ECE 5984: Power Distribution System Analysis

Lecture 7: Load Models

Reference: Textbook, Chapter 9

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VirginiaTech
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Outline

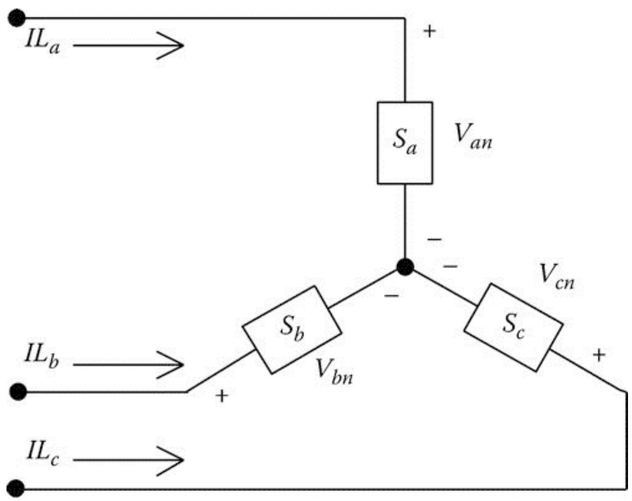
Approximate power flow analysis of Chapter 2

- Each loads described as a kVA/PF or kW/kVAR duplet
- Given feeder voltage, find voltages at all buses
- Found currents from complex power and assuming nominal voltage at load

Load models for detailed power flow analysis

- Wye or delta-connected loads
- Multiple-phase (V- or 1-phase as special cases)
- Power-voltage relationship (ZIP model) and combinations
- Static or dynamic loads (induction motor loads)

Constant-power loads

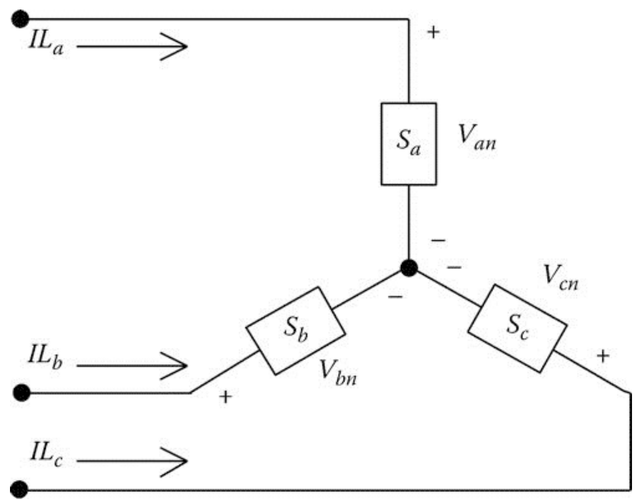


- Common in power flow analysis
- *Given:* complex three-phase power $\{s_\phi^0\}_\phi$
- *Backward sweep:* given voltages, update current

$$i_\phi = \left(\frac{s_\phi^0}{v_\phi}\right)^* \quad \forall\phi \quad \Longrightarrow \quad \mathbf{i} = \mathrm{dg}^{-1}(\mathbf{v}^*)(\mathbf{s}^0)^*$$

- *Forward sweep:* given substation voltage and currents, update voltages downstream
- Complex power remains constant at all $\mathbf{v}$: $s_\phi(v_\phi) = s_\phi^0, \quad \forall\phi$
- Single-/two-phase loads modeled by setting missing currents to zero

Constant-impedance loads



- *Given:* complex rated three-phase power at rated voltage $|v_\phi^{\text{rated}}|$

- Constant per-phase *impedance* found once as

$$z_\phi^0 = \frac{|v_\phi^{\text{rated}}|^2}{(s_\phi^{\text{rated}})^*}$$

- *Backward sweep:* given voltage, update current as

$$i_\phi = \frac{v_\phi}{z_\phi^0} \quad \forall \phi \quad \Longrightarrow \quad \mathbf{i} = \text{dg}^{-1}(\mathbf{z}^0)\mathbf{v}$$

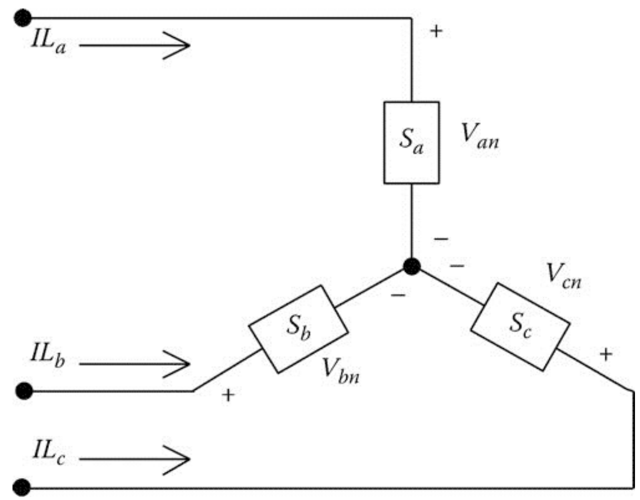
- Propagate currents upstream; update voltages downstream; and iterate

- Complex power $s_\phi(v_\phi) = v_\phi i_\phi^* = \dfrac{|v_\phi|^2}{(z_\phi^0)^*} \quad \forall \phi$

  magnitude (apparent power) depends *quadratically* on $|v_\phi|$

  power factor remains *constant*

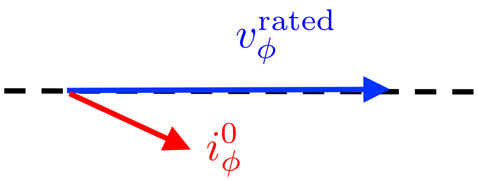
Constant-current loads



- *Given:* complex rated three-phase power at rated voltage $|v_\phi^{\text{rated}}|$

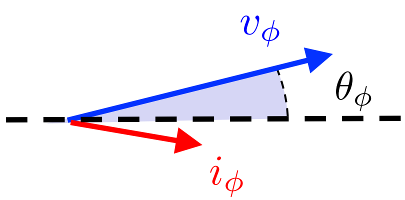
- Constant per-phase *current* found once as

$$i_\phi^0 = \frac{\left(s_\phi^{\text{rated}}\right)^*}{|v_\phi^{\text{rated}}|}$$



- *Backward sweep*: given voltage $v_\phi = |v_\phi| e^{j\theta_\phi}$, update current

$$i_\phi = i_\phi^0 e^{j\theta_\phi} = i_\phi^0 \frac{v_\phi}{|v_\phi|} \quad \forall\phi$$



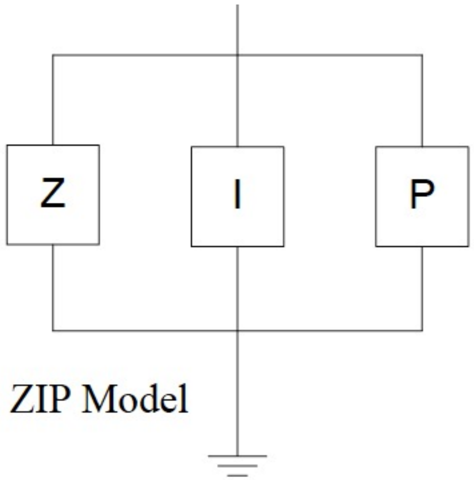
- Propagate currents upstream; update voltages downstream; and iterate

- Complex power $s_\phi(v_\phi) = |v_\phi| \cdot (i_\phi^0)^* \; \forall\phi$

  magnitude (apparent power) depends *linearly* on $|v_\phi|$

  power factor remains *constant*

ZIP model



[Kevin Schneider, PNNL]

- Loads typically described as combination of 3 types
- To specify a ZIP load:
  - complex three-phase power $\{s_\phi\}$
  - percentages for either two or all three of the load types (Z%, I%, P%)
  - percentages refer to *complex power at nominal voltage*
- Line current will be updated as the sum of the three respective currents
- Constant load types are special cases (100% Z, I, or P)
- ZIP is a time-invariant model; recent research on time-variant loads *(water heaters, HVAC, buildings)*

Example of ZIP load

Combination load with rated complex power

$$\mathbf{s} = \begin{bmatrix} 1.9 + j1.2 \\ 2.3 + j0.8 \\ 1.8 + j0.8 \end{bmatrix} \text{ MVA at } V_{LL} = 12.47 \text{ kV and } Z : 20\%;\ I : 30\%;\ P : 50\%$$

- Constant-impedance component $\mathbf{s}_Z^{\text{rated}} = 0.2\mathbf{s} \implies z_\phi^0 = \dfrac{7,200^2}{(s_{Z,\phi}^{\text{rated}})^*}$

- Constant-current component $\mathbf{s}_I^{\text{rated}} = 0.3\mathbf{s} \implies i_\phi^0 = \dfrac{\left(s_{I,\phi}^{\text{rated}}\right)^*}{7,200}$

- Constant-power component $\mathbf{s}_P^0 = 0.5\mathbf{s}$

- Once constants have been calculated, currents are iteratively updated as

$$\begin{aligned} i_\phi(v_\phi^t) &= i_{Z,\phi}(v_\phi^t) + i_{I,\phi}(v_\phi^t) + i_{P,\phi}(v_\phi^t) \\ &= \frac{v_\phi^t}{z_\phi^0} + \frac{v_\phi^t}{|v_\phi^t|} i_\phi^0 + \left(\frac{s_\phi^0}{v_\phi^t}\right)^* \end{aligned}$$

- Currents are propagated upstream; voltages are updated downstream

Voltage conversions (LN to LL)

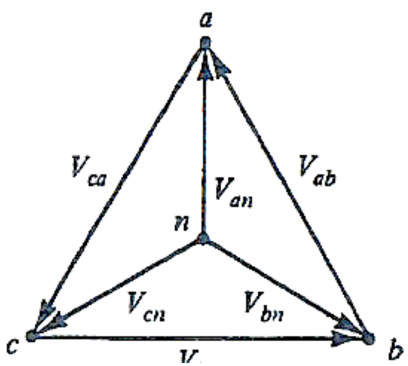
- *LN voltages* $\mathbf{v} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$ *LL voltages* $\tilde{\mathbf{v}} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$

- LN-to-LL conversion $\tilde{\mathbf{v}} = \mathbf{D}_f \mathbf{v}$ where $\mathbf{D}_f = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$ *singular matrix!*

- *Difference* matrix $\mathbf{D}_f$ not to be confused with matrix $\mathbf{D}$ in ABCD models

- LL voltages are zero-sum $\mathbf{1}^\top \tilde{\mathbf{v}} = V_{ab} + V_{bc} + V_{ca} = 0$ *even for unbalanced conditions*

- Usual conversion $\tilde{\mathbf{v}} = \sqrt{3} e^{j\pi/6} \mathbf{v}$
  holds only for balanced voltages



Voltage conversions (LL to LN)

- Without extra info, LN voltages cannot be uniquely recovered from LL voltages

$$\tilde{\mathbf{v}} = \mathbf{D}_f \mathbf{v} \quad \text{where} \quad \mathbf{D}_f = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

singular matrix!

- For example, if $\tilde{\mathbf{v}} = \mathbf{D}_f \mathbf{v} \;\Rightarrow\; \tilde{\mathbf{v}} = \mathbf{D}_f(\mathbf{v} + \beta \mathbf{1}) \;\; \forall \beta \in \mathbb{C}$

- We can recover a set of *'equivalent'* LN voltages by adding the extra info that they are zero sum $\mathbf{1}^\top \mathbf{v} = 0$

$$\tilde{\mathbf{v}} = \mathbf{D}_f \mathbf{v} = \mathbf{D}_f \mathbf{v} + \frac{1}{3}\mathbf{1}\mathbf{1}^\top \mathbf{v} = \left(\mathbf{D}_f + \frac{1}{3}\mathbf{1}\mathbf{1}^\top\right)\mathbf{v} \Rightarrow$$

$$\mathbf{v} = \mathbf{W}\tilde{\mathbf{v}} \quad \text{where} \quad \mathbf{W} = \left(\mathbf{D}_f + \frac{1}{3}\mathbf{1}\mathbf{1}^\top\right)^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

'Law of cosines'

- *Goal:* Given *LL* voltage magnitudes, find *LL* voltage phasors

- Determine their phase angles using the *law of cosines*

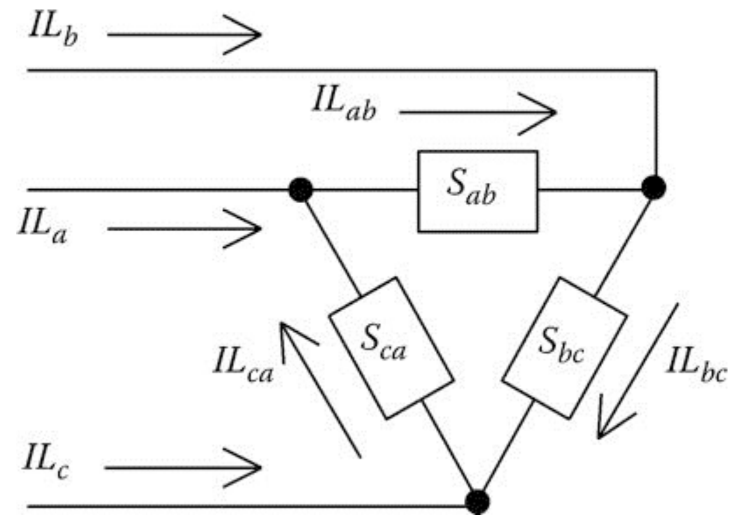
- Exploit zero-sum property $V_{ab} + V_{bc} + V_{ca} = 0$ and use V_{ab} as reference

$$\begin{aligned} V_{ab} + V_{bc} + V_{ca} &= 0 \Rightarrow \\ V_{ab} + V_{bc} &= -V_{ca} \Rightarrow \\ |V_{ab}|^2 + |V_{bc}|^2 + 2\text{Re}\{V_{ab}V_{bc}^*\} &= |V_{ca}|^2 \Rightarrow \\ |V_{ab}|^2 + |V_{bc}|^2 + 2|V_{ab}| \cdot |V_{bc}| \cos(\theta_{ab} - \theta_{bc}) &= |V_{ca}|^2 \Rightarrow \\ |V_{ab}|^2 + |V_{bc}|^2 + 2|V_{ab}| \cdot |V_{bc}| \cos(\theta_{bc}) &= |V_{ca}|^2 \end{aligned}$$

Delta-connected loads

line currents $\mathbf{i} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$

phase (delta) currents $\tilde{\mathbf{i}} = \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$



- We are given voltage and complex power ratings for each delta phase
- Phase (delta) currents are computed as in Y-connection based on ZIP parameters
- Line currents are computed as differences of phase currents

$$\mathbf{i} = \begin{bmatrix} I_{ab} - I_{ca} \\ I_{bc} - I_{ab} \\ I_{ca} - I_{bc} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tilde{\mathbf{i}} = \mathbf{D}_f^\top \tilde{\mathbf{i}}$$

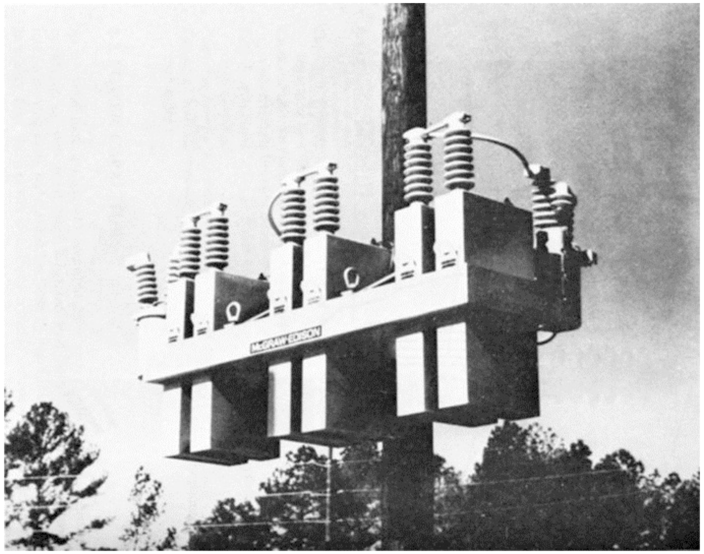
Shunt capacitors

- In Ch. 2, capacitors were approximated as constant-current
- To be precise, they are *constant-susceptance* loads
- Calculate susceptance from ratings

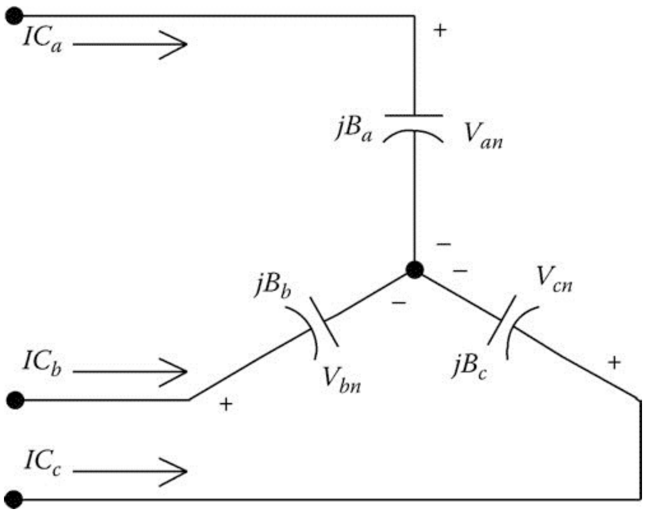
$$B_\phi = \frac{Q_\phi^{\text{rated}}}{|v_\phi^{\text{rated}}|^2} \text{ Siemens} > 0$$

- Update line currents as

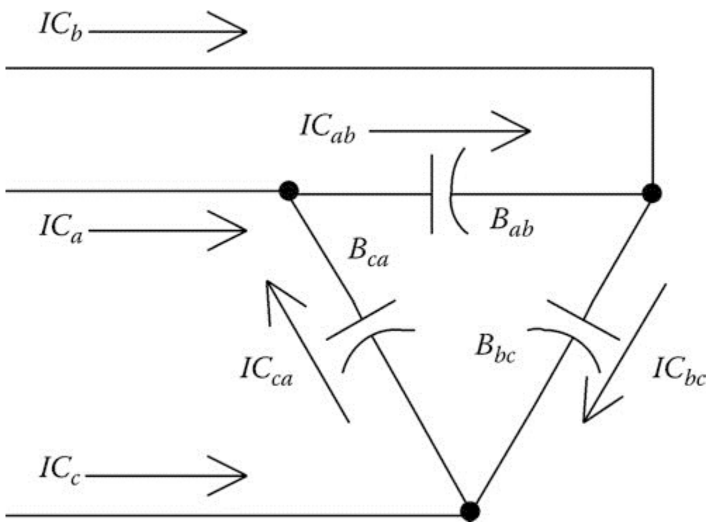
$$i_\phi = jB_\phi v_\phi \quad \forall \phi$$



capacitors on a switched pole-top rack [Gonen]



Delta-connected capacitors



- Phase susceptance $B_\phi = \dfrac{Q_\phi^{\text{rated}}}{V_{\text{LL}}^2}$ Siemens > 0

- Update phase (delta) currents as $\tilde{\mathbf{i}} = j\text{dg}(\mathbf{B})\tilde{\mathbf{v}}$

- Update line currents as $\mathbf{i} = \mathbf{D}_f^\top \tilde{\mathbf{i}}$

Induction machines under unbalanced conditions

- *Goal:* incorporate induction machines into our power flow analysis

- *Questions* to be answered

  a) Given terminal (stator) voltages, find currents

  b) Given stator voltage and current, find rotor voltage/current and losses

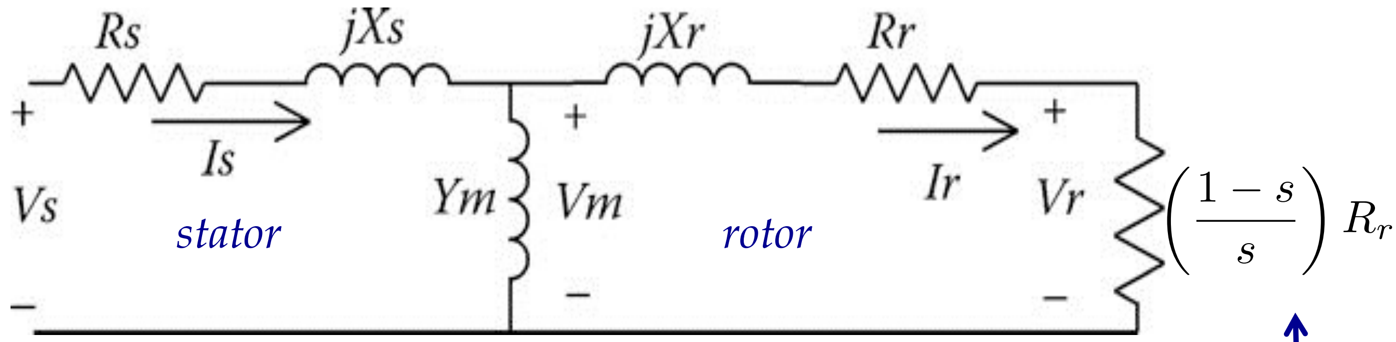
  c) Find the slip that yields a specified value of power

- *Methodology*

  Step 1) Use existing sequence model to find a phase impedance model $\mathbf{v} = \mathbf{Z}\mathbf{i}$

  Step 2) Use T-model to propagate voltages/currents from stator to rotor

Induction machine under balanced operation



virtual resistor models power converted from electrical to mechanical

- Per-phase IM model under balanced conditions

$$n_s = \frac{120 f_e}{P} \text{ synchronous speed [rpm]}$$

$$f_e : \text{electric frequency}$$

$$P : \text{number of poles}$$

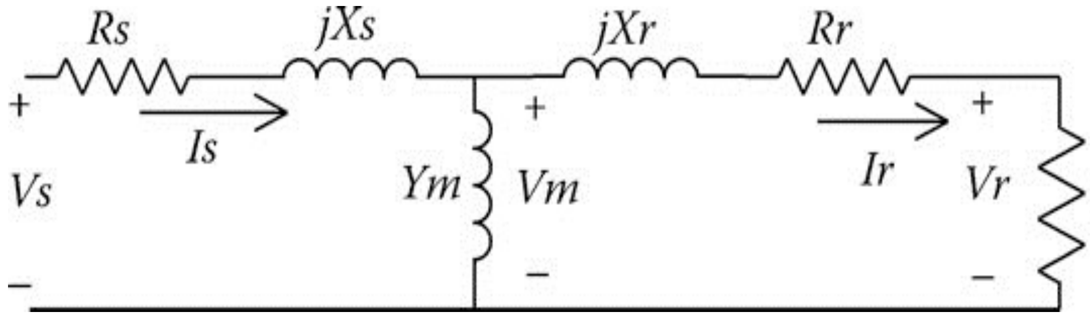
$$n_r : \text{mechanical (rotor) speed [rpm]}$$

$$s = \frac{n_\mathrm{s} - n_r}{n_\mathrm{s}}, \;\; s \in [0, 1]$$

- Stator windings connected as *ungrounded Wye* or *Delta*

Sequence-equivalent circuit

- Study unbalanced conditions through sequence domain



$$\left(\frac{1-s_i}{s_i}\right) R_r = R_{r,i}$$

- Zero-sequence current and voltage are zero

- Sequence circuits differ only in slip

$$s_1 = s$$

$$s_2 = 2 - s \;\Rightarrow\; \left(\frac{1-s_2}{s_2}\right) R_r < 0 \text{ negative!}$$

- Equivalent impedance

$$Z_0 = 1$$

$$Z_i = Z_s + \frac{(jX_m)(Z_r + R_{r,i})}{jX_m + Z_r + R_{r,i}}, \;\; i = 1, 2$$

J. E. Williams, "Operation of 3-phase induction motors on unbalanced voltages," *AIEE Trans. Power App. Syst.*, Apr. 1954.

Problem setup

- Consider ungrounded Wye, since sequence networks apply for Wye
- If Delta-connected, convert it to ungrounded Wye with $Z_Y = \frac{Z_\Delta}{3}$
- *Goal:* given unbalanced *LN* voltages, find line currents; and vice versa
- Line currents are zero-sum
- *LL* voltages are zero-sum
- *LN* voltages assumed zero-sum wlog ('equivalent')

Step 1: phase impedance matrix

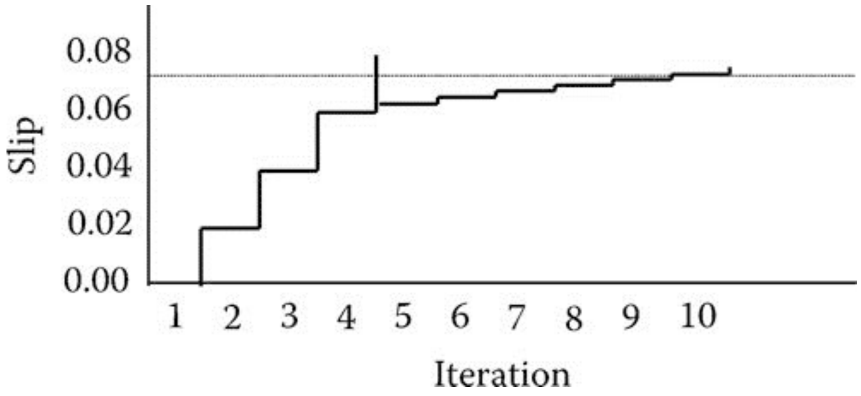
- Sequence circuits are decoupled $\mathbf{i}_s = \mathrm{dg}^{-1}(\mathbf{z})\mathbf{v}_s$ where $\mathbf{z} := [Z_0 \ Z_1 \ Z_2]^\top$
- Sequence-to-phase transformation $$\boxed{\mathbf{i} = \mathbf{A}_s \mathrm{dg}^{-1}(\mathbf{z})\mathbf{A}_s^{-1}\mathbf{v}}$$

voltages used must be zero-sum; not *LG* voltages that may be available

- Phase-frame admittance matrix $\mathbf{A}_s \mathrm{dg}^{-1}(\mathbf{z})\mathbf{A}_s^{-1}$ *depends on slip!*
- Relating line currents to *LL* voltages $$\boxed{\mathbf{i} = \mathbf{A}_s \mathrm{dg}^{-1}(\mathbf{z})\mathbf{A}_s^{-1}\mathbf{W}\tilde{\mathbf{v}}}$$
- Previous two boxed equations can be inverted
- Per-phase complex power $\mathbf{s} = \mathrm{dg}(\mathbf{v})\mathbf{i}^*$
- Total power consumed by motor $S = \mathbf{1}^\top \mathbf{s} = \mathbf{v}^\top \mathbf{i}^*$

Step 2: computing the slip

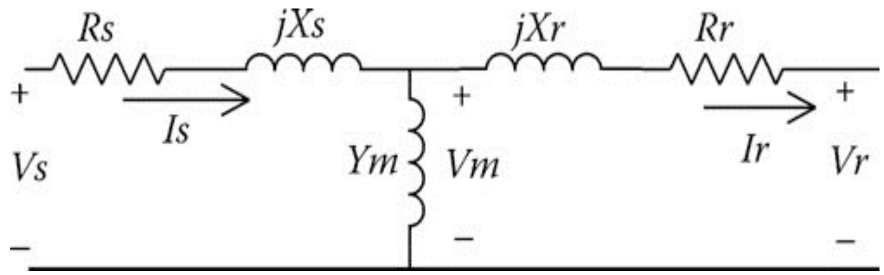
- So far: given input voltage and slip, find line currents to propagate upstream
- Motor has been assumed a *constant-impedance* load depending on given slip
- Usually the input power is specified (*constant-power* load) and the *slip varies*
- Given input voltage, iteratively update slip to match specified power



- Input *active power* is an increasing function of slip (at steady state)
- Use bisection or incremental updates to bring the error in input power to zero

Step 3: finding rotor voltages/currents

- What if we need to find the internal circuit quantities?



- ABCD model for each sequence circuit

$$\begin{bmatrix} V_s^{\text{st}} \\ I_s^{\text{st}} \end{bmatrix} = \begin{bmatrix} 1 + Z_s Y_m & Z_s + Z_r + Z_s Z_r Y_m \\ Y_m & 1 + Z_r Y_m \end{bmatrix} \begin{bmatrix} V_s^r \\ I_s^r \end{bmatrix}, \quad s = 0, 1, 2$$

- Inverted ABCD model for each sequence circuit

$$\begin{bmatrix} V_s^r \\ I_s^r \end{bmatrix} = \begin{bmatrix} 1 + Z_r Y_m & -(Z_s + Z_r + Z_s Z_r Y_m) \\ -Y_m & 1 + Z_s Y_m \end{bmatrix} \begin{bmatrix} V_s^{\text{st}} \\ I_s^{\text{st}} \end{bmatrix}, \quad s = 0, 1, 2$$

- Sequence circuits are independent; stack together; invert through A_s

$$\begin{bmatrix} \mathbf{v}_s^{\text{st}} \\ \mathbf{i}_s^{\text{st}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s^r \\ \mathbf{i}_s^r \end{bmatrix}$$

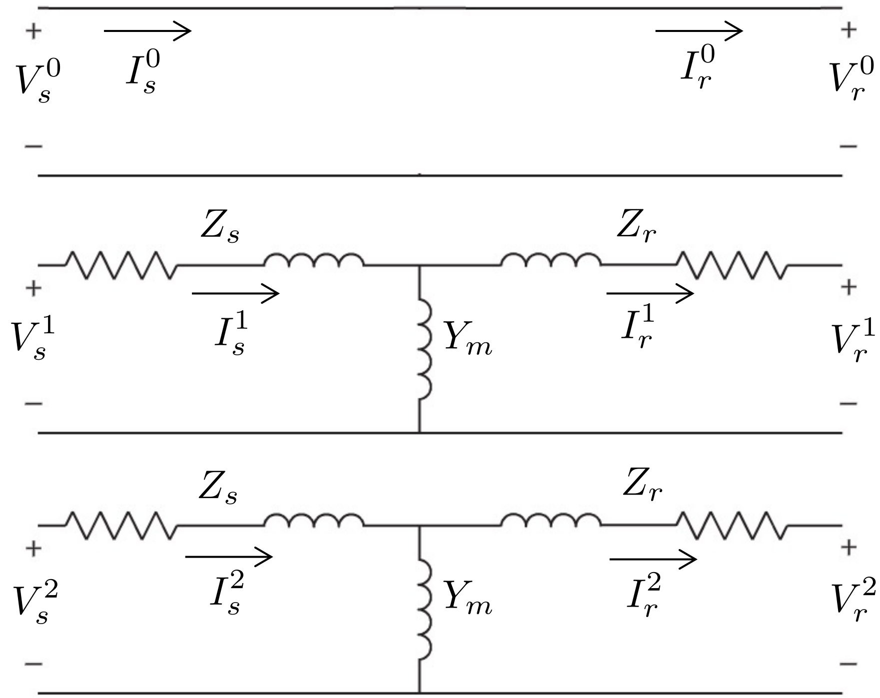
- Useful to calculate SCL, RCL, converted power, rotor current

Sequence networks of T model

$$\mathbf{Z}_s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.1 + j0.25 & 0 \\ 0 & 0 & 0.1 + j0.25 \end{bmatrix}$$

$$\mathbf{Z}_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.08 + j0.175 & 0 \\ 0 & 0 & 0.08 + j0.175 \end{bmatrix}$$

$$\mathbf{Y}_m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -j0.16 & 0 \\ 0 & 0 & -j0.16 \end{bmatrix}$$



- ABCD model for motor's T circuit

$$\begin{bmatrix} \mathbf{v}^{\text{st}} \\ \mathbf{i}^{\text{st}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{r} \\ \mathbf{i}^{r} \end{bmatrix}$$

$$\mathbf{A} = \mathbf{I} + \mathbf{A}_s \mathbf{Z}_s \mathbf{Y}_m \mathbf{A}_s^{-1}$$

$$\mathbf{B} = \mathbf{A}_s(\mathbf{Z}_s + \mathbf{Z}_r + \mathbf{Z}_s \mathbf{Y}_m \mathbf{Z}_r)\mathbf{A}_s^{-1}$$

$$\mathbf{C} = \mathbf{A}_s \mathbf{Y}_m \mathbf{A}_s^{-1}$$

$$\mathbf{D} = \mathbf{I} + \mathbf{A}_s \mathbf{Y}_m \mathbf{Z}_r \mathbf{A}_s^{-1}$$

- Inverse model through matrix inversion or circuit trick
- Zero-sequence can be selected identical to positive sequence to yield identity matrices!

Summary

- Time-invariant loads through ZIP model

- Induction motor under unbalanced conditions

    a) relate stator voltages to currents for a specific slip

    b) given stator voltages/currents, find rotor voltages/currents

    c) find slip to match power (consumed or generated)

- All models can be readily integrated to Forward-Backward Sweep

- *Not covered*

    - fitting ZIP model parameters

    - time-variant models (water heaters, HVAC, buildings)

    - dynamic modeling (induction machine was assumed in steady-state)