ECE 5984: Power Distribution System Analysis

Lecture 3: Approximate Feeder Analysis

Reference: Textbook, Chapter 3

Instructor: V. Kekatos

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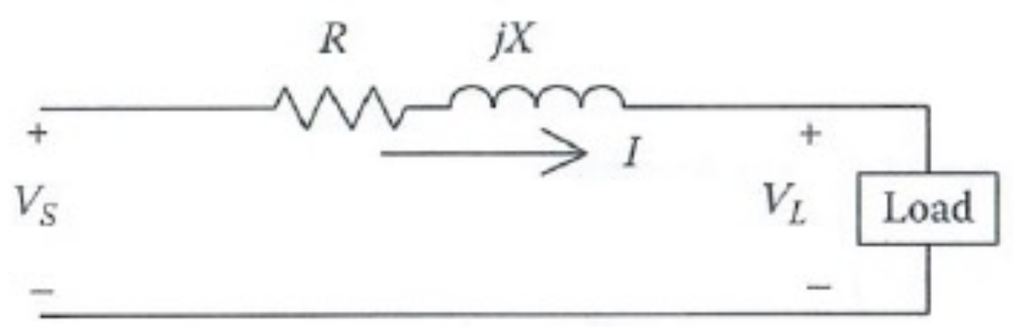
Approximate analysis

- Develop *approximate* methods for determining voltage drops and power losses

- *Assumptions*
  - loads are balanced three-phase
  - loads are constant-current (if constant-power, assume negligible voltage drop)
  - lines are transposed and three-phase

- Single-phase (line-to-neutral) equivalent



$$V_S = ZI + V_L$$

[Kersting]

Voltage drop: $\Delta V := |V_S| - |V_L|$

Power losses: $P_\ell := 3R|I|^2$

Line impedance

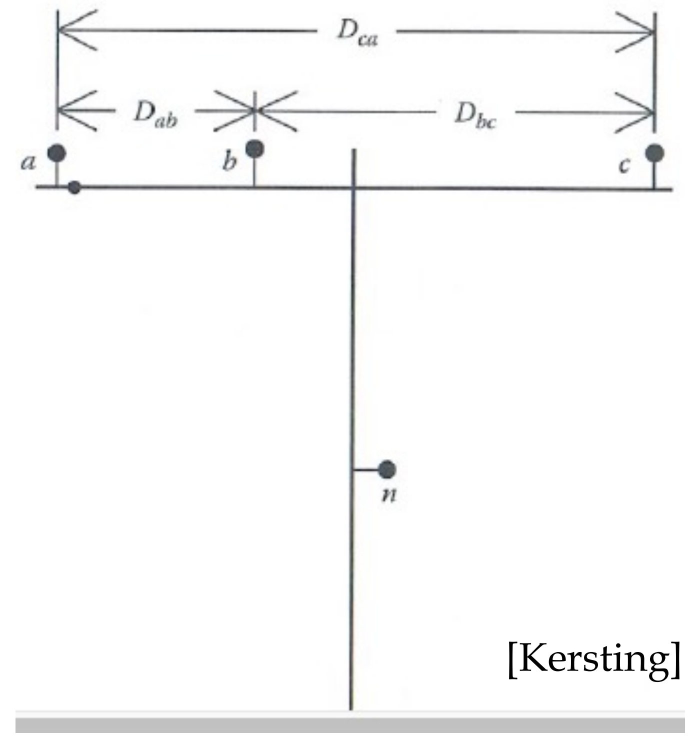
- Due to transposition and balanced loads, *positive-sequence impedance* suffices

$$z_1 = r + j\omega \cdot 2 \cdot 10^{-7} \cdot \ln\left(\frac{\bar{D}}{\bar{R}}\right) \;\; \Omega/\text{m}$$

$$= r + j \cdot 0.1213 \cdot \ln\left(\frac{\bar{D}}{\bar{R}}\right) \;\; \Omega/\text{mile}$$

GMD: $\bar{D} := (D_{ab} D_{bc} D_{ca})^{1/3}$

GMR: $\bar{R}$



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Voltage drop

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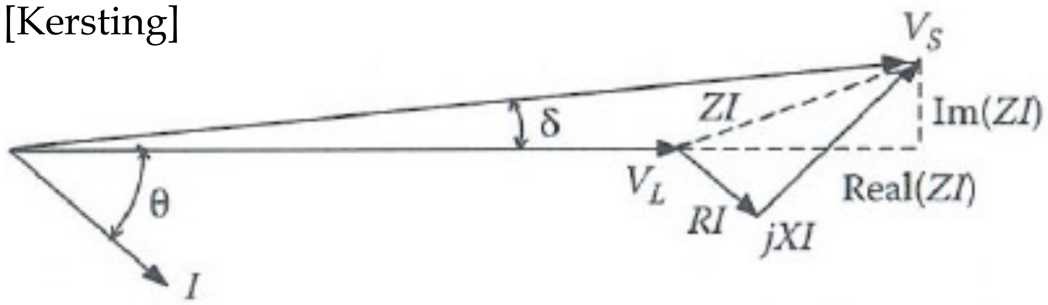


- Because δ is small, approximate $\Delta V \simeq \text{Re}(ZI)$ [Why?]

Example: $V_S = 2400$ V, $Z = 0.284 + j0.568\ \Omega$, $I = 43\angle -25^\circ$ A $\Rightarrow$
$V_L = 2378.41\angle -0.40^\circ$ V

exact voltage drop: $\Delta V = 21.59$ V $= 0.9\%$

approx. voltage drop: $\Delta V = 21.65$ V $= 0.9\%$

‘Voltage-square law’

Both relative voltage drop and power losses are (approximately) inversely proportional to the LL voltage level

$$\Delta V \simeq \mathrm{Re}(ZI) \simeq \mathrm{Re}\left(Z\frac{S^*}{\sqrt{3}V_{\mathrm{LL}}}\right) \quad \Rightarrow \quad \boxed{\frac{\Delta V}{V_{\mathrm{LN}}} = \mathrm{Re}\left(\frac{ZS^*}{V_{\mathrm{LL}}^2}\right)}$$

If you double the voltage level, you can transfer four times more power for the same relative voltage drop (or the same power for four times the distance)

$$P_\ell = 3R|I|^2 \simeq 3R\left|\frac{S}{\sqrt{3}V_{\mathrm{LL}}}\right|^2 \quad \Rightarrow \quad \boxed{P_\ell = \frac{R|S|^2}{V_{\mathrm{LL}}^2}}$$

If you double the voltage level, you can transfer two times more power for the same copper losses (or the same power for four times the distance)

K_{drop} factor

- Factors used for fast and approximate voltage drop calculations

Definition: *Voltage drop relative to nominal phase (not LL) voltage for serving 1 kVA load with given power factor located 1 mile away*

$$K_{\text{drop}} := \frac{\Delta V \ [\%]}{\text{kVA} \cdot \text{mile}}$$

- From previous approximation $\Delta V \simeq \text{Re}(ZI)$

- Impedance over a mile $Z = z \ [\Omega/\text{mile}]$

- Current for 1 kVA @ given PF $S = |S|e^{j\theta}, \ \theta = \pm \cos^{-1}(\text{PF}) : +/- \text{ for lagging/leading}$

  typical: lagging PF 0.9

  $\theta = +25.84^\circ$

$$I \simeq \frac{S^*}{\sqrt{3}V_{\text{LL}}} = \frac{1000 \cdot e^{-j\theta}}{\sqrt{3}V_{\text{LL}}}$$

- Simplified expression $K_{\text{drop}} = \frac{\Delta V}{V_{\text{LN}}} \times 100 = \frac{\text{Re}(z \cdot e^{-j\theta})}{V_{\text{LL}}^2} \times 10^5$

voltage-square law

K_{drop} factor (cont'd)

- Simplified expression $$K_{\text{drop}} = \frac{\text{Re}(z \cdot e^{-j\theta})}{V_{\text{LL}}^2} \times 10^5 \text{ [\% voltage drop (LN)/mile/kVA]}$$

- Utilities compute K factors for all combinations of voltage levels and line types

Example: For a line with $z = 0.306 + j0.627\ \Omega/\text{mile}$, compute K_{drop} assuming lagging PF 0.9 and nominal voltage of 12.47 kV

$$K_{\text{drop}} = 3.53 \cdot 10^{-4}$$

- Questions that can be easily answered with K factors:

How far can I serve a 7500 kVA load within the 3% voltage drop?

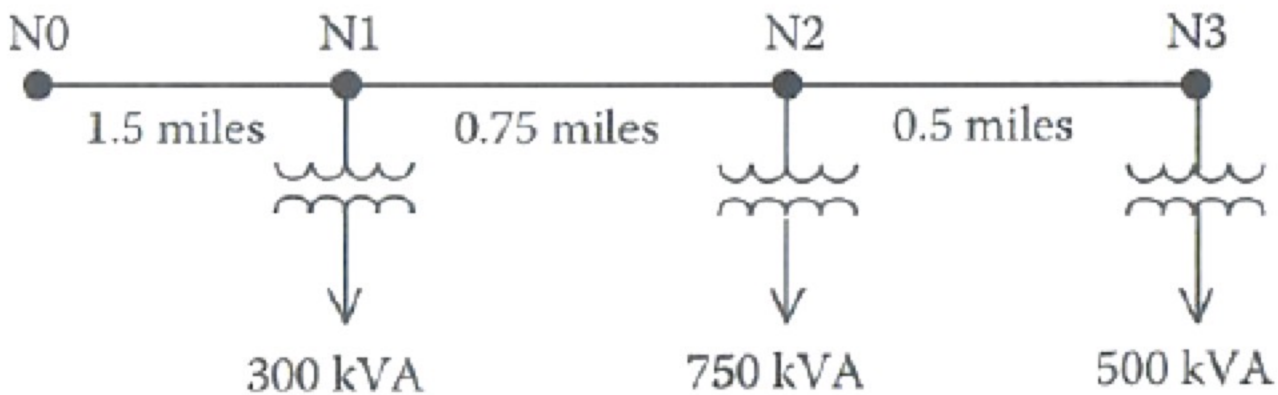
$$K_{\text{drop}} \cdot 7,500 \cdot d = 3 \quad \Rightarrow \quad d = 1.13 \text{ miles}$$

What is the maximum load I can serve within 1.5 miles from the substation?

$$K_{\text{drop}} \cdot L \cdot 1.5 = 3 \quad \Rightarrow \quad L = 5,668 \text{ kVA}$$

Line segments in series

- Voltage drops calculated through K factors apply additively



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Example

$$\Delta V_{01} = \frac{|V_0| - |V_1|}{V_{\text{LN}}}\% = K_{\text{drop}} \cdot (300 + 750 + 500) \cdot 1.5 = 0.82$$

$$\Delta V_{12} = \frac{|V_1| - |V_2|}{V_{\text{LN}}}\% = K_{\text{drop}} \cdot (750 + 500) \cdot 0.75 = 0.33$$

$$\Delta V_2 = \Delta V_{01} + \Delta V_{12} = 1.15$$

K_{rise} factor

- Factors for approximately calculating voltage rise incurred by capacitors

$$K_{\text{rise}} := \frac{\Delta V_{\text{rise}}\ [\%]}{\text{kVAR} \cdot \text{mile}}$$

Definition: *Voltage rise relative to nominal phase voltage by 1 kVAR capacitor located 1 mile away*

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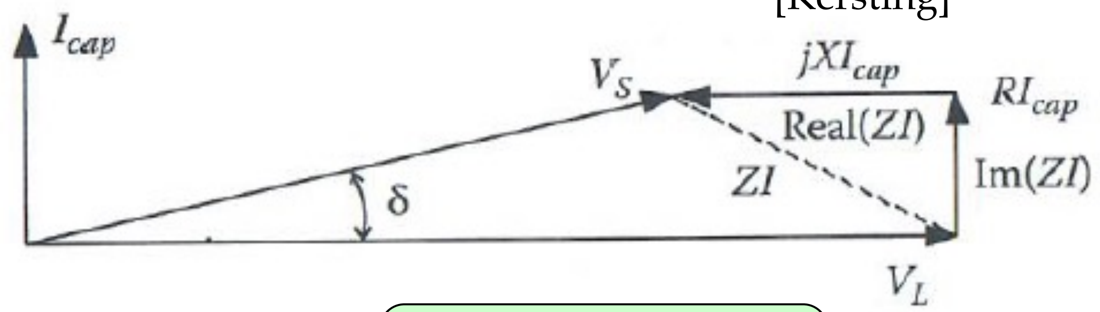

$$\Delta V_{\text{rise}} \simeq |\text{Re}(ZI_c)| = X|I_c|$$

$$K_{\text{rise}} = \frac{x}{V_{\text{LL}}^2} \times 10^5$$

Example: For the previous line, find the K_{rise} and the capacitor rating needed to fix a voltage drop of 3.5% to 2.5% located 1.5 miles away

$$K_{\text{rise}} = 4.03 \cdot 10^{-4} \qquad K_{\text{rise}} \cdot Q_c \cdot 1.5 = 1 \quad \Rightarrow \quad Q_c = 1,653 \text{ kVAR}$$

Uniformly distributed discrete loads

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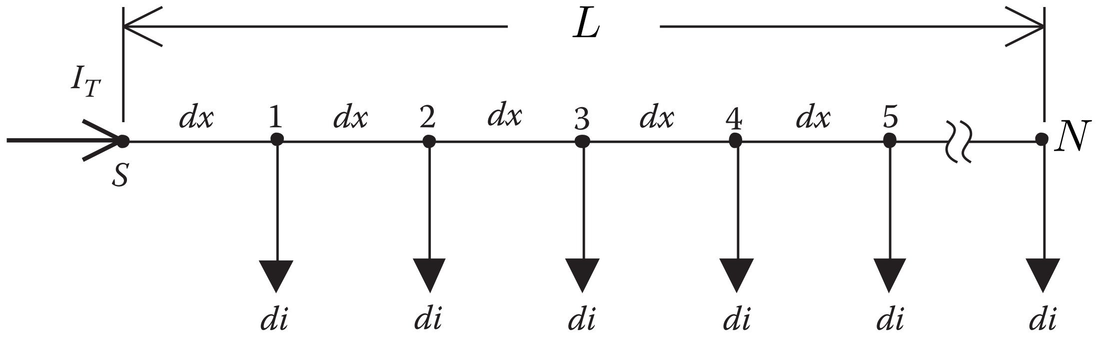
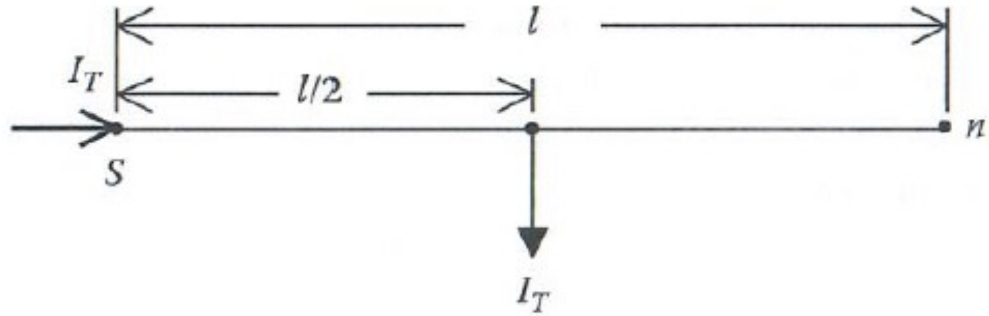


- N loads uniformly distributed over a line segment of length L
- Current loads drawing equal current
- Analyze voltage drop at *end of segment* and total power losses

Uniformly distributed discrete loads (cont'd)

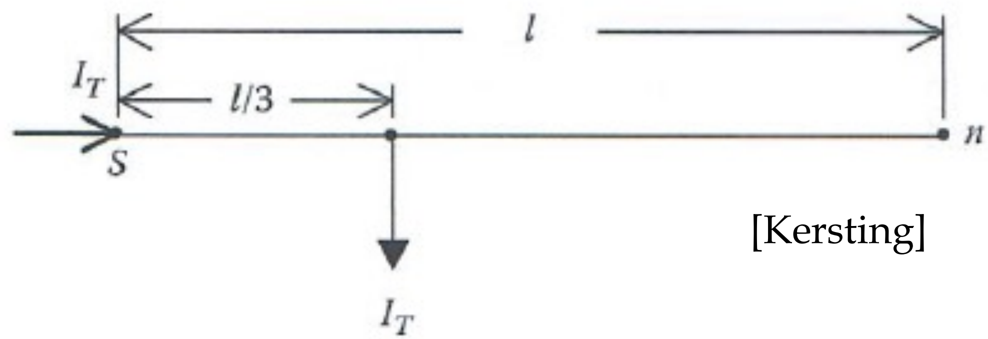
Voltage drop

$$\Delta V = \text{Re}\left\{\frac{1}{2}ZI\left(1+\frac{1}{N}\right)\right\}$$
$$\overset{N\to\infty}{=} \text{Re}\left\{\frac{1}{2}ZI\right\}$$



Power losses

$$P_\ell = 3R|I|^2\left[\frac{1}{3}+\frac{1}{2N}+\frac{1}{6N^2}\right]$$
$$\overset{N\to\infty}{=} 3\frac{R}{3}|I|^2$$



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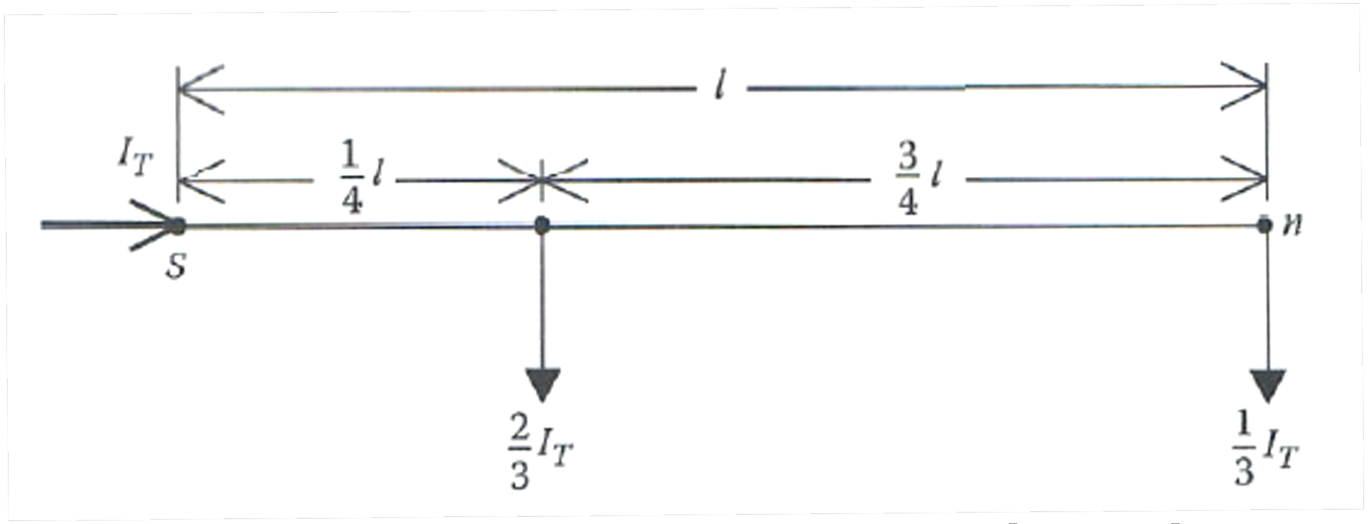
Uniformly distributed discrete loads (cont'd)

Can you replace all loads with two loads rather than one that agrees both in voltage drop and power losses?



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Loads uniformly distributed over areas

- Assume constant-current loads uniformly distributed over a geographic area

- A primary distribution line runs across and serves the area

- ***Given***
  - load density D [kVA/mile^2]
  - area shape and dimensions
  - geographical area A [mile^2]
  - load PF (assumed constant)
  - per-mile impedance of main line
  - nominal voltage V_{LL}

- ***Wanted*:** Find voltage drop at the end of the primary and total power losses

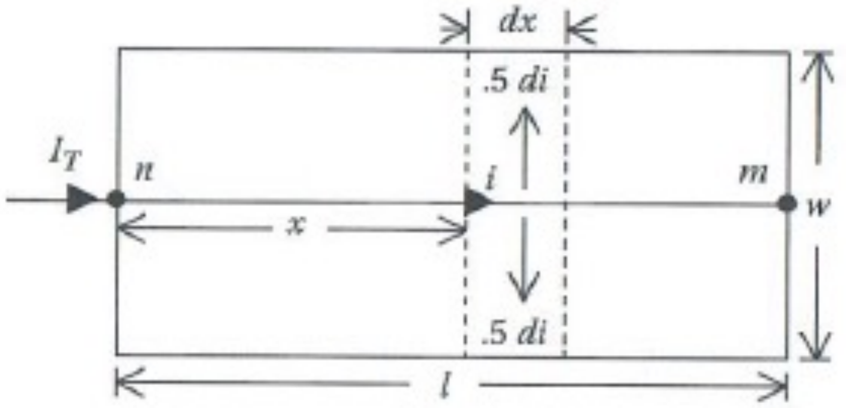
- Total current at the substation entering the primary

$$I \simeq \frac{D \cdot A \cdot e^{-j\theta}}{\sqrt{3} \cdot V_{LL}}$$

Rectangular area

Voltage drop $\Delta V = \text{Re}\left\{\frac{1}{2}ZI\right\}$

Power losses $P_\ell = 3\left(\frac{1}{3}R|I|^2\right)$



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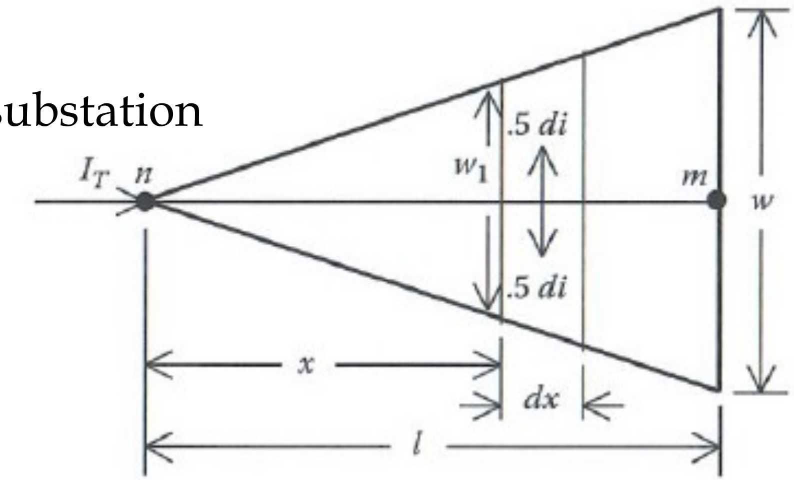
- Compare to uniformly distributed loads

Example: Find power losses and choose voltage level between 4.16 and 12.47 kV for rectangular area with

$$\ell = 10,000 \text{ ft};\ w = 6,000 \text{ ft};\ D = 2500 \text{ kVA/mile}^2;\ \text{PF} = 0.9;\ z = 0.306 + j0.627\ \Omega/\text{mile}$$

$$1 \text{ mile} = 5280 \text{ feet}$$

Triangular area



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- Load increases as moving away from substation

Voltage drop $\Delta V = \text{Re}\left\{\frac{2}{3} Z I\right\}$

Power losses $P_\ell = 3\left(\frac{8}{15} R |I|^2\right)$

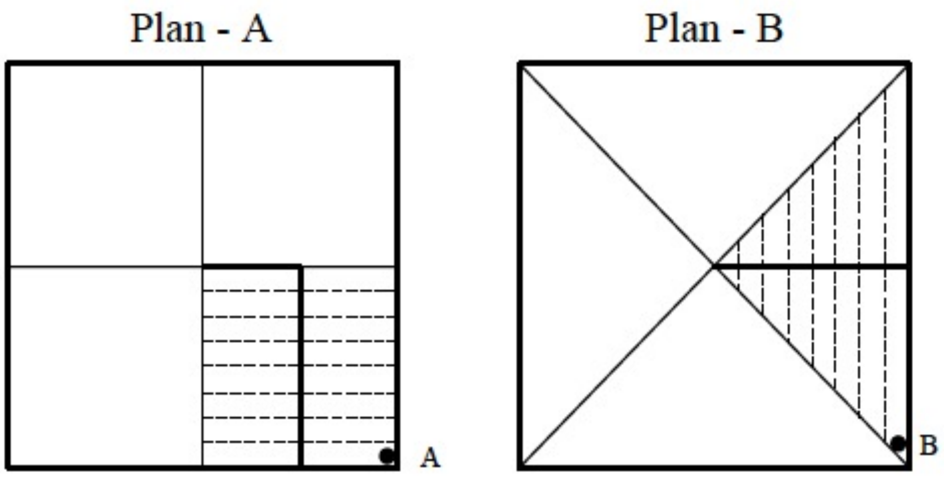
- *K* factors can be combined with formulas derived from distributed loads

***Example*:** Find voltage drop in triangular area and placement of a 1,800 kVAR capacitor to bring voltage within 3%

$$\ell = 15,000 \text{ ft}; \;\; w = 6,000 \text{ ft}; \;\; D = 3500 \text{ kVA/mile}^2; \;\; \text{PF} = 0.9$$

$$K_{\text{drop}} = 0.00035291 \; \frac{\Delta V \;\; [\%]}{\text{kVA} \cdot \text{mile}}; \;\; K_{\text{rise}} = 0.00040331 \; \frac{\Delta V \;\; [\%]}{\text{kVAR} \cdot \text{mile}}$$

Why rectangular and triangle areas?



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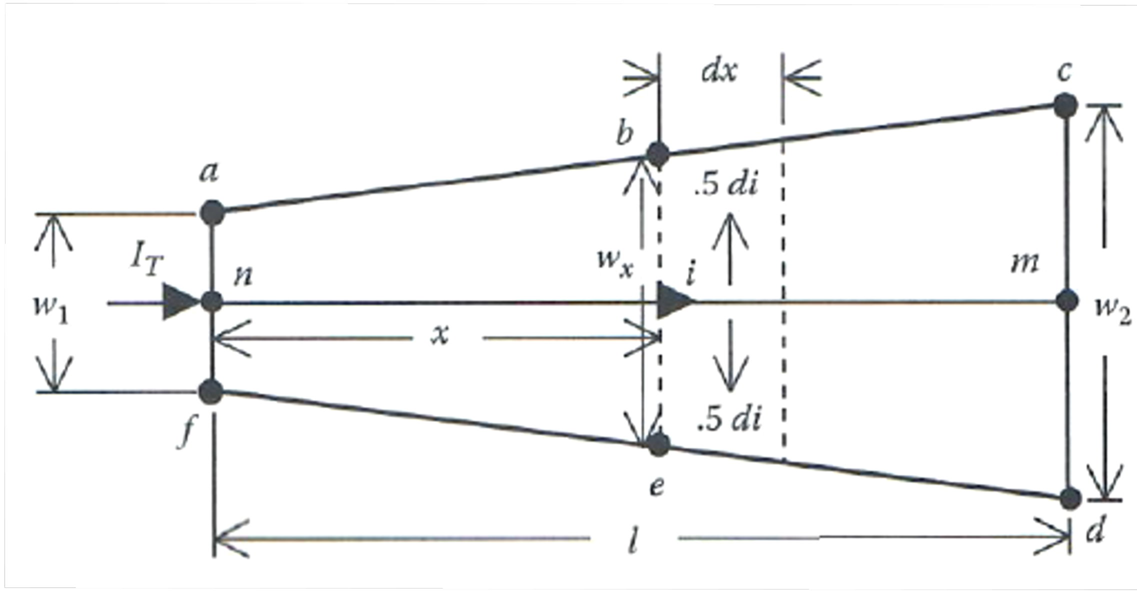
Trapezoidal area



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Voltage drop $$\Delta V = \mathrm{Re}\{ZI\} \frac{w_1 + 2w_2}{3(w_1 + w_2)}$$

Power losses $$P_\ell = 3R|I|^2 \frac{8w_2^2 + 9w_1w_2 + 3w_1^2}{15(w_1 + w_2)^2}$$

- Square area as special case $w_1 = w_2 = w$
- Triangular area as special case $w_1 = 0;\ w_2 = w$

Area coverage principle

$$\Delta V = \beta \cdot \mathrm{Re}\{ZI\} \quad \Rightarrow \quad \frac{\Delta V}{V_{\mathrm{LN}}} = \mathrm{Re}\left\{\frac{\beta z \ell S^*}{V_{\mathrm{LL}}^2}\right\} = \mathrm{Re}\left\{\frac{\beta z \ell D A e^{-j\theta}}{V_{\mathrm{LL}}^2}\right\}$$

depends on geometry (square, triangle, trapezoid)

- For the same line, geometry, and power factor $\quad \dfrac{\Delta V}{V_{\mathrm{LN}}} \propto \dfrac{\ell D A}{V_{\mathrm{LL}}^2}$
- Assume constant load density D
- For the same % voltage drop, service area can change as

$$\left.\begin{aligned} \ell' &= \alpha \ell \\ w' &= \alpha w \end{aligned}\right\} \Rightarrow \quad A' = \alpha^2 A \quad \text{where} \quad \alpha = \left(\frac{V'_{\mathrm{LN}}}{V_{\mathrm{LN}}}\right)^{2/3}$$

$\Delta V = 3\%,\ V_{\mathrm{LN}} = 1$

$\ell = 2,\ w = 0.5,\ A = 1$

$\alpha = 2^{2/3} = 1.587$

$\alpha^2 = 2^{4/3} = 2.52$

$\Delta V = 3\%,\ V_{\mathrm{LN}} = 2$

$\ell = 3.17,\ w = 0.79,\ A = 2.52$

Summary

- Approximated voltage drop as $|V_1| - |V_2| \simeq \mathrm{Re}\{ZI_{12}\}$
- Introduced K_{drop}/K_{rise} for fast calculations of voltage drops
- Replaced uniformly distributed loads with discrete loads to match losses and/or voltage drop
- Derived formulas for voltage drop and power losses for basic area shapes
- Aforementioned tools are useful for planning/design purposes
  - sizing transformers and capacitors
  - deciding voltage levels
  - assigning customers to feeders