ECE 5984: Power Distribution System Analysis

Lecture 15: Center-Tapped Transformers and Secondaries

Reference: Textbook, Chapter 11

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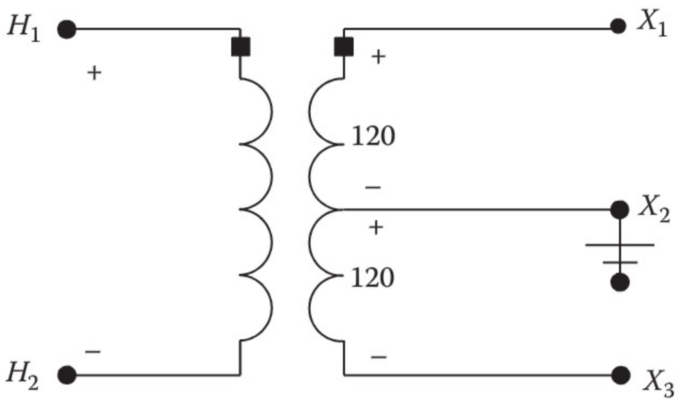
VirginiaTech
Invent the Future

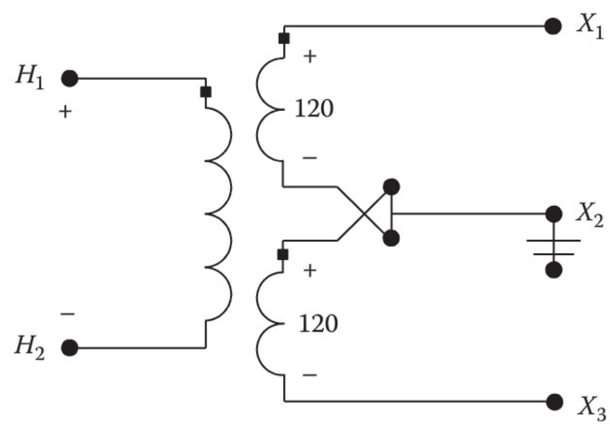
Center-tapped and three-winding transformers

terminology: 240/120 V



center-tapped (two-winding) transformer

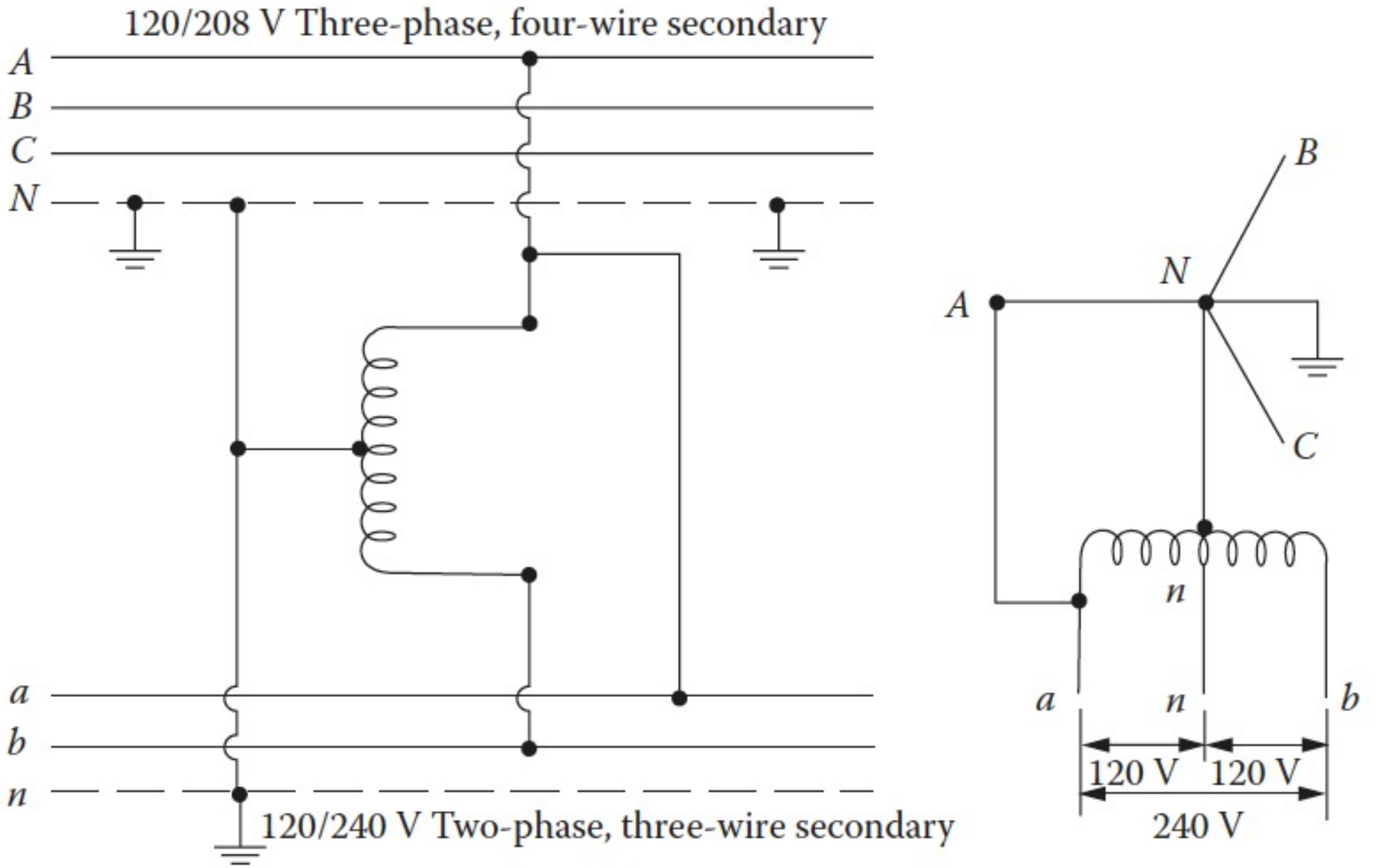
terminology: 120/240 V



three-winding transformer

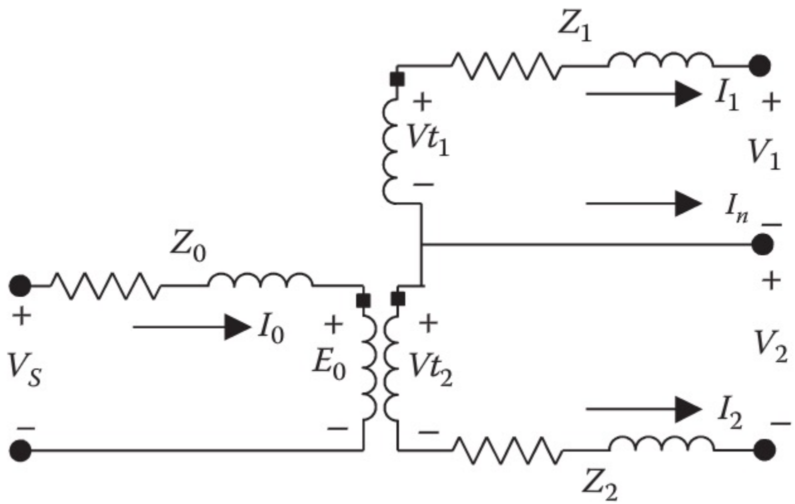
- Either known as *split-phase*
- Oven, washing machine, dryer connected on 240V; else on 120V
- Secondary windings in 120/240 can be connected in series or parallel

Autotransformer implementation



[Gonen: Electric Power Distribution Engineering]

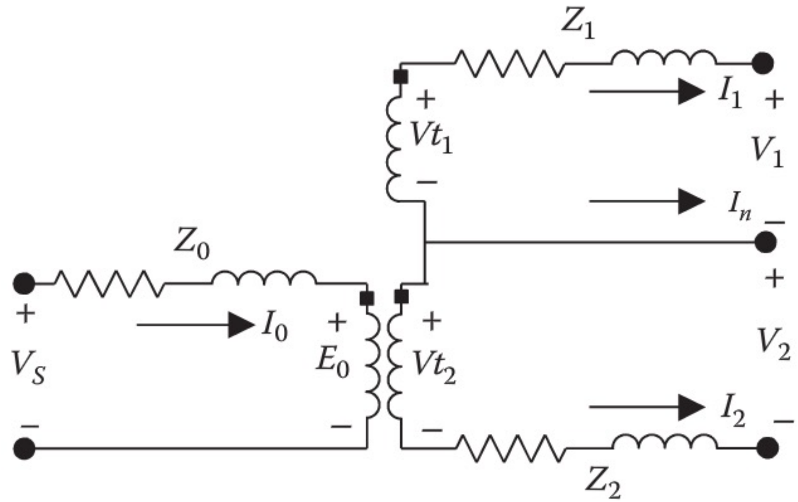
Center-tapped transformer model



$$n_t = \frac{\text{primary voltage}}{\text{secondary (full) voltage}}$$

- Backward model $\quad \mathbf{D} = \frac{1}{2n_t}[+1 \ \ -1]$
- Forward model $\quad \mathbf{E} = \frac{1}{2n_t}\mathbf{1}, \qquad \mathbf{F} = \begin{bmatrix} Z_1 + \frac{Z_0}{4n_t^2} & -\frac{Z_0}{4n_t^2} \\ +\frac{Z_0}{4n_t^2} & -\left(Z_2 + \frac{Z_0}{4n_t^2}\right) \end{bmatrix}$

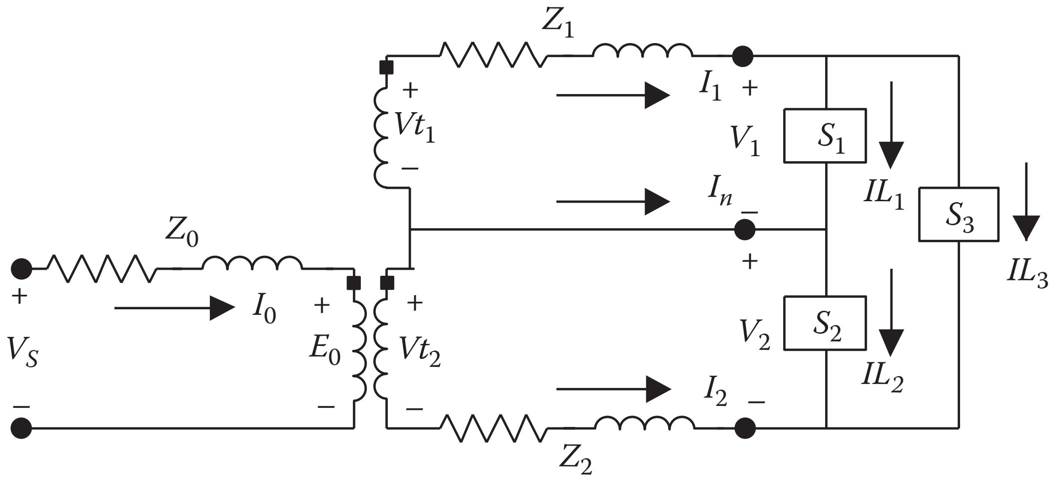
Finding impedances



- Typically given the per-unit transformer impedance; found via a short-circuit test
- It is empirically partitioned across windings as

$$Z_0 = 0.5R_M + j0.8X_M \text{ pu}$$
$$Z_1 = Z_2 = R_M + j0.4X_M \text{ pu}$$

Example



- Constant-power loads

$S_1 = 10\,\text{kVA at 95\% lagging}$
$S_2 = 15\,\text{kVA at 90\% lagging}$
$S_3 = 25\,\text{kVA at 85\% lagging}$

Source voltage: $7200\angle 0\,\text{V}$

- 50 kVA, 7200-240/120 V center-tapped transformer $R_M + jX_M = 0.011 + j0.018$ pu

- Split impedances

$$Zpu_0 = 0.5 \cdot R_A + j0.8 \cdot X_A = 0.0055 + j0.0144 \text{ pu}$$
$$Zpu_1 = R_A + j0.4 \cdot X_A = 0.011 + j0.0072 \text{ pu}$$

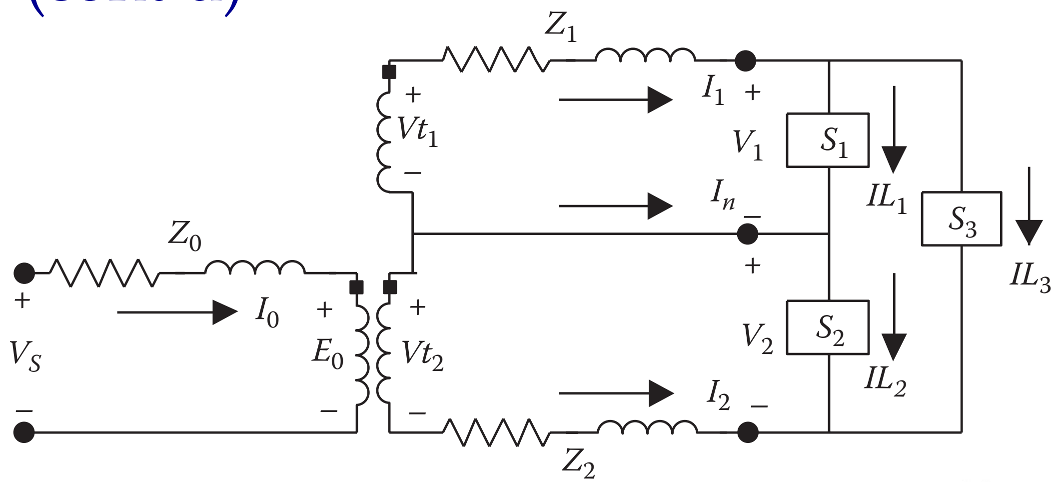
- Convert to ohms

$$Z_{\text{base,hi}} = \frac{7,200^2}{50,000} = 1,036.8$$
$$Z_{\text{base,lo}} = \frac{120^2}{50,000} = 0.288$$

$$Z_0 = Zpu_0 \cdot Zbase_{hi} = 5.7024 + j14.9299\,\Omega$$
$$Z_1 = Z_2 = Zpu_1 \cdot Zbase_{lo} = 0.0032 + j0.0024\,\Omega$$

- Construct matrices using above and turns ratio $n_t = \dfrac{7,200}{240} = 30$

Example (cont'd)



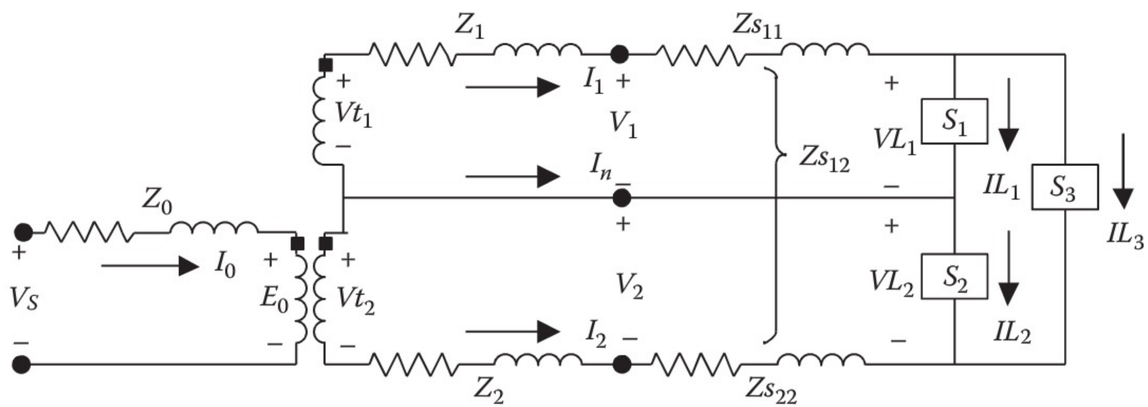
- Initialize load voltages (unloaded system) $[V_{ld}] = \begin{bmatrix} V_1 \\ V_2 \\ V_1 + V_2 \end{bmatrix} = \begin{bmatrix} 120\underline{/0} \\ 120\underline{/0} \\ 240\underline{/0} \end{bmatrix}$

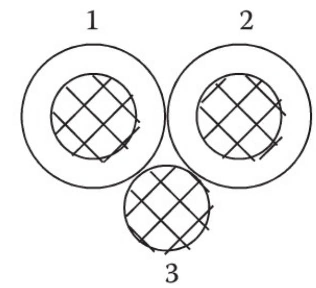
- Update load currents $Id_i = \left(\frac{SL_i \cdot 1000}{Vld_i} \right)^* = \begin{bmatrix} 83.3\underline{/-18.2} \\ 125.0\underline{/-25.8} \\ 104.2\underline{/-31.8} \end{bmatrix}$

- Update secondary line currents $[I_{12}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} IL_1 \\ IL_2 \\ IL_3 \end{bmatrix} = \begin{bmatrix} 186.2\underline{/-25.8} \\ 228.9\underline{/151.5} \end{bmatrix}$

- Forward/backward sweep converged within four iterations $[V_{ld}] = \begin{bmatrix} 117.88\underline{/-0.64} \\ 117.71\underline{/-0.63} \\ 235.60\underline{/-0.64} \end{bmatrix}$

Triplex secondary





- Two insulated phase conductors and one uninsulated neutral conductor

- Apply Carson's equations to get a 3x3 primitive impedance matrix

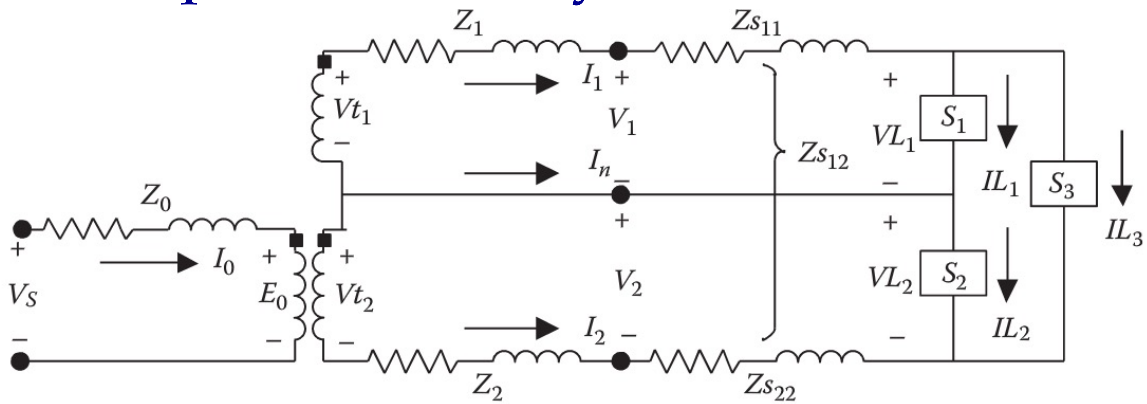
$$D_{12} = D + 2T \qquad D : \text{conductor diameter}$$
$$D_{13} = D_{23} = D + T \qquad T : \text{insulation thickness}$$

- Kron reduction (grounded neutral) to get a 2x2 phase impedance matrix **Z**

- Backward/forward updates $\quad \mathbf{v}_n := \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \ \mathbf{v}_m := \begin{bmatrix} V_{L1} \\ V_{L2} \end{bmatrix}, \ V_{L3} = V_{L1} + V_{L2}$

$$\mathbf{i}_n = \mathbf{i}_m$$
$$\mathbf{v}_m = \mathbf{v}_n - \mathbf{Z}\mathbf{i}_m$$

Example with triplex secondary



- Phase impedance matrix of triplex $[Zs] = \begin{bmatrix} 0.0271 + j0.0146 & 0.0087 + j0.0081 \\ 0.0087 + j0.0081 & 0.0271 + j0.0146 \end{bmatrix} \Omega$

- Secondary voltages $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 117.89\angle -0.64 \\ 117.75\angle -0.62 \end{bmatrix}$

- Load voltages $\begin{bmatrix} VL_1 \\ VL_2 \\ VL_3 \end{bmatrix} = \begin{bmatrix} 114.63\angle -0.45 \\ 122.58\angle -0.96 \\ 237.21\angle -0.72 \end{bmatrix}$

voltage rise!

- Line currents $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 190.2\angle -26.2 \\ 227.5\angle 150.6 \end{bmatrix}$

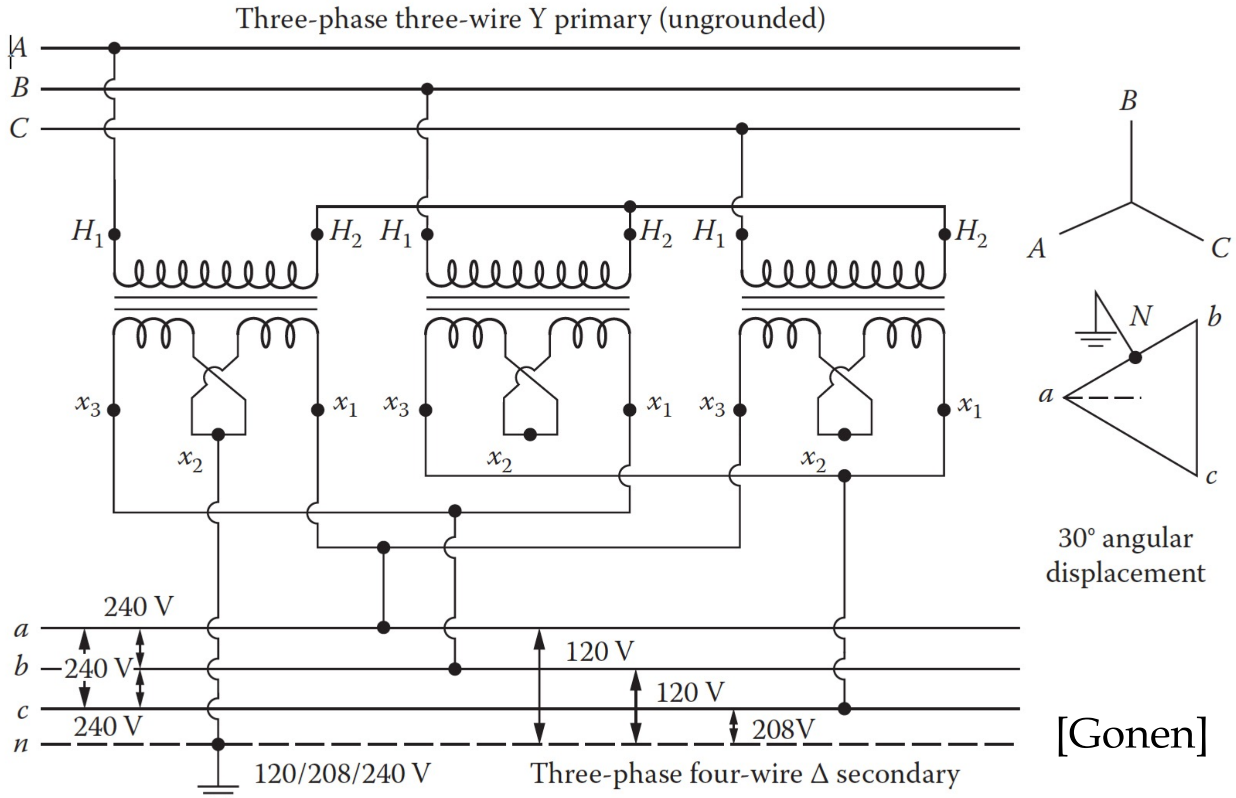
- Primary current $I_0 = 6.98\angle -28.0$

- Neutral current $I_n = [t_n] \cdot [I_{12}] = 25.4\angle -15.0$

- Ground current $I_g = -(I_n + I_1 + I_2) = 20.8\angle -84.5$

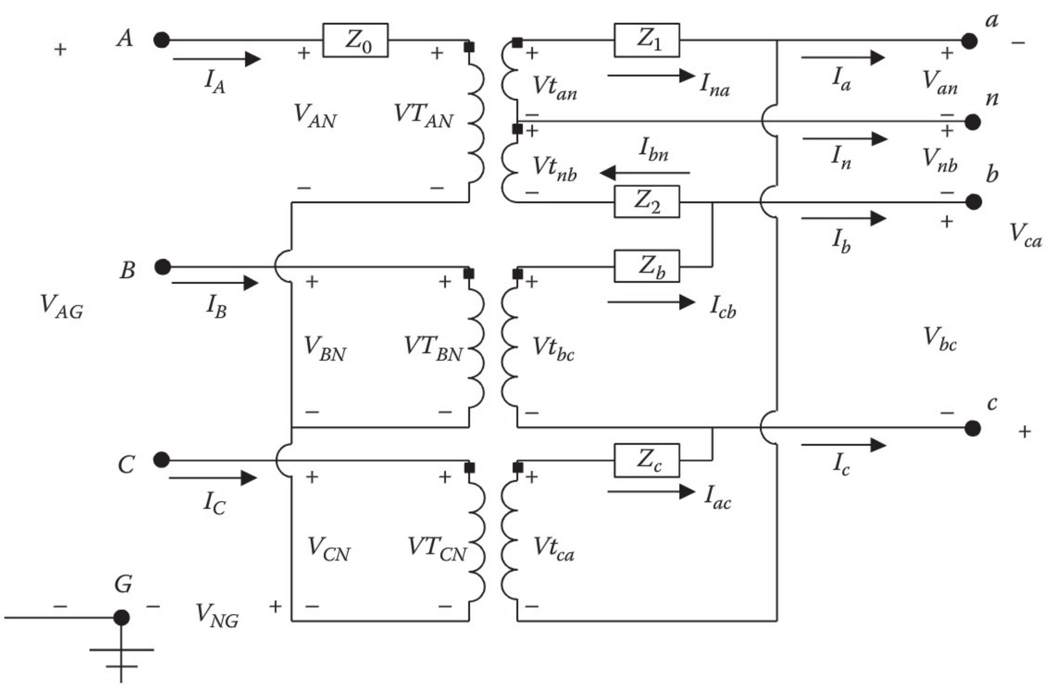
So far, considered 3W 1-phase service. How about 4W 3-phase service?

Wye-delta center-tapped transformer



- Common connection for combination of single- and three-phase loads
- One center-tapped *'lighting' transformer* for three-wire service to single-phase loads
- Two regular *'power' transformers* for three-phase loads

Wye-delta center-tapped transformer



- Use zero-sum primary currents to convert secondary delta to line currents

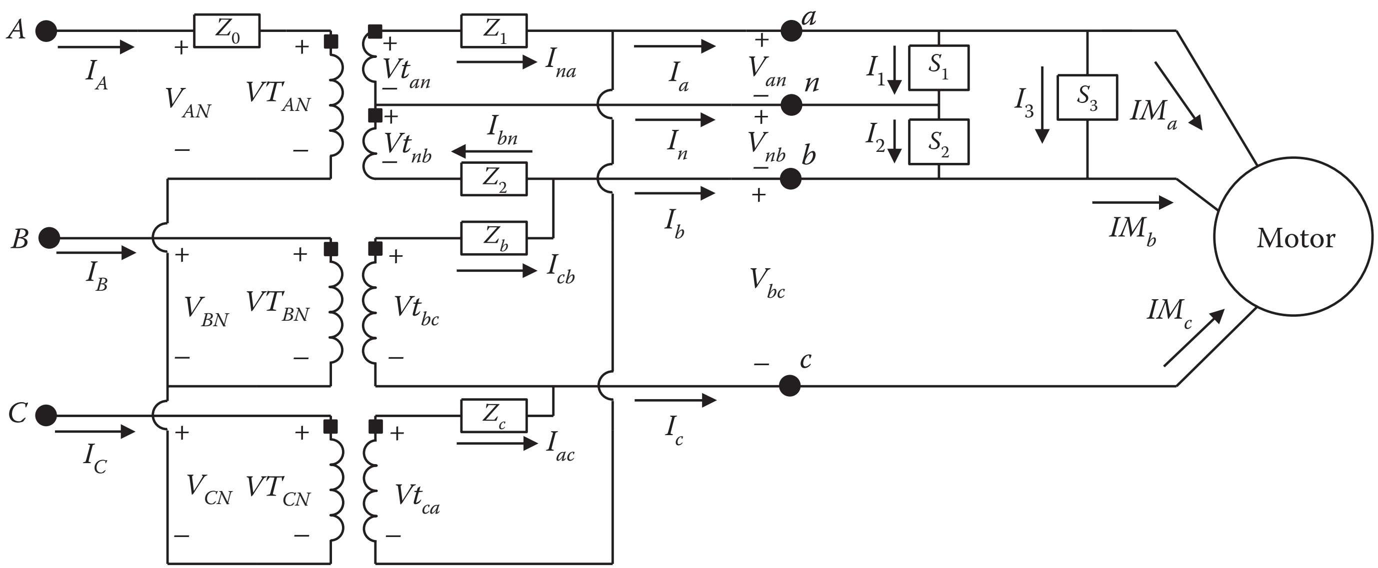
- Backward sweep $\mathbf{D} = \frac{1}{6n_t}\begin{bmatrix} 2 & -2 & 0 & 0 \\ -1 & 1 & -3 & 0 \\ -1 & 1 & 3 & 0 \end{bmatrix}$

neutral current can be ignored

- Forward sweep $\mathbf{E} = \frac{1}{2n_t}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{F} = \frac{1}{6}\begin{bmatrix} 5z_1 + \frac{z_0}{n_t^2} & z_1 - \frac{z_0}{n_t^2} & 3z_1 & 0 \\ -z_2 + \frac{z_0}{n_t^2} & -5z_2 - \frac{z_0}{n_t^2} & -3z_2 & 0 \\ -z_b & z_b & -3z_b & 0 \\ -z_c & z_c & 3z_c & 0 \end{bmatrix}$

$\tilde{\mathbf{v}}_m = \mathbf{E}\mathbf{v}_n - \mathbf{F}\mathbf{i}_m$

Example with wye-delta center-tapped transformer



single-phase loads

$SL_1 = 3\,\text{kVA}, 120\,\text{V}, 0.95$ lagging power factor

$SL_2 = 5\,\text{kVA}, 120\,\text{V}, 0.90$ lagging power factor

$SL_3 = 8\,\text{kVA}, 240\,\text{V}, 0.85$ lagging power factor

induction motor

$Zs = 0.0774 + j0.1843\,\Omega$

$Zr = 0.0908 + j0.1843\,\Omega$

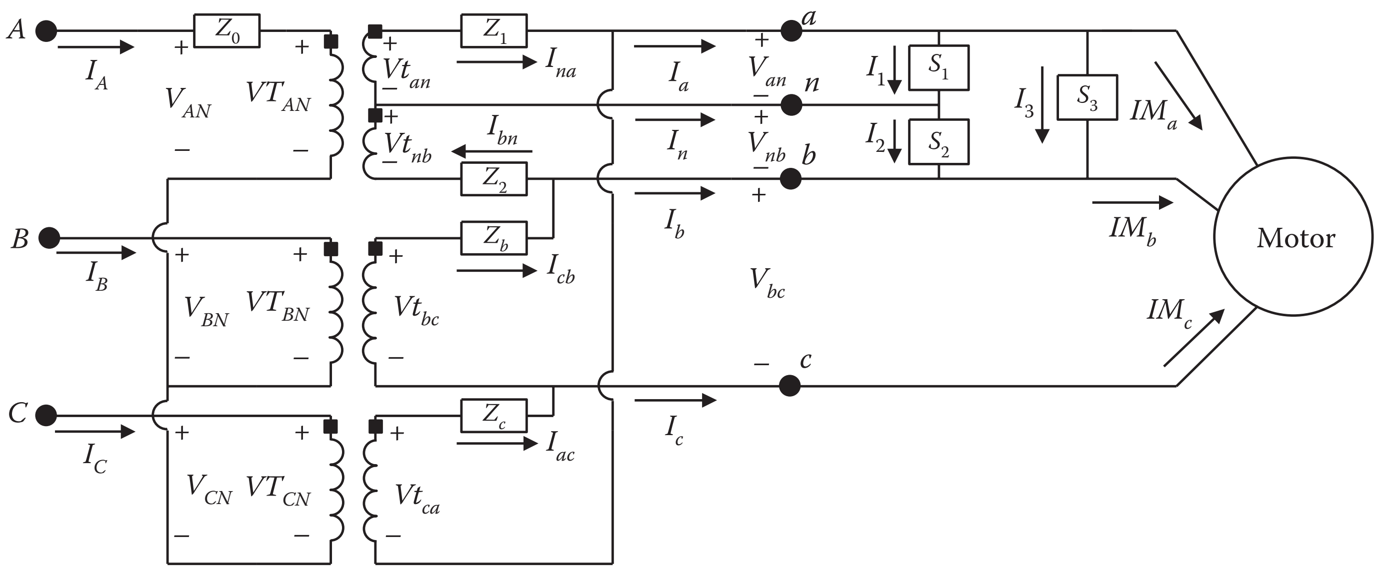
$Zm = 0 + j4.8385\,\Omega$

slip $= 0.035$

transformers and source

Lighting transformer: 25 kVA, 7200–240/120 V, $ZL_{pu} = 0.012 + j0.017$
Power transformers: 10 kVA, 7200–240 V, $ZP_{pu} = 0.016 + j0.014$
Source voltage: Balanced line-to-neutral 7200 V

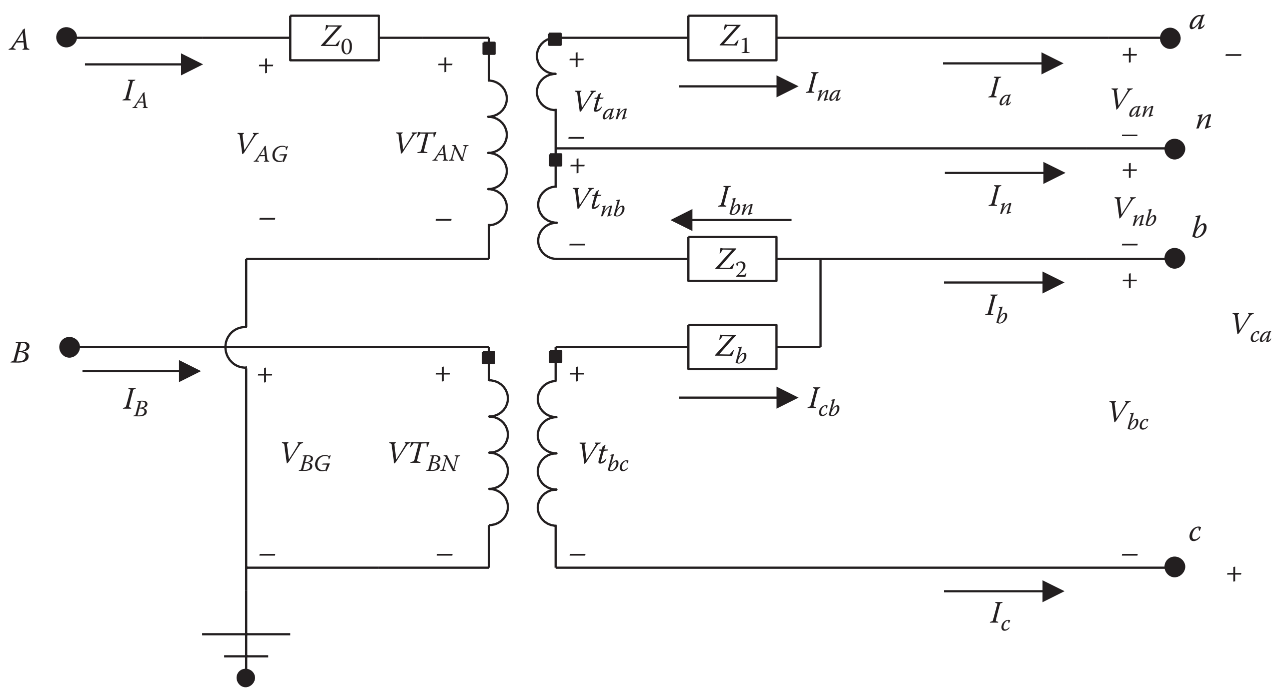
Example with wye-delta center-tapped transformer (cont'd)



motor

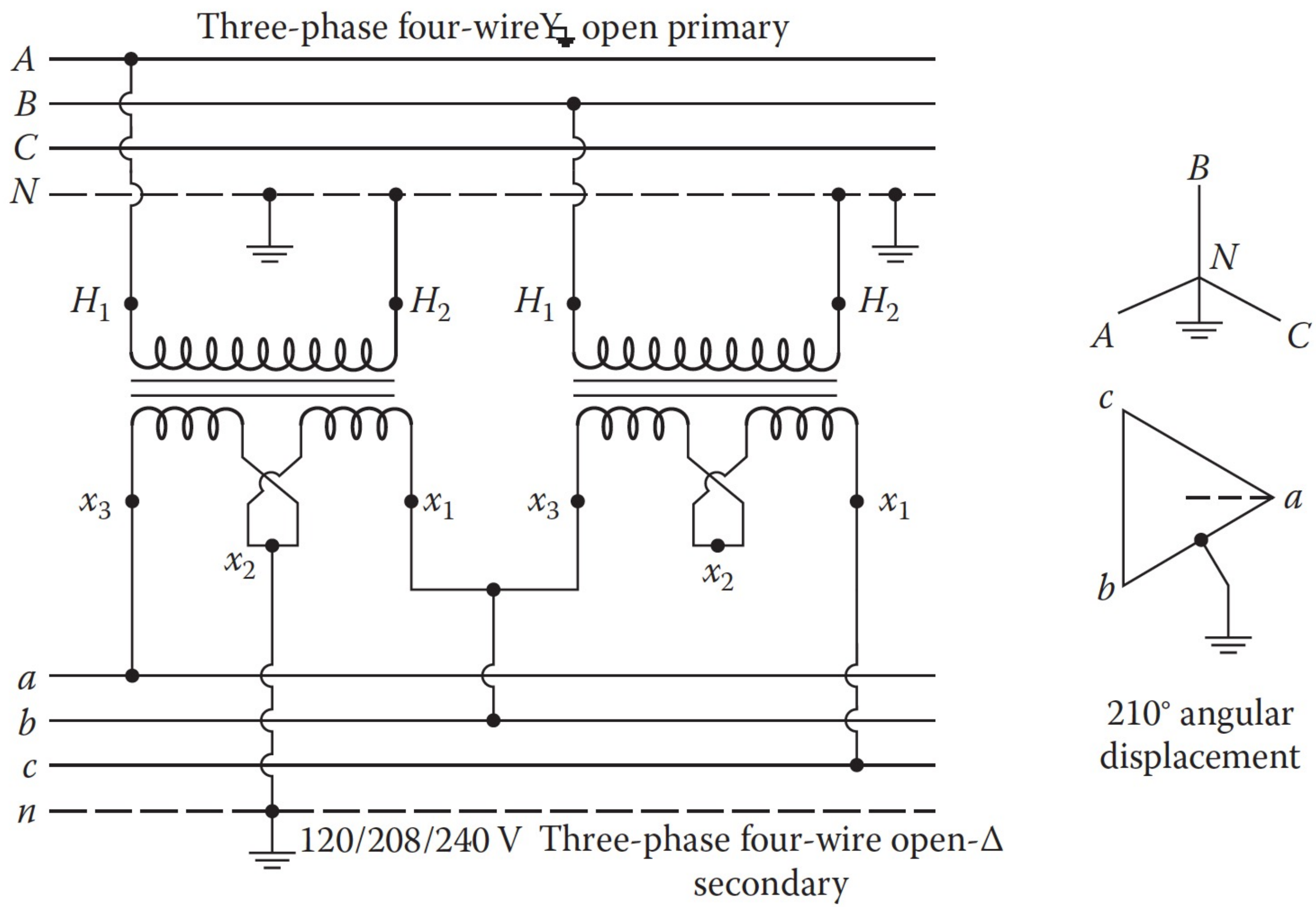
$$[VM] = \begin{bmatrix} 234.7\underline{/-0.39} \\ 235.1\underline{/-120.1} \\ 236.1\underline{/119.7} \end{bmatrix} \qquad V_{unbalance} = 0.3382\% \qquad [I_M] = \begin{bmatrix} 56.3\underline{/-65.6} \\ 56.1\underline{/176.6} \\ 58.1\underline{/54.6} \end{bmatrix}$$

Open Wye – open Delta



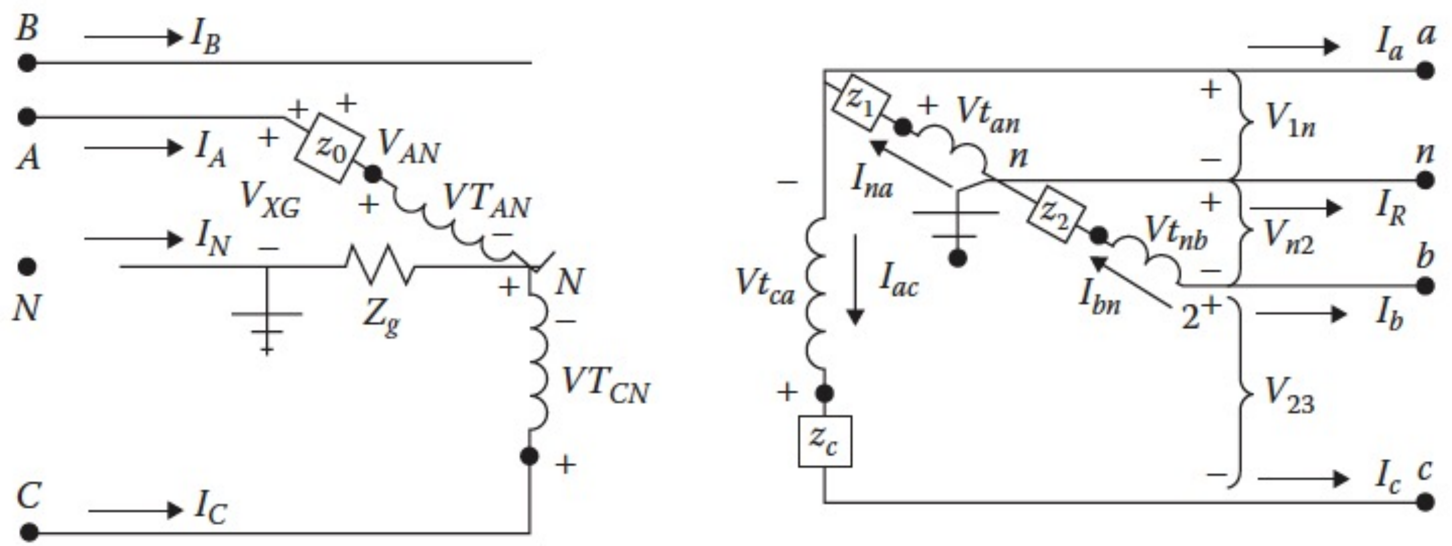
- Two sending currents and two sending voltages
- Four receiving currents and three receiving voltages
- *Leading*: lighting transformer on A; power transformer on B
- *Lagging*: lighting transformer on A; power transformer on C

Leading open Wye – open Delta



- Leading because lighting (resp. power) transformer is connected on A (resp. B)

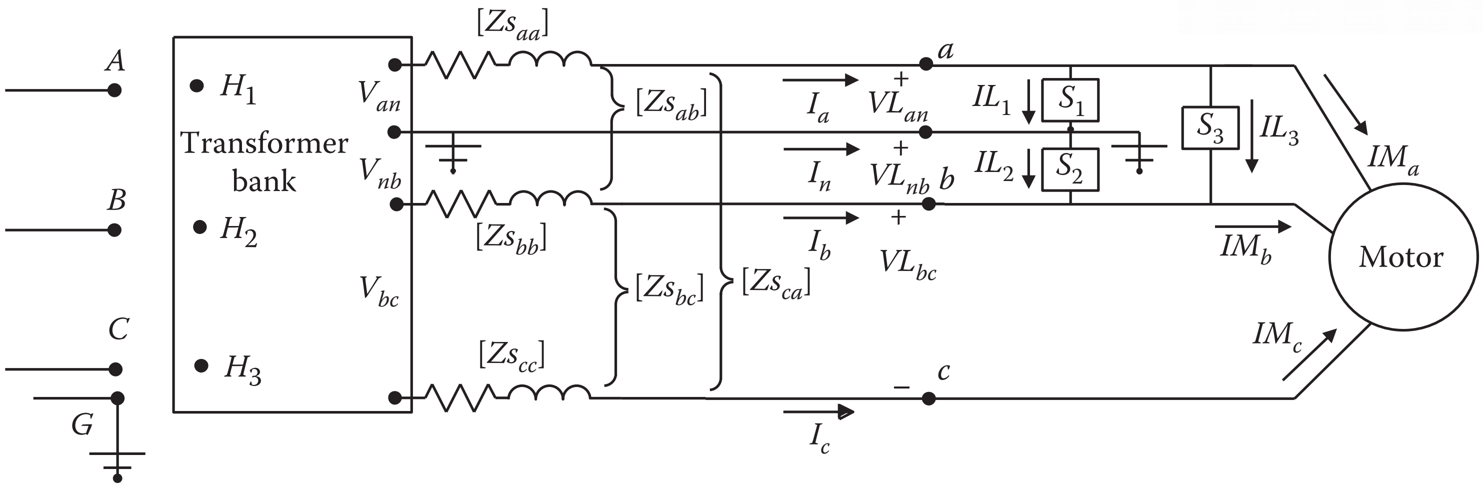
Lagging open Wye – open Delta



- No trick in deriving the CDEF model; similar model for leading connections
- Backward model $$\mathbf{D} = \frac{1}{2n_t}\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$
- Forward model

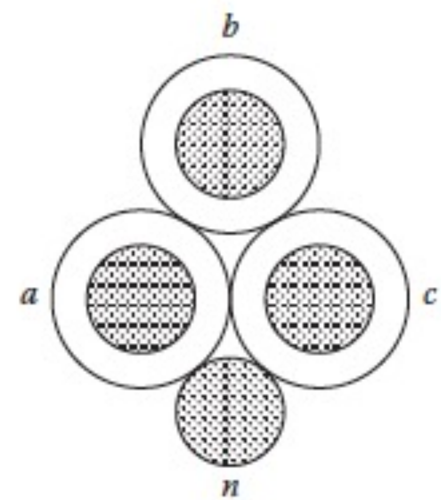
$$\mathbf{E} = \frac{1}{2n_t}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{F} = \begin{bmatrix} z_1 + \frac{z_0+z_g}{4n_t^2} & -\frac{z_0+z_g}{4n_t^2} & z_1 + \frac{z_0+3z_g}{4n_t^2} & 0 \\ \frac{z_0+z_g}{4n_t^2} & -z_2 - \frac{z_0+z_g}{4n_t^2} & \frac{z_0+3z_g}{4n_t^2} & 0 \\ \frac{z_g}{2n_t^2} & -\frac{z_g}{2n_t^2} & z_c + \frac{3z_g}{2n_t^2} & 0 \end{bmatrix}$$

Quadruplex secondary

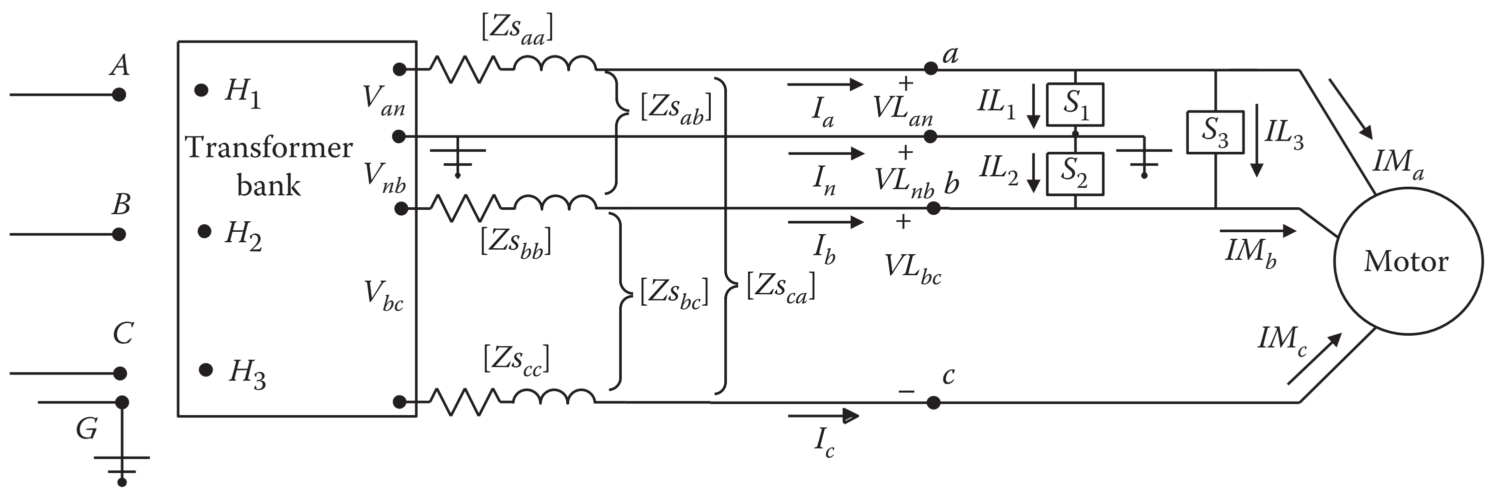


- Kron reduction on primitive (4x4) to get the 3x3 phase impedance matrix **Z**

$$\begin{bmatrix} \tilde{v}_a \\ \tilde{v}_b \\ \tilde{v}_c \end{bmatrix} = \begin{bmatrix} v_{an} - v_{L,an} \\ v_{bn} - v_{L,bn} \\ v_{cn} - v_{L,cn} \end{bmatrix} = \mathbf{Z} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$



Incorporating line drop



- Relate transformer-load voltage drops to line voltage drops

$$\underbrace{\begin{bmatrix} v_{an} \\ v_{nb} \\ v_{bc} \\ v_{ca} \end{bmatrix}}_{\mathbf{v}_n} - \underbrace{\begin{bmatrix} v_{L,an} \\ v_{L,nb} \\ v_{L,bc} \\ v_{L,ca} \end{bmatrix}}_{\mathbf{v}_m} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{v}_a \\ \tilde{v}_b \\ \tilde{v}_c \end{bmatrix}$$

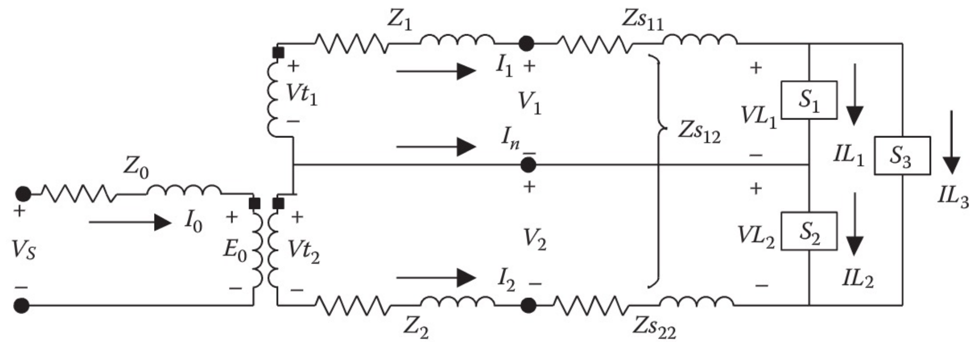
- Forward update

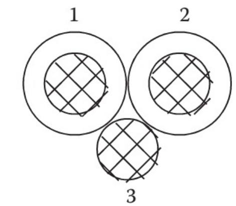
$$\mathbf{v}_m = \mathbf{v}_n - \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ -z_{ab} & -z_{bb} & -z_{bc} \\ z_{ab} - z_{ac} & z_{bb} - z_{bc} & z_{bc} - z_{cc} \\ z_{ac} - z_{aa} & z_{bc} - z_{ab} & z_{cc} - z_{ac} \end{bmatrix} \mathbf{i}_m$$

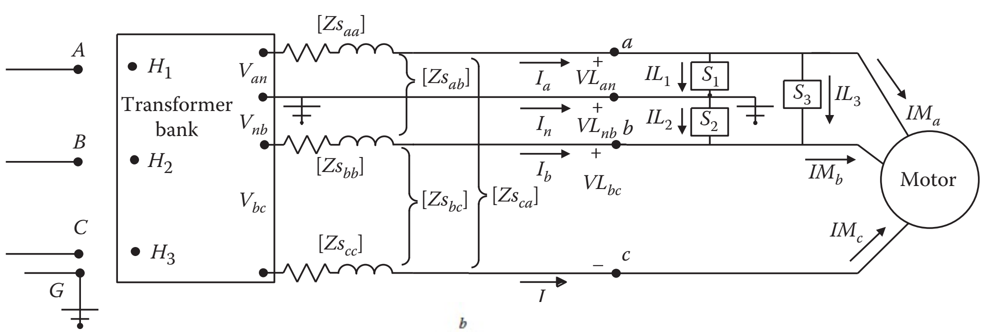
Summary

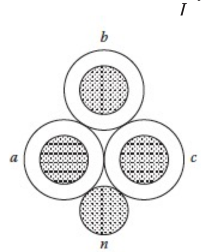
3W single-phase service

- split-phase transformer
- triplex cable









4W three-phase service

- Wye-Delta transformer
- open Wye-open delta transformer
- quadruplex cable