ECE 5984: Power Distribution System Analysis

Lecture 2: Load Allocation across a Feeder

Reference: Textbook, Chapter 2

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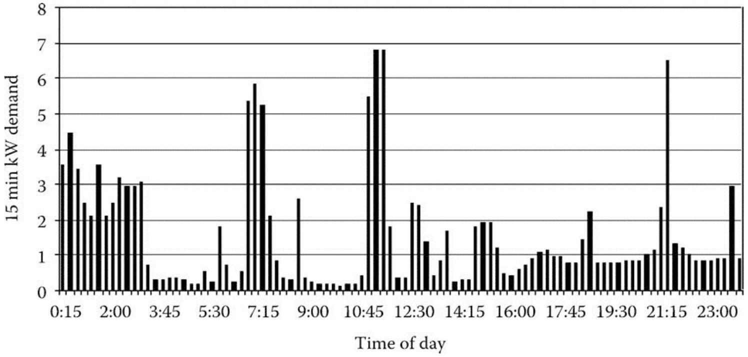
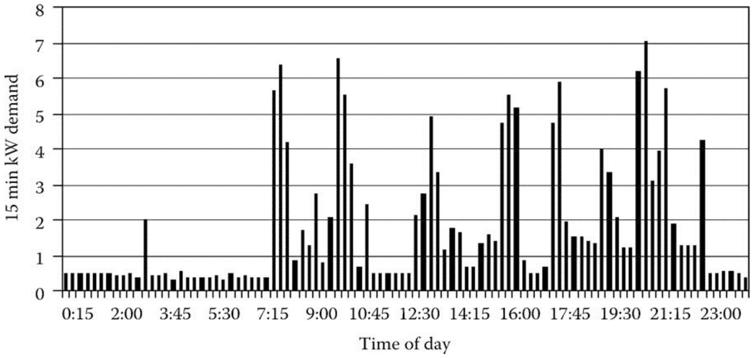
Motivation for allocating load

Facts

- In transmission systems, load forecasting is pretty accurate (law of large numbers)
- In distribution systems, load exhibits higher variability





Problem

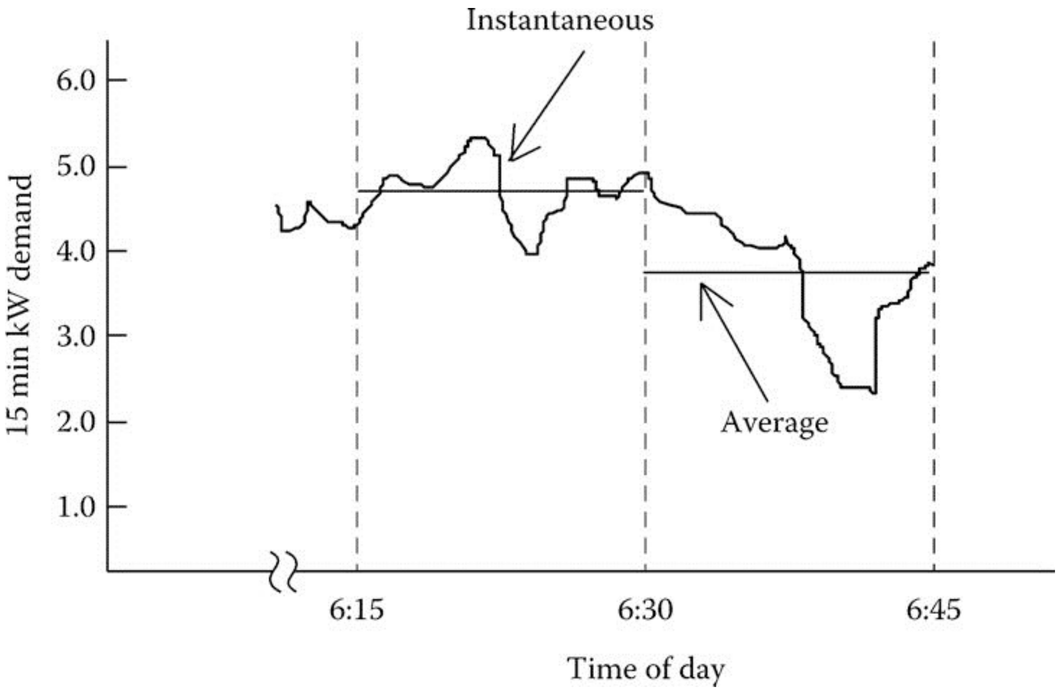
- Study variations of individual and aggregate loads, in particular their max values

Why?

- To size equipment (transformers) and study voltage drops

Individual customer load

- Demand: load (kW, kVA, kVAR, A) averaged over a time period (e.g., 15 min)

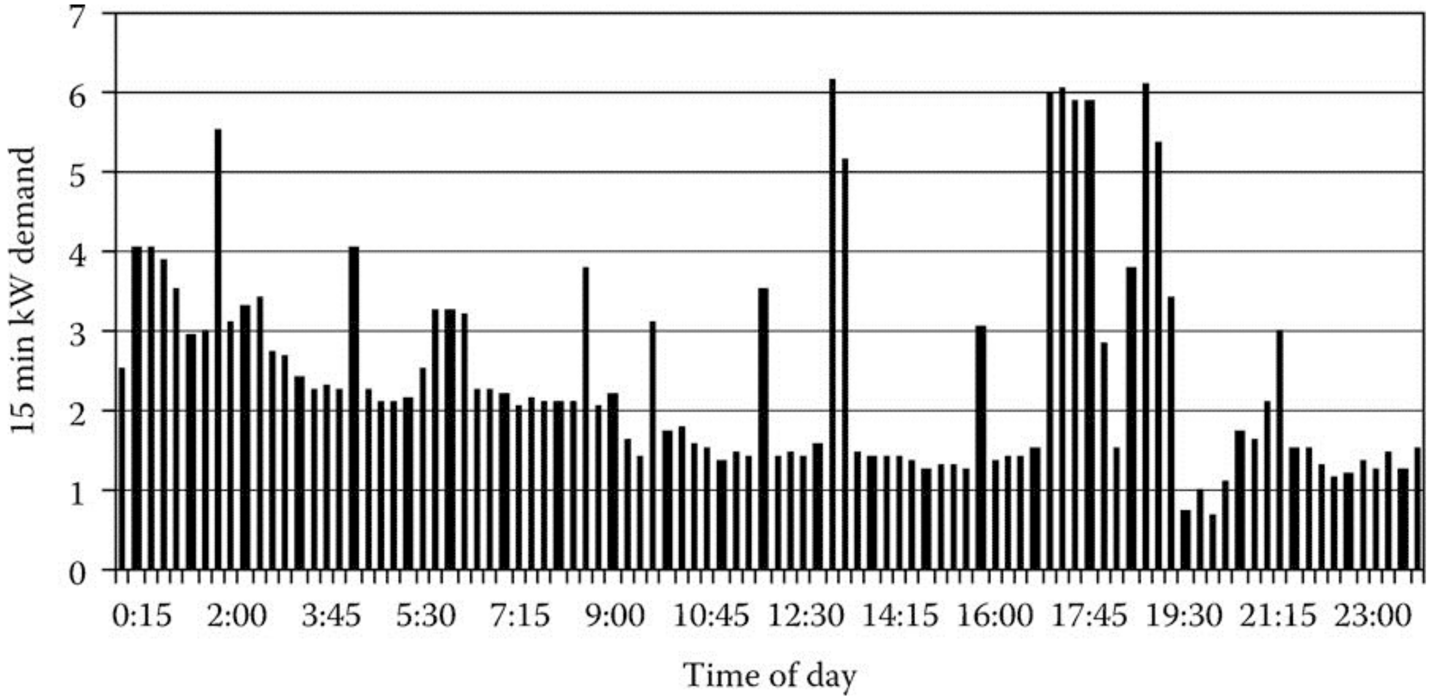


example: the 15-min demand ending at 6:30 pm is 4.75 kW

- Individual loads vary significantly; interested in the maximum
- Characterize variability for transformer sizing; load allocation; voltage drops

Average and maximum demand

- Collect demands across time for customer *n:* $\{d_{n,t}\}_{t=1}^{T}$



- *Average demand* $\tilde{d}_n := \frac{1}{T}\sum_{t=1}^{T} d_{n,t}$ example : $\tilde{d}_1 = 2.46 \text{ kW}$

  How to calculate monthly energy consumption?

- *Maximum demand* $\bar{d}_n := \max_t \{d_{n,t}\}$ example : $\bar{d}_1 = 6.19 \text{ kW at } 13{:}15$

Load factor and demand factor

- *Load factor* $$\mathrm{LF}_n := \frac{\tilde{d}_n}{\bar{d}_n} \leq 1$$

$$\text{example : } \ \mathrm{LF}_1 = \frac{\tilde{d}_1}{\bar{d}_1} = \frac{2.46 \text{ kW}}{6.19 \text{ kW}} = 0.40$$

LF=1 is ideal for better utilization of facilities

- *Demand factor*

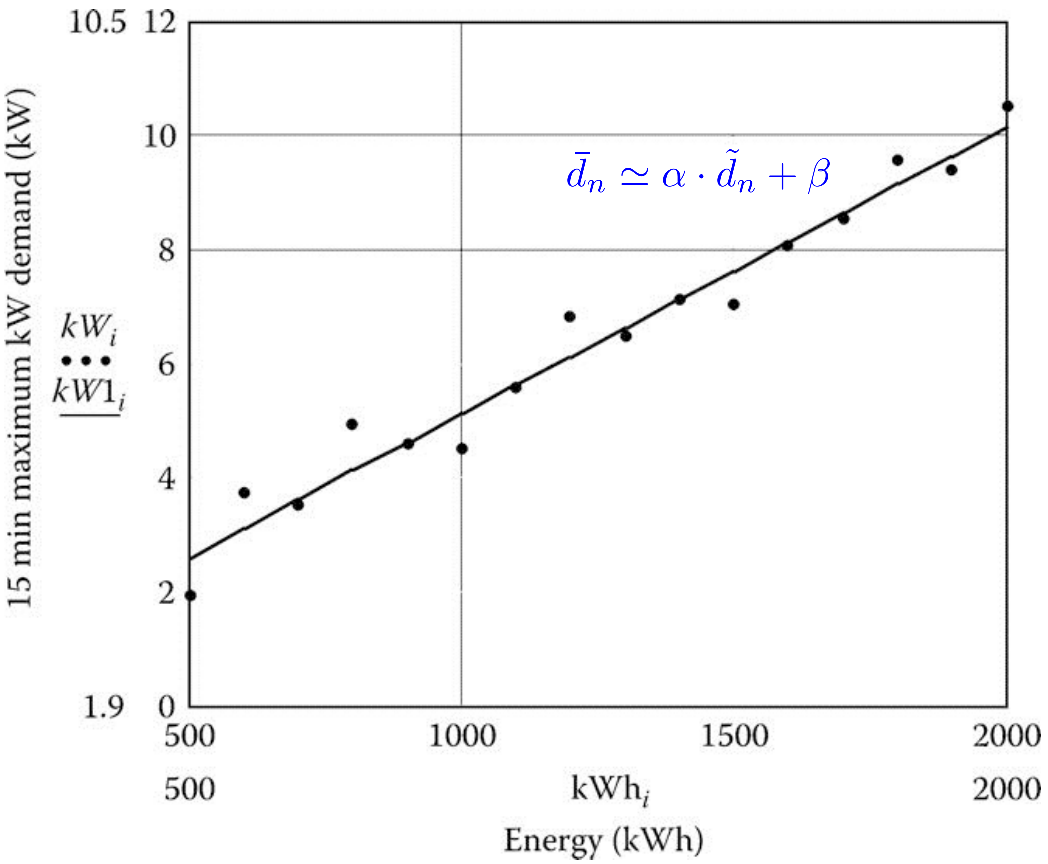
$$\text{Demand factor} := \frac{\text{maximum customer demand}}{\text{sum of device ratings}} = \frac{6.19 \text{ kW}}{35 \text{ kW}} = 0.18 \leq 1$$

percentage of electrical devices that are on when maximum demand occurs

Characterized load for one customer. How about aggregated load in a distribution transformer serving 5-50 customers?

Load surveys

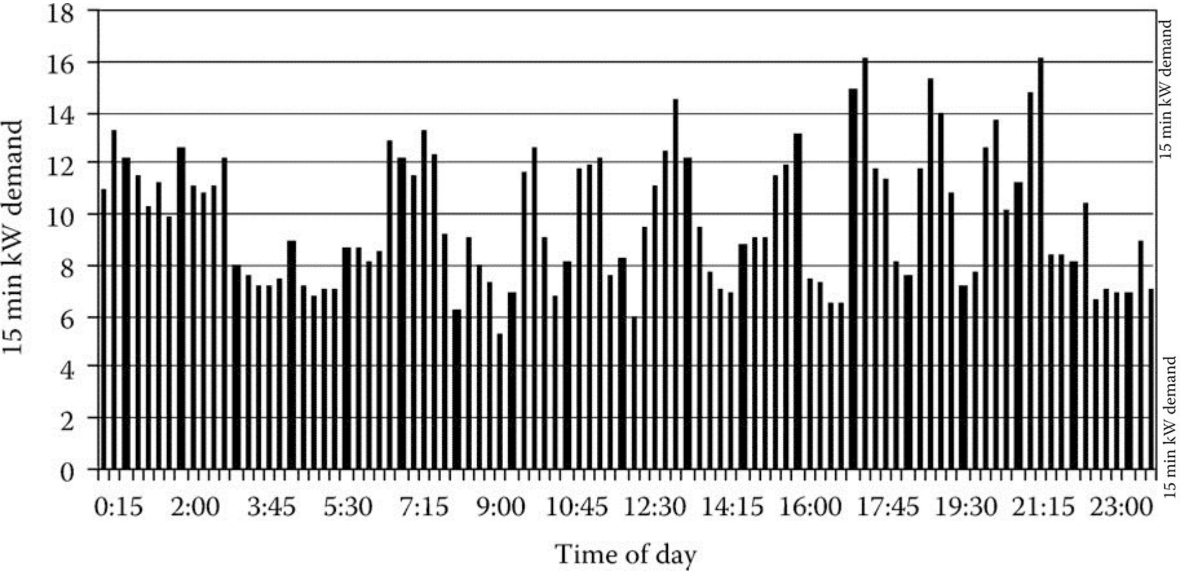
- We are metering energy monthly (billing); but do not know user maximum
- Interested in maxima to determine transformer ratings



such studies are now much easier with smart meter data

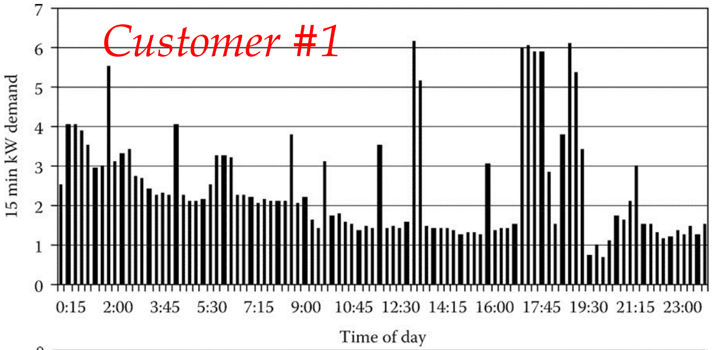
- Install meters over selected customers and use linear regression to find a linear transformation between monthly energy consumption and maximum demand

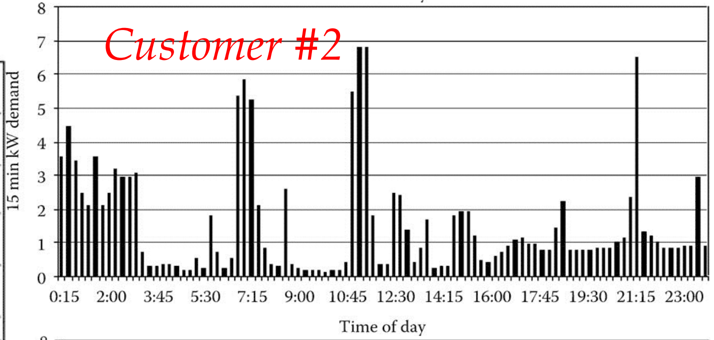
Load at distribution transformer

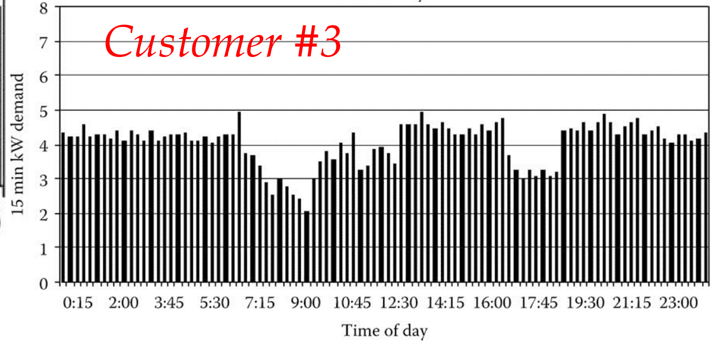


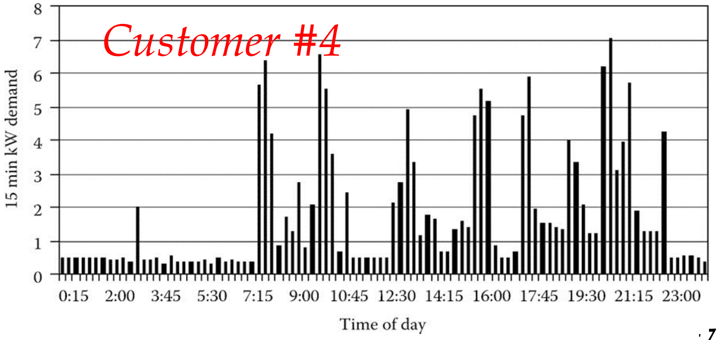
Diversified demand: sum of customer demands

$$\mathbf{d} = \sum_{n=1}^{N} \mathbf{d}_n$$

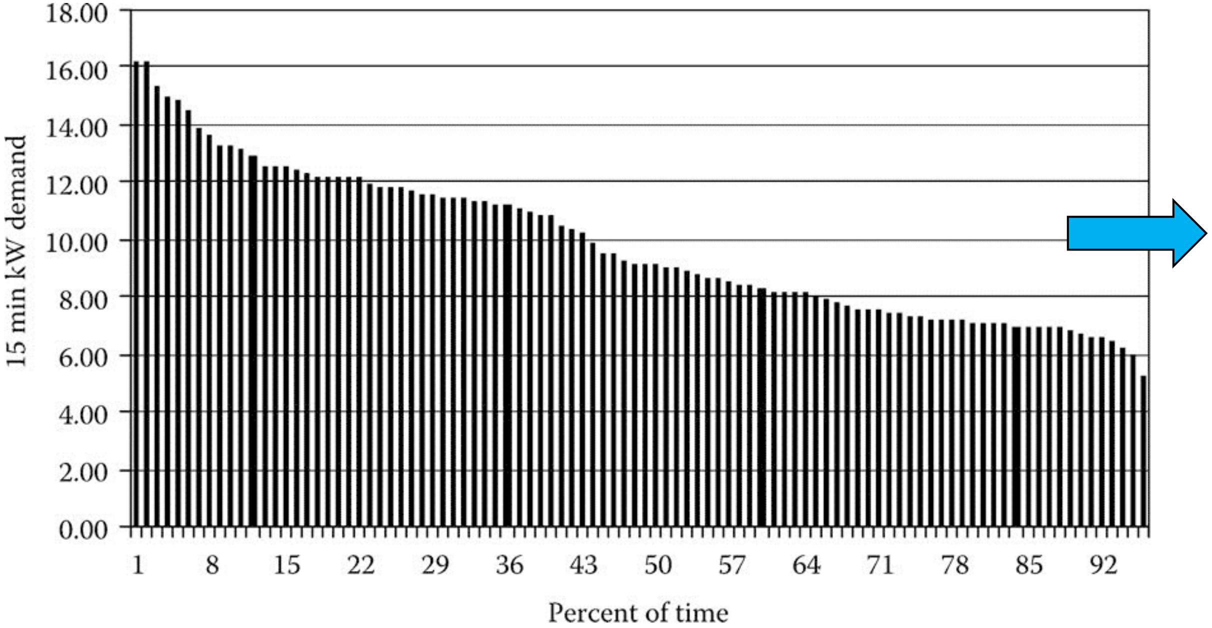


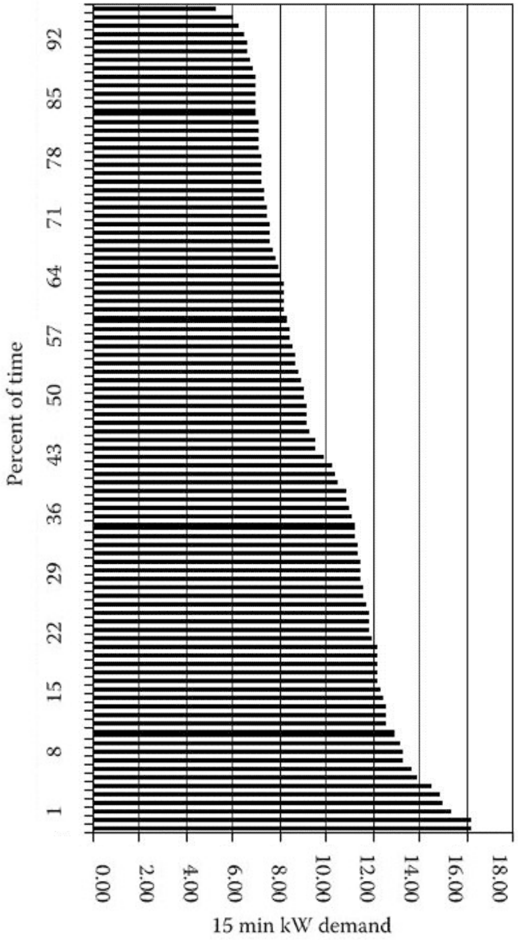






Load duration curve





- Sort diversified demand to evaluate transformer stress

$$\begin{aligned}\Pr[D \geq d] &= 1 - \Pr[D \leq d] \\ &= 1 - \mathrm{CDF}(d)\end{aligned}$$

- Each bar corresponds to 15min/24h=1.04%
- *Example:* 22% of the time, the transformer serves more than 12 kW

Maximum demand for load aggregations

- *Maximum diversified demand* (max of sum)

$$\bar{d} := \max_t \{d_t\} = \max_t \left\{ \sum_{n=1}^{N} d_{n,t} \right\} \qquad \text{example : } \ \bar{d} = 16.16 \text{ kW at } 17{:}30$$

- *Maximum non-coincident diversified demand* (sum of max)

$$\hat{d} := \sum_{n=1}^{N} \bar{d}_n = \sum_{n=1}^{N} \max_t \{d_{n,t}\} \qquad \text{example : } \ \hat{d} = 6.18 + 6.82 + 4.93 + 7.05 = 24.98 \text{ kW}$$

Per-customer maxima do not necessarily occur at the same time …

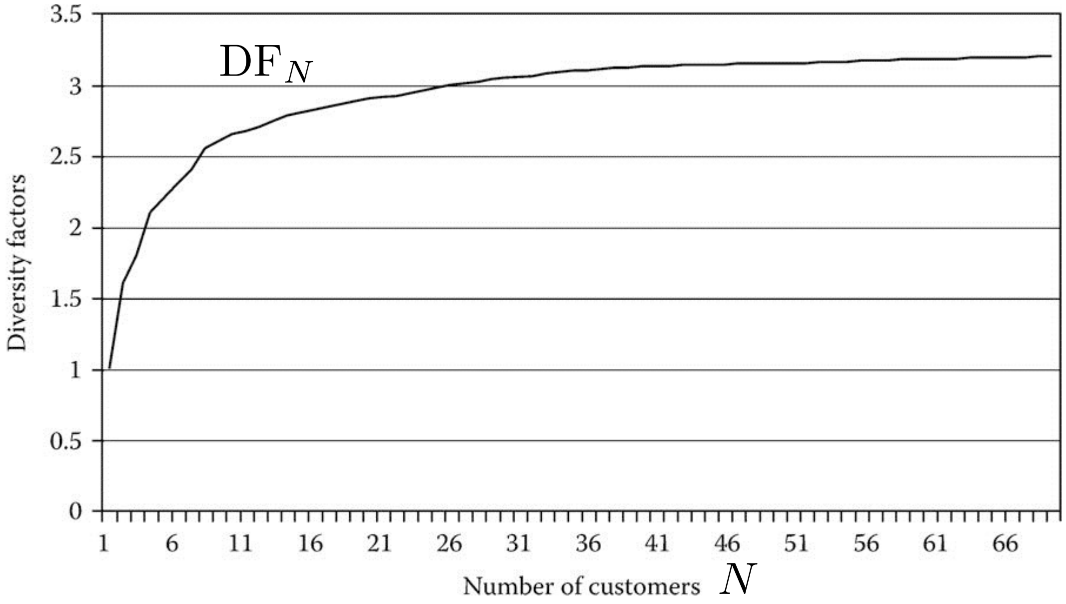
- *Diversity factor* $\mathrm{DF} := \frac{\hat{d}}{\bar{d}} \geq 1 \qquad \text{example : } \ \mathrm{DF} = \frac{\hat{d}}{\bar{d}} = 1.55$

[why?]

Why do we care about the max non-coincident diversified demand?

Diversity factors

- Maximum diversified demand is *hard to measure*
- We usually know max demand per customer *(metered or inferred from bill)*
- How to translate from $\hat{d} \rightarrow \bar{d}$?
- Calculate diversity factors DF experimentally
  - record $\bar{d}$ at specific network locations
  - record $\{\bar{d}_n\}$ for all customers downstream these locations



Recap

1. For each customer, either directly meter max demand $\tilde{d}_n$, or use linear regression from customer's total demand (monthly bills) $\bar{d}_n$

2. Sum per-customer maxima to find

$$\hat{d}^N = \sum_{n=1}^{N} \bar{d}_n$$

3. Use experimentally estimated DF_N to find max diversified demand

$$\bar{d}^N = \frac{\hat{d}^N}{\mathrm{DF}_N}$$

We will next see three methods to allocate loads and calculate voltage drops

1) Load allocation using diversity factors

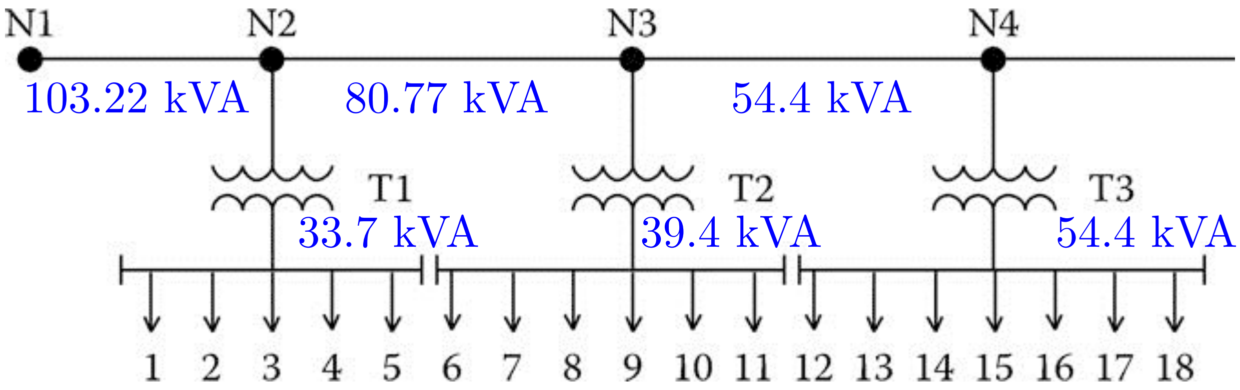
Example 2.1: Analyze a single-phase lateral

Given: a) monthly energy usage per customer; assume 0.9 PF lagging

b) load survey shows that $\bar{d}_n = 0.2 + 0.008 \cdot \text{monthly energy bill [kWh]}$

c) diversity factors (see graph two slides earlier)

Wanted: max diversified demand for each transformer and line segment



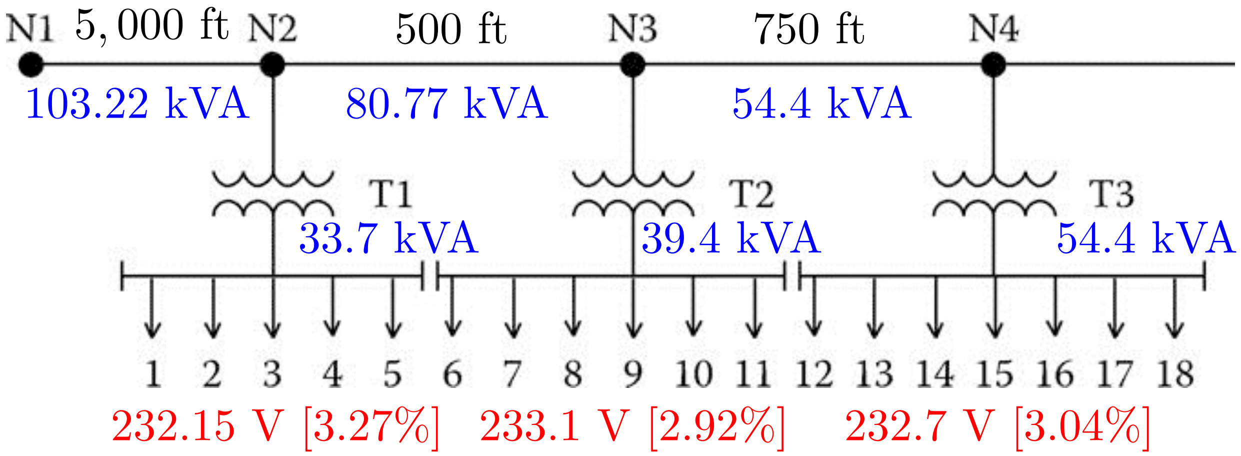
	Customer	#1	#2	#3	#4	#5
$\tilde{d}_n$	kWh	1523	1645	1984	1590	1456
$\bar{d}_n$	kW	12.4	13.4	16.1	12.9	11.9

Transformer ratings:
T1: 25 kVA
T2: 37.5 kVA
T3: 50 kVA

KCL load does not apply with max aggregated (real or apparent) powers!

Voltage drop using diversity factors

Example: Find voltage at each transformer assuming V_{N1}=2.4 kV



1 mile = 5,280 ft

$$z_{\text{line}} = 0.3 + j0.6\ \Omega/\text{mile}$$

$$\text{T1} : 25\ \text{kVA}, 2400 - 240\ \text{V},\ \ Z = 1.8\angle 40^\circ\%$$

$$\text{T2} : 37.5\ \text{kVA}, 2400 - 240\ \text{V},\ \ Z = 1.9\angle 45^\circ\%$$

$$\text{T3} : 50\ \text{kVA}, 2400 - 240\ \text{V},\ \ Z = 2.0\angle 50^\circ\%$$

2) Transformer load management

- The method of diversity factors requires knowing

  d1) DF_N's for different N

  d2) customer maxima; and

  d3) customer assignment to transformers

- *Transformer load management* is a simpler but less accurate method
  - relies on the fact that transformers are metered in greater detail

- *Training stage*
  - Collect historical data from transformers
  - Fit a linear regression model matching monthly energy served to peak demand

- *Operational stage*
  - Knowing *d3)* and customer bills, find total energy to be served
  - Using the trained model, predict max demand

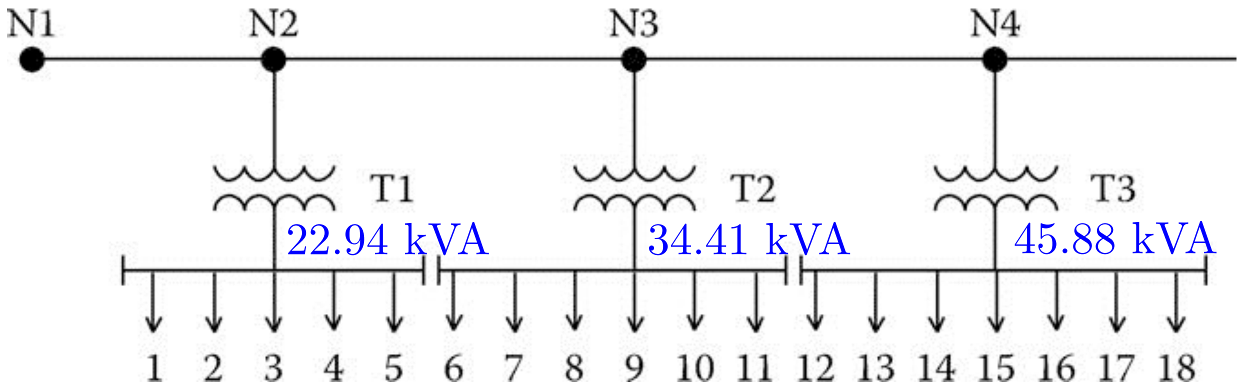
3) Allocation factors

Distribute maximum feeder demand on transformers based on their rating

$$\bar{d}_{\mathrm{T}_k} = \bar{d}_{\text{feeder}} \times \frac{\mathrm{kVA}_{T_k}}{\sum_m \mathrm{kVA}_{T_m}}$$

allocation factor

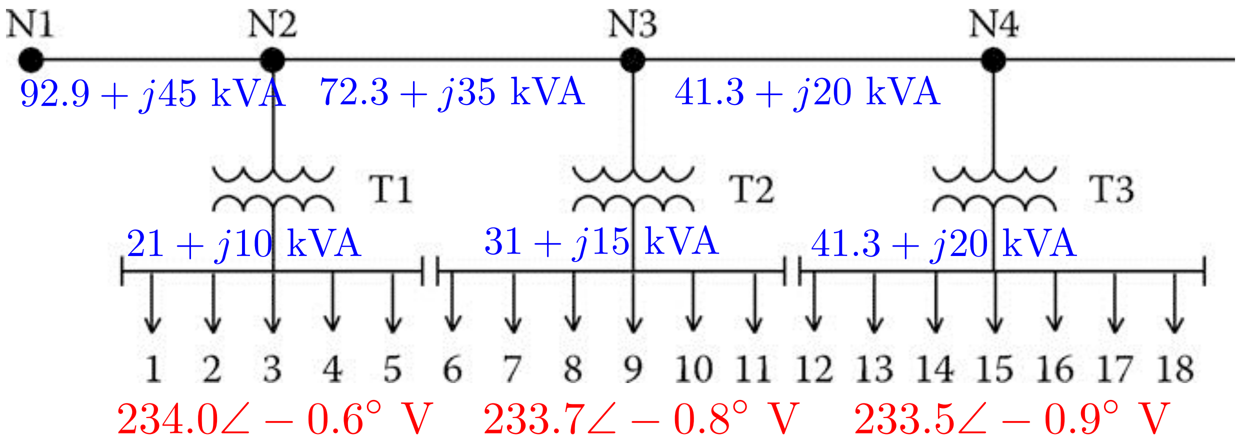
Example: Re-allocate load assuming maximum demand at feeder is 92.9 kW



Easier to implement, but good only for voltage drop calculations; since it assumes load has been allocated to XFMs properly

Voltage drop using allocation factors

Example: Find XFM voltages using allocation factors

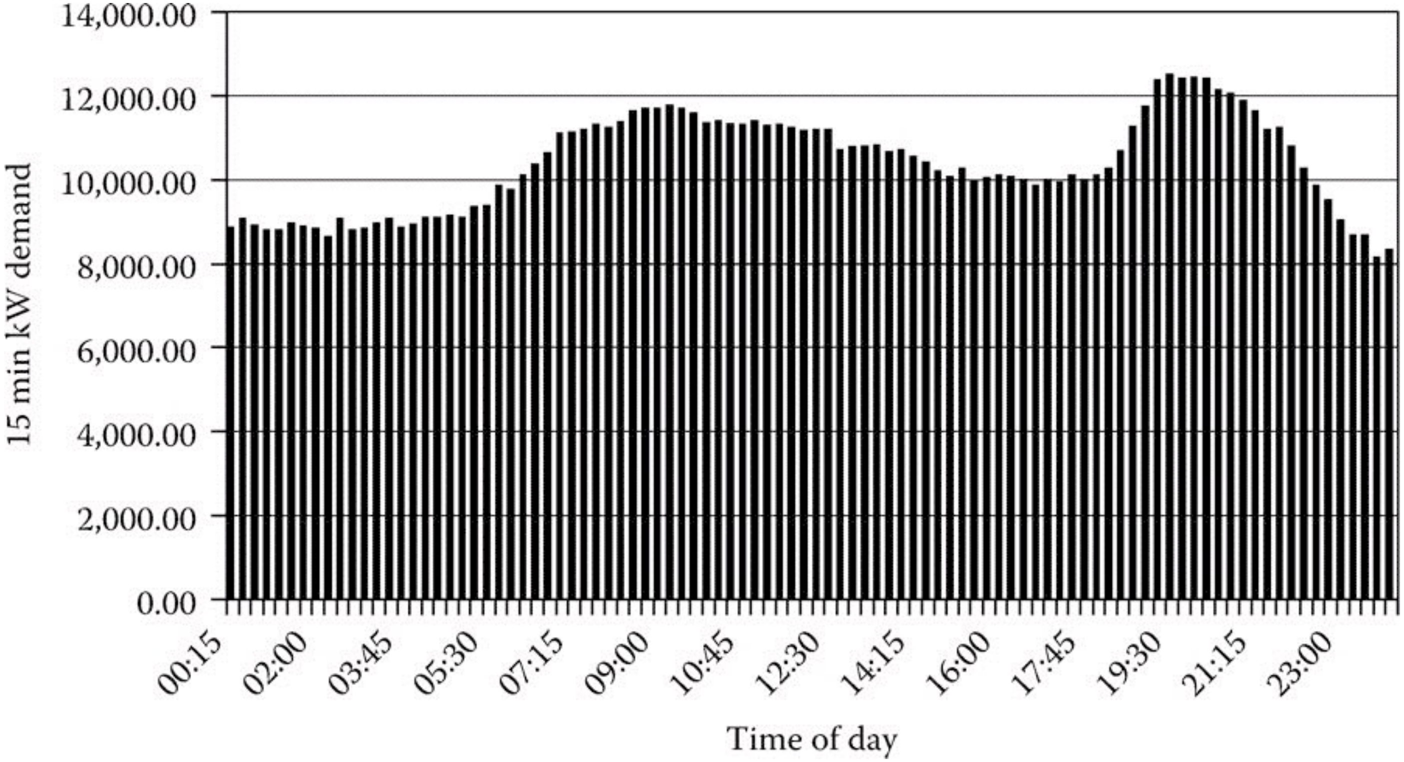


- Due to *linearity* of allocation factors, KCL now holds for line segments
- Results obtained are similar; this method may be less conservative
- Note *small voltage angles*

Does this method yield the actual voltages assuming demands are correct?

Feeder load

- Smoother demand curve
- Define statistics similar to transformers (maximum diversified demand)



Summary

- Allocate load to size transformer and study voltage drops
- *Method 1*: diversity factors $\tilde{d}_n \quad \xrightarrow[\text{regression}]{\text{linear}} \quad \bar{d}_n \quad \longrightarrow \quad \hat{d}_{T_k} = \sum_{n \in T_k} \bar{d}_n \quad \xrightarrow{DF_N} \quad \bar{d}_{T_k}$
- *Method 2*: transformer load management $\tilde{d}_n \quad \longrightarrow \quad \sum_{n \in T_k} \tilde{d}_n \xrightarrow[\text{regression}]{\text{linear}} \quad \bar{d}_{T_k}$
- *Method 3*: allocation factors $\bar{d}_{T_k} = \frac{\mathrm{kVA}_{T_k}}{\sum_m \mathrm{kVA}_{T_m}} \bar{d}_{\text{feeder}}$
- Requirements for every method
- M3 features linearity in flows; none of the methods considers losses