ECE 5984: Power Distribution System Analysis

Lecture 5: Shunt Admittance of Distribution Lines

Reference: Textbook, Chapter 5, and Glover-Sarma-Overbye

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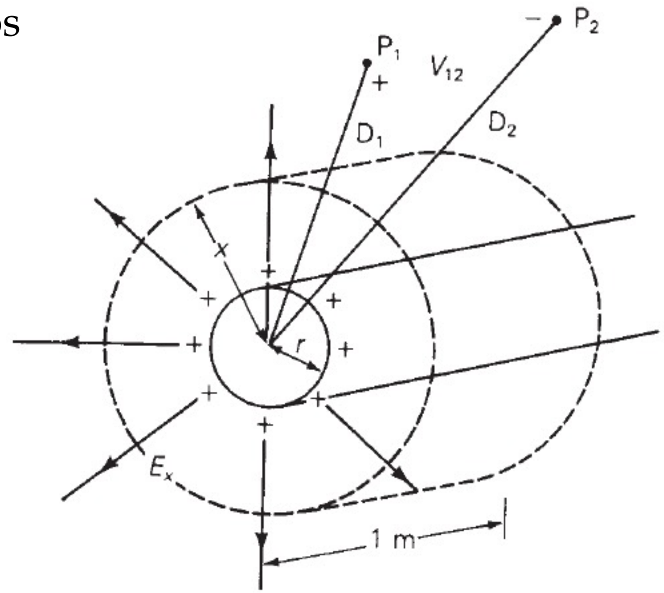
Invent the Future

Line capacitance

- Conductors separated by insulating medium modeled as capacitance
- Electric field between conductors induces electric charges (currents) [Gauss' law]
- Capacitance calculated as charge-to-voltage ratios



[Glover-Sarma-Overybye]

- Voltage difference due to charge q [Cb/m]

$$V_{12} = \frac{q}{2\pi\epsilon} \ln \frac{D_2}{D_1}$$

permittivity $\epsilon = \epsilon_r \epsilon_0$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ [F/m]}$$

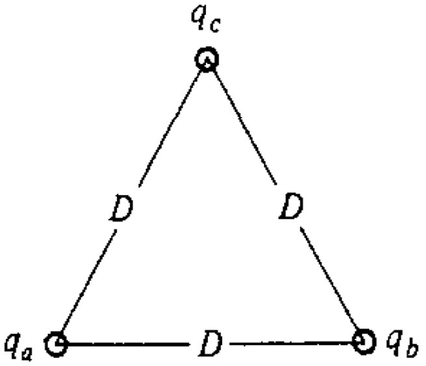
- Superposition for multiple conductors

$$V_{ij} = \frac{1}{2\pi\epsilon} \sum_{k=1}^{N} q_k \ln \frac{D_{kj}}{D_{ki}} \text{ [V]}$$

- Electric charges are *not* related to loads; exist due to voltage even with unloaded line

Transmission lines

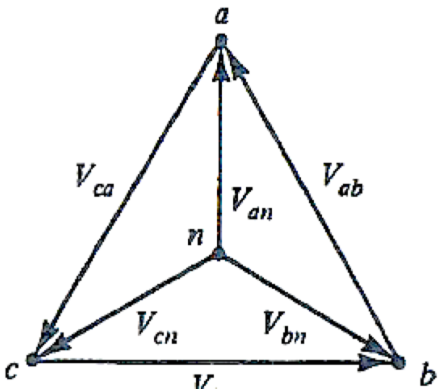
- Equidistant conductors (D) of radius r



$$V_{ab} = \frac{1}{2\pi\epsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{D}{D}\right]$$
$$= \frac{1}{2\pi\epsilon}(q_a - q_b)\ln\frac{D}{r}$$

- Interested in phase voltages

- Assuming *(a1)* symmetry (or transposition); and *(a2)* balanced charges $q_a = -q_b - q_c$

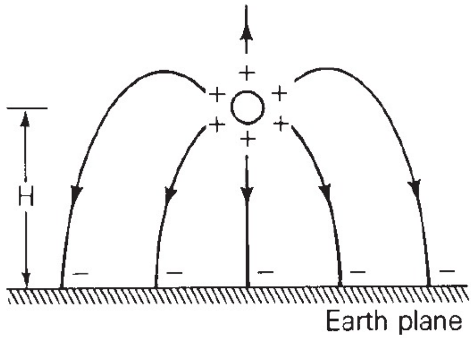


$$V_{an} = \frac{V_{ab} + V_{ac}}{3} = \frac{1}{2\pi\epsilon}\left[\frac{q_a - q_b + q_a - q_c}{3}\right]\ln\frac{D}{r} = \frac{1}{2\pi\epsilon}q_a \ln\frac{D}{r}$$

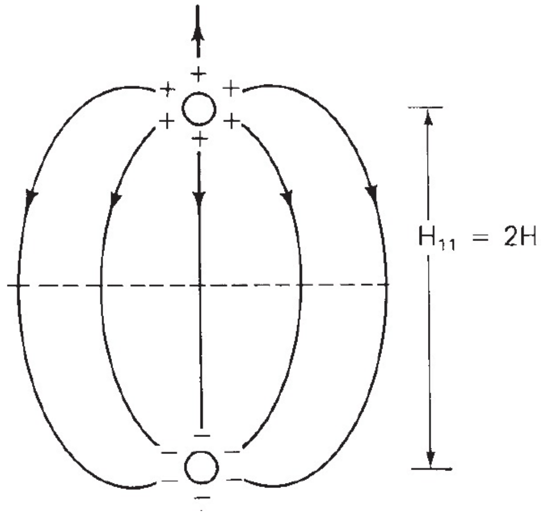
- Shunt capacitance for phase conductor *a:* $C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\epsilon}{\ln\frac{D}{r}}$ [F/m]

Method of images

- Earth is modeled by mirror conductor carrying opposite charges



[Glover-Sarma-Overybye]

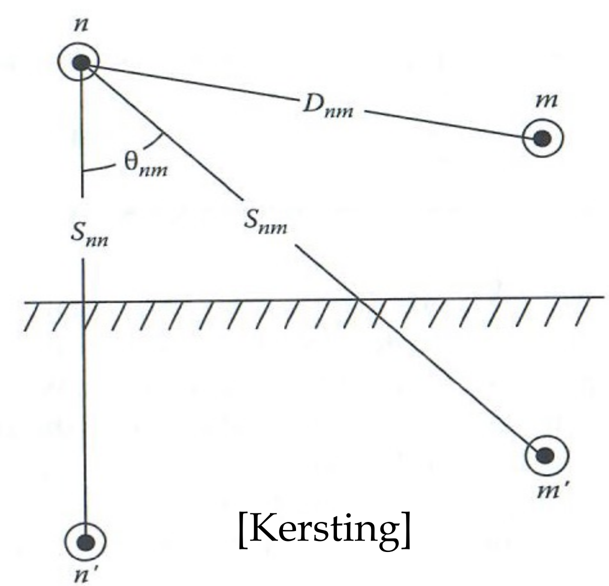


- Voltage difference across conductor and mirror

$$V_{ii'} = \frac{1}{2\pi\epsilon}\left[\sum_{k=1}^{N} q_k \ln\frac{S_{ki}}{D_{ki}} - \sum_{k=1}^{N} q_k \ln\frac{D_{ki}}{S_{ki}}\right]$$

- Due to symmetry

$$V_{in} = \frac{V_{ii'}}{2} = \frac{1}{2\pi\epsilon}\sum_{n=1}^{N} q_k \ln\frac{S_{ki}}{D_{ki}}$$



[Kersting]

Primitive potential coefficient matrix

- Phase voltages are linear combinations of charges at all non-dirt conductors

$$V_{in} = \sum_{k=1}^{N} P_{ik} q_k, \qquad \text{where} \quad P_{ik} := 11.177 \cdot \ln \frac{S_{ki}}{D_{ki}} \ [\text{mile}/\mu\text{F}]$$

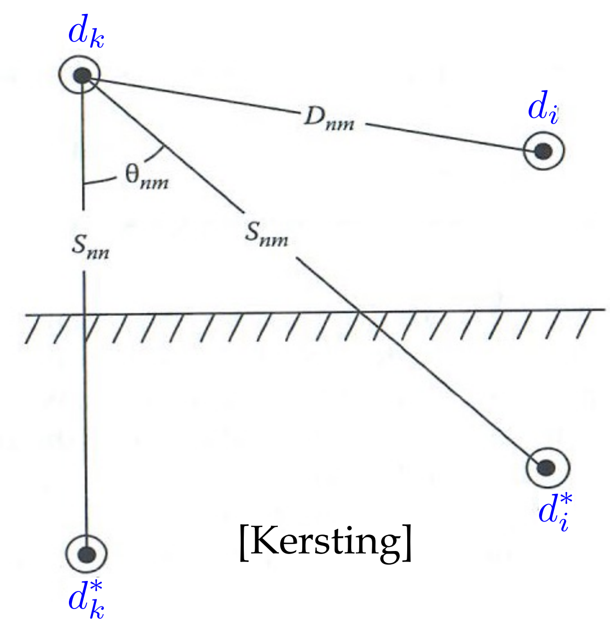
- *Primitive potential coefficient matrix* $\mathbf{P}$ [mile/μF]

not phasors
$$\begin{bmatrix} \mathbf{v}_\phi \\ \mathbf{v}_n \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{\phi\phi} & \mathbf{P}_{\phi n} \\ \mathbf{P}_{\phi n}^\top & \mathbf{P}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{q}_\phi \\ \mathbf{q}_n \end{bmatrix}$$

with P_{ik} as matrix entries

- *Trick:* If d_i is a complex number denoting the location of conductor i

$$D_{ki} = |d_k - d_i| \qquad \text{and} \qquad S_{ki} = |d_k - d_i^*|$$



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Phase potential and capacitance matrices

- Primitive potential coefficient matrix $\begin{bmatrix} \mathbf{v}_\phi \\ \mathbf{v}_n \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{\phi\phi} & \mathbf{P}_{\phi n} \\ \mathbf{P}_{\phi n}^\top & \mathbf{P}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{q}_\phi \\ \mathbf{q}_n \end{bmatrix}$

- Kron reduction to eliminate $\boldsymbol{q}_n$ since $\mathbf{v}_n = \mathbf{0}$

- *Phase potential coefficient matrix*

$$\mathbf{v}_\phi = \mathbf{P}_\phi \mathbf{q}_\phi \quad \text{where} \quad \mathbf{P}_\phi = \mathbf{P}_{\phi\phi} - \mathbf{P}_{\phi n}\mathbf{P}_{nn}^{-1}\mathbf{P}_{\phi n}^\top \ [\text{mile}/\mu\text{F}]$$

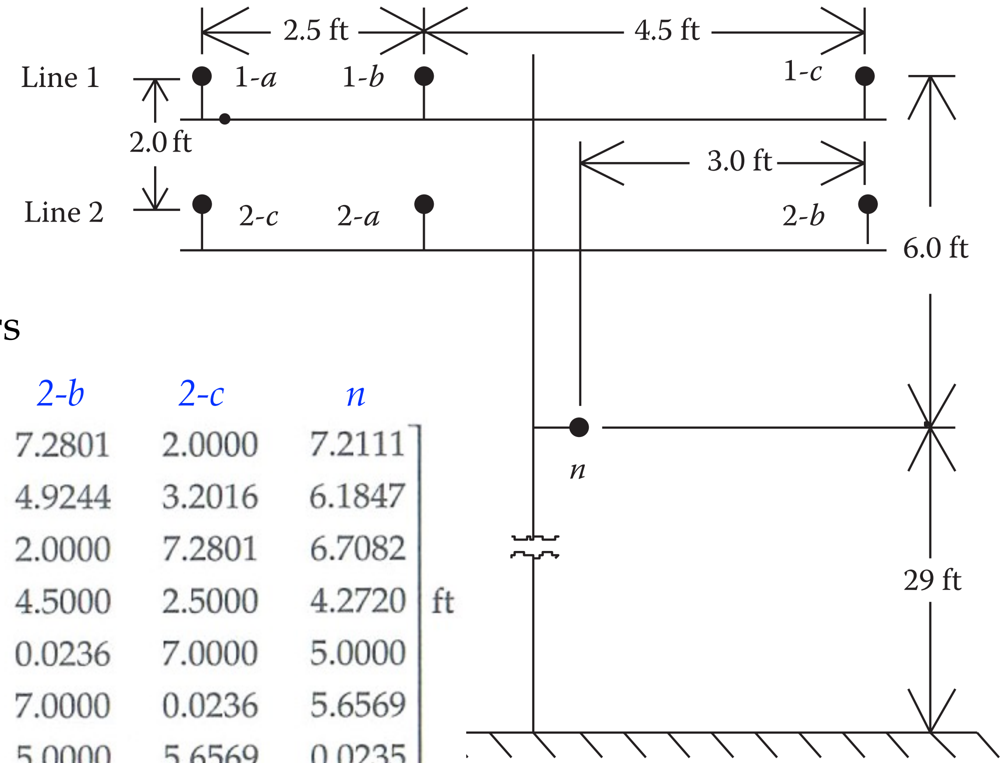
- *Phase capacitance matrix* $\mathbf{q}_\phi = \mathbf{C}_\phi \mathbf{v}_\phi \quad \text{where} \quad \mathbf{C}_\phi = \mathbf{P}_\phi^{-1} \ \ \mu\text{F}/\text{mile}$

positive diagonal and negative off-diagonal entries

- *Phase shunt admittance matrix* $\mathbf{Y}_{\text{sh}} = j\omega \mathbf{C}_\phi \ \mu\text{S}/\text{mile}$

Parallel lines

Procedure generalizes to parallel lines



Step 1: Find distances between conductors

$$\mathbf{D} = \begin{array}{c} \begin{array}{ccccccc} \textit{1-a} & \textit{1-b} & \textit{1-c} & \textit{2-a} & \textit{2-b} & \textit{2-c} & \textit{n} \end{array} \\ \begin{bmatrix} 0.0300 & 2.5000 & 7.0000 & 3.2016 & 7.2801 & 2.0000 & 7.2111 \\ 2.5000 & 0.0300 & 4.5000 & 2.0000 & 4.9244 & 3.2016 & 6.1847 \\ 7.0000 & 4.5000 & 0.0300 & 4.9244 & 2.0000 & 7.2801 & 6.7082 \\ 3.2016 & 2.0000 & 4.9244 & 0.0236 & 4.5000 & 2.5000 & 4.2720 \\ 7.2801 & 4.9244 & 2.0000 & 4.5000 & 0.0236 & 7.0000 & 5.0000 \\ 2.0000 & 3.2016 & 7.2801 & 2.5000 & 7.0000 & 0.0236 & 5.6569 \\ 7.2111 & 6.1847 & 6.7082 & 4.272 & 5.0000 & 5.6569 & 0.0235 \end{bmatrix} \end{array} \text{ ft}$$

Step 2: Find distances between conductors and mirror conductors

$$\mathbf{S} = \begin{bmatrix} 70.000 & 70.045 & 70.349 & 68.046 & 68.359 & 68.000 & 64.125 \\ 70.045 & 70.000 & 70.145 & 68.000 & 68.149 & 68.046 & 64.018 \\ 70.349 & 70.145 & 70.000 & 68.149 & 68.000 & 68.359 & 64.070 \\ 68.046 & 68.000 & 68.149 & 66.000 & 66.153 & 66.047 & 62.018 \\ 68.359 & 68.149 & 68.000 & 66.153 & 66.000 & 66.370 & 62.073 \\ 68.000 & 68.046 & 68.359 & 66.047 & 66.370 & 66.000 & 62.129 \\ 64.125 & 64.018 & 64.070 & 62.018 & 62.073 & 62.129 & 60.000 \end{bmatrix} \text{ ft}$$

Parallel lines (cont'd)

Step 3: Form primitive potential coefficient matrix $P_{ij} := 11.177 \cdot \ln \frac{S_{ji}}{D_{ji}}$ [mile/μF]

Step 4: Kron reduction to get the phase potential coefficient matrix

$$\begin{bmatrix} \mathbf{v}_{\phi_1} \\ \mathbf{v}_{\phi_2} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{\phi_1\phi_1} & \mathbf{P}_{\phi_1\phi_2} \\ \mathbf{P}^\top_{\phi_1\phi_2} & \mathbf{P}_{\phi_1\phi_2} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{\phi_1} \\ \mathbf{q}_{\phi_2} \end{bmatrix}$$

Step 5: Invert to get the (coupled) phase capacitance matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{P}_{\phi_1\phi_1} & \mathbf{P}_{\phi_1\phi_2} \\ \mathbf{P}^\top_{\phi_1\phi_2} & \mathbf{P}_{\phi_1\phi_2} \end{bmatrix}^{-1}$$

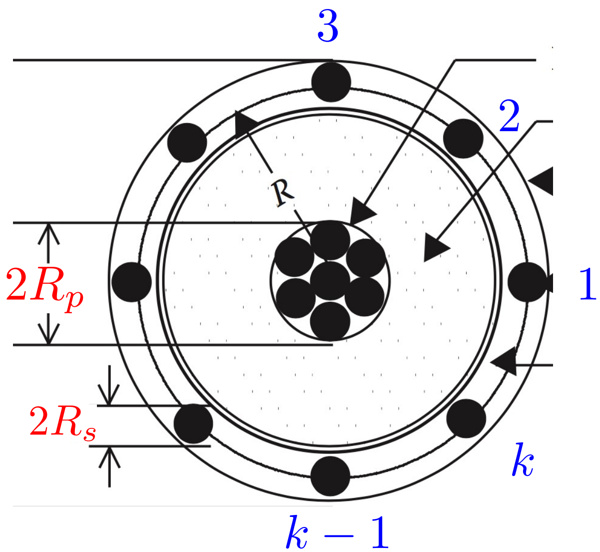
Parallel lines (cont'd)

Step 6: Convert capacitance to (coupled) phase shunt admittance matrix

$$\mathbf{Y}_{\mathrm{sh},12} = j\omega\mathbf{C} = \begin{bmatrix} j6.2992 & -j1.3413 & -j0.4135 & -j0.7889 & -j0.2992 & -j1.6438 \\ -j1.3413 & j6.5009 & -j0.8038 & -j1.4440 & -j0.5698 & -j0.7988 \\ -j0.4135 & -j0.8038 & j6.0257 & -j0.5553 & -j1.8629 & -j0.2985 \\ -j0.7889 & -j1.4440 & -j0.5553 & j6.3278 & -j0.6197 & -j1.1276 \\ -j0.2992 & -j0.5698 & -j1.8629 & -j0.6197 & j5.9016 & -j0.2950 \\ -j1.6438 & -j0.7988 & -j0.2985 & -j1.1276 & -j0.2950 & j6.1051 \end{bmatrix} \mu\mathrm{S/mile}$$

- Compare shunt admittance values to series impedance from Lecture 5…

Concentric neutral underground cables



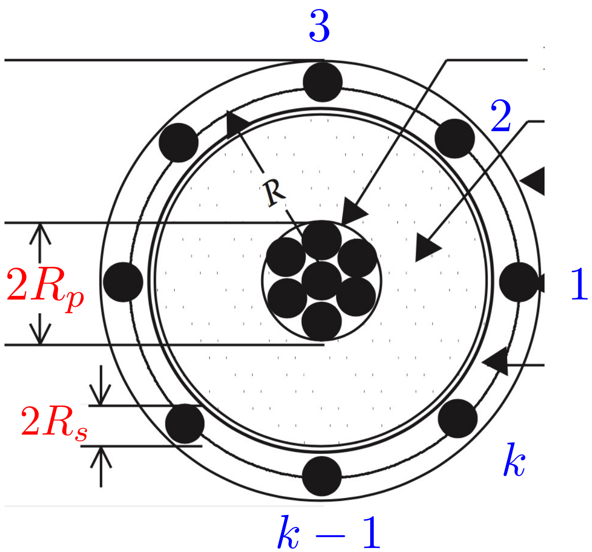
k : # concentric neutrals
R : radius of concentric arrangement
R_p : radius of phase conductor
R_s : radius of neutral strand
D_{1n} : distance between strand 1 and strand n

- Shielding confines electric fields within cables
- No coupling between phase cables, and between cables and earth
- All neutral strands are at the same potential (ground)
- Voltage between phase conductor and ground (e.g., neutral strand #1)

$$V_{pg} = \frac{1}{2\pi\epsilon}\left[q_p \ln\frac{R}{R_p} + \sum_{n=1}^{k} q_n \ln\frac{D_{1n}}{R}\right]$$

- Shielding does not confine magnetic fields; hence we still have mutual impedances

Concentric neutral underground cables (cont'd)



$$V_{pg} = \frac{1}{2\pi\epsilon}\left[q_p \ln \frac{R}{R_p} + \sum_{n=1}^{k} q_n \ln \frac{D_{1n}}{R}\right]$$

- Equal charge on neutral strands

$$q_n = -\frac{q_p}{k}, \ \forall n = 1, \ldots, k$$

- Distances between strands

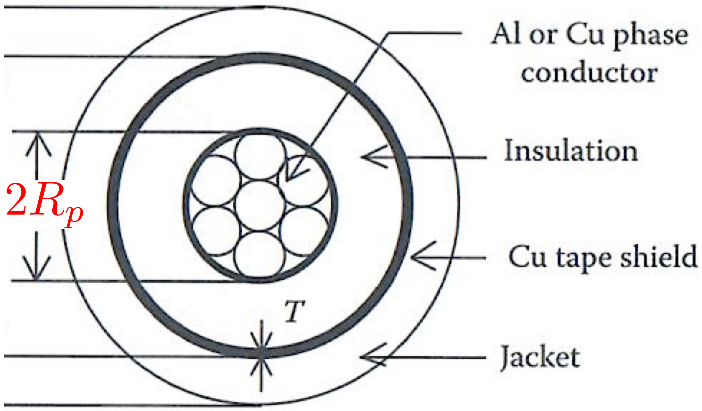
$$D_{1n} = |R - Re^{j\frac{2\pi(n-1)}{k}}|, \ n = 2, \ldots, k$$

- Using formula for bundled conductors uniformly spaced on the perimeter

$$C_{pg} = \frac{q_p}{V_{pg}} = \frac{2\pi\epsilon}{\ln \frac{R}{R_p} - \frac{1}{k}\ln \frac{kR_s}{R}}$$

Material	Range of Values of Relative Permittivity
Polyvinyl Chloride (PVC)	3.4–8.0
Ethylene-Propylene Rubber (EPR)	2.5–3.5
Polyethylene (PE)	2.5–2.6
Cross-Linked Polyethlyene (XLPE)	2.3–6.0

Tape-shielded cables



- Limiting case of concentric neutrals for $k \to \infty$

$$C_{pg} = \frac{q_p}{V_{pg}} = \frac{2\pi\epsilon}{\ln \frac{R}{R_p}}$$

- For either cables, *no capacitive coupling across phases* or circuits in parallel lines
- Phase admittance matrix is diagonal $$\mathbf{Y}_{\text{sh}} = \begin{bmatrix} j96.5569 & 0 & 0 \\ 0 & j96.5569 & 0 \\ 0 & 0 & j96.5569 \end{bmatrix} \mu\text{S/mile}$$
- *For both overhead and underground lines, shunt admittances are typically ignored*

Sequence admittance

- Similarly to series impedances

$$\mathbf{i}_\phi = \mathbf{Y}_{\mathrm{sh},\phi}\mathbf{v}_\phi \quad \Longleftrightarrow \quad \mathbf{i}_s = \mathbf{Y}_{\mathrm{sh},s}\mathbf{v}_s$$

- Sequence shunt admittance matrix

$$\mathbf{Y}_{\mathrm{sh},s} := \mathbf{A}_s^{-1}\mathbf{Y}_{\mathrm{sh},\phi}\mathbf{A}_s = \begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{01} & y_{11} & y_{12} \\ y_{02} & y_{12} & y_{22} \end{bmatrix}$$

- Diagonal for underground or transposed overhead lines

- In fact, for underground lines with three identical cables $\quad \mathbf{Y}_{\mathrm{sh},s} = \mathbf{Y}_{\mathrm{sh},\phi}$

Summary

- Find distances between (mirror) conductors
- Find primitive potential coefficient matrix
- Kron reduction to get the phase potential coefficient matrix
- Inversion to get the phase capacitance matrix
- For underground cables, the capacitance matrix is a scaled identity
- Shunt admittance is typically ignored