ECE 5984: Power Distribution System Analysis

Lecture 6: Distribution Line Models

Reference: Textbook, Chapter 6

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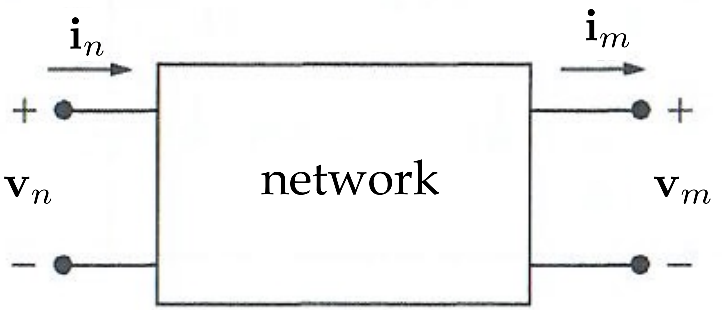
Virginia Tech
Invent the Future

Outline

- Use line charecteristics to obtain a two-port network for distribution lines



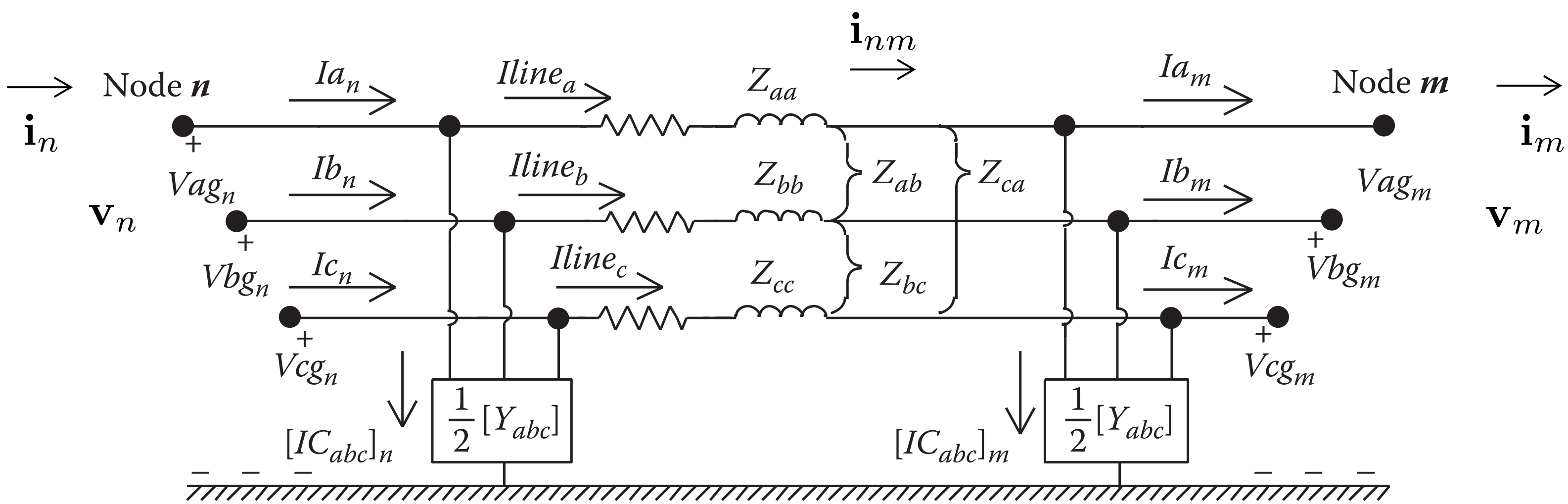
- Derive convenient *matrix expressions* for voltages and (line, neutral, ground) currents

- See why expressions are need in power flow solvers

- Study special cases of transposed and parallel lines

Multiphase line segment model

- Wave propagation equations to model long, medium, and short lines



KCL at receiving node:
$$\mathbf{i}_{nm} = \mathbf{i}_m + \tfrac{1}{2}\mathbf{Y}_{nm}\mathbf{v}_m \tag{1}$$

Ohm's law:
$$\mathbf{v}_n - \mathbf{v}_m = \mathbf{Z}_{nm}\mathbf{i}_{nm} \tag{2}$$

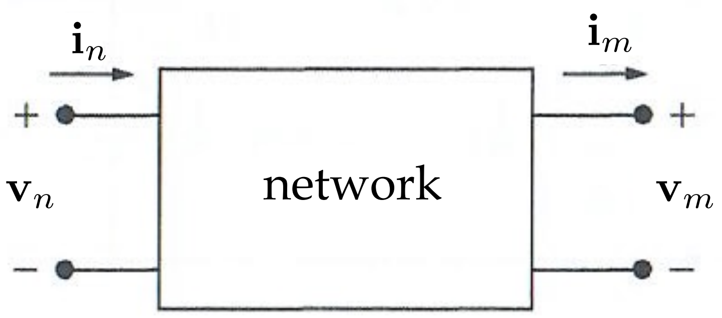
KCL at sending node:
$$\mathbf{i}_n = \tfrac{1}{2}\mathbf{Y}_{nm}\mathbf{v}_n + \mathbf{i}_{nm} \tag{3}$$

- We will express the line as a two-port network

Two-port network

- Plug (1) in (2) to get $$\mathbf{v}_n = \left(\mathbf{I} + \tfrac{1}{2}\mathbf{Z}\mathbf{Y}\right)\mathbf{v}_m + \mathbf{Z}\mathbf{i}_m \tag{4}$$

- Plug (2) and (4) in (3) $$\mathbf{i}_n = \mathbf{Y}\left(\mathbf{I} + \tfrac{1}{4}\mathbf{Z}\mathbf{Y}\right)\mathbf{v}_m + \left(\mathbf{I} + \tfrac{1}{2}\mathbf{Y}\mathbf{Z}\right)\mathbf{i}_m \tag{5}$$



- Matrix generalization of the transmission line model

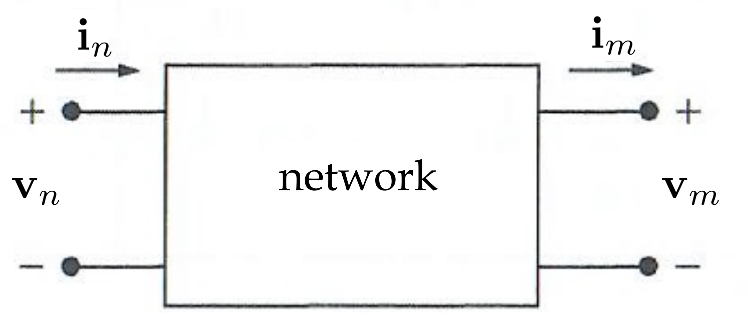
$$\begin{aligned} \mathbf{v}_n &= \mathbf{A}\mathbf{v}_m + \mathbf{B}\mathbf{i}_m & \mathbf{A} &= \mathbf{I} + \tfrac{1}{2}\mathbf{Z}\mathbf{Y} & \mathbf{B} &= \mathbf{Z} \\ \mathbf{i}_n &= \mathbf{C}\mathbf{v}_m + \mathbf{D}\mathbf{i}_m & \mathbf{C} &= \mathbf{Y}\left(\mathbf{I} + \tfrac{1}{4}\mathbf{Z}\mathbf{Y}\right) & \mathbf{D} &= \mathbf{I} + \tfrac{1}{2}\mathbf{Y}\mathbf{Z} = \mathbf{A}^\top \end{aligned}$$

typo in (6.15): $\mathbf{D} \neq \mathbf{I} + \frac{1}{2}\mathbf{Z}\mathbf{Y}$

Reversing network sides

- Previous model expresses sending side in terms of receiving side

$$\begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix}$$



- Express receiving side in terms of sending side

$$\begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ -\mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix}$$

argument and formula in (6.21)-(6.22) are not correct

- Show it either through matrix inversion or exploit circuit symmetry

Additional line matrices

- Express receiving voltage in terms of its current and sending voltage

$$\mathbf{v}_m = \mathbf{E}\mathbf{v}_n - \mathbf{F}\mathbf{i}_m \quad \text{where} \quad \mathbf{E} := \mathbf{A}^{-1}, \mathbf{F} := \mathbf{A}^{-1}\mathbf{B}$$

follows from top block of ABCD model

- Similar to the expression used in *transmission systems* $\mathbf{Y}_{\text{series}} := \mathbf{Z}_{\text{series}}^{-1}$

$$\mathbf{i}_n = \left(\mathbf{Y}_{\text{series}} + \tfrac{1}{2}\mathbf{Y}_{\text{shunt}}\right)\mathbf{v}_n - \mathbf{Y}_{\text{series}}\mathbf{v}_m$$

if you want to build the Y-bus matrix

- Neutral currents can be found via the *neutral transformation matrix*

$$\begin{bmatrix} \mathbf{v}_\phi - \mathbf{v}'_\phi \\ \mathbf{v}_n - \mathbf{v}'_n \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\phi\phi} & \mathbf{Z}_{\phi n} \\ \mathbf{Z}_{\phi n}^\top & \mathbf{Z}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{mn,\phi} \\ \mathbf{i}_{mn,\text{neutral}} \end{bmatrix} \Rightarrow$$

$$\mathbf{i}_{nm,\text{neutral}} = -\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top \mathbf{i}_{nm,\phi}$$

$$\mathbf{v}_\phi - \mathbf{v}'_\phi = \left(\mathbf{Z}_{\phi\phi} - \mathbf{Z}_{\phi n}\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top\right)\mathbf{i}_\phi$$

phase impedance matrix

- Ground current $i_{nm,g} = -\mathbf{1}^\top \mathbf{i}_{nm,\text{neutral}} - \mathbf{1}^\top \mathbf{i}_{nm,\phi}$

Example

An untrasposed distribution line with the given matrices connects unbalanced load to a balanced three-phase 12.47 kV source.

Given sending currents, find receiving voltages, complex load, neutral and ground current.

$$\mathbf{Z}_\phi = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix} \ \Omega$$

$$\mathbf{Y}_{\mathrm{sh},\phi} = \begin{bmatrix} j10.7409 & -j3.4777 & -j1.3322 \\ -j3.4777 & j11.3208 & -j2.2140 \\ -j1.3322 & -j2.2140 & j10.2104 \end{bmatrix} \ \mu\mathrm{S}$$

$$\mathbf{v}_n = \begin{bmatrix} 7199.56\,\underline{/0} \\ 7199.56\,\underline{/-120} \\ 7199.56\,\underline{/120} \end{bmatrix} \ \mathrm{V} \qquad \mathbf{i}_n = \begin{bmatrix} 249.97\,\underline{/-24.5} \\ 277.56\,\underline{/-145.8} \\ 305.54\,\underline{/95.2} \end{bmatrix} \ \mathrm{A}$$

balanced source

Modified line model

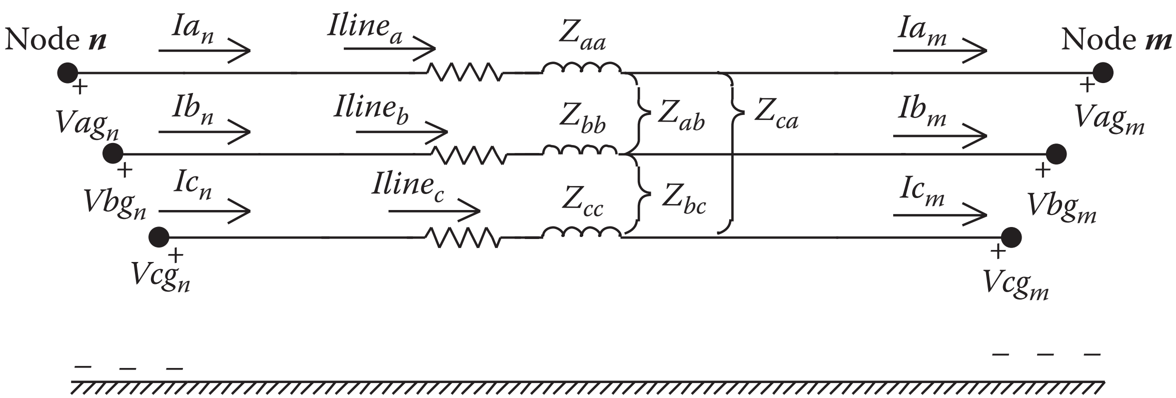
- Modulo long rural lines or underground cables, shunt admittance can be ignored

$$\begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix} \Longrightarrow \begin{aligned} \mathbf{v}_n - \mathbf{v}_m &= \mathbf{Z}\mathbf{i}_m \\ \mathbf{i}_n &= \mathbf{i}_m \end{aligned}$$

$$\mathbf{A} = \mathbf{I} + \tfrac{1}{2}\mathbf{Z}\mathbf{Y} \simeq \mathbf{I} \qquad \mathbf{B} = \mathbf{Z}$$

$$\mathbf{C} = \mathbf{Y}\left(\mathbf{I} + \tfrac{1}{4}\mathbf{Z}\mathbf{Y}\right) \simeq \mathbf{0} \qquad \mathbf{D} = \mathbf{I} + \tfrac{1}{2}\mathbf{Y}\mathbf{Z} \simeq \mathbf{I}$$

- Relating receiving voltage to its current and sending voltage $\mathbf{v}_m = \mathbf{v}_n - \mathbf{Z}\mathbf{i}_m$



- Vectorized form of *short* transmission line model

Example (cont'd)

- Receiving voltage $\mathbf{v}_m = \mathbf{v}_n - \mathbf{Z}\mathbf{i}_m = \begin{bmatrix} 6942.53\,\underline{/-1.47} \\ 6918.35\,\underline{/-121.55} \\ 6887.71\,\underline{/117.31} \end{bmatrix}$ V *unbalanced!*

- Receiving voltage would be unbalanced even if load were balanced! [*why?*]

- ANSI/NEMA standards for unbalance

$$V_{\text{unbalance}} = \frac{\text{max deviation from average magnitude}}{\text{average magnitude}} \cdot 100\%$$

- Example

$$|V_{\text{average}}| = \frac{6942.53 + 6918.35 + 6887.71}{3} = 6916.20 \text{ V}$$

$$V\,deviation_{\max} = |6887.71 - 6916.20| = 28.49$$

$$V_{\text{unbalance}} = \frac{28.49}{6916.20} \cdot 100 = 0.4119\%$$

imbalance should be lower than 1% for an induction motor

Example (cont'd)

- Complex load $$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \begin{bmatrix} V_{ag} \cdot I_a^* \\ V_{bg} . I_b^* \\ V_{cg} \cdot I_c^* \end{bmatrix} = \begin{bmatrix} 1597.2 + j678.8 \\ 1750.8 + j788.7 \\ 1949.7 + j792.0 \end{bmatrix} \text{kVA}$$

- Neutral transformation matrix

$$-\frac{1}{z_{nn}} \mathbf{z}_{\phi n}^{\top} = [-0.4292 - j0.1291 \quad -0.4476 - j0.1373 \quad -0.4373 - j0.1327]$$

- Neutral current $$i_{n,\text{neutral}} = -\frac{1}{z_{nn}} \mathbf{z}_{\phi n}^{\top} \mathbf{i}_{\phi} = 26.2 / \underline{-29.5} \ \text{A}$$

- Ground current $$I_g = -(I_a + I_b + I_c + I_n) = 32.5 / \underline{-77.6} \ \text{A}$$

Transposed lines

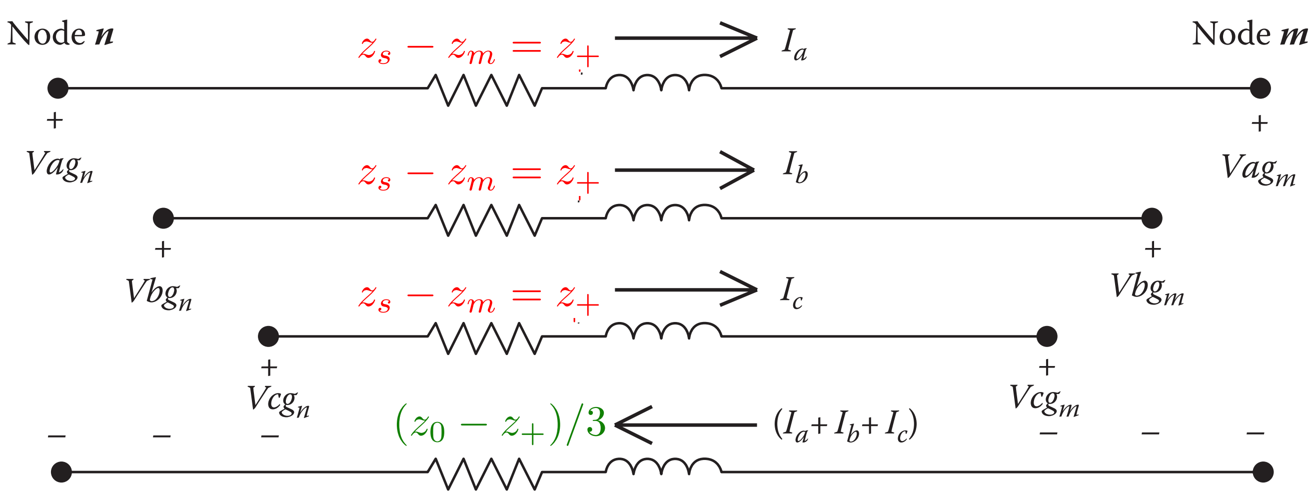
- Line model (ignoring shunt) $\mathbf{Z}_\phi = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} = (z_s - z_m)\mathbf{I}_3 + z_m \mathbf{1}\mathbf{1}^\top$

- Voltage drop $\mathbf{v}_n - \mathbf{v}_m = (z_s - z_m)\mathbf{i} + z_m \mathbf{1}\left(\mathbf{1}^\top \mathbf{i}\right)$

- Relation between sequence and (off)diagonal phase impedances

$$\mathbf{Z}_s = \begin{bmatrix} z_s + 2z_m & 0 & 0 \\ 0 & z_s - z_m & 0 \\ 0 & 0 & z_s - z_m \end{bmatrix}$$

z_0 ($z_s + 2z_m$), z_+ ($z_s - z_m$)



- Apply KVL to see equivalence of circuit

- What if phase currents are balanced?

Three-wire Delta lines

- Need to express voltage drop in LL voltages and line currents

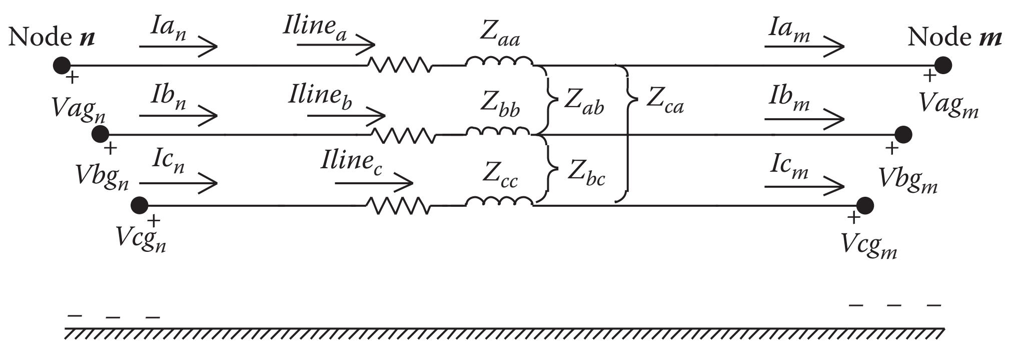
- Use 'equivalent' LN voltages so equations derived so far can be used

$$\textit{LL voltages} \quad \tilde{\mathbf{v}} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \qquad \textit{LN voltages} \quad \mathbf{v} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

- LN-to-LL conversion $\tilde{\mathbf{v}} = \mathbf{D}_f \mathbf{v}, \quad \mathbf{D}_f = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

$e.g., \; V_{ab} = V_{an} - V_{bn}$

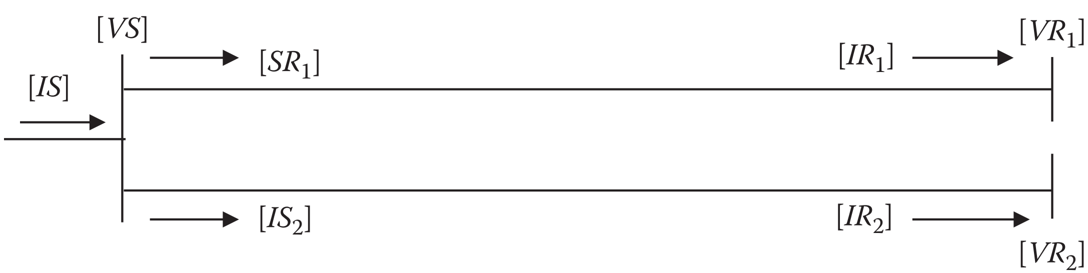
$$\begin{aligned} \tilde{\mathbf{v}}_n - \tilde{\mathbf{v}}_m &= \mathbf{D}_f(\mathbf{v}_n - \mathbf{v}_m) \\ &= \mathbf{D}_f \mathbf{Z}_\phi \mathbf{i}_{\text{line}} \end{aligned}$$



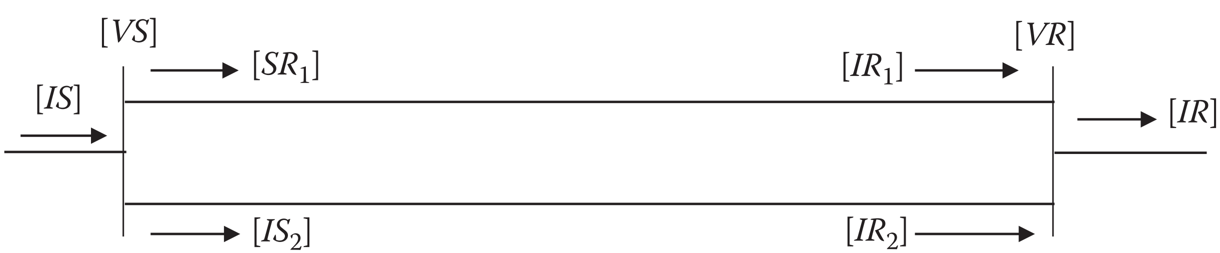
Parallel lines

- Two-port network model holds for coupled impedance matrices
- Three scenarios can be identified

lines sharing no end (physically parallel)



lines sharing one end (physically parallel)



lines sharing both ends (electrically parallel)

Lines sharing no end

- Except for having vectors of double dimensions, no other difference

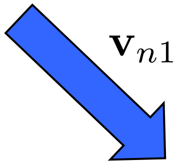
$$\begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{v}_{n1} \\ \mathbf{v}_{n2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{12}^\top & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{m1} \\ \mathbf{v}_{m2} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{12}^\top & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix}$$

Lines sharing one end

- Sending voltages coincide

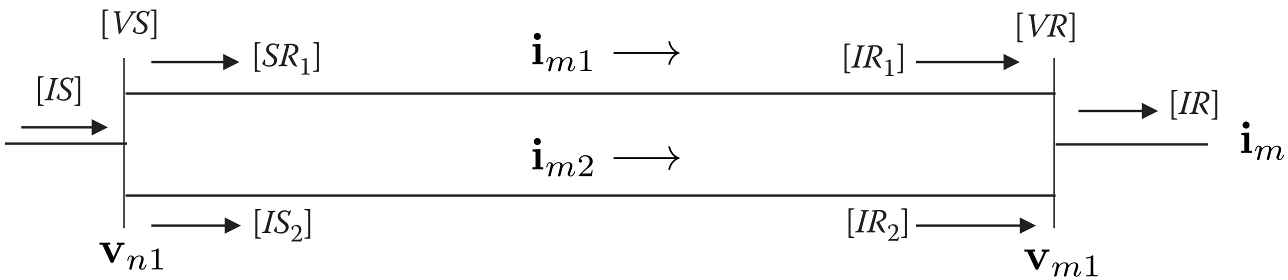
$$\begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ -\mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix}$$



$\mathbf{v}_{n1} = \mathbf{v}_{n2}$

$$\mathbf{v}_{m1} = (\mathbf{A}_{11} + \mathbf{A}_{12})\mathbf{v}_n - \mathbf{B}_{11}\mathbf{i}_1 - \mathbf{B}_{12}\mathbf{i}_2$$
$$\mathbf{v}_{m2} = (\mathbf{A}_{12}^\top + \mathbf{A}_{22})\mathbf{v}_n - \mathbf{B}_{12}^\top\mathbf{i}_1 - \mathbf{B}_{22}\mathbf{i}_2$$

Lines sharing both ends



- Sending and receiving voltages coincide

- Given receiving voltage and net current, find per-line currents

$$\begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{v}_m \\ \mathbf{i}_m \end{bmatrix}$$

unknown

$$\mathbf{v}_{n1} = (\mathbf{A}_{11} + \mathbf{A}_{12})\mathbf{v}_{m1} + \mathbf{B}_{11}\mathbf{i}_{m1} + \mathbf{B}_{12}\mathbf{i}_{m2}$$
$$\mathbf{v}_{n1} = (\mathbf{A}_{12}^\top + \mathbf{A}_{22})\mathbf{v}_{m1} + \mathbf{B}_{12}^\top\mathbf{i}_{m1} + \mathbf{B}_{22}\mathbf{i}_{m2}$$

$$\mathbf{i}_{m2} = \mathbf{i}_m - \mathbf{i}_{m1}$$

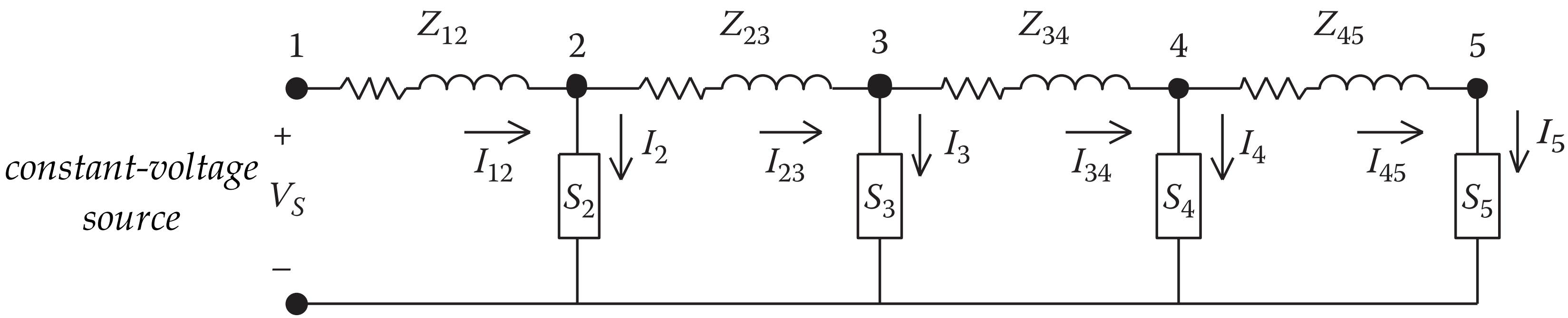
$$\mathbf{i}_{m1} = \left(\mathbf{B}_{12} + \mathbf{B}_{12}^\top - \mathbf{B}_{11} - \mathbf{B}_{22}\right)^{-1} \left[\left(\mathbf{A}_{11} + \mathbf{A}_{12} - \mathbf{A}_{12}^\top - \mathbf{A}_{22}\right) \mathbf{v}_{m1} + \left(\mathbf{B}_{12} - \mathbf{B}_{22}\right) \mathbf{i}_m\right]$$

- How can you find sending current and voltage?

Power flow solvers

- Giver a feeder and loads, we would like to find the voltages at all buses
- If we know the voltages, we can specify everything else
- Two-port networks relate *linearly* voltages and currents (sending/receiving)
- However, *load currents depend on load voltages* (unless load is an impedance)
- Nonlinear power flow equations are solved by iterative techniques
- *Forward-backward solver*: technique tailored to distribution grids (radial)

Forward-backward solver



- Toy example on line feeder with constant-power loads

- *Backward sweep* updates currents given voltages

$$I_i^{(t+1)} = \left(\frac{S_i}{V_i^{(t)}}\right)^* \quad \text{[load]}$$

$$I_{i-1,i}^{(t+1)} = I_i^{(t+1)} + I_{i,i+1}^{(t+1)} \quad \text{[line]}$$

feeder end to feeder head

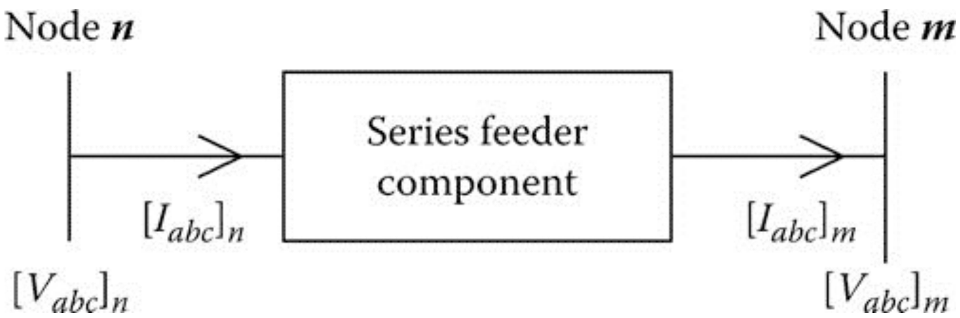
- *Forward sweep* updates voltages given currents

$$V_{i+1}^{(t+1)} = V_i^{(t+1)} + Z_{i,i+1} I_{i,i+1}^{(t+1)}$$

feeder head to feeder end

- Several iterations needed to converge; initialize voltages at nominal

Series components



- Distribution lines, voltage regulators (Lecture 8), and transformers (Lecture 9)
- *Backward sweep* $\mathbf{i}_n^{(t+1)} = \mathbf{C}\mathbf{v}_m^{(t)} + \mathbf{D}\mathbf{i}_m^{(t+1)}$
- *Forward sweep* $\mathbf{v}_m^{(t+1)} = \mathbf{E}\mathbf{v}_n^{(t+1)} - \mathbf{F}\mathbf{i}_m^{(t+1)}$
- Voltages are *LN* ones
- Matrix $\mathbf{C} = \mathbf{0}$ except for
  - long underground lines (due to shunt admittance)
  - grounded Wye-Delta transformers

Shunt components (Lecture 7)

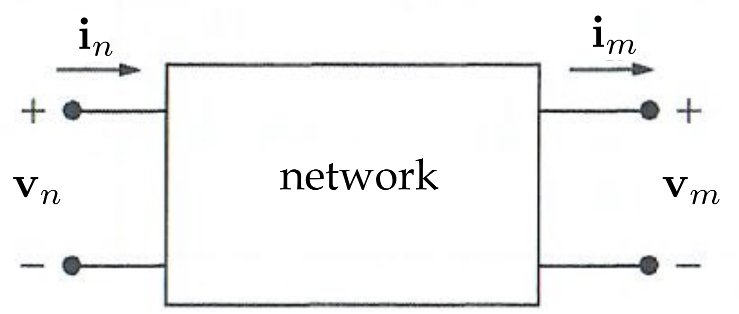
- Spot static loads:
  for ZIP loads, compute currents for each component separately

- Spot induction motors:
  constant-impedance (fixed speed/slip)
  constant-power: compute slip value first

- Capacitor banks:
  constant-impedance loads
  susceptance computed using rated voltage/VAR values

Summary

- Developed ABCD model for multiphase untransposed distribution lines



- Untransposed lines entail unbalanced currents even for balanced loads

- Model simplifies except for long rural or underground lines

- Studied the special cases of transposed and parallel lines

- Explained how the obtained matrices are used in FB solver