ECE 5984: Power Distribution System Analysis

Lecture 8: Voltage Regulation

Reference: Textbook, Chapter 7

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VirginiaTech
Invent the Future

Outline

- Need for voltage regulation in distribution systems
- Voltage control mechanisms and standards
- Single-phase autotransformers and ABCD models
- Controlling single-phase voltage regulators
- Three-phase voltage regulators and models

Voltage regulation

- In distribution systems, voltage control is more challenging

$$\Delta V \simeq \mathrm{Re}\{zI\} = \mathrm{Re}\left\{z\left(\frac{s}{V}\right)^*\right\}$$
$$\simeq \mathrm{Re}\{zs^*\} = rp + xq$$

- Voltage drops due to both active and reactive loads (significant line resistances)

Effects of over-voltage

reduced light bulb life and electronic devices

Effects of under-voltage

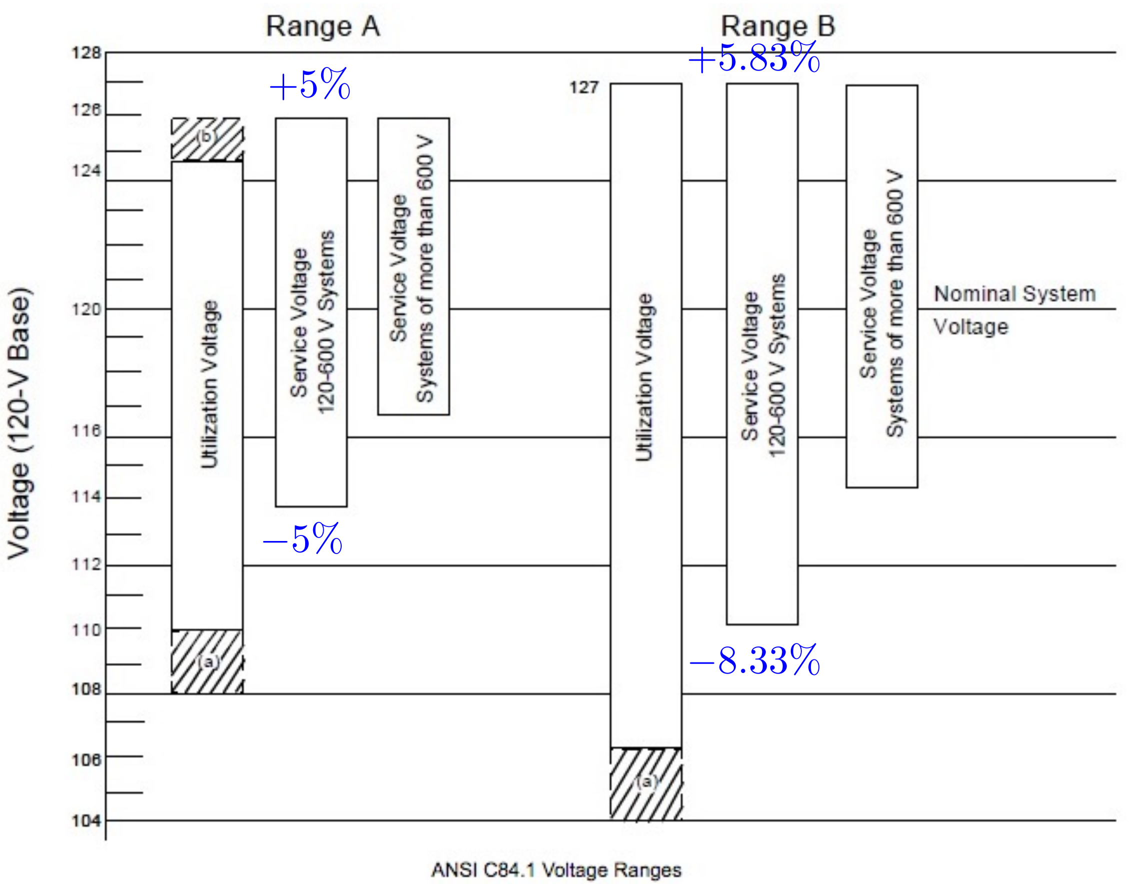
lower illumination

constant-z heating devices (e.g., water heaters) operate slower

higher starting currents on motors and overheating

Standard voltage ratings

- American National Standards Institute (ANSI) C84.1-1995 on *5-min RMS voltages*



Range A must be the basis for equipment's design and rating for satisfactory performance. Range B necessarily results from the practical operating conditions on supply and/or user systems. Such conditions should be limited in extent, duration and frequency. Corrective measures shall be undertaken within a reasonable time to bring back voltages within Range A limits.

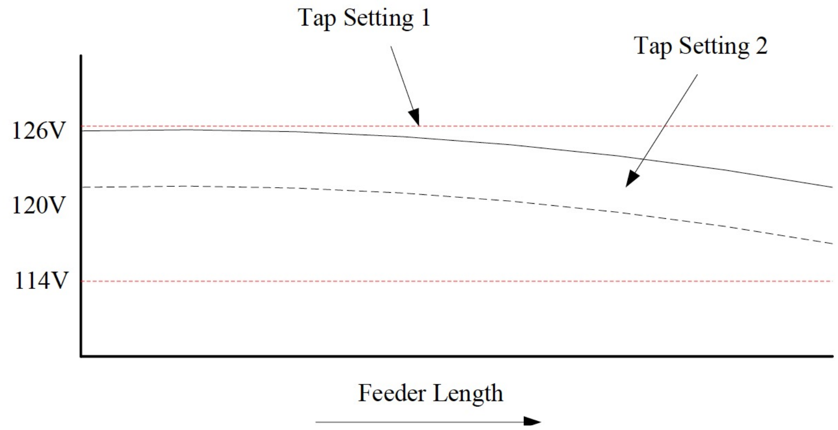
- *Service/delivery* vs. *utilitization* voltage (~4V drop within customer's wiring)
- Imbalance less than 3% at the utility meter

Means to improve voltage profile

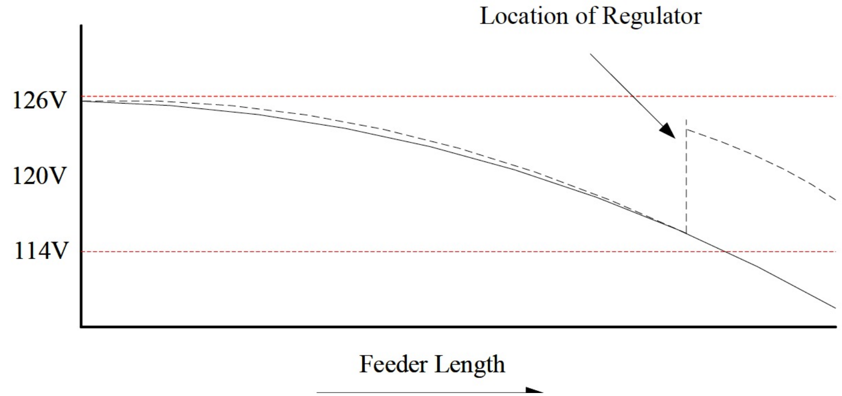
- Increase feeder conductor size (reconducting)
- Increase primary voltage level
- Converting feeder sections from single-phase to multiphase
- Load balancing across feeders
- Build new substations and feeders
- Install regulators at substation and primary feeders
- Install capacitors at substation and primary feeders

Voltage regulation mechanisms

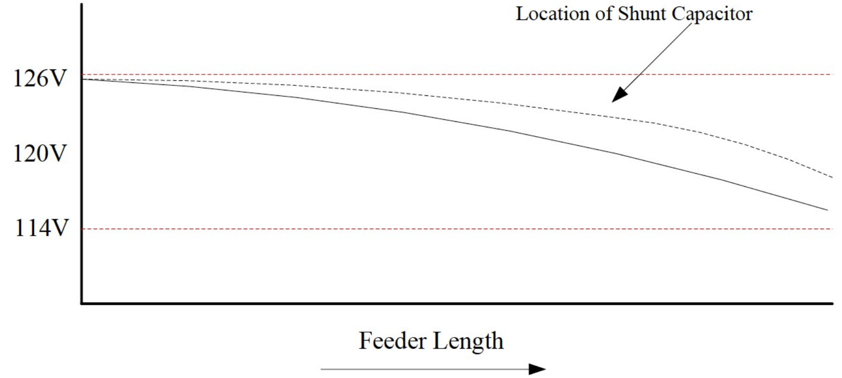
LTC transformer (substation regulator)



step-voltage regulator



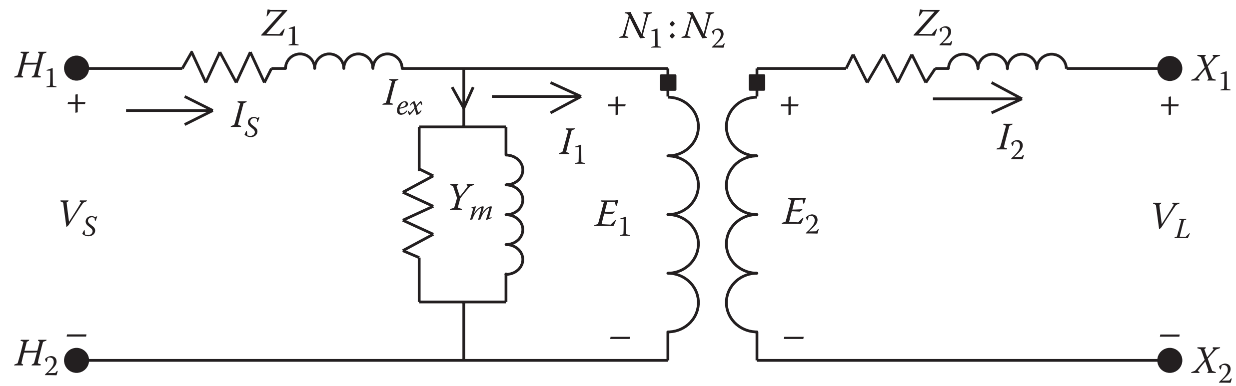
shunt capacitors



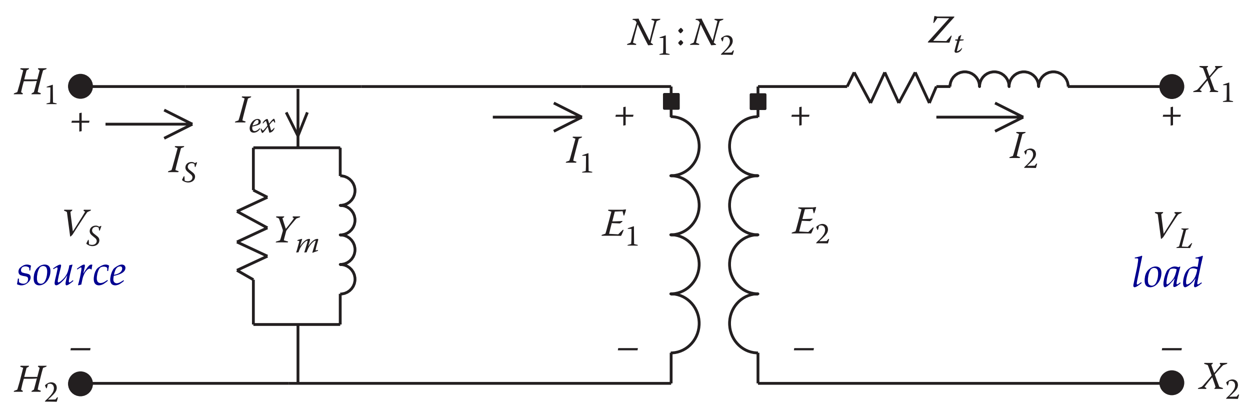
Single-phase transformer

- Detailed model

turns ratio $n_t = \dfrac{N_2}{N_1}$



- Approximate model

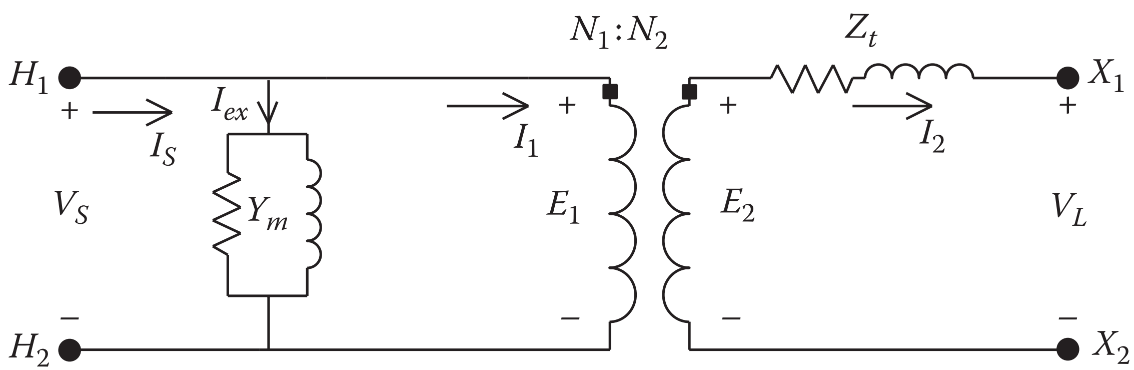


- High-voltage side impedance referred to low-voltage side

$$Z_t = n_t^2 Z_1 + Z_2$$

- The dot convention for voltages and currents

Single-phase transformer ABCD



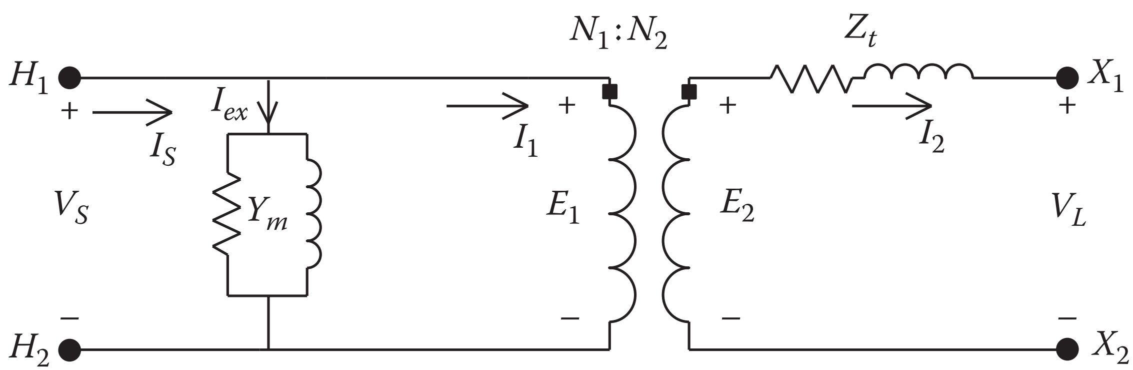
- Transformation ratios $\quad \dfrac{E_1}{E_2} = \dfrac{N_1}{N_2} = \dfrac{1}{n_t} \quad \text{and} \quad \dfrac{I_1}{I_2} = n_t$

- AB model $\quad V_S = E_1 = \dfrac{E_2}{n_t} = \dfrac{1}{n_t} V_L + \dfrac{Z_t}{n_t} I_L$

- CD model

$$\begin{aligned} I_S &= Y_m V_S + I_1 \\ &= \frac{Y_m}{n_t} V_L + \frac{Y_m Z_t}{n_t} I_L + n_t I_L \\ &= \frac{Y_m}{n_t} V_L + \left(n_t + \frac{Y_m Z_t}{n_t} \right) I_L \end{aligned}$$

Single-phase transformer ABCD (cont'd)



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

$$a = \frac{1}{n_t}; \quad b = \frac{Z_t}{n_t}$$

$$c = \frac{Y_m}{n_t}; \quad d = \frac{Y_m Z_t}{n_t} + n_t$$

- Ideal transformer $\quad Y_m = Z_t = 0$

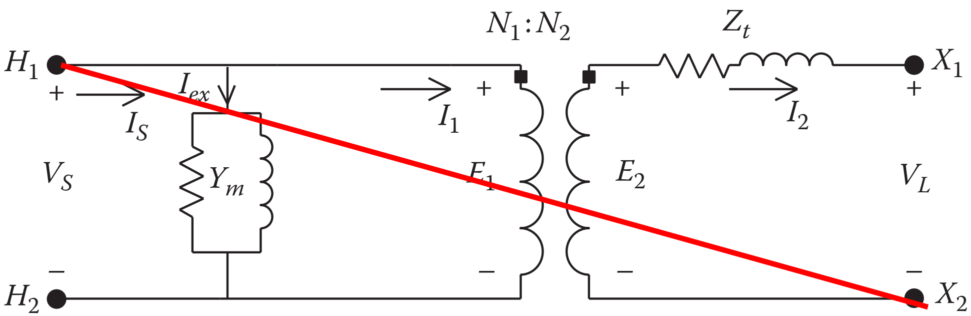
- Receiving voltage in terms of its current and sending voltage (E/F model)

$$V_L = n_t V_S - Z_t I_L$$

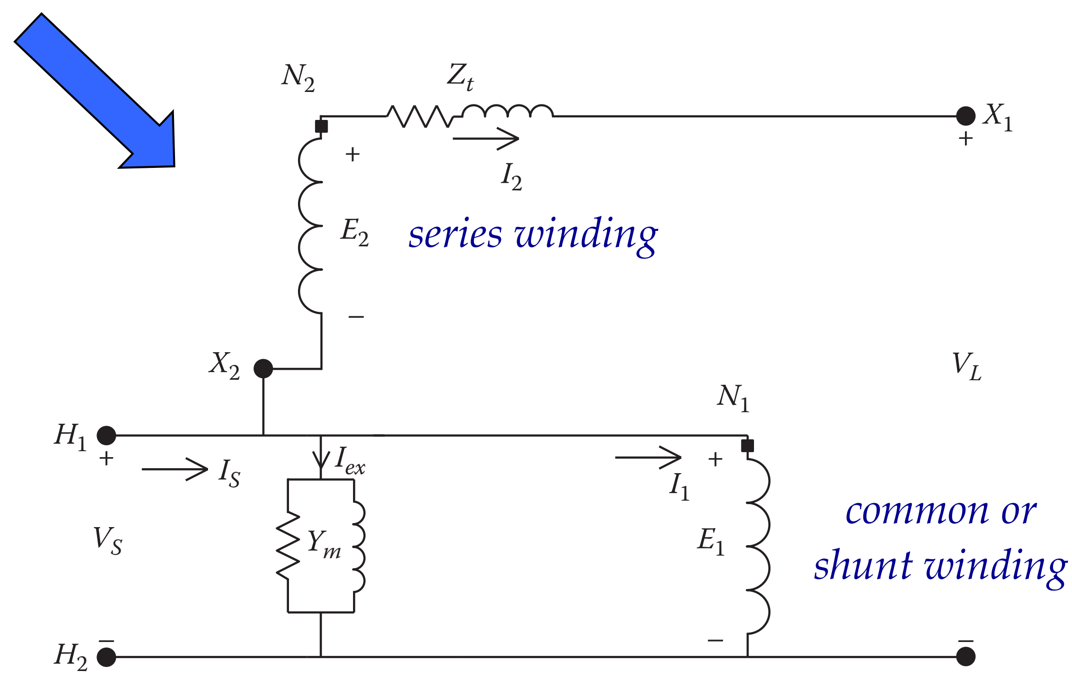
- *Lecture 9* covers network models for all transformer connections

Autotransformer

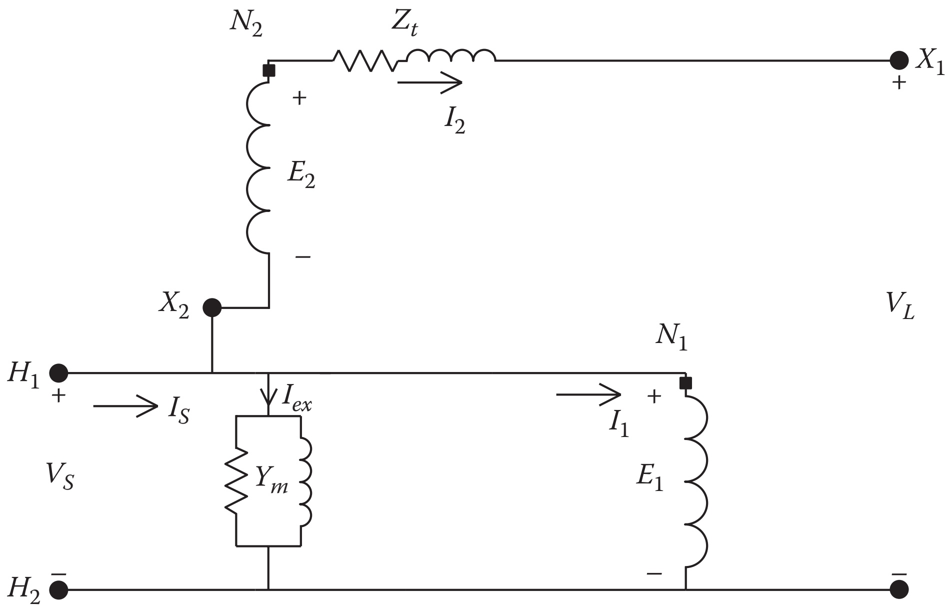
- A single-winding tapped, or a two-winding transformer reconnected



- Windings magnetically and electrically coupled



Step-up autotransformer ABCD



- Transformation ratios

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{n_t} \quad \text{and} \quad \frac{I_1}{I_2} = n_t$$

- AB model

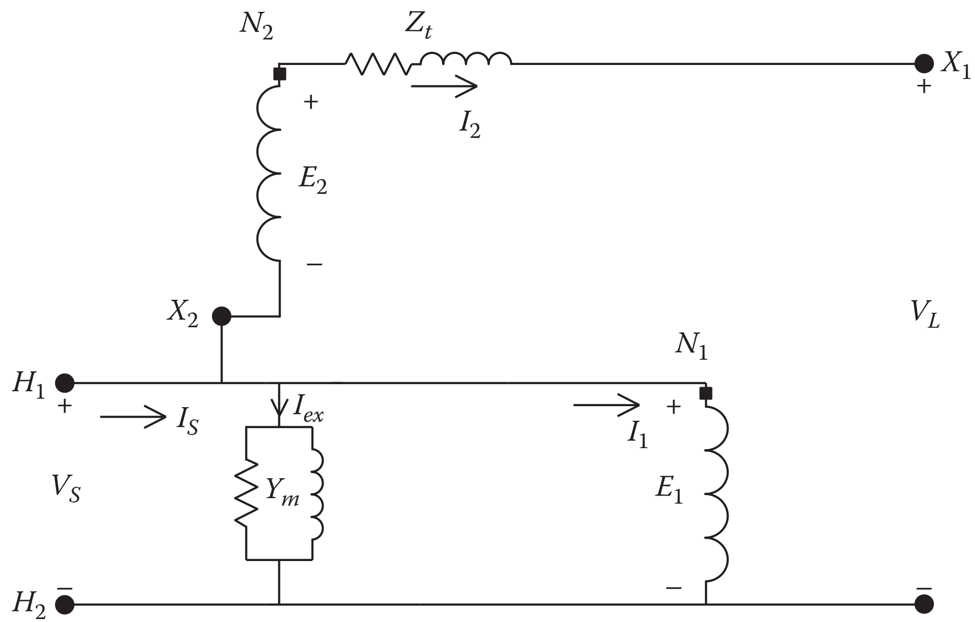
$$\begin{aligned} V_L &= E_1 + E_2 - Z_t I_L \\ &= E_1(1+n_t) - Z_t I_L \\ &= V_S(1+n_t) - Z_t I_L \;\Rightarrow\; V_S = \frac{1}{1+n_t} V_L + \frac{Z_t}{1+n_t} I_L \end{aligned}$$

- *Step-up* because for small $Z_t : \; V_L \simeq (1+n_t) V_S$

- CD model

$$\begin{aligned} I_S &= Y_m V_S + I_1 + I_2 \\ &= \frac{Y_m}{1+n_t} V_L + \frac{Y_m Z_t}{1+n_t} I_L + (1+n_t) I_L \\ &= \frac{Y_m}{1+n_t} V_L + \left(1 + n_t + \frac{Y_m Z_t}{1+n_t}\right) I_L \end{aligned}$$

Step-up autotransformer ABCD (cont'd)



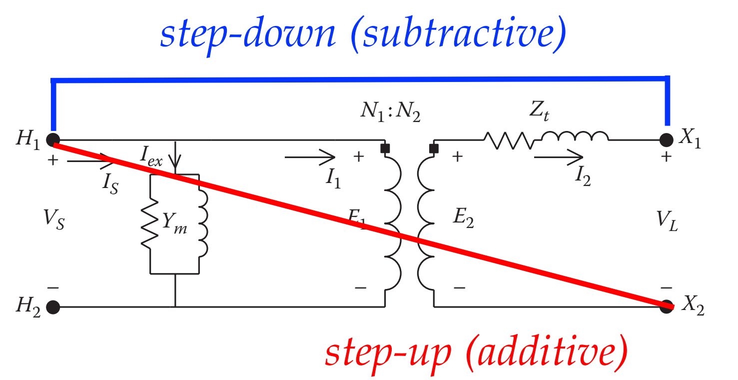
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix} \qquad a = \frac{1}{1+n_t}; \quad b = \frac{Z_t}{1+n_t}$$

$$c = \frac{Y_m}{1+n_t}; \quad d = 1 + n_t + \frac{Y_m Z_t}{1+n_t}$$

- Different from autotransformers, regulators have small ratios

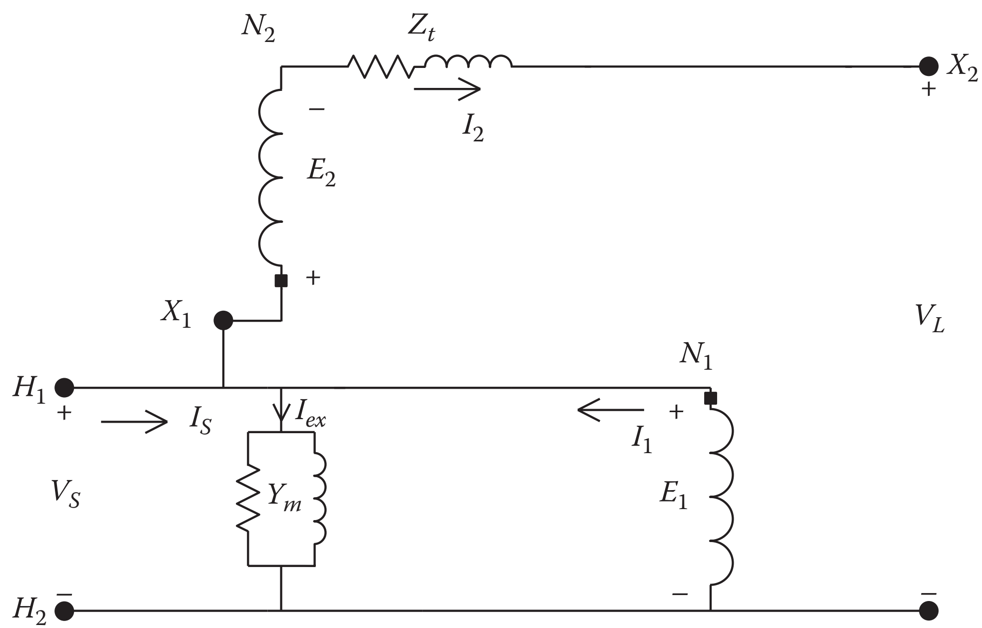
Step-down autotransformer ABCD (cont'd)

- Series winding voltage is subtracted from common winding voltage



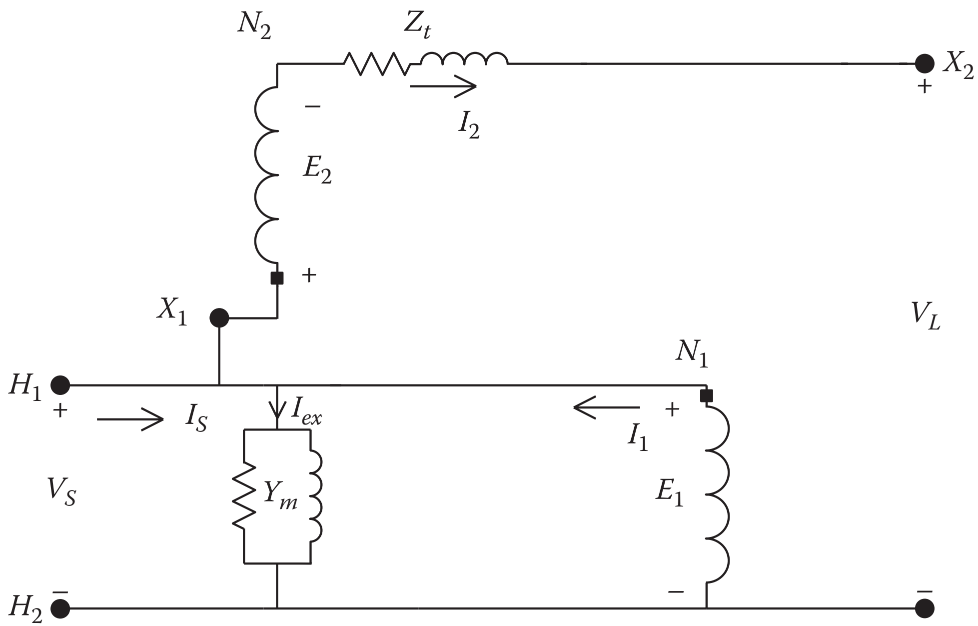
- ABCD model can be obtained similarly using

$$V_L = E_1 - E_2 - Z_t I_L$$
$$I_S = Y_m V_s - I_1 + I_2$$



- Current polarities have been reversed, but still satisfy dot convention

Step-down autotransformer ABCD (cont'd)



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

$$a = \frac{1}{1-n_t}; \quad b = \frac{Z_t}{1-n_t}$$

$$c = \frac{Y_m}{1-n_t}; \quad d = 1 - n_t + \frac{Y_m Z_t}{1-n_t}$$

- Receiving voltage in terms of its current and sending voltage

$$V_L = (1 \pm n_t)V_S - Z_t I_L$$

- Step-down model looks like step-up model with $n_t' = -n_t$

Rating advantage

- *Voltage ratings* $\quad V_S = E_1 \;$ and $\; V_L = E_1 \pm E_2 = \dfrac{E_2}{n_t} \pm E_2 = \dfrac{1 \pm n_t}{n_t} E_2$

- *Power ratings* for autotransformer and two-winding transformer

$$\begin{aligned} S_A &= V_S I_S = V_L I_L \\ S_W &= E_1 I_1 = E_2 I_2 \end{aligned} \quad \Longrightarrow \quad \frac{S_A}{S_W} = \frac{V_L I_L}{E_2 I_2} = \frac{1 \pm n_t}{n_t}$$

- Significant power rating advantage for small turns ratios

$$\text{If } V_L = (1 + n_t) V_S = 1.1 \cdot V_S \quad \Rightarrow \quad n_t = 0.1 \quad \Rightarrow \quad S_A = 11 \cdot S_W$$

$$\text{If } V_L = (1 + n_t) V_S = 10 \cdot V_S \quad \Rightarrow \quad n_t = 9 \quad \Rightarrow \quad S_A = 1.11 \cdot S_W$$

Per-unit impedances

- *Base impedances* significantly larger than those of two-winding transformers

load side

$$Z_{A,\text{base}} = \frac{V_L^2}{S_A} = \frac{(E_1 \pm E_2)^2}{S_A} = \left(\frac{1 \pm n_t}{n_t}\right)^2 \frac{E_2^2}{S_A}$$

$$Z_{W,\text{base}} = \frac{E_2^2}{S_W}$$

$$\frac{Z_{A,\text{base}}}{Z_{W,\text{base}}} = \left(\frac{1 \pm n_t}{n_t}\right)^2 \frac{S_W}{S_A} = \frac{1 \pm n_t}{n_t}$$

source side

$$Y_{A,\text{base}} = \frac{S_A}{V_S^2} = \left(\frac{1 \pm n_t}{n_t}\right)^2 \frac{S_A}{E_1^2}$$

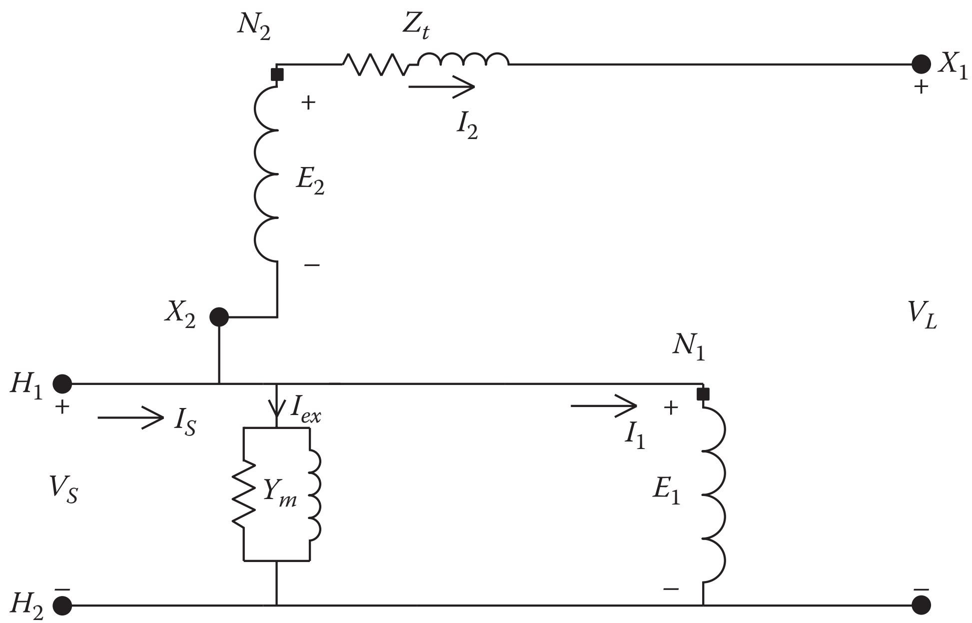
$$Y_{W,\text{base}} = \frac{S_W}{E_1^2}$$

$$\frac{Y_{A,\text{base}}}{Y_{W,\text{base}}} = \frac{1 \pm n_t}{n_t}$$

- *Per-unit impedances* are significantly smaller than those of two-winding transformer

- Hence, autotransformers are oftentimes modeled as ideal (impedances ignored)

Approximate model of ideal transformer



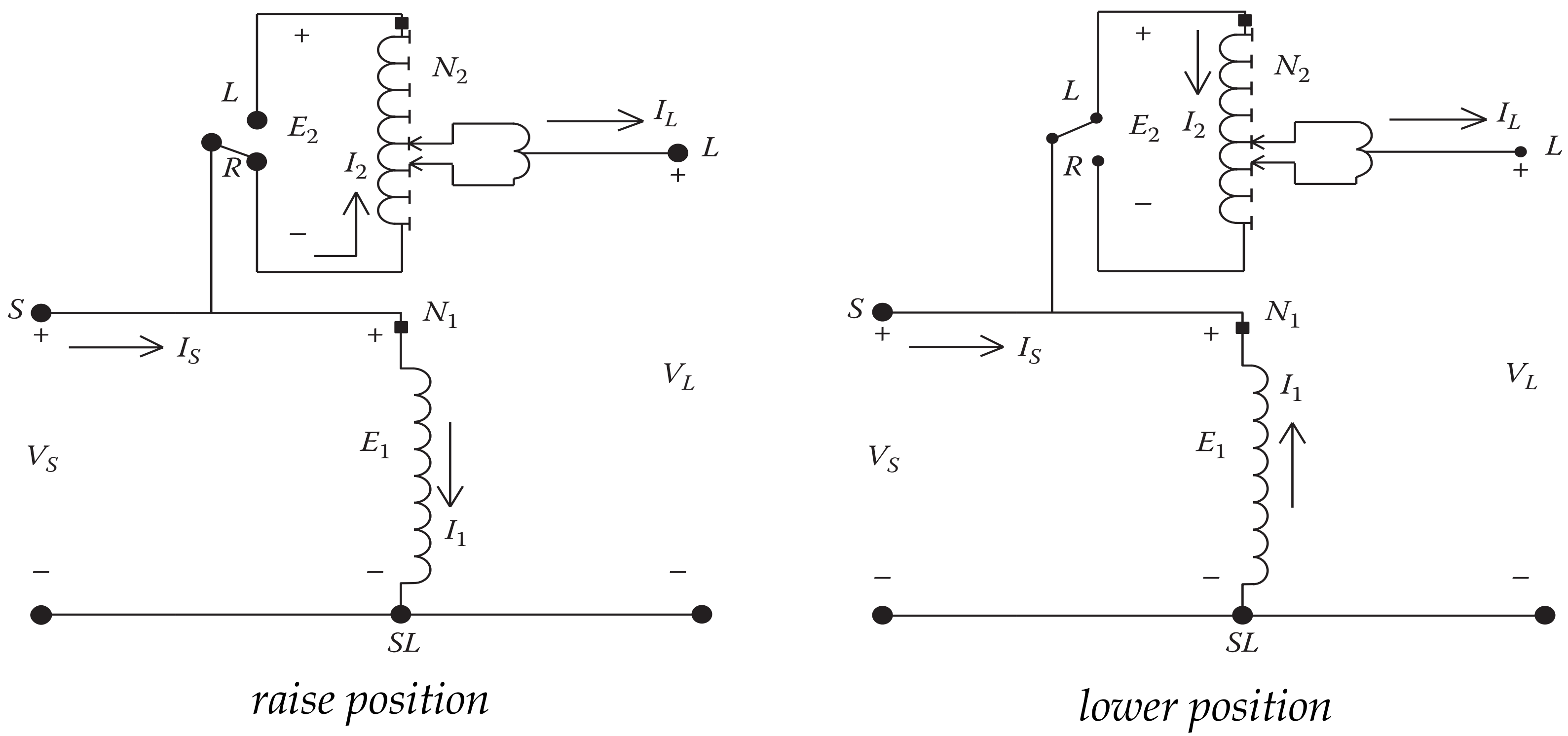
- Distribution auto-transformers can be approximated as *ideal ones* with little error

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix} \qquad \begin{array}{l} a = \dfrac{1}{1 \pm n_t}; \quad b \simeq 0 \\ c \simeq 0; \quad d \simeq 1 \pm n_t \end{array} \qquad \begin{array}{l} + \text{ for raise} \\ - \text{ for lower} \end{array}$$

SVR of Type A

- Step-voltage regulator (SVR) = autotransformer + load tap changer (LTC) mechanism

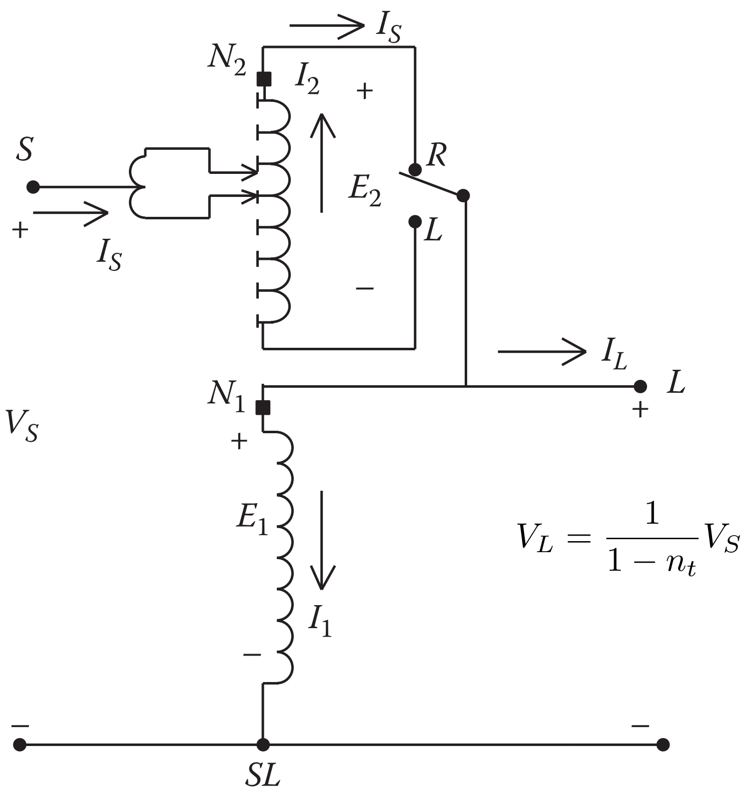
Type A: load connected to series winding



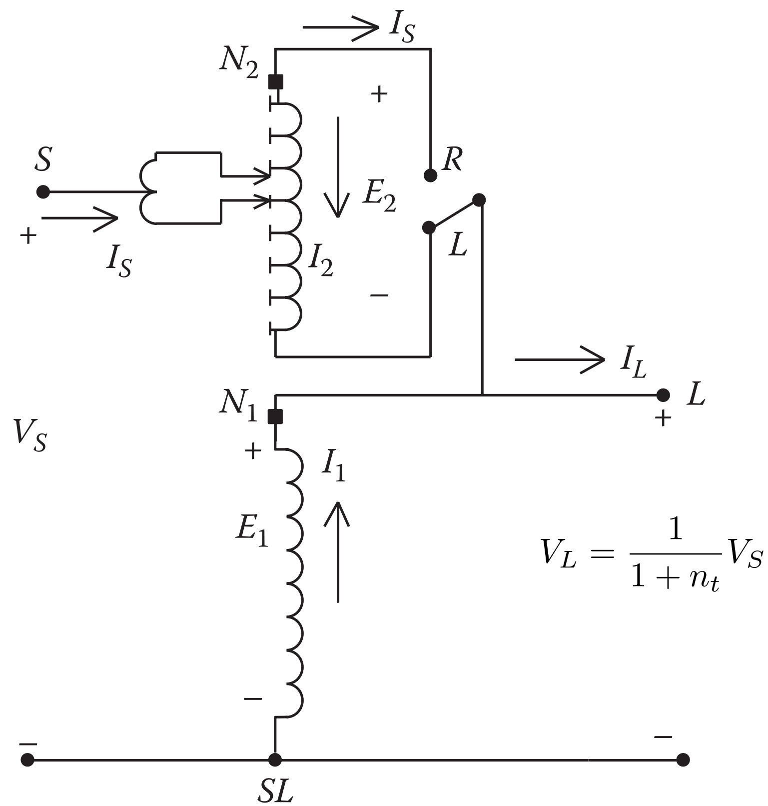
- Reversing switch changes directions of currents
- ±10% range in 32 steps of 20/32=0.625% (approx. 0.75V for nominal V_S=120V)

SVR of Type B

Type B: source connected to series winding (more popular)



raise position



lower position

$$V_S = E_1 - E_2 = (1-n_t)E_1 = (1-n_t)V_L$$
$$I_L = I_2 - I_1 = (1-n_t)I_S$$

$$V_S = E_1 + E_2 = (1+n_t)E_1 = (1+n_t)V_L$$
$$I_L = I_2 + I_1 = (1+n_t)I_S$$

Regulator model

- Neglect series impedances and shunt admittances

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

- Type B (shown here)

$$a = a_R; \quad b = c = 0; \quad d = \frac{1}{a_R}; \quad a_R = 1 \mp \frac{N_2}{N_1}$$

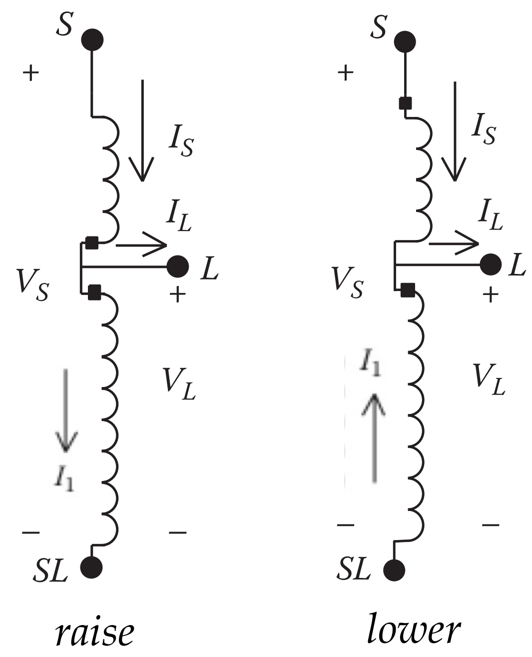
$$a_R = 1 \mp 0.00625 \cdot \text{Tap}$$

- Type A (derived earlier)

$$a = \frac{1}{a_R}; \quad b = c = 0; \quad d = a_R; \quad a_R = 1 \pm \frac{N_2}{N_1}$$

- Example of raise position

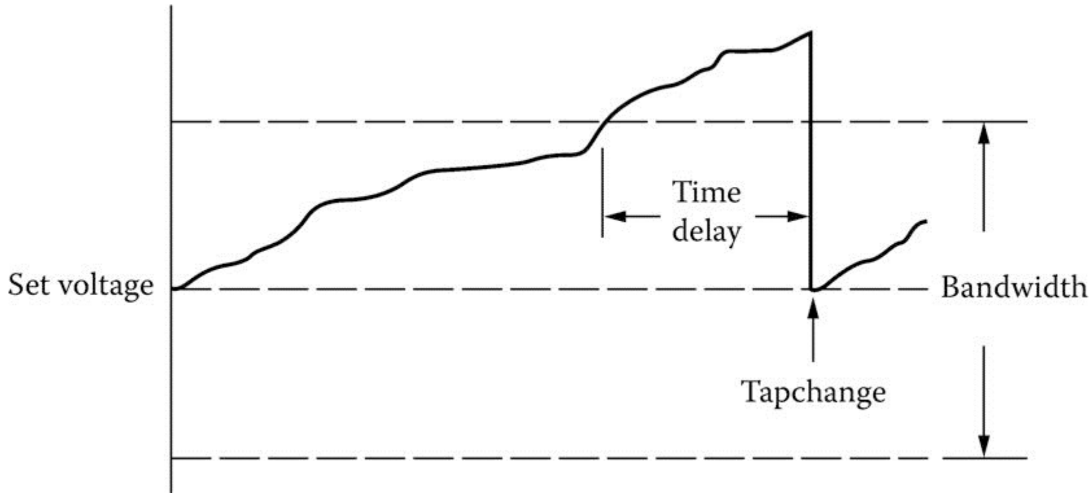
$$V_L = (1 + n_t)V_S \text{ (type A)} \quad \text{or} \quad V_L = \frac{1}{1 - n_t} V_S \text{ (type B)}$$



	Type A	Type B
Raise	+	–
Lower	–	+

Control settings

- *Set voltage* (e.g., 124 V)



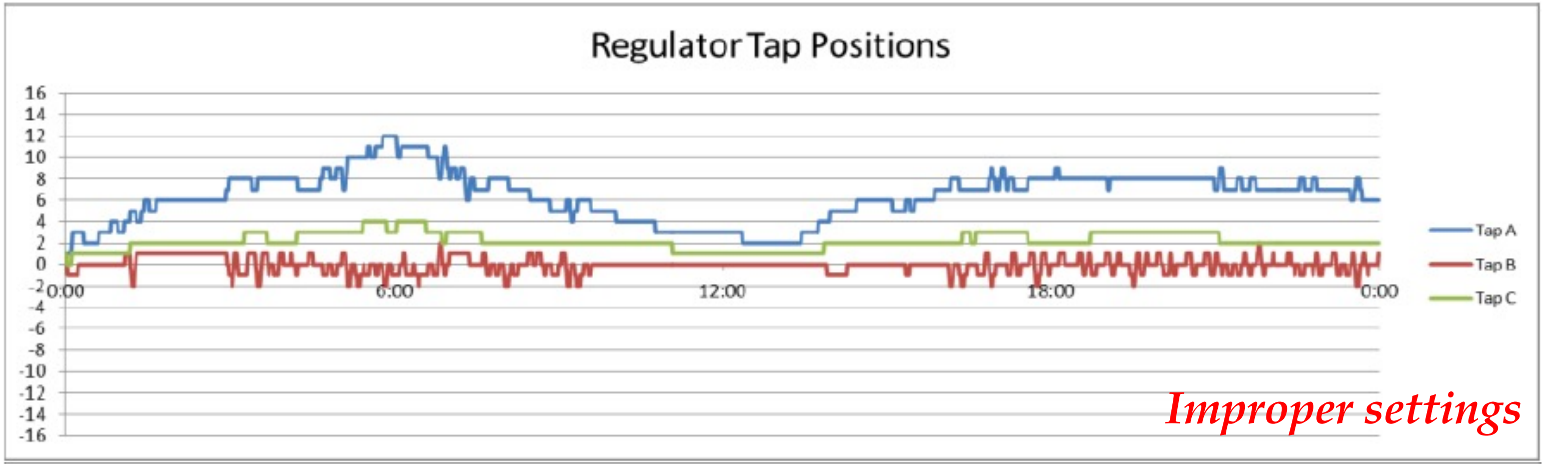
- *Bandwidth:* allowed deviation around set voltage level (e.g., ±1V); specified as full (two-sided) bandwidth

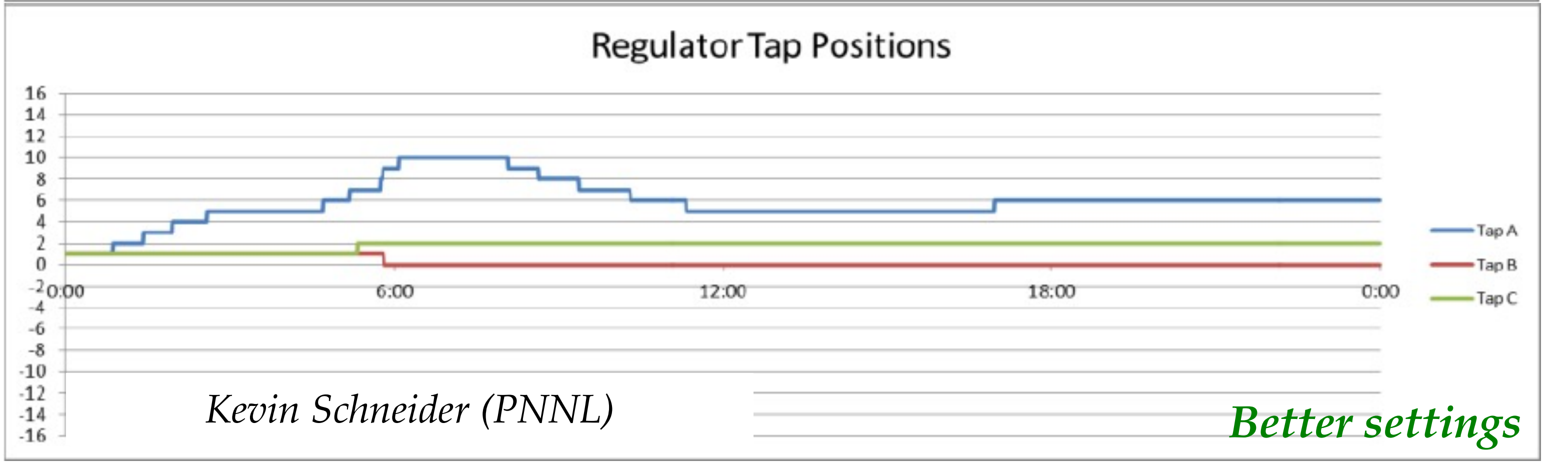
  Deadband should be wider than the voltage change obtained by a single tap, otherwise tap may be flipping back-and-forth without ever falling in the deadband

- *Time delay* before taking action (to avoid fluctuations due to e.g., motor starting)

Tuning bandwidth and delay

- Avoid excessive regulator operation and oscillations (reduces lifetime)

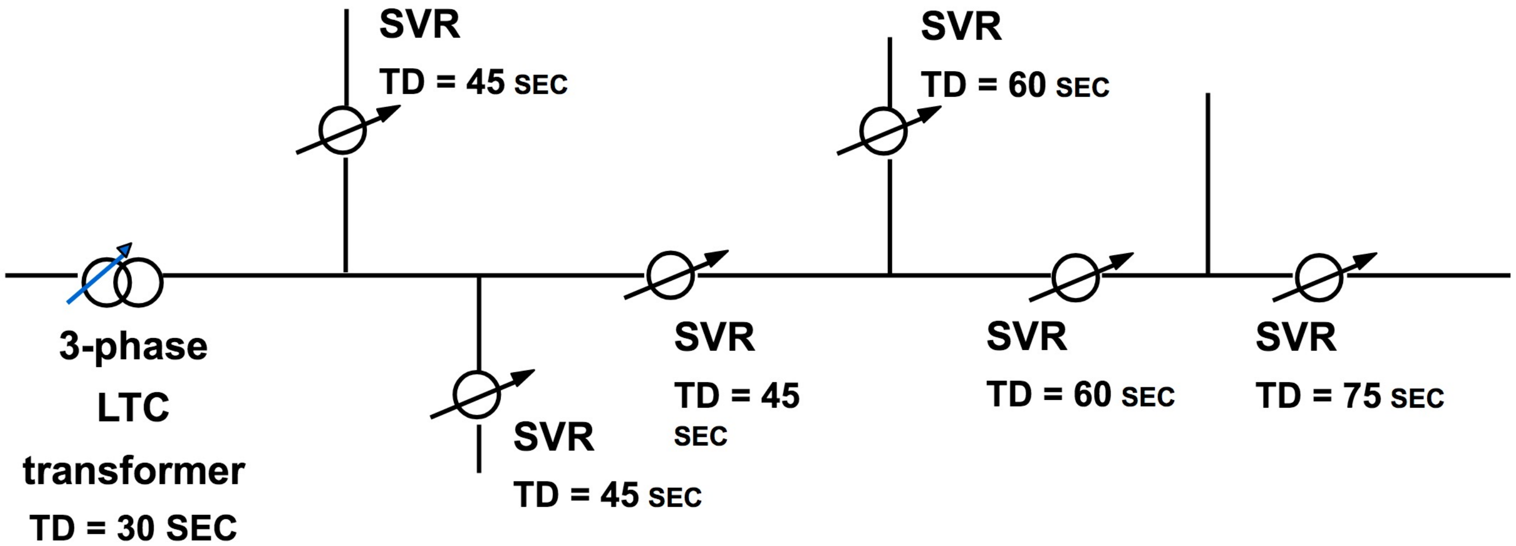




- Tuning is harder (impossible) with fluctuations in solar irradiance and PVs
- Smart inverters can handle fluctuations at faster timescale

Time delays

- Usually, in the range of 10-120 seconds



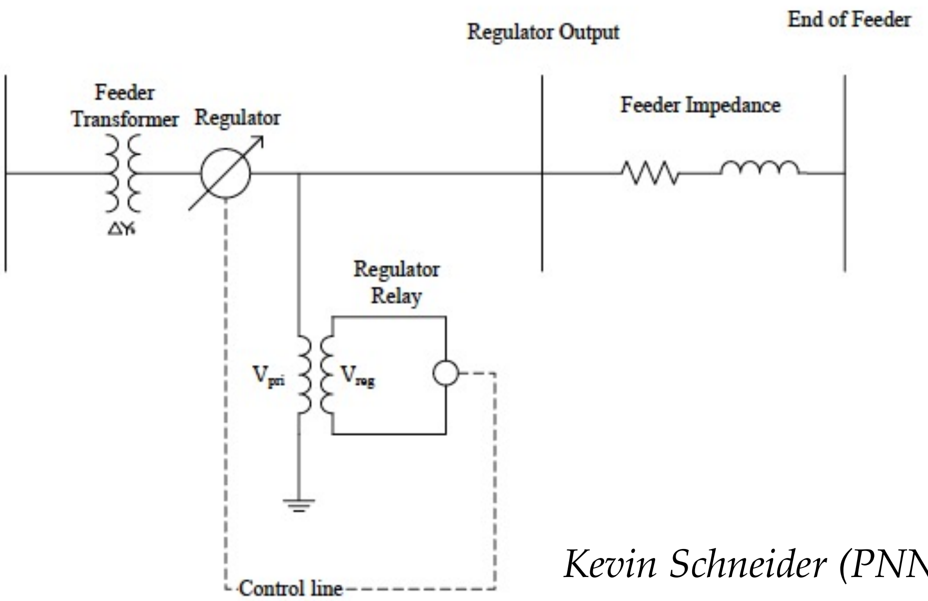
Eaton – Cooper Power Systems

- *Rule 1:* Each regulator in series downstream requires a longer time delay
- *Rule 2:* Minimum time delay from one SVR to the next in cascade is 15 seconds

Control of voltage regulators

- Not all feeders have SVRs: no need for short and/or lightly-loaded feeders

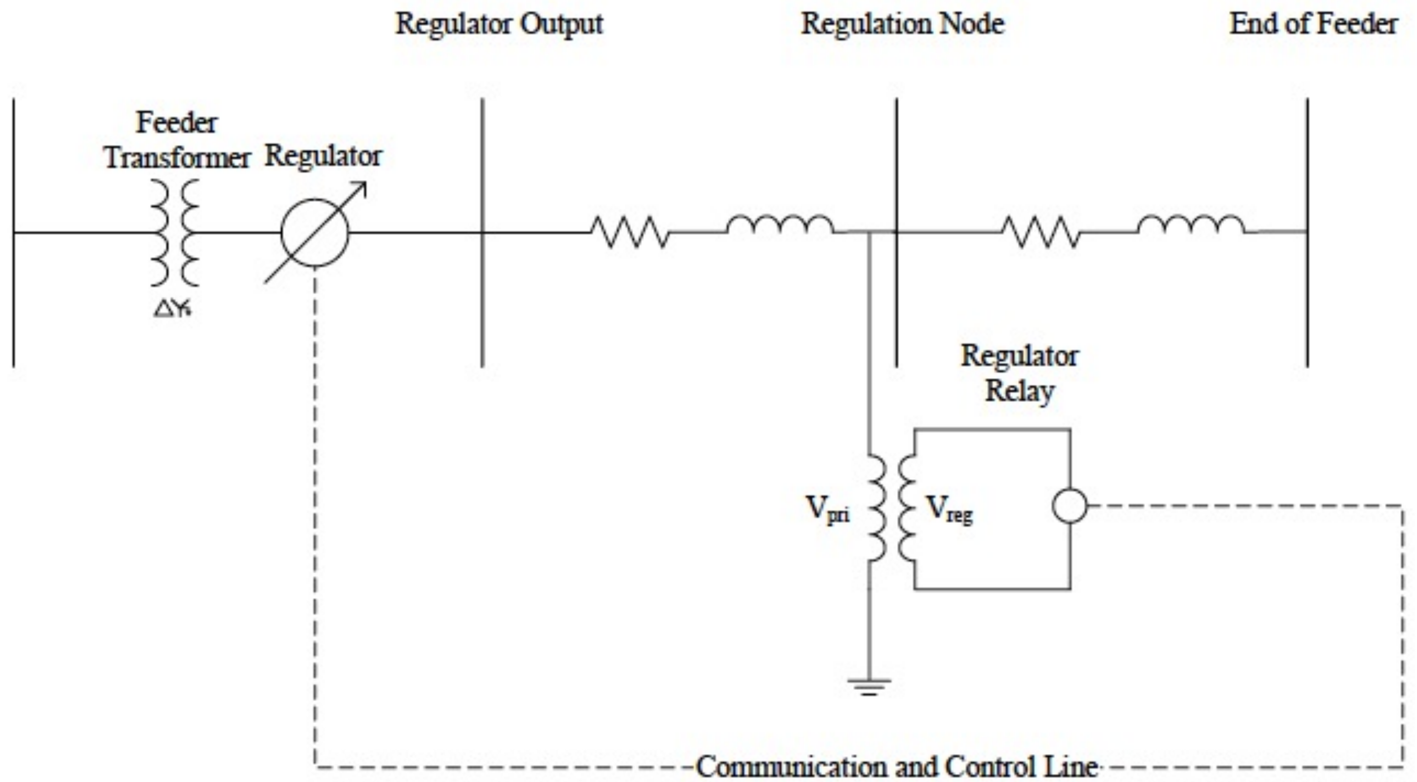
1. Control based on *local voltage output*



Kevin Schneider (PNNL)

Control of voltage regulators

2. Control based on *remote voltage*



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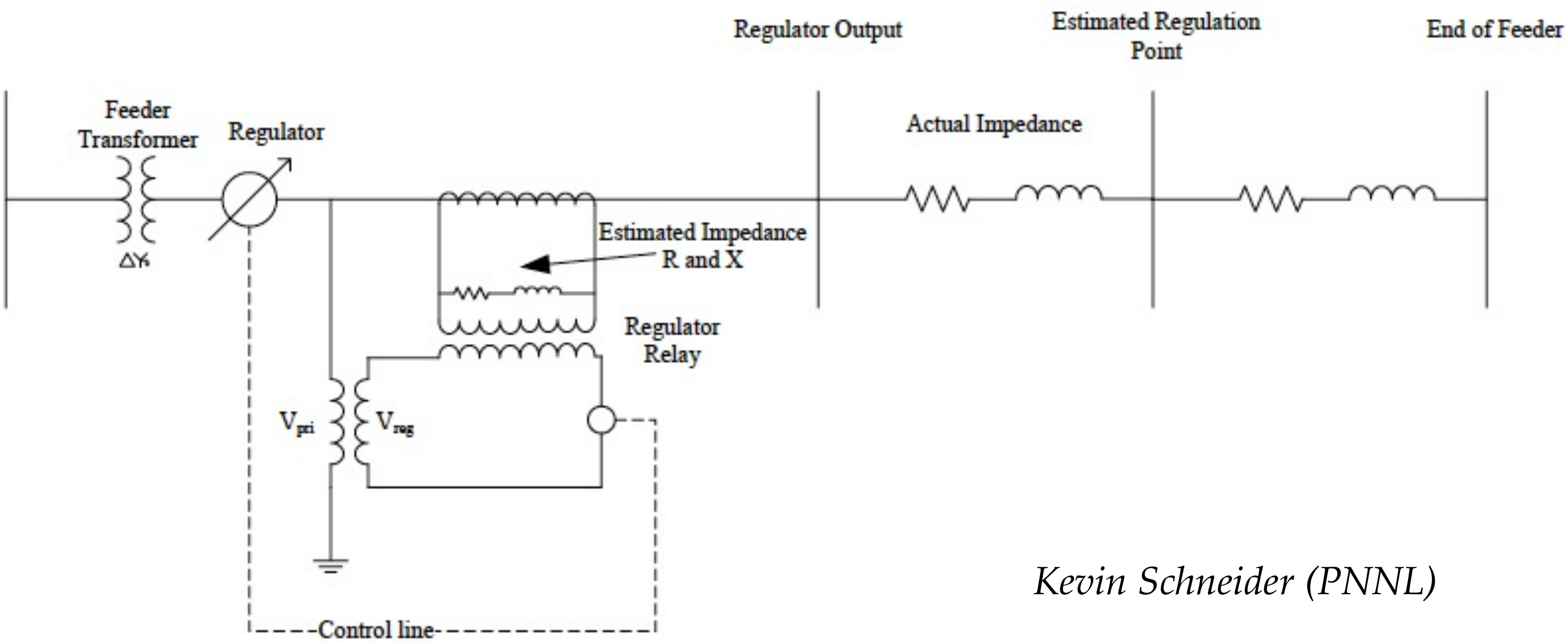
Control of voltage regulators

3. Control based on *voltage predicted at a pseudo-point*

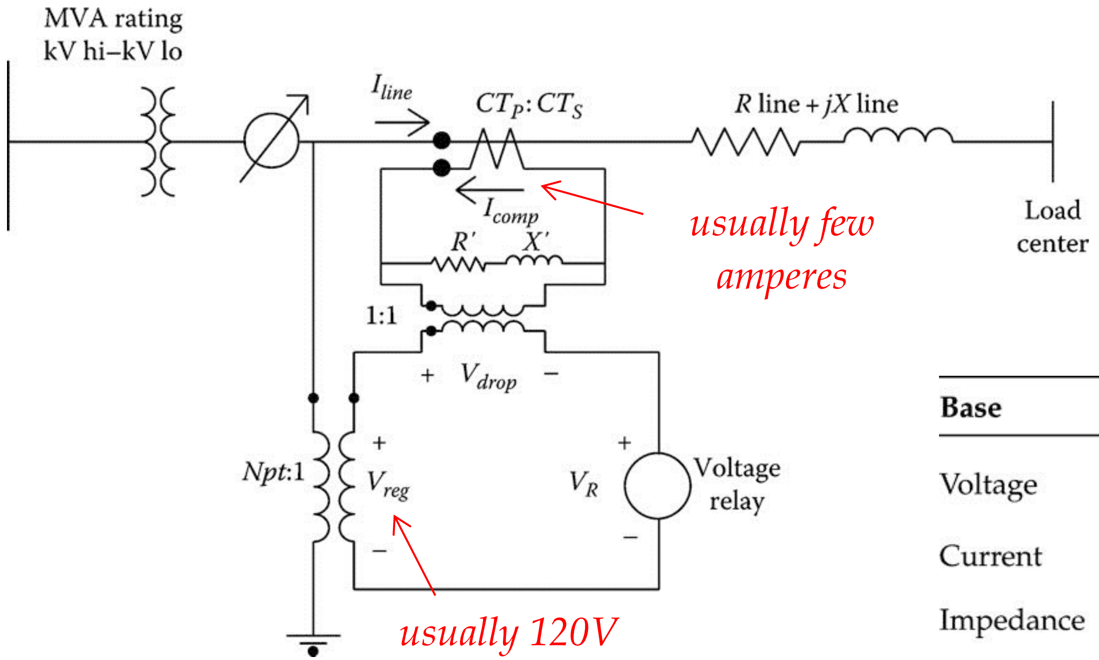


Kevin Schneider (PNNL)

- *Line drop compensator (LDC)* models the voltage drop at a remote point (load center)

Line drop compensator (LDC)

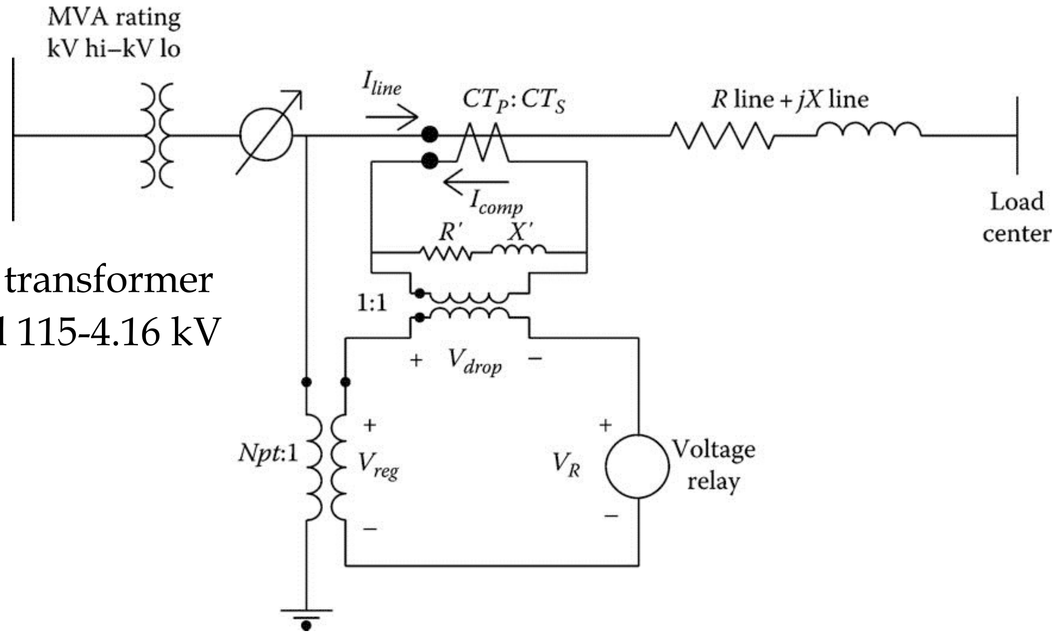
- Equivalent impedance is an analog (or digital) model of the actual line



Base	Line Circuit	Compensator Circuit
Voltage	V_{LN}	$\frac{V_{LN}}{N_{PT}}$
Current	CT_P	CT_S
Impedance	$Zbase_{line} = \frac{V_{LN}}{CT_P}$	$Zbase_{comp} = \frac{V_{LN}}{N_{PT} \cdot CT_S}$

- LDC impedance $Z_{\text{LDC}} = Z_{\text{line}} \frac{Z_{\text{LDC,base}}}{Z_{\text{line,base}}} = Z_{\text{line}} \left(\frac{V_{\text{LDC,base}}}{V_{\text{line,base}}} \right) \left(\frac{I_{\text{line,base}}}{I_{\text{LDC,base}}} \right) = Z_{\text{line}} \frac{\text{CT}_P}{N_{PT}\text{CT}_S} \ \Omega$
- LDC impedance oftentimes expressed in Volts $Z_{\text{LDC,V}} = Z_{\text{LDC}}\text{CT}_S = Z_{\text{line}} \frac{\text{CT}_P}{N_{PT}} \ \text{V}$ (voltage drop under rated current)

Examples 7.4-7.6



substation transformer
5 MVA and 115-4.16 kV

1. Find PT/CT ratings if the LDC ratings are 120V and 5A

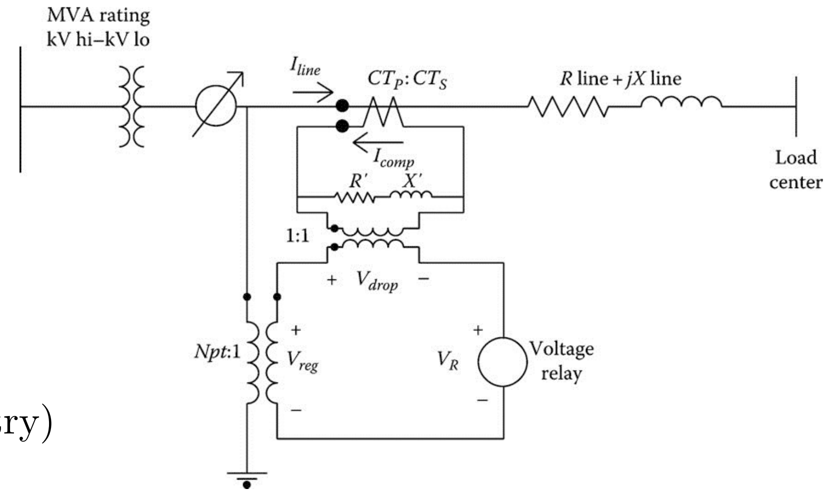
- LDC potential transformer

$$V_{\mathrm{LL,base}} = 4.16\mathrm{kV} \Rightarrow \quad V_{\mathrm{LN,base}} = 2,401.8\mathrm{V} \simeq 2.4\mathrm{kV} \quad \Rightarrow \quad N_{\mathrm{PT}} = \frac{2,400\mathrm{V}}{120\mathrm{V}} = 20$$

- LDC current transformer

$$I_{\mathrm{base}} = \frac{5\mathrm{MVA}}{\sqrt{3} \cdot V_{\mathrm{LL,base}}} = 693.9\mathrm{A} \simeq 700\mathrm{A} \quad \Rightarrow \quad \mathrm{CT} = \frac{\mathrm{CT}_P}{\mathrm{CT}_S} = \frac{700\mathrm{A}}{5\mathrm{A}} = 140$$

Examples 7.4-7.6 (cont'd)



2. Find LDC setting in ohms and volts if the equivalent impedance between SVR and load center is $Z_{\text{line}} = 0.3 + j0.9\ \Omega$

$$Z_{\text{LDC}} = Z_{\text{line}} \frac{\text{CT}_P}{N_{PT}\text{CT}_S} \Omega = 2.1 + j6.3\Omega$$

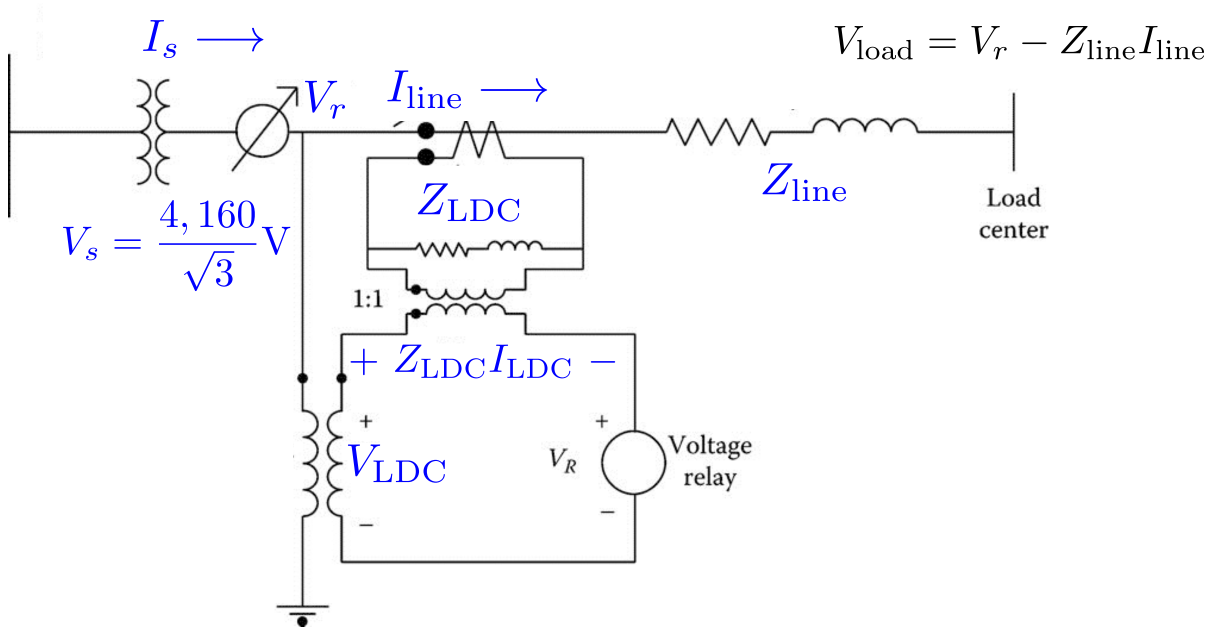
$$Z_{\text{LDC,V}} = (2.1 + j6.3)\Omega \cdot 5\text{A} = 10.5 + j31.5\text{V (data entry)}$$

3. 2.5 MVA load is to be served at 4.16 kV with 0.9 PF lagging. Voltage level to be held within 2V around 120V. Determine tap position so relay voltage is within [119:121]V

- Line current (assuming constant-current load)

$$I_{\text{line}} = \frac{2.5\text{MVA}}{\sqrt{3} \cdot V_{\text{LL,base}}} = 346.97\angle - 25.84^{\text{o}} \Rightarrow I_{\text{LDC}} = 2.478\angle - 25.84^{\text{o}}\ \text{A}$$

Examples 7.4-7.6 (cont'd)



- Relay voltage models load voltage $V_{\text{relay}} = V_{\text{LDC}} - Z_{\text{LDC}} I_{\text{LDC}} = \frac{V_r - Z_{\text{line}} I_{\text{line}}}{N_{\text{PT}}} = \frac{V_{\text{load}}}{N_{\text{PT}}}$
- Regulator output (type-B) $V_r = \frac{V_s}{a_R(\tau)}$ where $a_R(\tau) = 1 - 0.00625 \cdot \tau$

  and $\tau \in \{-16, \ldots, -1, 0, +1, \ldots, +16\}$
- At neutral position $a_R(0) = 1$, relay voltage is $|V_{\text{relay}}| = 109.24\text{V}$
- Raise tap τ so that relay voltage goes above 119V

Examples 7.4-7.6 (cont'd)

- Relay voltage magnitude for different tap settings

- Working assumptions
  - constant-current load
  - VR started from neutral position

- Can you repeat the analysis for a constant-Z load?

- For specific loading, the SVR tap depends also on prior loading conditions

✓ e.g., if the SVR was at +16 before (due to heavy load earlier), then it will now stop at tap +14, not +13 simply because it started *'from above'*

	-16.0000	98.3929
	-15.0000	99.0122
	-14.0000	99.6388
	-13.0000	100.2727
	-12.0000	100.9139
	-11.0000	101.5628
	-10.0000	102.2193
	-9.0000	102.8837
	-8.0000	103.5560
	-7.0000	104.2365
	-6.0000	104.9252
	-5.0000	105.6223
	-4.0000	106.3279
	-3.0000	107.0423
	-2.0000	107.7656
	-1.0000	108.4979
neutral position	0	109.2394
	1.0000	109.9903
	2.0000	110.7508
	3.0000	111.5210
	4.0000	112.3012
	5.0000	113.0914
	6.0000	113.8921
	7.0000	114.7032
	8.0000	115.5251
	9.0000	116.3579
	10.0000	117.2019
	11.0000	118.0573
	12.0000	118.9243
final position	13.0000	119.8031
	14.0000	120.6941
	15.0000	121.5974
	16.0000	122.5133

Examples 7.4-7.6 (textbook solution)

- Book's solution uses three approximations

$$
\begin{aligned}
V_{\text{relay}} &= \frac{V_s}{(1-0.00625\cdot\tau)\,N_{\text{PT}}} - \frac{Z_{\text{line}} I_{\text{line}}}{N_{\text{PT}}} \\
&\simeq (1+0.00625\cdot\tau)\,\frac{V_s}{N_{\text{PT}}} - \frac{Z_{\text{line}} I_{\text{line}}}{N_{\text{PT}}} \\
&= \left(\frac{V_s}{N_{\text{PT}}} - \frac{Z_{\text{line}} I_{\text{line}}}{N_{\text{PT}}}\right) + \left(0.00625\frac{V_s}{N_{\text{PT}}}\right)\tau \\
&\simeq \hat{V}_s + 0.75\tau \Rightarrow
\end{aligned}
$$

$$
\tau = \frac{V_{\text{relay}} - \hat{V}_s}{0.75} \simeq \frac{|V_{\text{relay}}| - |\hat{V}_s|}{0.75} = \frac{119-109.24}{0.75} = 13.02
$$

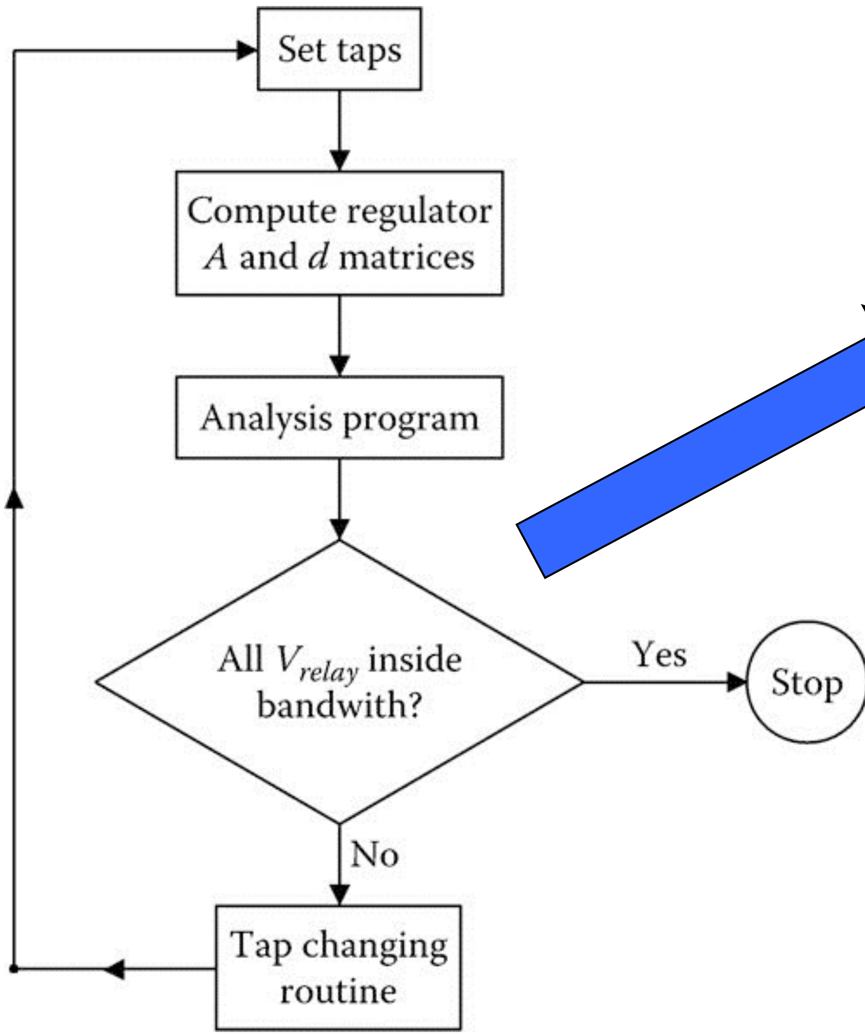
approx. 1: $\frac{1}{1-0.00625\cdot\tau} \simeq 1+0.00625\cdot\tau$

approx. 2: 0.00625 x (~120) = 0.75

approx. 3: small phase differences

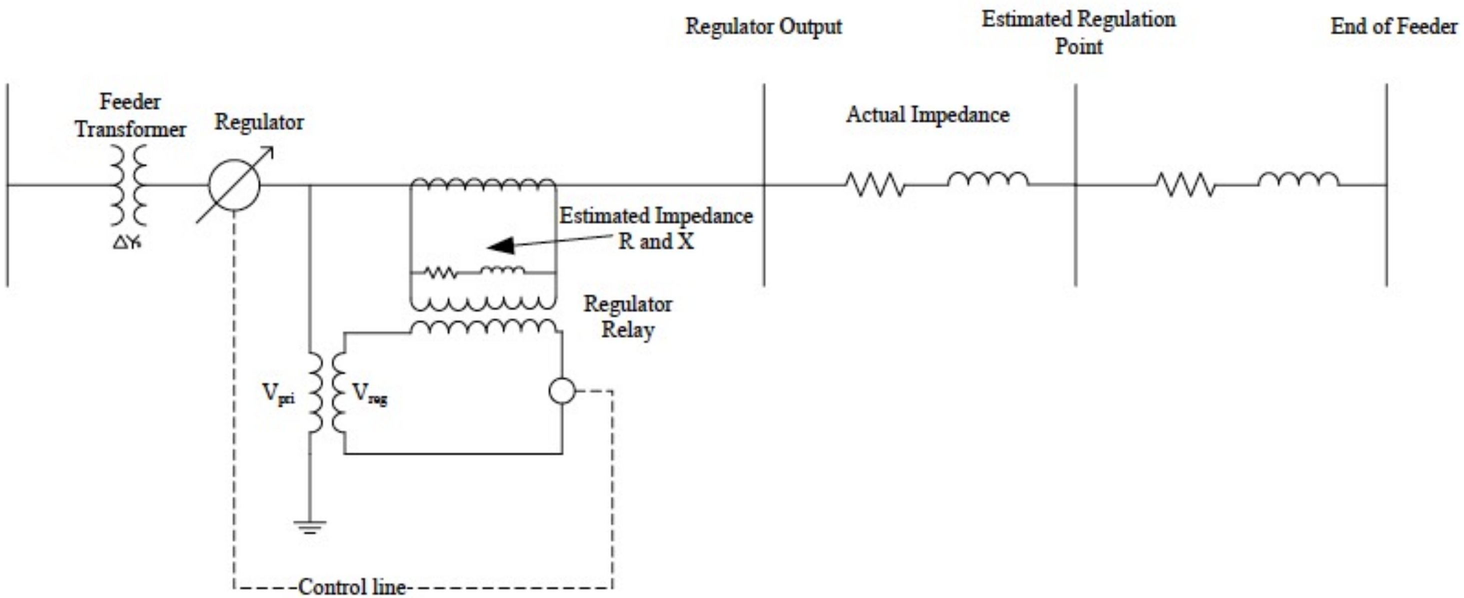
- Textbook also repeats analysis for constant power and finds final tap position to be +12

Finding taps using a PF solver



$$
\begin{aligned}
Y := \; & \text{for } i \in 1..3 \\
& \quad V_{reg_i} \leftarrow \frac{VLN_{3_i}}{N_{pt}} \\
& \quad I_{reg_i} \leftarrow \frac{I_{abc_i}}{CT} \\
& V_{relay} \leftarrow V_{reg} - Z_{comp} \cdot I_{reg} \\
& Tap_1 \leftarrow Tap_1 + 1 \text{ if } \left|V_{relay_1}\right| < 120 \\
& Tap_2 \leftarrow Tap_2 + 1 \text{ if } \left|V_{relay_2}\right| < 120 \\
& Tap_3 \leftarrow Tap_3 + 1 \text{ if } \left|V_{relay_3}\right| < 120 \\
& Out_1 \leftarrow Tap \\
& Out
\end{aligned}
$$

LDC impedance



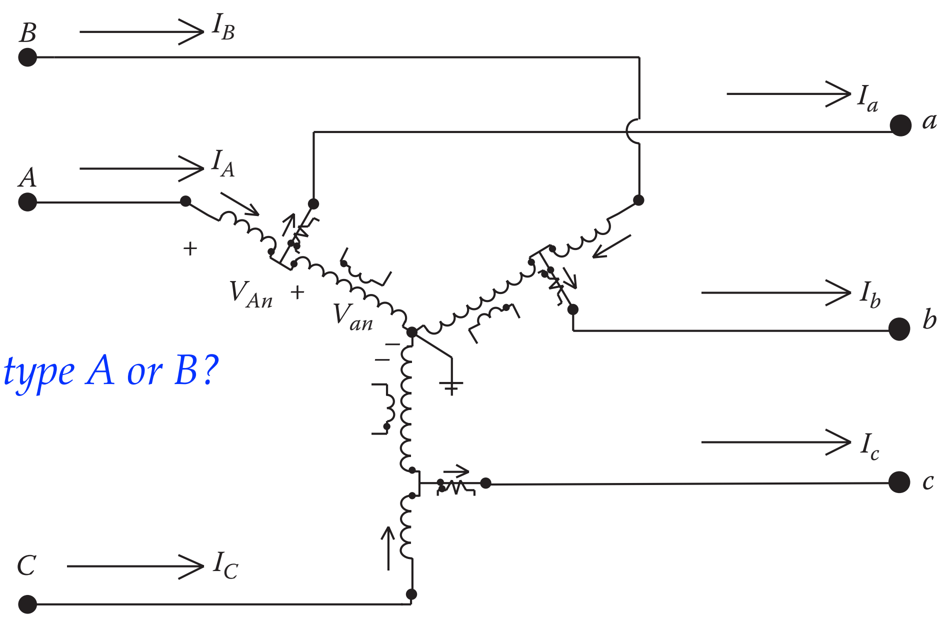
- So far, assumed Z_{LDC} models voltage drop on actual line
- However, usually Z_{LDC} is only an *equivalent impedance* to model the voltage drop between the SVR and a load center located several buses and laterals downstream
- Z_{LDC} is then computed using PF simulations to compute the three needed quantities under different conditions

$$Z_{\text{line,eq}} = \frac{V_r - V_{\text{load}}}{I_{\text{reg}}}$$

Let's move on to three-phase SVRs...

Note: this current may not be reaching the load center

Wye SVR



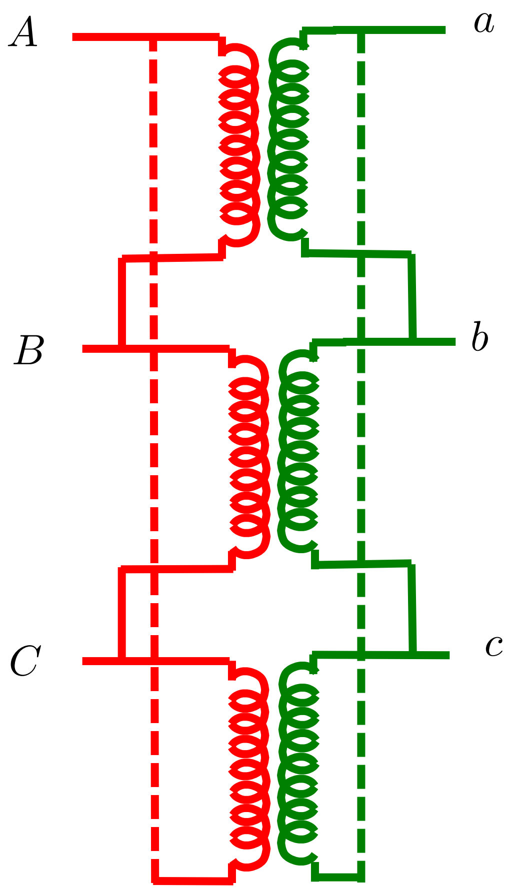
type A or B?

- Each regulator may have its own tap changer/LDC
- Advantage over a single three-phase SVRs that are 'gang'-operated
- Problem if LDCs share the same line impedance
- Generalized matrices

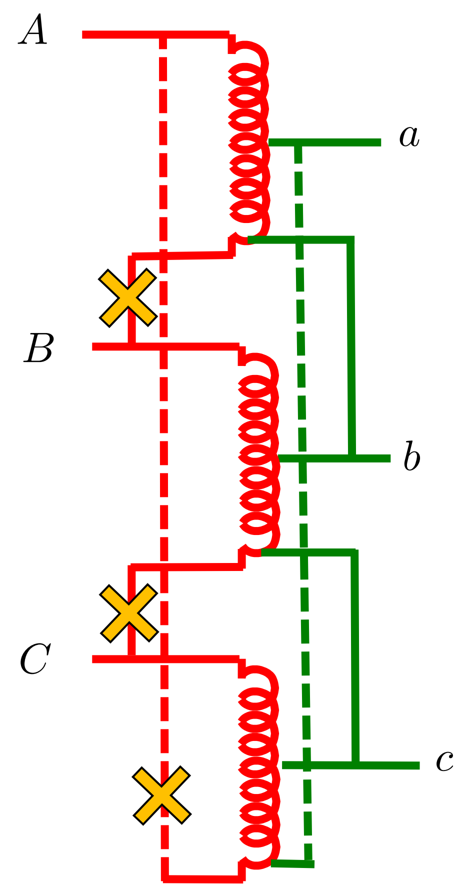
$$\mathbf{v}_\Phi = \mathbf{A}\mathbf{v}_\phi + \mathbf{B}\mathbf{i}_\ell \qquad \mathbf{A} = \begin{bmatrix} a_R^a & 0 & 0 \\ 0 & a_R^b & 0 \\ 0 & 0 & a_R^c \end{bmatrix}; \quad \mathbf{B} = \mathbf{C} = \mathbf{0}; \quad \mathbf{D} = \mathbf{A}^{-1}$$
$$\mathbf{i}_L = \mathbf{C}\mathbf{v}_\phi + \mathbf{D}\mathbf{i}_\ell$$

- Model holds for raise/lower position; extends trivially to open Wye connection

Delta-connected regulators



3 single-phase ordinary
transformers connected in delta



3 single-phase voltage regulators
(type B) connected in delta

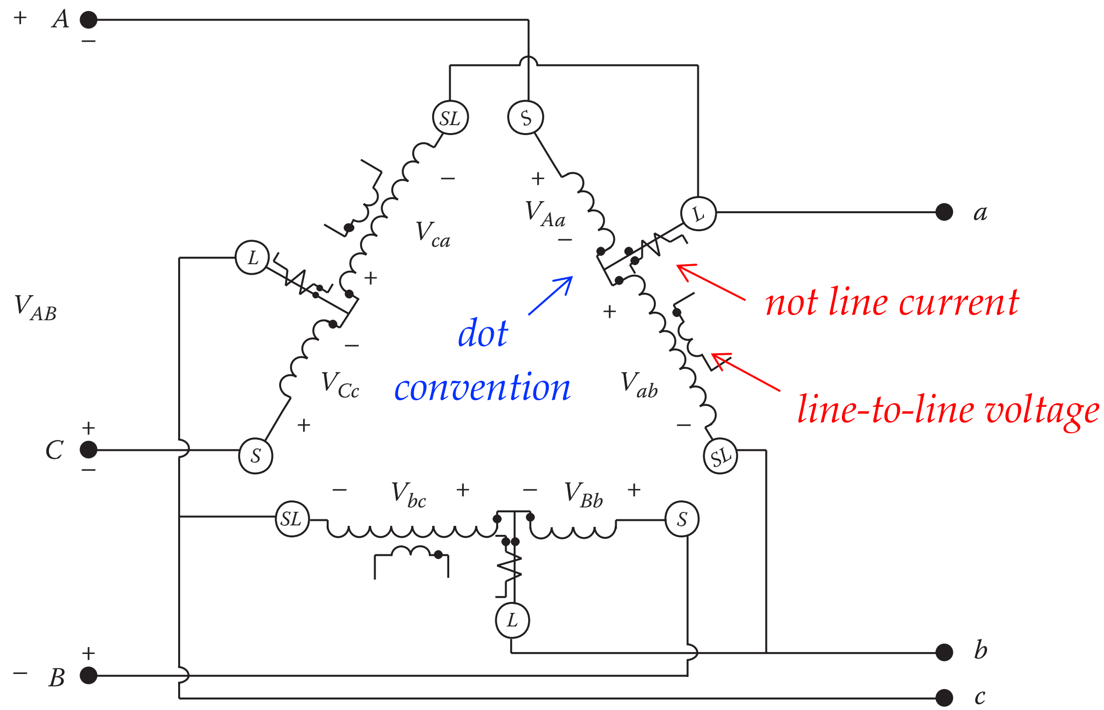
- *E.g.*, point b cannot be connected at both ends of the series winding of second VR

Delta-connected regulators (voltages)

- Used in three-wire delta feeders

- Voltage transformations

$$
\begin{aligned}
V_{AB} &= V_{Aa} + V_{ab} - V_{Bb} \\
&= -\frac{N_2^{ab}}{N_1^{ab}} V_{ab} + V_{ab} - \left(-\frac{N_2^{bc}}{N_1^{bc}} V_{bc} \right) \\
&= \left(1 - \frac{N_2^{ab}}{N_1^{ab}} \right) V_{ab} + \frac{N_2^{bc}}{N_1^{bc}} V_{bc} \\
&= a_R^{ab} V_{ab} + (1 - a_R^{bc}) V_{bc}
\end{aligned}
$$



recall that $a_R^{ab} = 1 \mp \frac{N_2^{ab}}{N_1^{ab}}$

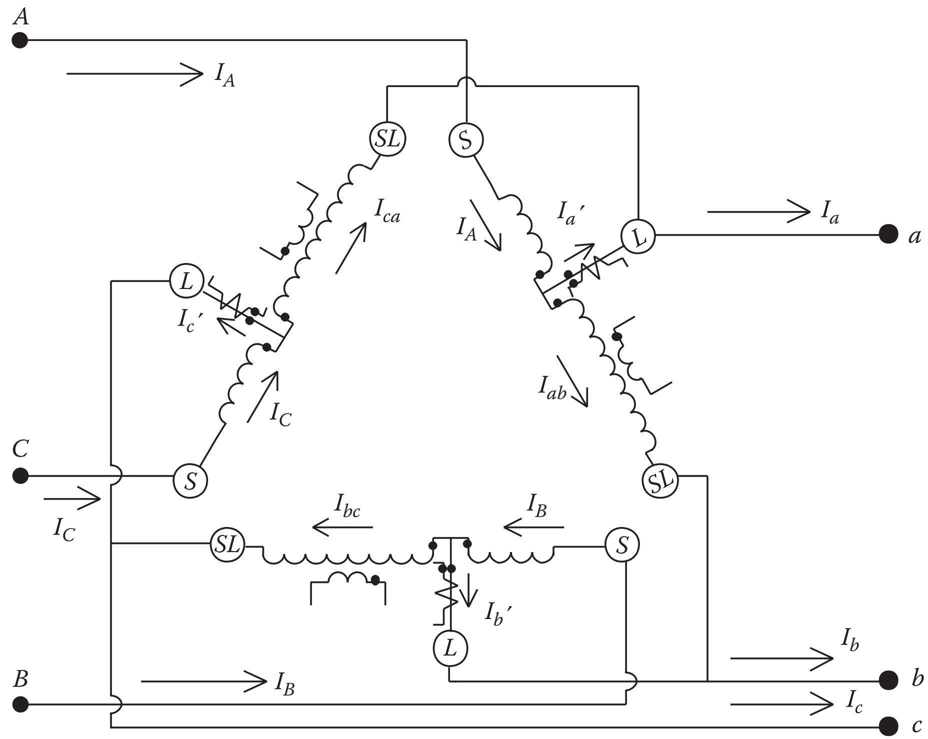
- Repeat for other LL loops to get AB model

$$
\mathbf{v}_{\Phi,LL} = \begin{bmatrix} a_R^{ab} & 1 - a_R^{bc} & 0 \\ 0 & a_R^{bc} & 1 - a_R^{ca} \\ 1 - a_R^{ab} & 0 & a_R^{ca} \end{bmatrix} \mathbf{v}_{\phi,LL}
$$

Delta-connected regulators (currents)

- Current transformations

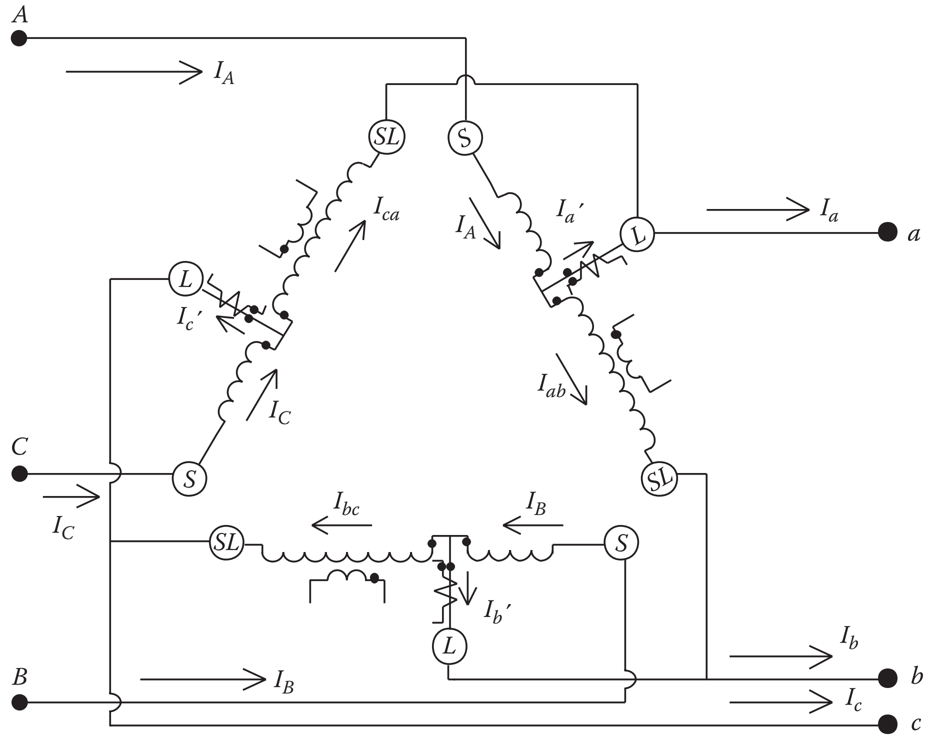
$$
\begin{aligned}
I_a &= I_{ca} + I_A - I_{ab} \\
&= \frac{N_2^{ca}}{N_1^{ca}} I_C + I_A - \frac{N_2^{ab}}{N_1^{ab}} I_A \\
&= \left(1 - \frac{N_2^{ab}}{N_1^{ab}}\right) I_A + \frac{N_2^{ca}}{N_1^{ca}} I_C \\
&= a_R^{ab} I_A + (1 - a_R^{ca}) I_C
\end{aligned}
$$



- Repeat for other load nodes to get the CD *after inversion*

$$
\mathbf{i}_\ell = \begin{bmatrix} a_R^{ab} & 0 & 1 - a_R^{ca} \\ 1 - a_R^{ab} & a_R^{bc} & 0 \\ 0 & 1 - a_R^{bc} & a_R^{ca} \end{bmatrix} \mathbf{i}_L
$$

Closed delta SVR (ABCD)



- Generalized matrices $\quad \mathbf{v}_{\Phi,LL} = \mathbf{A}\mathbf{v}_{\phi,LL} + \mathbf{B}\mathbf{i}_\ell$

$$\mathbf{i}_L = \mathbf{C}\mathbf{v}_{\phi,LL} + \mathbf{D}\mathbf{i}_\ell$$

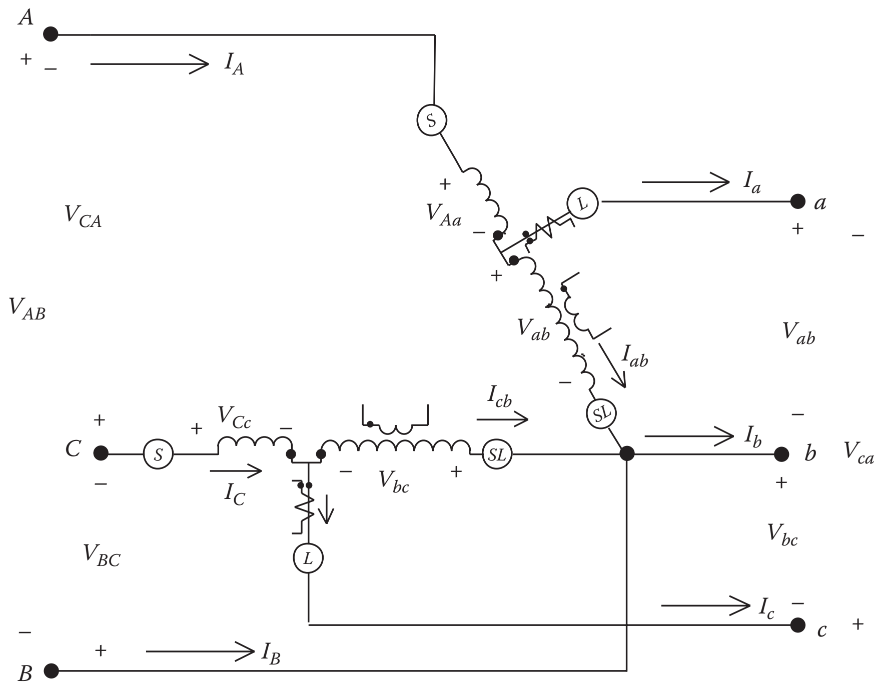
$$\mathbf{A} = \begin{bmatrix} a_R^{ab} & 1-a_R^{bc} & 0 \\ 0 & a_R^{bc} & 1-a_R^{ca} \\ 1-a_R^{ab} & 0 & a_R^{ca} \end{bmatrix}; \mathbf{B} = \mathbf{C} = \mathbf{0}; \mathbf{D} = \begin{bmatrix} a_R^{ab} & 0 & 1-a_R^{ca} \\ 1-a_R^{ab} & a_R^{bc} & 0 \\ 0 & 1-a_R^{bc} & a_R^{ca} \end{bmatrix}^{-1}$$

- Harder to control due to coupling in voltages and currents

Open delta SVR

- Regulate voltage in 3-wire delta system with only two regulators

- Voltage transformations



$$V_{AB} = V_{Aa} + V_{ab}$$

$$= \left(1 - \frac{N_2^{ab}}{N_1^{ab}}\right) V_{ab} = a_R^{ab} V_{ab}$$

$$V_{BC} = a_R^{bc} V_{bc}$$

$$V_{CA} = -(V_{AB} + V_{BC})$$

$$= -a_R^{ab} V_{ab} - a_R^{bc} V_{bc}$$

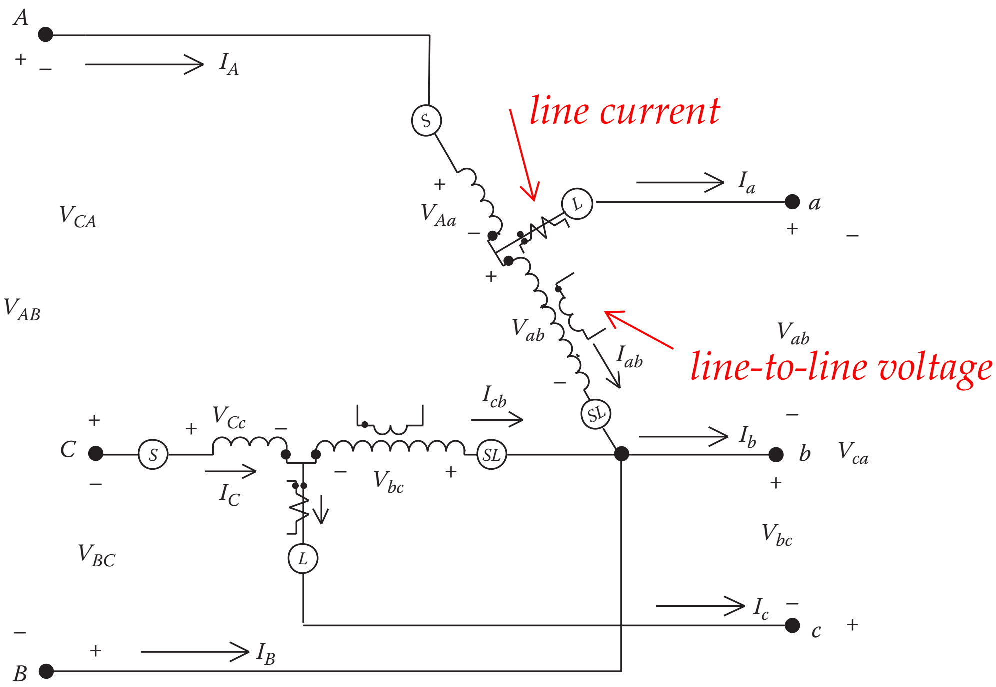
- Current transformations

$$I_A = I_a + I_{ab} = I_a + \frac{N_2^{ab}}{N_1^{ab}} I_A \quad \Rightarrow \quad I_A = \frac{1}{a_R^{ab}} I_a$$

similarly: $$I_C = \frac{1}{a_R^{cb}} I_c$$

$$I_B = -(I_A + I_B)$$

Open delta SVR (cont'd)



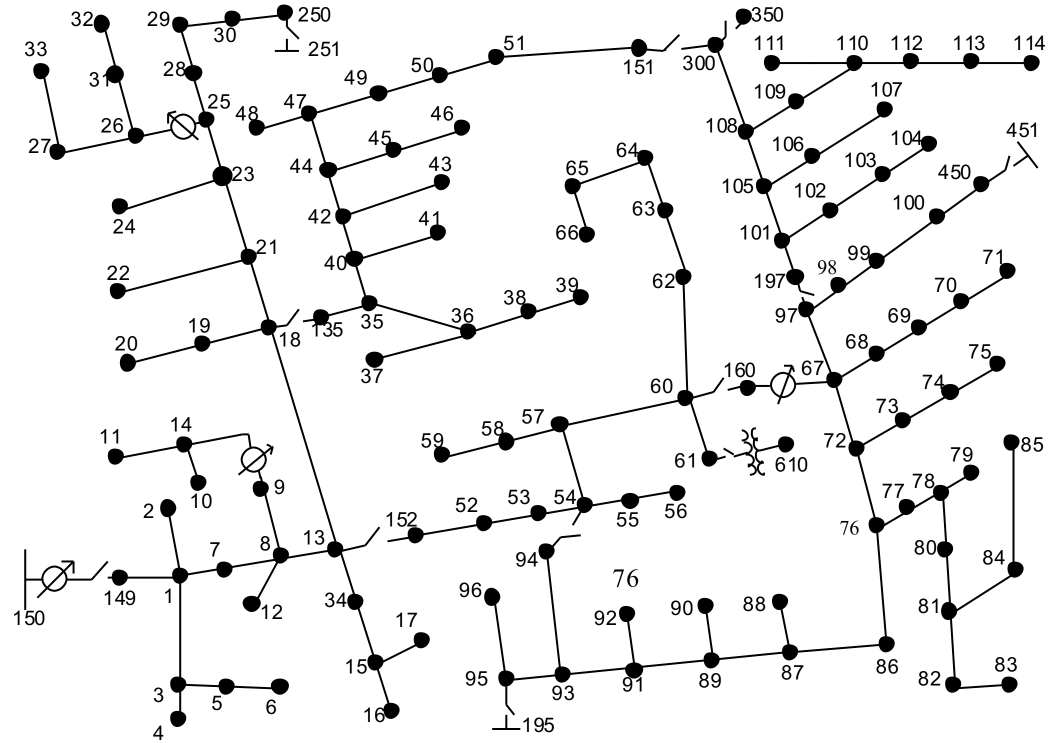
- Generalized matrices $\mathbf{v}_{\Phi,LL} = \mathbf{A}\mathbf{v}_{\phi,LL} + \mathbf{B}\mathbf{i}_{\ell}$

$$\mathbf{i}_L = \mathbf{C}\mathbf{v}_{\phi,LL} + \mathbf{D}\mathbf{i}_{\ell}$$

$$\mathbf{A} = \begin{bmatrix} a_R^{ab} & 0 & 0 \\ 0 & a_R^{cb} & 0 \\ -a_R^{ab} & -a_R^{cb} & 0 \end{bmatrix}; \mathbf{B} = \mathbf{C} = \mathbf{0}; \mathbf{D} = \begin{bmatrix} 1/a_R^{ab} & 0 & 0 \\ -1/a_R^{ab} & 0 & -1/a_R^{cb} \\ 0 & 0 & 1/a_R^{cb} \end{bmatrix}$$

Examples from IEEE 123- bus feeder

Regulator ID:	3	
Line Segment:	25 - 26	
Location:	25	
Phases:	A-C	
Connection:	2-Ph,L-G	
Monitoring Phase:	A & C	
Bandwidth:	1	
PT Ratio:	20	
Primary CT Rating:	50	
Compenator:	Ph-A	Ph-C
R - Setting:	0.4	0.4
X - Setting:	0.4	0.4
Voltage Level:	120	120



Regulator ID:	2
Line Segment:	9 - 14
Location:	9
Phases:	A
Connection:	1-Ph, L-G
Monitoring Phase:	A
Bandwidth:	2.0 volts
PT Ratio:	20
Primary CT Rating:	50
Compensator:	Ph-A
R - Setting:	0.4
X - Setting:	0.4
Voltage Level:	120

Regulator ID:	1
Line Segment:	150 - 149
Location:	150
Phases:	A-B-C
Connection:	3-Ph, Wye
Monitoring Phase:	A
Bandwidth:	2.0 volts
PT Ratio:	20
Primary CT Rating:	700
Compensator:	Ph-A
R - Setting:	3
X - Setting:	7.5
Voltage Level:	120

Regulator ID:	4		
Line Segment:	160 - 67		
Location:	160		
Phases:	A-B-C		
Connection:	3-Ph, LG		
Monitoring Phase:	A-B-C		
Bandwidth:	2		
PT Ratio:	20		
Primary CT Rating:	300		
Compensator:	Ph-A	Ph-B	Ph-C
R - Setting:	0.6	1.4	0.2
X - Setting:	1.3	2.6	1.4
Voltage Level:	124	124	124

Summary

- Voltage regulation is challenging in distribution systems

- Voltage control mechanisms
  - OLTC
  - in-line voltage regulators
  - capacitors
  - (smart inverters)

- Control of SVR can be performed based on
  - local readings
  - remote readings
  - LDC circuit
  - centrally computed voltage setpoints

- Derived ABCD models for single- and three-phase SVRs