ECE 5984: Power Distribution System Analysis

Lecture 4: Series Impedance of Distribution Lines

Reference: Textbook, Chapter 4

Instructor: V. Kekatos

Motivation

- Different from transmission, in distribution systems
  (a) lines are not transposed; and
  (b) loads (currents) are unbalanced

- Study the effect of earth based on Carson's equations

- Model untransposed distribution lines with any number of conductors

- Derive two-port multi-phase network with the minimum information required

- Cover the cases of multiphase and multi-neutral (overhead or underground) lines

Flux linkage

- Current flowing through a wire induces magnetic field [Ampere's law]

- Time-varying magnetic field induces voltage drop at ends of wire [Faraday's law]

- *Inductance*: flux linkage produced per ampere of current

$$v = \frac{d\lambda}{dt} = L\frac{di}{dt} \quad \Rightarrow \quad \boxed{L = \frac{\lambda}{i}}$$

- Flux linkage induced from wire center till point at distance *R*

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Henry/m}$$

$$\lambda = \lambda_{\text{int}} + \lambda_{\text{ext}} = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{R}{\tilde{D}_{11}}\right) \cdot i \quad \Longrightarrow \quad \lambda = 2 \cdot 10^{-7} \ln\frac{R}{D_{11}} \cdot i \text{ [Wb-turns/m]}$$

$\nearrow$ *wire radius* ($\tilde{D}_{11}$)

$$D_{11} = \tilde{D}_{11} e^{-\mu_r/4}$$

scaled wire radius

Self and mutual inductance

- *Self inductance*

$$L = \frac{\lambda}{i} = 2 \cdot 10^{-7} \cdot \ln \frac{R}{D_{11}} \text{ [Henry]}$$

D_{11} : wire radius

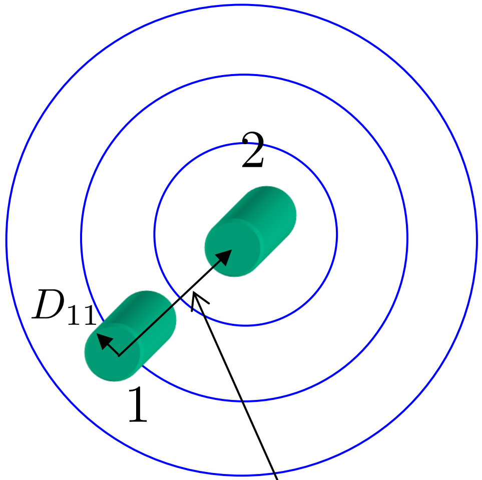
- Reactance at 60 Hz

$$x = \omega L = 2\pi \cdot 60 \cdot 2 \cdot 10^{-7} \cdot \ln \frac{R}{D} \cdot \frac{1609 \text{ m}}{\text{mile}}$$

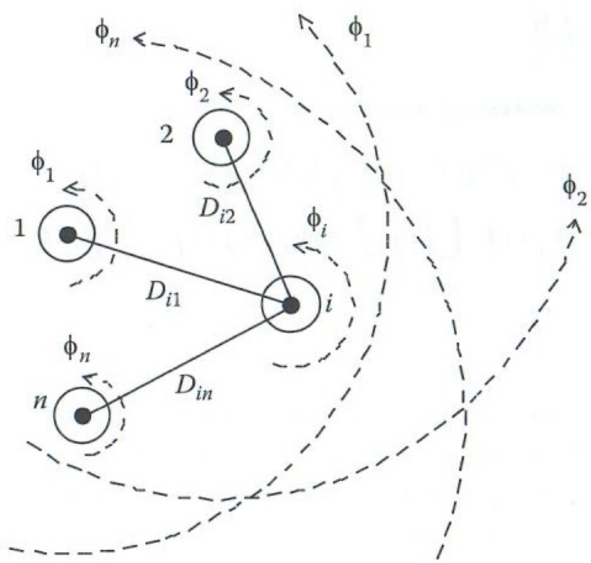
$$= 0.1213 \cdot \ln \frac{R}{D} \ \Omega/\text{mile}$$

- *Mutual inductance:*
  Only external flux from wire 2 beyond wire distance is captured by wire 1

$$\lambda_{12} = \int_{D_{12}}^{R} d\lambda_2(x)dx = \frac{\mu_0}{2\pi} \ln \frac{R}{D_{12}} i_2$$



Total flux linkage due to multiple conductors



- Due to superposition of magnetic fields

$$\lambda_n = \sum_{k=1}^{N} \lambda_{nk} = 2 \cdot 10^{-7} \sum_{k=1}^{N} \ln \frac{R}{D_{nk}} i_k$$

as $R \to \infty$ all actually different R_k become equal

$$D_{nk} = \begin{cases} \text{radius of conductor } n & , \; k = n \\ \text{distance between conductors } n \text{ and } k & , \; k \neq n \end{cases}$$

- Linkage goes to *infinity* as distant point moves away from conductors…

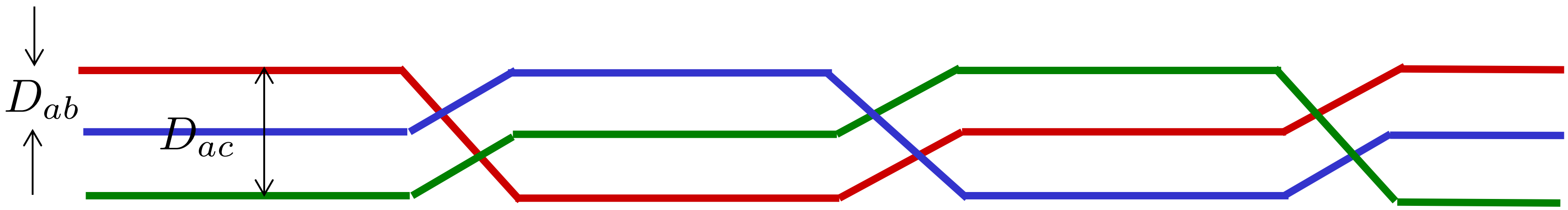
- For zero-sum currents (not necessarily balanced), linkage simplifies

0

$$\lambda_n = 2 \cdot 10^{-7} \ln R \sum_{k=1}^{N} i_k + 2 \cdot 10^{-7} \sum_{k=1}^{N} \ln \frac{1}{D_{nk}} i_k$$

- Problem with infinity resolved; but self and mutual inductances persist

Three-phase transposed line



- If *(a1)* currents sum up to zero; and *(a2)* lines are transposed

$$\lambda_a = 2 \cdot 10^{-7} \left(\ln \frac{1}{D_{aa}} i_a + \ln \frac{1}{\bar{D}} (i_b + i_c) \right) \qquad [i_b + i_c = -i_a]$$

$$= 2 \cdot 10^{-7} \ln \frac{\bar{D}}{D_{aa}} i_a \qquad \bar{D} := \sqrt[3]{D_{ab}^2 D_{ac}}$$

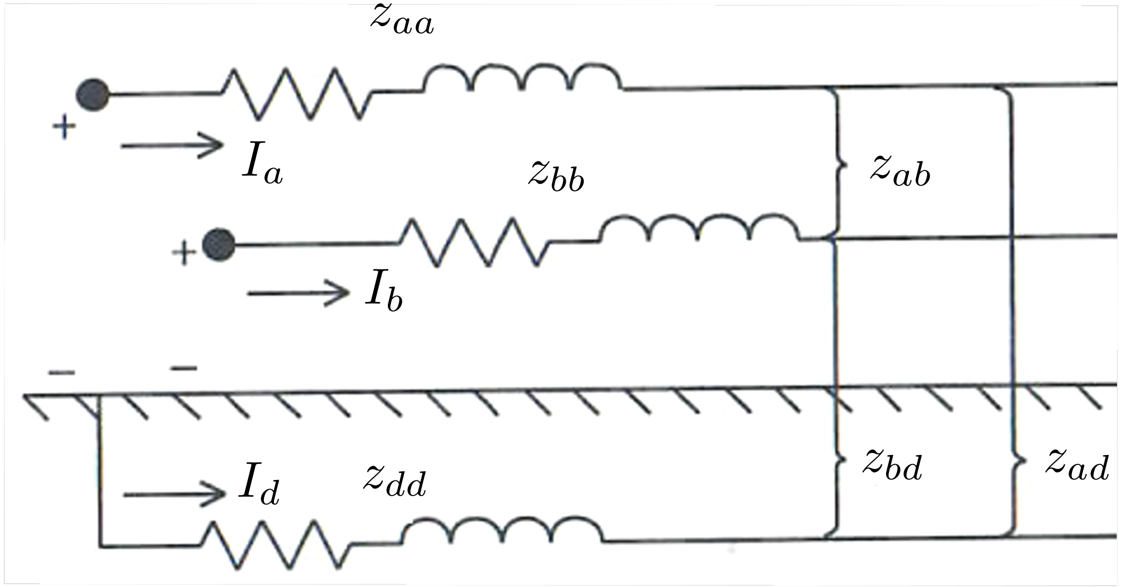
- Mutual linkages vanish allowing for per-phase (separable and identical) analysis

- Assumption *(a1)* does not hold under faults

- Symmetry in transmission infrastructure allows for symmetrical components

- None of the *(a1)*-*(a2)* holds in distribution grids

Ground return



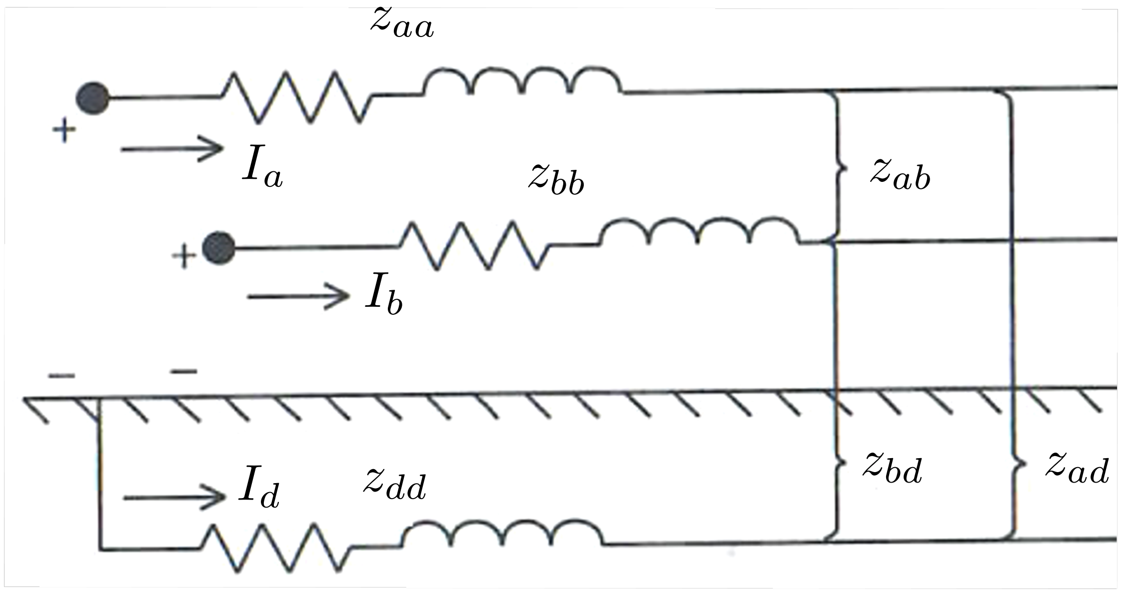
- Current flows through ground return or fictitious 'dirt' conductor
- Assumption *(a1)* on zero-sum currents re-established

$$i_d = -\sum_{n=1}^{N} i_n$$

may have multiple non-dirt conductors (neutrals, phases)

- Previous analysis holds with self and mutual inductances present

Self and mutual impedances



$$z_{mn} = \begin{cases} r_m + j0.1213 \cdot \ln \frac{1}{D_{mm}} \; [\Omega/\text{mile}], & m = n \\ j0.1213 \cdot \ln \frac{1}{D_{mn}} \; [\Omega/\text{mile}], & m \neq n \end{cases}$$

self impedance (non-dirt)

mutual impedance (non-dirt)

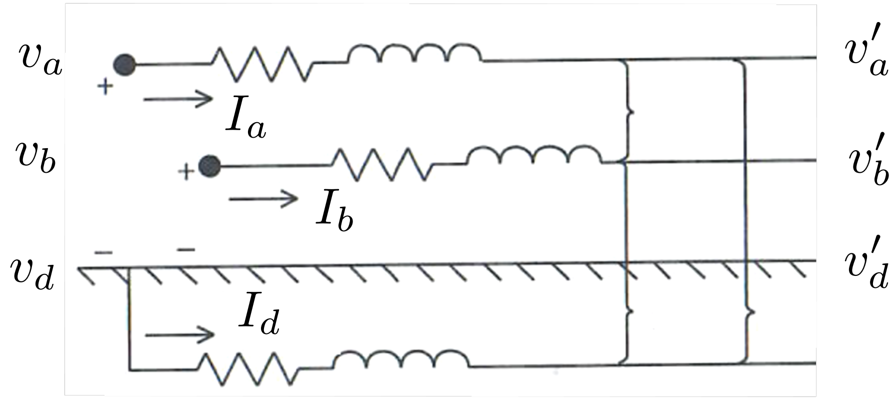
$$z_{md} = j0.1213 \cdot \ln \frac{1}{D_{md}} \quad [\Omega/\text{mile}]$$

mutual impedance (dirt/non-dirt)

$$z_{dd} = r_d + j0.1213 \cdot \ln \frac{1}{D_{dd}} \quad [\Omega/\text{mile}]$$

self impedance (dirt)

Primitive impedance matrix



- Partition into non-dirt and dirt parts

$$\begin{bmatrix} \mathbf{v} - \mathbf{v}' \\ v_d - v'_d \end{bmatrix} = \begin{bmatrix} \mathbf{Z} & \mathbf{z}_d \\ \mathbf{z}_d^\top & z_{dd} \end{bmatrix} \begin{bmatrix} \mathbf{i} \\ i_d \end{bmatrix}$$

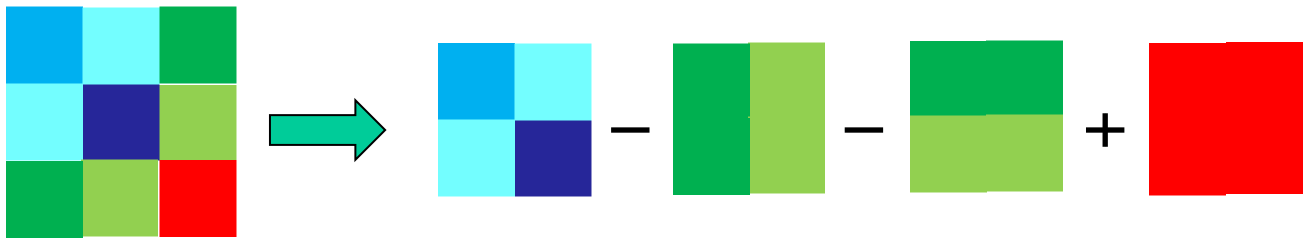
- Voltages are referenced to v_d
- Zero-sum currents yield $i_d = -\mathbf{1}^\top \mathbf{i}$

- Subtract last row from all other rows to refer voltages to their related grounds

$$\Delta\mathbf{v} = (\mathbf{v} - v_d\mathbf{1}) - (\mathbf{v}' - v'_d\mathbf{1}) = \underbrace{\left(\mathbf{Z} - \mathbf{1}\mathbf{z}_d^\top - \mathbf{z}_d\mathbf{1}^\top + z_{dd}\mathbf{1}\mathbf{1}^\top\right)}_{\mathbf{Z}'}\mathbf{i}$$

Primitive impedance matrix (cont'd)

- Structure of *primitive matrix* $\mathbf{Z}'$? $\mathbf{Z}' := \mathbf{Z} - \mathbf{1}\mathbf{z}_d^\top - \mathbf{z}_d\mathbf{1}^\top + z_{dd}\mathbf{1}\mathbf{1}^\top$



- Entries of primitive matrix

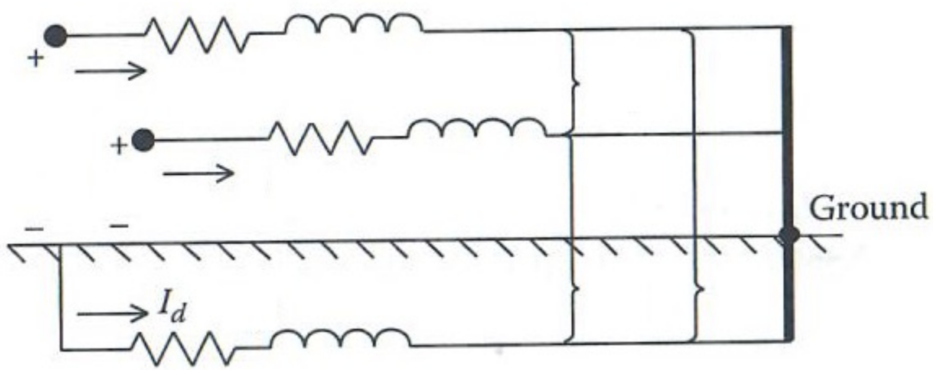
$$
\begin{aligned}
z'_{mn} &= z_{mn} - j0.1213 \cdot \ln \frac{1}{D_{nd}} - j0.1213 \cdot \ln \frac{1}{D_{md}} + r_d + j0.1213 \cdot \ln \frac{1}{D_{dd}} \\
&= z_{mn} + r_d + j0.1213 \cdot \ln \frac{D_{nd}D_{md}}{D_{dd}} \quad [\Omega/\text{mile}]
\end{aligned}
$$

- Replace 0.1213 with $\omega \cdot 2 \cdot 10^{-7}$ for general formula [Ohms/meter]

- Dirt resistance and a reactance term are added on all entries of original ***Z***!

- However, we do not know $r_d, D_{nd}, D_{md}, D_{dd} \ldots$

Experimental approach



- Ground all conductors on right-hand side $v'_a = v'_b = \ldots = v'_d$

$$
\begin{aligned}
v_a - v'_a &= z_{aa} I_a + z_{ab} I_b + z_{ad} I_d \\
v_b - v'_b &= z_{ab} I_a + z_{bb} I_b + z_{bd} I_d \\
v_d - v'_d &= z_{ad} I_a + z_{bd} I_b + z_{dd} I_d
\end{aligned}
$$

- Subtract last row from first to get

$$
\begin{aligned}
v_a - (v'_a - v'_d) &= z_{aa} I_a + z_{ab} I_b + z_{ad} I_d - z_{ad} I_a - z_{bd} I_b - z_{dd} I_d \Rightarrow \\
v_a &= (z_{aa} + z_{dd} - 2 z_{ad}) I_a + (z_{ab} + z_{dd} - z_{bd} - z_{ad}) I_b
\end{aligned}
$$

Carson's equations

- Carson developed detailed physical model to capture effect of earth [1926]

- In practice, use *modified* (approximate) Carson's equations to account for earth

$$z'_{mn} - z_{mn} = 0.00158836 \cdot f + j\omega \left(7.6786 + \frac{1}{2} \ln \frac{\rho}{f}\right) \ \Omega/\text{mile}$$

ρ : resistivity

f : system frequency

- For typical values $f = 60$ Hz and $\rho = 100$ Ω-m

$$z'_{mn} - z_{mn} = 0.0953 + j \cdot 0.12134 \times 7.93402 \ \Omega/\text{mile}$$

- Carson's equations provided unknown terms

$$\begin{aligned} z'_{mn} &= z_{mn} + r_d + j0.1213 \cdot \ln \frac{D_{nd} D_{md}}{D_{dd}} \\ &= r_{mn} + \underbrace{r_d}_{=0.0953} + j \cdot 0.12134 \cdot \left(\ln \frac{1}{D_{mn}} + \underbrace{\ln \frac{D_{nd} D_{md}}{D_{dd}}}_{\ln D_e = 7.93402} \right) \ [\Omega/\text{mile}] \end{aligned}$$

- Because D_e is in *ft* for 7.93402, all other distances under *ln* should be in *ft*!

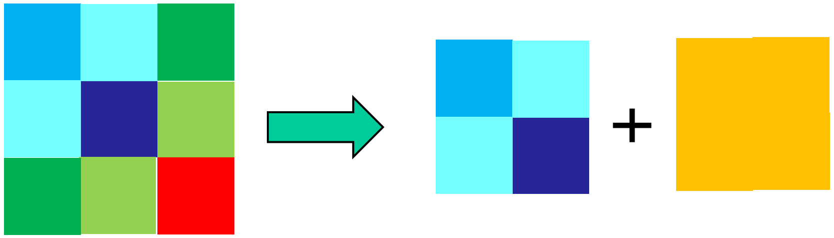
Primitive impedance matrix

- Modified Carson's equations showed that

Primitive impedance matrix

$$\mathbf{Z}' = \mathbf{Z} + z_d \mathbf{1}\mathbf{1}^\top$$

$$z_d = r_d + j\omega \cdot 2 \cdot 10^{-7} \ln D_e$$



- From now on, we will be using symbol **Z** to denote **Z**′

- Primitive impedance matrix is *N*x*N*, where *N* is # of conductors
  - Overhead 4-wire grounded line: *4x4*
  - Underground cable with three concentric neutral cables: *6x6*

Phase impedance matrix

- Reduce primitive matrix to a *3x3* matrix capturing only phase conductors

- Partition voltage drop equations as [check dimensions]

$$\begin{bmatrix} \mathbf{v}_\phi - \mathbf{v}'_\phi \\ \mathbf{v}_n - \mathbf{v}'_n \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\phi\phi} & \mathbf{Z}_{\phi n} \\ \mathbf{Z}_{\phi n}^\top & \mathbf{Z}_{nn} \end{bmatrix} \begin{bmatrix} \mathbf{i}_\phi \\ \mathbf{i}_n \end{bmatrix}$$

- *Key point:* neutrals are grounded $\mathbf{v}_n = \mathbf{v}'_n = \mathbf{0}$

- *Kron reduction:* linear system where the given vector has a zero block

$$\mathbf{i}_n = -\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top \mathbf{i}_\phi \qquad \textit{neutral transformation matrix}$$

$$\mathbf{v}_\phi - \mathbf{v}'_\phi = \left(\mathbf{Z}_{\phi\phi} - \mathbf{Z}_{\phi n}\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top\right)\mathbf{i}_\phi \qquad \textit{phase impedance matrix}$$

Special cases

$$\mathbf{Z}_\phi := \mathbf{Z}_{\phi\phi} - \mathbf{Z}_{\phi n}\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ab} & z_{bb} & z_{bc} \\ z_{ac} & z_{bc} & z_{cc} \end{bmatrix}$$

symmetric matrix

- *Transposed three-phase line*

$$\mathbf{Z}_\phi := \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} = (z_s - z_m)\mathbf{I}_3 + z_m\mathbf{1}\mathbf{1}^\top$$

  due to averaging across segments $z_m = (z_{ab} + z_{ac} + z_{bc})/3$

- *two-phase line*
  - primitive is *3x3*
  - Kron-reduced is *2x2*
  - zero-padded is *3x3*

$$\mathbf{Z}_\phi := \begin{bmatrix} z_{aa} & 0 & z_{ac} \\ 0 & 0 & 0 \\ z_{ac} & 0 & z_{cc} \end{bmatrix}$$

- *single-phase line*

$$\mathbf{Z}_\phi := \begin{bmatrix} 0 & 0 & 0 \\ 0 & z_{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- *three-wire delta connection:*
  no need for Kron reduction, but Carson's correction is still needed

Transposed three-phase line

- Primitive impedance matrix

$$\mathbf{Z} = \begin{bmatrix} z_{ii} & z_{ij} & z_{ij} & z_{in} \\ z_{ij} & z_{ii} & z_{ij} & z_{in} \\ z_{ij} & z_{ij} & z_{ii} & z_{in} \\ z_{in} & z_{in} & z_{in} & z_{nn} \end{bmatrix}$$

$$z_{ii} = r_i + r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{\bar{R}_i}$$

$$z_{ij} = r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{\bar{D}_{ij}}$$

$$z_{in} = r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{\bar{D}_{in}}$$

$$z_{nn} = r_n + r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{R_n}$$

- Phase impedance matrix

$$\begin{aligned} \mathbf{Z}_\phi &= \begin{bmatrix} z_{ii} & z_{ij} & z_{ij} \\ z_{ij} & z_{ii} & z_{ij} \\ z_{ij} & z_{ij} & z_{ii} \end{bmatrix} - \frac{1}{z_{nn}} \begin{bmatrix} z_{in} \\ z_{in} \\ z_{in} \end{bmatrix} \begin{bmatrix} z_{in} \\ z_{in} \\ z_{in} \end{bmatrix}^\top \\ &= \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \\ &= (z_s - z_m)\mathbf{I}_3 + z_m \mathbf{1}\mathbf{1}^\top \end{aligned}$$

where $z_s = z_{ii} - \frac{z_{in}^2}{z_{nn}}$ and $z_m = z_{ij} - \frac{z_{in}^2}{z_{nn}}$

Transposed three-phase line (cont'd)

- Voltage drop under transposed lines and zero-sum currents $(\mathbf{1}^\top \mathbf{i}_\phi = 0)$

$$\begin{aligned} \mathbf{v}_\phi - \mathbf{v}'_\phi &= \mathbf{Z}_\phi \mathbf{i}_\phi \\ &= (z_s - z_m)\mathbf{i}_\phi + z_m \mathbf{1}\underbrace{\mathbf{1}^\top \mathbf{i}_\phi}_{0} \\ &= (z_s - z_m)\mathbf{i}_\phi \end{aligned}$$

- Single-phase equivalent and line impedance

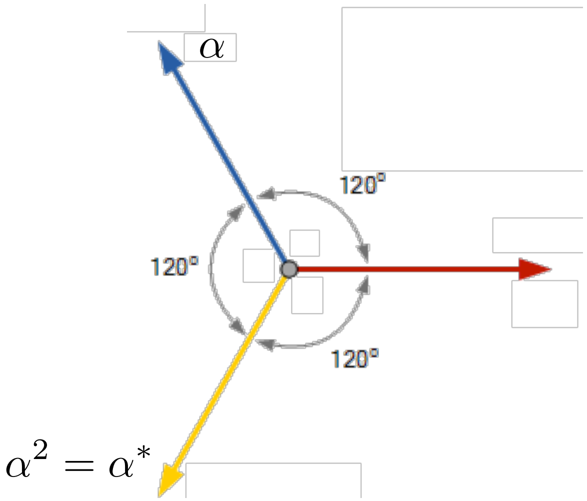
$$\begin{aligned} z_s - z_m &= \left(z_{ii} - \frac{z_{in}^2}{z_{nn}}\right) - \left(z_{ij} - \frac{z_{in}^2}{z_{nn}}\right) \\ &= z_{ii} - z_{ij} \\ &= \left(r_i + r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{\bar{R}_i}\right) - \left(r_d + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{D_e}{\bar{D}_{ij}}\right) \\ &= r_i + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{\bar{D}_{ij}}{\bar{R}_i} \end{aligned}$$

- Effect of earth disappears; no Carson's modification needed of line impedances!
- That is why approx. analysis of Chapter 3 involved single z for a line

From phase to sequence domains

- Phase to sequence domain $\mathbf{v}_\phi = \mathbf{A}_s \mathbf{v}_s \quad \Leftrightarrow \quad \mathbf{v}_s = \mathbf{A}_s^{-1} \mathbf{v}_\phi$

- The angle of 120º $\alpha = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$



- Transformation matrix

$$\mathbf{A}_s := \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}, \quad \mathbf{A}_s^{-1} := \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} = \frac{1}{3} \mathbf{A}_s^H$$

conjugate transpose

- Useful properties

$(p1)\ \alpha^2 = \alpha^*$

$(p2)\ 1 + \alpha + \alpha^2 = 0$

$(p3)\ \mathbf{A}_s \mathbf{1} = 3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 3\mathbf{e}_1$

$(p4)\ \mathbf{A}_s^{-1} \mathbf{1} = \mathbf{e}_1$

Sequence impedance matrix

- Voltage drop in two domains

$$\begin{aligned} \mathbf{v}_\phi - \mathbf{v}'_\phi &= \mathbf{Z}_\phi \mathbf{i}_\phi \Rightarrow \\ \mathbf{A}_s(\mathbf{v}_s - \mathbf{v}'_s) &= \mathbf{Z}_\phi \mathbf{A}_s \mathbf{i}_s \Rightarrow \\ \mathbf{A}_s^{-1}\mathbf{A}_s(\mathbf{v}_s - \mathbf{v}'_s) &= \mathbf{A}_s^{-1}\mathbf{Z}_\phi \mathbf{A}_s \mathbf{i}_s \Rightarrow \\ \mathbf{v}_s - \mathbf{v}'_s &= \mathbf{Z}_s \mathbf{i}_s \end{aligned}$$

- Sequence impedance matrix

$$\mathbf{Z}_s = \mathbf{A}_s^{-1}\mathbf{Z}_\phi\mathbf{A}_s = \begin{bmatrix} z_{00} & z_{01} & z_{02} \\ z_{01} & z_{11} & z_{12} \\ z_{02} & z_{12} & z_{22} \end{bmatrix}$$

zero, positive, negative sequence self/mutual impedances

- Special case: *Transposed lines* $\mathbf{Z}_\phi = (z_s - z_m)\mathbf{I}_3 + z_m \mathbf{1}\mathbf{1}^\top$

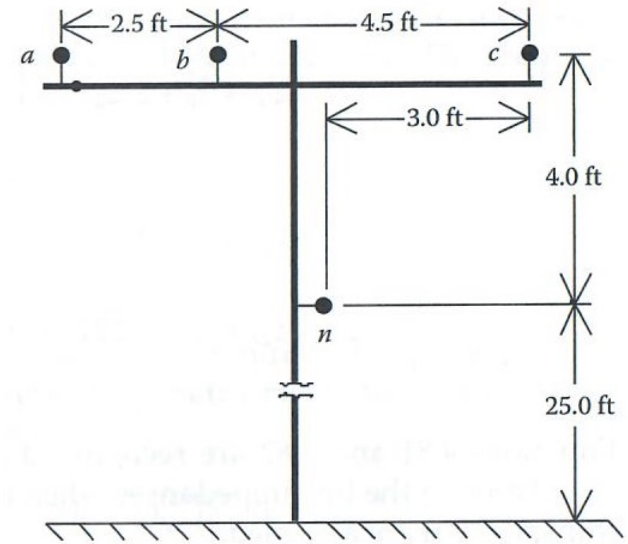
$$\begin{aligned} \mathbf{Z}_s &= (z_s - z_m)\mathbf{I}_3 + z_m \mathbf{A}_s^{-1}\mathbf{1}\mathbf{1}^\top \mathbf{A}_s \\ &= (z_s - z_m)\mathbf{I}_3 + 3z_m \mathbf{e}_1\mathbf{e}_1^\top \\ &= \begin{bmatrix} z_s + 2z_m & 0 & 0 \\ 0 & z_s - z_m & 0 \\ 0 & 0 & z_s - z_m \end{bmatrix} \end{aligned}$$

$$r_i + j\omega \cdot 2 \cdot 10^{-7} \ln \frac{\bar{D}_{ij}}{\bar{R}_i}$$

- Neutral impedance and ground effect appear only on $z_{00} = z_{ii} + 2z_{ij} - 3\frac{z_{in}^2}{z_{nn}}$

Example

Find phase impedance matrix and zero-/positive-sequence impedances for the untransposed three-phase line



336,400 26/7 ACSR: $GMR = 0.0244\,\text{ft}$
$Resistance = 0.306\,\Omega/\text{mile}$

4/0 6/1 ACSR: $GMR = 0.00814\,\text{ft}$
$Resistance = 0.5920\,\Omega/\text{mile}$

$$\mathbf{Z} = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix} \Omega/\text{mile}$$

$$\mathbf{Z}_\phi = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

- Compare r/x ratios; value range; and (off)-diagonal entries

Example (cont'd)

$$\mathbf{Z}_s = \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & -0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & -0.0723 - j0.0060 \\ 0.0256 + j0.0115 & 0.0723 - j0.0059 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

$z_{11} = z_{22}$

- Assume now the line has been transposed

- Average the (off)-diagonal entries of the phase impedance matrix

- Find the related sequence impedance matrix

$$\mathbf{Z}_s^{\text{tr}} = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

- How does $\mathbf{Z}_s^{tr}$ compare to $\mathbf{Z}_s$?

Discussion on symmetrical components

- Symmetrical components were used in older power distribution software (for computational efficiency)

- Analysis with symmetrical components (diagonal $\mathbf{Z}_s^{\mathrm{tr}}$) is equivalent to replacing diagonal and off-diagonal elements of $\mathbf{Z}_\phi$ with their averages

- *Example:* balanced voltage source, untransposed 3-phase line; and unbalanced load

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 155.24 \\ 156.86 \\ 98.167 \end{bmatrix} \mathrm{V} \qquad \begin{bmatrix} \hat{V}_a \\ \hat{V}_b \\ \hat{V}_c \end{bmatrix} = \begin{bmatrix} 157.97 \\ 155.52 \\ 93.25 \end{bmatrix} \mathrm{V} \qquad \begin{matrix} a : -1.75\% \\ b : +0.85\% \\ c : +5.00\% \end{matrix}$$

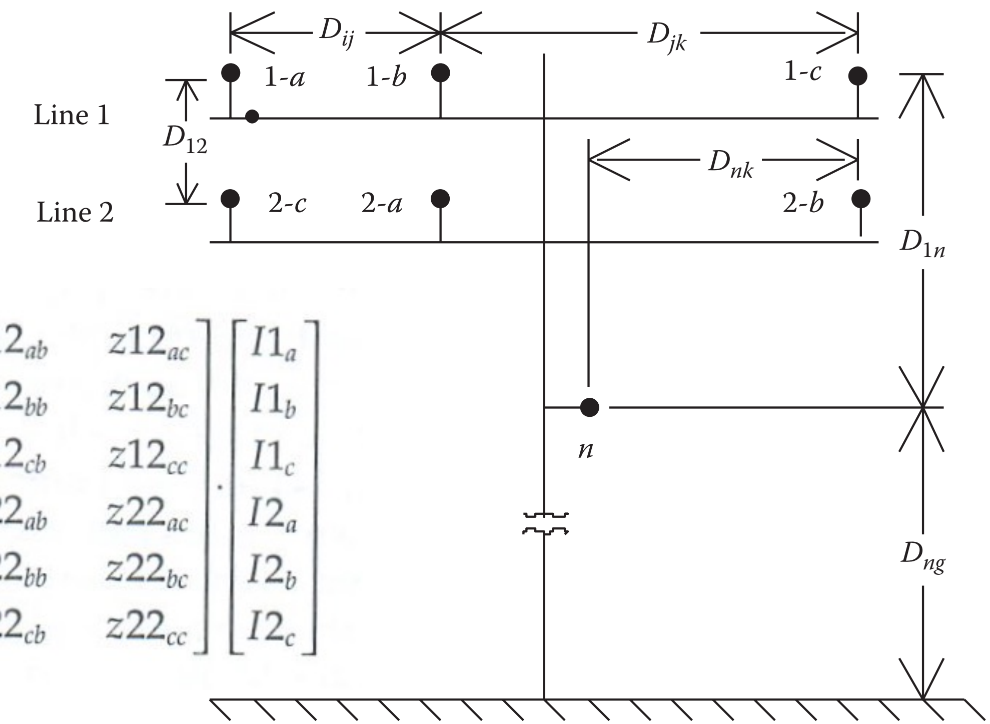
exact voltages — *voltages approximated with symmetrical components* — *errors*

- Symmetrical components cannot be applied for 1- or 2-phase lines

- Full phase impedance analysis needed and is currently implemented in software

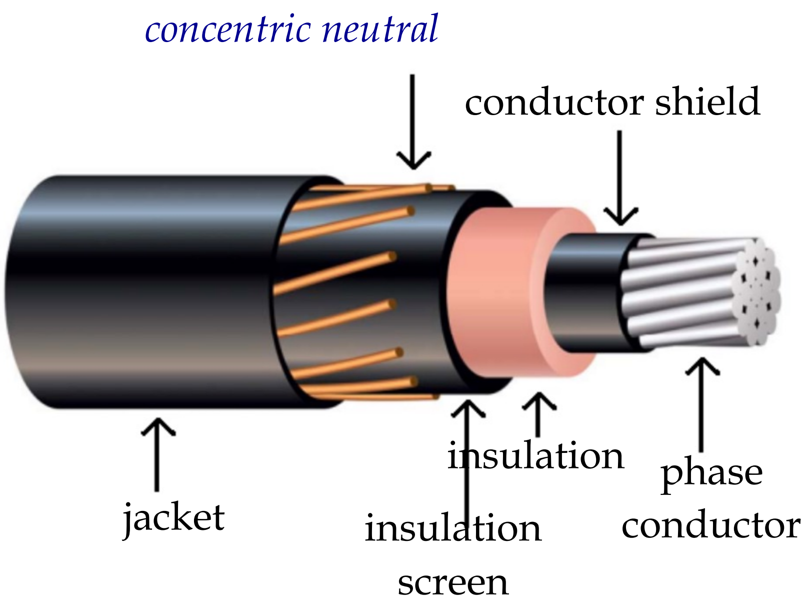
Parallel lines

- Lines may run parallel sharing a *pole*, *right of way (different poles), or trench*
- Usually, one neutral per pole
- Use modified Carson's equations to find the primitive impedance matrix
- Use Kron reduction to find the coupled phase impedance matrix

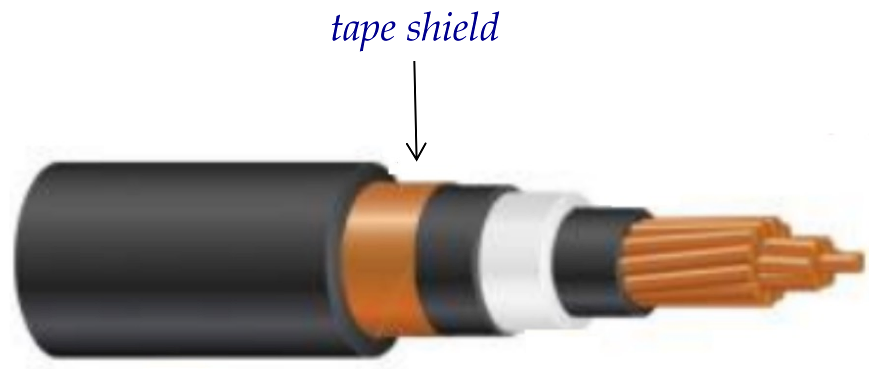


$$\begin{bmatrix} v1_a \\ v1_b \\ v1_c \\ v2_a \\ v2_b \\ v2_c \end{bmatrix} = \begin{bmatrix} z11_{aa} & z11_{ab} & z11_{ac} & z12_{aa} & z12_{ab} & z12_{ac} \\ z11_{ba} & z11_{bb} & z11_{bc} & z12_{ba} & z12_{bb} & z12_{bc} \\ z11_{ca} & z11_{cb} & z11_{cc} & z12_{ca} & z12_{cb} & z12_{cc} \\ z21_{aa} & z21_{ab} & z21_{ac} & z22_{aa} & z22_{ab} & z22_{ac} \\ z21_{ba} & z21_{bb} & z21_{bc} & z22_{ba} & z22_{bb} & z22_{bc} \\ z21_{ca} & z21_{cb} & z21_{cc} & z22_{ca} & z22_{cb} & z22_{cc} \end{bmatrix} \cdot \begin{bmatrix} I1_a \\ I1_b \\ I1_c \\ I2_a \\ I2_b \\ I2_c \end{bmatrix}$$

Underground cables



usually for unbalanced residential

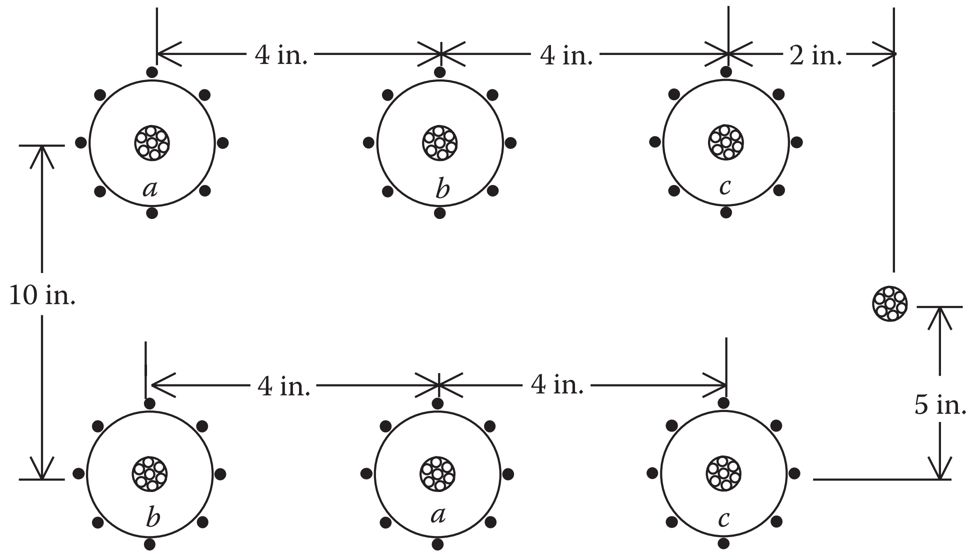


[PEguru.com]

usually for relatively balanced industrial

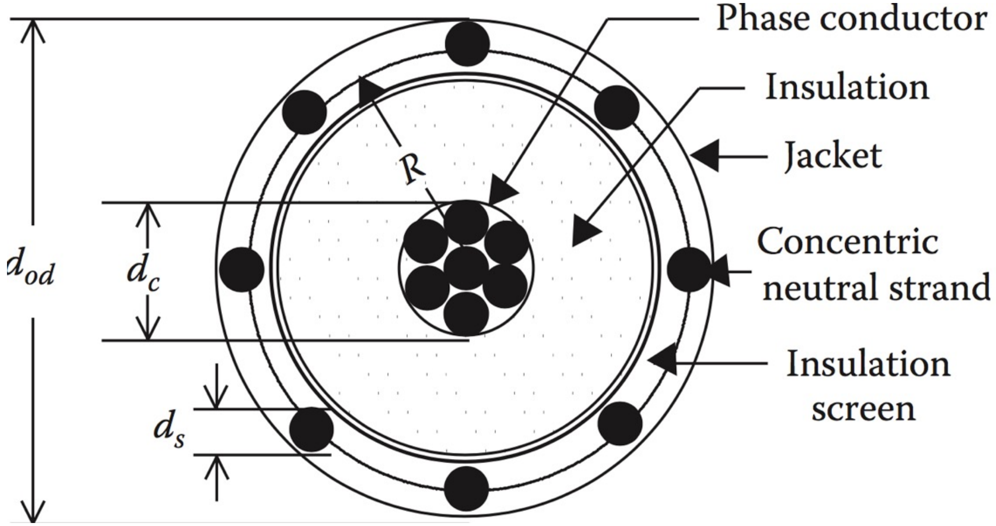
- Above 1kV, concentric neutrals/shields needed to reduce electrical stress on insulation
- Additional neutrals may also be present; all grounded

Primitive matrix for underground cables



- Apply Carson's equations and Kron reduction as before
- Resistances, GMRs, and GMDs are given for phase and regular neutral conductors
- Need to find resistances, GMRs, and GMDs for concentric & tape-shielded neutrals

Concentric neutrals



k : # concentric neutrals

r_s : neutral strand resistance

$$R = \frac{d_{od} - d_s}{2}$$

- Resistance $r_{cn} = \frac{r_s}{k}\ \Omega/\text{mile}$

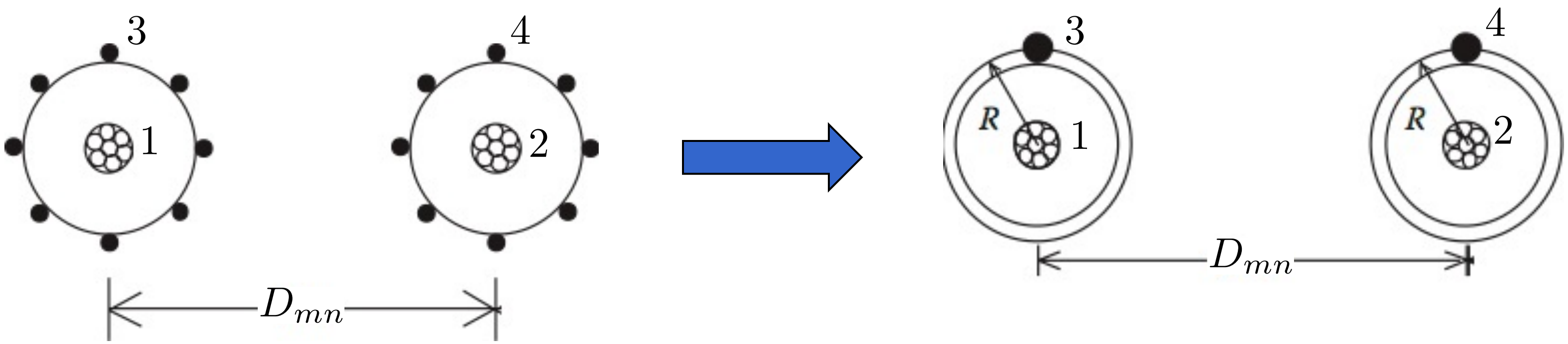
- GMR follows from formula for *bundled* transmission lines

$$\text{GMR}_{cn} = \sqrt[k]{\text{GMR}_s \cdot k \cdot R^{k-1}}$$

see Problem 4.16 in Glover, Sarma, & Overbye

- Textbook switches often between *ft=12 in*, *in*, and *mil=0.001 in*

Concentric neutrals (cont'd)



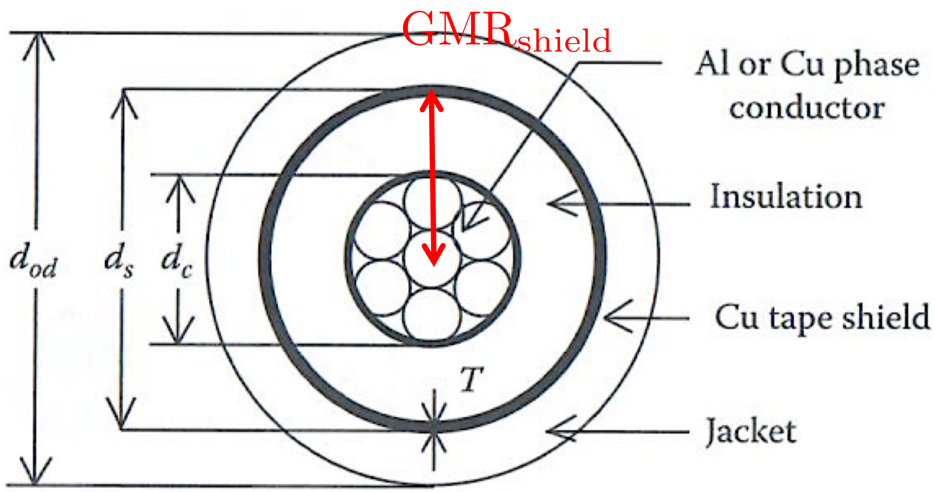
- GMD between neutral and related phase conductor $D_{13} = \sqrt[k]{R^k} = R$

- GMD between neutral and other phase conductor can be approximated by replacing the concentric neutral with one strand at the top

$$D_{14} = \sqrt{D_{12}^2 + R^2}$$

- In this way, GMD between neutrals captured correctly too

Tape-shielded cables



- Resistance of tape $r_{\text{shield}} = 7.93 \cdot 10^8 \cdot \dfrac{\rho}{d_s T}\ \Omega/\text{mile}$

- GMR of tape shield is radius from center to midpoint of tape [symmetry or limiting case for concentric neutrals]

- GMD between tape shields or a tape shield and another conductor equals distance between conductors

Summary

- Find impedances from geometric configurations

- Modify impedances to capture the effect of earth

- Convert primitive to phase impedance matrix via Kron reduction

- Two-port model for multiphase lines

$$\mathbf{v}_\phi - \mathbf{v}'_\phi = \left(\mathbf{Z}_{\phi\phi} - \mathbf{Z}_{\phi n}\mathbf{Z}_{nn}^{-1}\mathbf{Z}_{\phi n}^\top\right)\mathbf{i}_\phi$$

- Connection with special case of transposed lines and symmetric components

- Studied the cases of parallel lines and underground cables