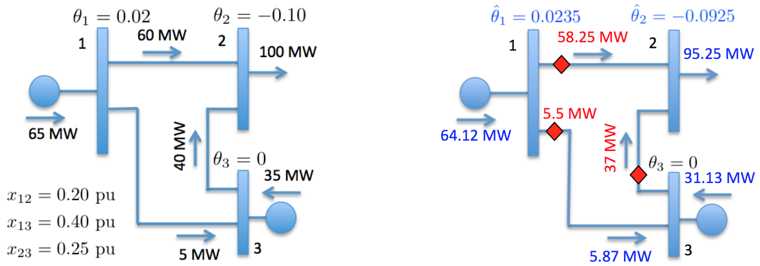


Figure: *Left*: actual state. *Right*: Measurements in red; use only $\{P_{12}, P_{32}\}$ to find state $(\hat{\theta}_1, \hat{\theta}_2)$ via PF equations; estimated quantities in blue.

Why PSSE and not PF?

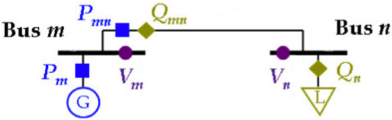
1. measurements are noisy
2. electric quantities are not measured at all grid locations

Measurement model

- **System state**: vector of voltages $\mathbf{v}$ in polar or rectangular coordinates

$$z_m = h_m(\mathbf{v}) + \epsilon_m, \quad m = 1, \ldots, M$$

- Function $h_m(\mathbf{v})$ can be linear or non-linear



- M: number of measurements
- ϵ_m noise of m-th measurement
- *Zero-injection buses* handled via constraints $p_n = q_n = 0$ or $\mathcal{I}_n = 0$
- If actual measurements are not enough, operators may use *pseudo-measurements* (scheduled generation, historical loads)

Introduction to estimation theory

- If ϵ_m is random and $\mathbf{v}$ deterministic, then $z_m = h_m(\mathbf{v}) + \epsilon_m$ is random

$$\text{e.g., if } \epsilon_m \sim \mathcal{N}\left(0, \sigma_m^2\right) \quad \Longrightarrow \quad z_m \sim \mathcal{N}\left(h_m(\mathbf{v}), \sigma_m^2\right)$$

$$p(z_m) = \frac{1}{\sqrt{2\pi\sigma_m^2}} \cdot \exp\left[-\frac{(z_m - h_m(\mathbf{v}))^2}{2\sigma_m^2}\right]$$

- If $\{\epsilon_m\}_{m=1}^M$ are independent, then $\{z_m\}_{m=1}^M$ are independent

$$p(\mathbf{z}; \mathbf{v}) = \prod_{m=1}^{M} p(z_m; \mathbf{v})$$

$\mathbf{z}$ is measured; $h_m(\cdot)$'s and σ_m's are known; and $\mathbf{v}$ is unknown

- For fixed $\mathbf{v}$, the function $p(\mathbf{z}; \mathbf{v})$ is the **likelihood** of observing $\mathbf{z}$

Maximum Likelihood Estimator (MLE)

Find the $\mathbf{v}$ that maximizes the **likelihood function** for the observed $\mathbf{z}$

$$\max_{\mathbf{v}} \; p(\mathbf{z};\mathbf{v}) = \prod_{m=1}^{M} p(z_m;\mathbf{v})$$

Equivalently [why?], minimize the **negative log-likelihood function**

$$\min_{\mathbf{v}} \; -\sum_{m=1}^{M} \log p(z_m;\mathbf{v})$$

For Gaussian noise $\epsilon_m \sim \mathcal{N}(0,\sigma_m^2)$, the MLE becomes

$$\hat{\mathbf{v}} = \arg\min_{\mathbf{v}} \; \sum_{m=1}^{M} \frac{1}{2\sigma_m^2}\left(z_m - h_m(\mathbf{v})\right)^2$$

First-order optimality conditions

$$\nabla_{\mathbf{v}} \sum_{m=1}^{M} \log p(z_m;\mathbf{v}) = \mathbf{0}$$

Weighted least-squares (WLS)

- Statistical formulation of PSSE by seminal work of Schweppe
- Linear or non-linear weighted least-squares problem

$$\hat{\mathbf{v}}_{\text{WLS}} = \arg\min_{\mathbf{v}} \sum_{m=1}^{M} \frac{1}{2\sigma_m^2} \left(z_m - h_m(\mathbf{v})\right)^2$$

- Non-convex problem unless measurement model is *linear*

$$\hat{\mathbf{v}} := \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{h}(\mathbf{v})\|^2$$

with $\mathbf{h}(\mathbf{v}) := [h_1(\mathbf{v}) \ \ldots \ h_M(\mathbf{v})]^\top$

- Let us ignore weights wlog; replace $z_m \rightarrow \frac{z_m}{\sigma_m}$ and $h_m(\mathbf{v}) \rightarrow \frac{h_m(\mathbf{v})}{\sigma_m}$

F. C. Schweppe et al, "Power System Static-State Estimation, Part I-III," *IEEE Trans. Power Apparatus and Systems*, Vol. PAS-89, No. 1, Jan. 1970.

Phasor measurement units (PMUs)

PMUs are *linear* functions of the state $\mathbf{v}$ (if state in rectangular coordinates)!

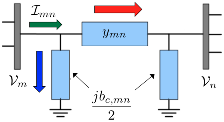
- Measurement of voltage at bus n

$$z_k = \mathcal{V}_n + \epsilon_k = \mathbf{e}_n^\top \mathbf{v} + \epsilon_k$$

- Measurement of line current

$$\mathcal{I}_{mn} = y_{mn}(\mathcal{V}_m - \mathcal{V}_n) + j\frac{b_{mn}^c}{2}\mathcal{V}_m$$

$$z_k = \mathbf{h}_k^\top \mathbf{v} + \epsilon_k$$



- Measurement of injection current $\mathcal{I}_m = \sum_{n\sim m} \mathcal{I}_{mn}$

$$z_k = \mathbf{h}_k^\top \mathbf{v} + \epsilon_k$$

- Vectors $\mathbf{h}_k$'s are known

A. Phadke – J. Thorp, 'Synchronized phasor measurements and their applications.' Springer, 2008.

Linear WLS estimator

- Collect measurement vectors $\mathbf{h}_k$'s as rows of matrix $\mathbf{H}$

$$\mathbf{z} = \mathbf{H}\mathbf{v} + \boldsymbol{\epsilon}$$

complex- or real-valued model $\mathbf{H}$ is $\mathbb{C}^{M \times N}$ or $\mathbb{R}^{2M \times 2N}$

- MLE is the (Weighted) Least-Squares estimator found by solving

$$\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{H}\mathbf{v}\|_2^2$$

(unconstrained) convex quadratic program

- Minimizer is *unique* if $\mathbf{H}$ is full column-rank

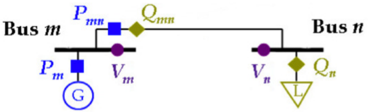
$$\hat{\mathbf{v}}_{\text{WLS}} = (\mathbf{H}^\top \mathbf{H})^{-1}\mathbf{H}\mathbf{z}$$

need at least $M \geq N$; typically $M \sim 1.7 - 2.3N$

SCADA measurements

Supervisory Control And Data Acquisition (SCADA) system measurements:

- line power flows $\{P_{mn}, Q_{mn}\}$
- line current magnitudes $\{|\mathcal{I}_{mn}|\}$
- bus voltage magnitudes $\{V_n\}$
- bus power injections $\{p_n, q_n\}$



- Functions h_m's for SCADA are *nonlinear*; quadratic if $\mathbf{v}$ in Cartesian

- MLE on SCADA data yields a non-linear WLS fit (non-convex problem)

$$\hat{\mathbf{v}}_{\text{WLS}} := \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{h}(\mathbf{v})\|_2^2$$

Gauss-Newton method

- Applies to non-linear LS; do not confuse with Newton-Raphson's method

- Approximate $\mathbf{h}(\mathbf{v})$ with its first-order Taylor's series expansion at $\mathbf{v}_k$

$$\mathbf{h}(\mathbf{v}) \simeq \hat{\mathbf{h}}_k(\mathbf{v}) = \mathbf{h}(\mathbf{v}_k) + \mathbf{J}_k(\mathbf{v} - \mathbf{v}_k), \qquad \mathbf{J}_k : \text{Jacobian matrix}$$

- Gauss-Newton updates: $\mathbf{v}_{k+1} = \arg\min_{\mathbf{v}} \|\mathbf{z} - \hat{\mathbf{h}}_k(\mathbf{v})\|_2^2$

- Since $\hat{\mathbf{h}}_k(\mathbf{v})$ is linear, GN solves a sequence of *linear* LS problems

$$\mathbf{v}_{k+1} = \mathbf{v}_k + (\mathbf{J}_k^\top \mathbf{J}_k)^{-1}\mathbf{J}_k^\top(\mathbf{z} - \mathbf{h}(\mathbf{v}_k))$$

- GN method converges to a local minimum

Solvers using QR decomposition*

Inverting $\mathbf{J}_k^\top \mathbf{J}_k$ is sensitive to its condition number

Solvers based on the QR decomposition are numerically more robust

- QR decomposition: $\mathbf{J}_k = \mathbf{Q}_k \tilde{\mathbf{R}}_k = \mathbf{Q}_k \begin{bmatrix} \mathbf{R}_k \\ \mathbf{0} \end{bmatrix}$

  for orthogonal $\mathbf{Q}_k^\top \mathbf{Q}_k = \mathbf{I}_M$ and upper triangular $\mathbf{R}_k$

- Find the minimizer of

$$\mathbf{v}^{k+1} = \arg\min_{\mathbf{v}} \ \|\mathbf{z}_k - \mathbf{J}_k \mathbf{v}\|_2^2 = \|\mathbf{Q}_k^\top \mathbf{z}_k - \tilde{\mathbf{R}}_k \mathbf{v}\|_2^2$$

  by solving a linear system with upper triangular form

- Upper part of error vector can be made zero
- Same complexity (due to QR), yet better numerical properties

Semidefinite program relaxation

- As in PF and OPF, introduce $\mathbf{V} = \mathbf{v}\mathbf{v}^H$
- Measurements are quadratic functions of $\mathbf{v}$; linear functions of $\mathbf{V}$

$$h_m(\mathbf{V}) = \mathrm{Tr}(\mathbf{M}_m \mathbf{V}) + \epsilon_m, \qquad m = 1, \ldots, M$$

- Express PSSE using SDP variable

$$\min_{\mathbf{V}} \quad \sum_{m=1}^{M} (z_m - \mathrm{Tr}(\mathbf{M}_m \mathbf{V}))^2$$

$$\text{s.to} \quad \mathbf{V} \succeq \mathbf{0}, \; \text{rank}(\mathbf{V}) = 1$$

- Drop rank constraint to get a convex problem

H. Zhu and G. B. Giannakis, "Power system nonlinear state estimation using distributed semidefinite programming," *IEEE J. of Sel. Topics in Signal Processing*, Vol. 8, No. 6, Dec. 2014.

More on semidefinite program relaxation

- Problem transformed to SDP via epigraph trick and Schur's complement

$$
\begin{aligned}
\min_{\mathbf{V},\mathbf{t}} \quad & \sum_{m=1}^{M} t_m \\
\text{s.to} \quad & \begin{bmatrix} t_m & z_m - \mathrm{Tr}(\mathbf{M}_m\mathbf{V}) \\ z_m - \mathrm{Tr}(\mathbf{M}_m\mathbf{V}) & 1 \end{bmatrix} \succeq \mathbf{0}, \;\; m = 1, \ldots, M \\
& \mathbf{V} \succeq \mathbf{0}
\end{aligned}
$$

- Relaxation is NOT exact for noisy measurements

- Heuristics based on minimizer $\hat{\mathbf{V}}$ can give good state estimates
  - Select $\hat{\mathbf{v}}$ as eigenvector corresponding to largest eigenvalue of $\hat{\mathbf{V}}$
  - Draw random states $\hat{\mathbf{v}} \sim \mathcal{CN}(\mathbf{0}, \hat{\mathbf{V}})$ and keep the one with the smallest fit

R. Madani, A. Ashraphijuo, J. Lavaei, and R. Baldick, "Power System State Estimation with a Limited Number of Measurements," in Proc. *IEEE Conf. on Decision and Control*, Dec. 2016.

Fast decoupled solver

To avoid evaluating $\mathbf{J}_k$ and inverting $\mathbf{J}_k^\top \mathbf{J}_k$

- resort to similar tricks as in power flow solvers
- decoupling between $P - \theta$ and $Q - V$
- approximate $(\mathbf{J}_k^\top \mathbf{J}_k)^{-1}$ at flat voltage profile $\mathbf{v} = \mathbf{1} + j\mathbf{0}$

A. Abur and A. Gomez-Exposito, "Power system state estimation: theory and implementation," *CRC Press*, 2004.

SCADA vs. PMU measurements

SCADA:

- readings every 4 secs
- no grid-wide angles due to lack of timing $\{|V_m|, P_m, Q_m, P_{mn}, Q_{mn}, I_{mn}\}$
- non-linear model: $\mathbf{z} = \mathbf{h}(\mathbf{v}) + \boldsymbol{\epsilon}$
- MLE via non-convex problem: $\hat{\mathbf{v}} = \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{h}(\mathbf{v})\|_2^2$

PMUs:

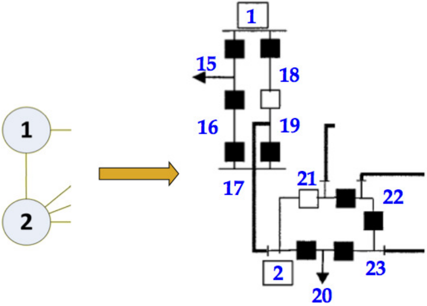
- 30 readings per second
- time-synchronized via GPS signaling: complex currents and voltages
- linear model ($\mathbf{v}$ in rectangular coordinates): $\mathbf{z} = \mathbf{H}\mathbf{v} + \boldsymbol{\epsilon}$
- MLE via convex problem: $\hat{\mathbf{v}} = \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{H}\mathbf{v}\|_2^2 = (\mathbf{H}^\top\mathbf{H})^{-1}\mathbf{H}\mathbf{z}$

Circuit breakers

Devices used for protection and grid reconfiguration

CBs are zero-impedance elements

- closed $\text{CB}_{15,16}$: $V_{15} = V_{16}$
- open $\text{CB}_{18,19}$: $I_{18,19} = 0$



Bus-branch model (left); bus section-switch model (right)

Generalized state estimation

- Network topology processor collects circuit breaker (CB) statuses
- Topology errors easily detected, but hardly identified by PSSE
- **Generalized state estimation (GSE)** seeks state and grid topology

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \quad \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x}\|_2^2$$

$$\text{s.to} \quad \mathbf{C}\mathbf{x} = \mathbf{0}$$

- State augmented to include section voltages and CB currents
- CB statutes effect linear equality constraints to ensure identifiability
- Solution via linear system

$$\begin{bmatrix} \mathbf{H}^\top\mathbf{H} & \mathbf{C}^\top \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^\top\mathbf{z} \\ \mathbf{0} \end{bmatrix}$$

Observability (identifiability) analysis

Given measurement set and grid parameters, assess state identifiability. If non-identifiable, find observable islands (maximally connected subgrids).

Q: Why bother? **A**: To select locations for pseudo-measurements

Observable: if different states $\mathbf{v}_1 \neq \mathbf{v}_2$ yield different outputs $\mathbf{h}(\mathbf{v}_1) \neq \mathbf{h}(\mathbf{v}_2)$

- runs online to cope with meter failures, meter delays, and grid changes
- uses the DC power flow model (pairs of active-reactive meters)

$$P_m = \sum_{n \neq m} b_{mn}(\theta_m - \theta_n)$$

- numerical and topological observability

K. A. Clements, P. R. Krumpholz, and G. W. Davis, "PSSE with measurement deficiency: An observability/measurement placement algorithm," *IEEE Trans. Power App. Syst.*, July 1983.
A. Monticelli, "Electric power system state estimation," *Proc. of the IEEE*, Feb. 2000.

Numerical observability

- Recall from DC grid model (state is $\boldsymbol{\theta}$):

$$\mathbf{p} = \mathbf{A}^\top \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta} \quad \text{and} \quad \mathbf{f} = \mathbf{X}^{-1}\mathbf{A}\boldsymbol{\theta}$$

- If measurements $\mathbf{z} = \mathbf{H}\boldsymbol{\theta}$ are only some p_n's and f_ℓ's, then $\mathbf{H} = \mathbf{SA}$ for a wide matrix $\mathbf{S} \in \mathbb{R}^{M \times L}$

- In general, system $\mathbf{z} = \mathbf{H}\boldsymbol{\theta}$ is identifiable iff $\mathbf{H}$ is full column-rank

- But, in PSSE there is an angle shift ambiguity: $\mathbf{A}\boldsymbol{\theta} = \mathbf{A}(\boldsymbol{\theta} + c\mathbf{1})$

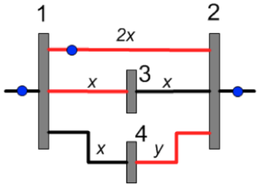
- Power system is observable iff $\text{null}(\mathbf{H}) = \text{null}(\mathbf{A}) = \{c\mathbf{1}, c \in \mathbb{R}\}$

- If not, for $\mathbf{u} \in \text{null}(\mathbf{H})$, the non-zeros of $\mathbf{Au}$ define unobservable lines

- Systematic removal of unobservable branches reveals observable islands

Topological observability

- Graph-theoretic approach that studies the sparsity pattern of $\mathbf{H}$ rather than the values of its entries
- Builds spanning tree (forest) by selecting branches (lines)
  - a flow measurement is assigned to its branch
  - an injection measurement can be assigned to an incident branch
  - at the end, reject injection meas. if they have a non-selected incident branch that does not form a loop with existing branches

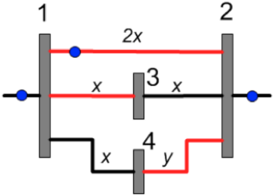


- Different algorithms implement the above

[R2] Gomez-Exposito, Conejo, Canizares, *Electric Energy Systems*, Chapter 4.6

Topological vs. numerical observability

- If a system is numerically observable, it is also topologically observable
- Converse does not hold



- *Counter-example*: shown system with $x = y$
  - topologically is observable
  - but numerically, it is not

A. Monticelli, "Electric power system state estimation," *Proc. of the IEEE*, Feb. 2000.

Bad data

Sources: time-skews, uncalibrated meters, reverse wiring

- Preprocessing (polarity and range tests)
- Assume linear model (DC, linearized AC, or PMU measurements)

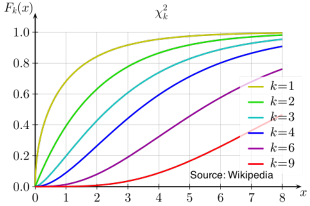
$$\mathbf{z} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon}, \quad \mathbf{H} \in \mathbb{R}^{M \times N}$$

- **Least-squares estimator (LSE)** solves $\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x}\|_2^2$
- LSE: $\hat{\mathbf{x}} = \left(\mathbf{H}^\top \mathbf{H}\right)^{-1} \mathbf{H}^\top \mathbf{z}$
- LSE residual: $\mathbf{r} := \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} = \mathbf{P}_{\mathbf{H}}^{\perp}\mathbf{z} = \mathbf{P}_{\mathbf{H}}^{\perp}\boldsymbol{\epsilon}$

  where $\mathbf{P}_{\mathbf{H}} := \mathbf{H}(\mathbf{H}^\top \mathbf{H})^{-1}\mathbf{H}^\top$ is a projection matrix onto $\mathrm{range}(\mathbf{H})$ and $\mathbf{P}_{\mathbf{H}}^{\perp} := \mathbf{I}_M - \mathbf{P}_{\mathbf{H}}$ is a projection matrix onto $\mathrm{range}(\mathbf{H})^\perp$
- For $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, it holds that $\mathbf{r} \sim \mathcal{N}(\mathbf{0}, \mathbf{P}_{\mathbf{H}}^{\perp})$, $\mathrm{rank}(\mathbf{P}_{\mathbf{H}}^{\perp}) = M - N$

Revealing outliers

Chi-squared test (detection test)



- Because $\mathbf{r} \sim \mathcal{N}(\mathbf{0}, \mathbf{P}_{\mathbf{H}}^{\perp})$, then $\|\mathbf{r}\|_2^2 \sim \chi^2_{M-N}$
- Find α for which $\Pr(\|\mathbf{r}\|_2^2 \leq \alpha) \geq 0.95$
- If $\|\mathbf{r}\|_2^2 > \alpha = F^{-1}_{\chi^2_{M-N}}(0.95)$, declare bad data

Largest normalized residual test (LNRT) (identification test)

- $r_i/\sqrt{P_{ii}} \sim \mathcal{N}(0, 1)$, P_{ii} is the i-th diagonal entry of $\mathbf{P}_{\mathbf{H}}^{\perp}$
- If $\max_i \frac{|r_i|}{\sqrt{P_{ii}}} > t$, measurement z_i for maximizing i is deemed *bad*
- Remove bad datum and re-compute LSE
- LSE can be computed efficiently using the matrix inversion lemma

Least Absolute Deviations (LAV) for robustness

- **LSE** is sensitive to outliers: single bad datum can deteriorate estimate
- **Least-median of squares (LMS)** is an NP-hard problem

$$\hat{\mathbf{x}}_{\mathrm{LMS}} := \arg\min_{\mathbf{x}} \operatorname*{med}_{i} (z_i - \mathbf{h}_i^\top \mathbf{x})^2$$

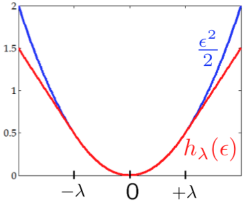
- **Least-absolute deviations (LAV)** can be expressed as a linear program

$$\hat{\mathbf{x}}_{\mathrm{LAV}} := \arg\min_{\mathbf{x}} \|\mathbf{z} - \mathbf{H}\mathbf{x}\|_1 \quad \Longleftrightarrow \quad \begin{aligned} \min_{\mathbf{x},\mathbf{t}} \quad & \mathbf{t}^\top \mathbf{1} \\ \text{s.to} \quad & -\mathbf{t} \leq \mathbf{z} - \mathbf{H}\mathbf{x} \leq \mathbf{t} \end{aligned}$$

- LAV with SDP relaxation for robust nonlinear PSSE

$$\begin{aligned} \min_{\mathbf{V}\succeq \mathbf{0},\mathbf{t}} \quad & \mathbf{t}^\top \mathbf{1} \\ \text{s.to} \quad & -t_m \leq z_m - \mathrm{Tr}(\mathbf{M}_m \mathbf{V}) \leq t_m, \quad m = 1, \ldots, M \end{aligned}$$

Huber estimator



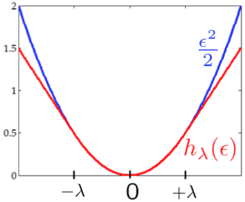
Huber's and quadratic functions

Huber estimator: combination of LS and LAV

$$\hat{\mathbf{x}}_{\mathrm{H}} := \arg\min_{\mathbf{x}} \sum_{i=1}^{M} h_\lambda(z_i - \mathbf{h}_i^\top \mathbf{x})$$

L. Mili, M. G. Cheniae, N. S. Vichare, and P. J. Rousseeuw, "Robust state estimation based on projection statistics of power systems," *IEEE Trans. on Power Systems*, Vol. 11, No. 2, May 1996.

Reformulating Huber's estimator



- Convex problem that can be reformulated as

$$\min_{\mathbf{x},\mathbf{o}} \ \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x} - \mathbf{o}\|_2^2 + \lambda\|\mathbf{o}\|_1$$

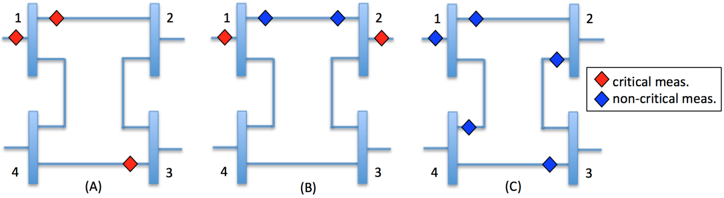
- Interpretation via compressed sensing and the outlier model

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon} + \mathbf{o}$$

V. Kekatos and G. B. Giannakis, "Distributed Robust Power System State Estimation," *IEEE Trans. on Power Syst.*, Vol. 28, No. 2, May 2013.

Critical measurements

Measurement i is critical if once removed from measurement set, the power system becomes unobservable



- **Claim**: column i of $\mathbf{P}_{\mathbf{H}}^{\perp}$ is zero; hence, $r_i = 0 \quad (\mathbf{r} = \mathbf{P}_{\mathbf{H}}^{\perp}\boldsymbol{\epsilon})$
- For critical measurement: undefined $\frac{|r_i|}{\sqrt{P_{ii}}}$, impossible cross-validation
- Bad data processing vulnerable to critical measurements
- Multiple corrupted readings: communication link failures, cyber-attacks

O. Kosut, L. Jia, R. J. Thomas, and L. Tong, "Malicious data attacks on the smart grid," *IEEE Trans. on Smart Grid*, Vol. 2, No. 4, Dec. 2011.

Cyber attacks

- Numbers of cyber-incidents on power SCADA: 3 (2009), 25 (2011)
  - Challenged by increased sensing and networking
  - GPS spoofing, generator controls, CB tripping
- Cyber-attacks on PSSE (situational awareness, billing, trading)

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon} + \mathbf{a}$$

  compromised meters at non-zero entries of $\mathbf{a}$

- **Stealth attacks** can arbitrarily mislead PSSE by $\mathbf{a} = \mathbf{H}\mathbf{v}$
- Deleting related rows of $\mathbf{H}$ deems the system unobservable

  another perspective of multiple critical measurements

Y. Liu, P. Ning, and M. K. Reiter, "False data injection attacks against state estimation in electric power grids," *ACM Trans. Info. and System Security*, May 2011.

Topics not covered

- Dynamic state estimation
- Decentralized solvers
- Attacks to energy markets
- Phasor measurement units (internal estimation algorithms)
- Measurement placement
- Estimating line and transformer parameters