ECE 5984: Power Distribution System Analysis

Lecture 14: Mixed-Integer Optimization Models for Distribution Grids

Reference: see publications list at the end

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Invent the Future

Outline

1) Motivation

2) McCormick linearization

3) Inverter curve modeling

4) Topology reconfiguration and restoration

5) Enforcing connectivity and radiality

6) Topology identification via probing

Motivation

- Inverter and capacitor control for loss minimization

$$
\begin{aligned}
\min_{\mathbf{q}_g,\gamma_n} \quad & \mathbf{q}^\top \mathbf{R}\mathbf{q} \\
\text{s.to} \quad & \mathbf{q} = \mathbf{q}_g + \mathbf{q}_d + q_c \mathbf{e}_n \\
& \mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + v_0 \mathbf{1} \\
& q_c = \gamma_n v_n b_c \\
& \underline{\mathbf{v}} \leq \mathbf{v} \leq \bar{\mathbf{v}}, \ \ \underline{\mathbf{q}}_g \leq \mathbf{q}_g \leq \bar{\mathbf{q}}_g \\
& \gamma_n \in \{0, 1\}
\end{aligned}
$$

switchable capacitor at bus n
with known susceptance b_c

- Technically, squared voltage magnitude, but can be linearized

- Binary variables needed for discrete decision-variables and *"if-else"* constraints

- *Other instances*: line switching, regulator taps, discrete load control

- Rule-of-thumb for better formulations: *avoid products of variables*

The big-M trick

- To avoid variable products, constrained can be reformulated *linearly*

$$q_c = \gamma_n v_n b_c \quad \Rightarrow \quad q_c = \begin{cases} v_n b_c & , \ \gamma_n = 1 \\ 0 & , \ \gamma_n = 0 \end{cases} \quad \Rightarrow \quad \begin{gathered} -M\gamma_n \leq q_c \leq M\gamma_n \\ -(1-\gamma_n)M \leq q_c - v_n b_c \leq (1-\gamma_n)M \end{gathered}$$

M is a ``large" constant, say 10^6

- Binary variable *selects* between the two cases

- Numerically, choice of M significantly affects performance

- Small M may destroy equivalence, large M slows solvers down

McCormick linearization

- For bounded continuous variables $x \in [\underline{x}, \overline{x}]$ and $y \in [\underline{y}, \overline{y}]$, if $z = xy$, then

$$(\overline{x} - x)(\overline{y} - y) \geq 0$$
$$(x - \underline{x})(y - \underline{y}) \geq 0$$
$$(\overline{x} - x)(y - \underline{y}) \geq 0$$
$$(x - \underline{x})(\overline{y} - y) \geq 0$$

~~$z = xy$~~ *relaxation*

$$z \geq \overline{x}y + x\overline{y} - \overline{x}\overline{y}$$
$$z \geq \underline{x}y + x\underline{y} - \underline{x}\underline{y}$$
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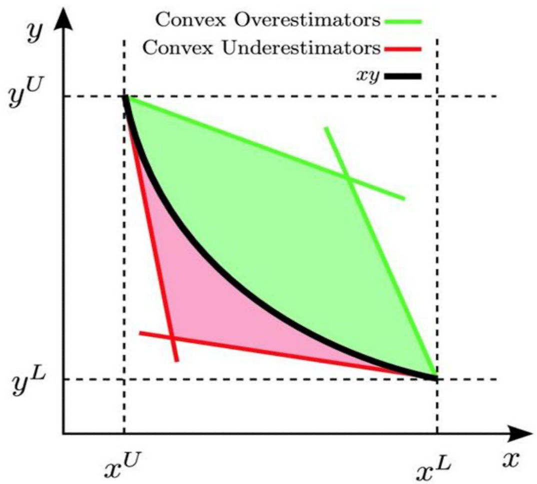


McCormick envelopes [1]

- The relaxation is *exact* if (at least) one variable, say x is binary $(\underline{x} = 0, \overline{x} = 1)$

$$x\underline{y} \leq z \leq x\overline{y}$$
$$y + (x-1)\overline{y} \leq z \leq y + (x-1)\underline{y}$$

- Compare to big-M?

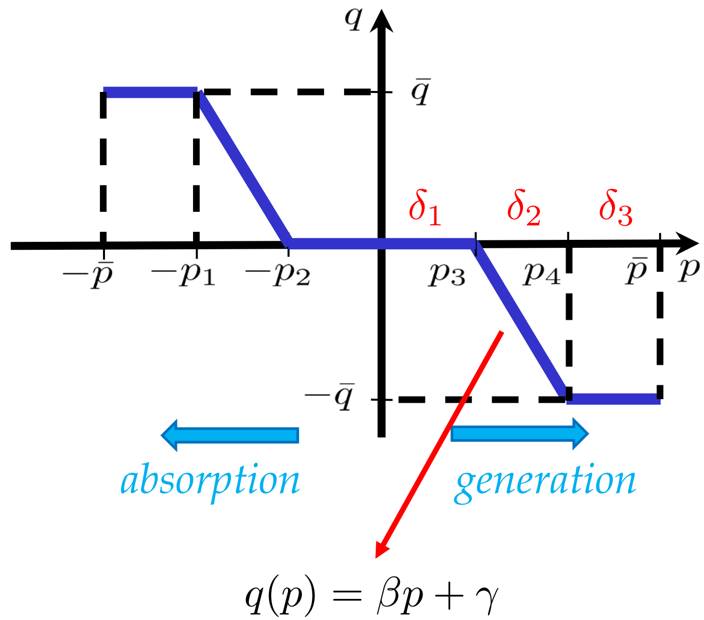
- Tight bounds yield superior numerical performance

Inverter curve modeling

- *Goal*: include inverter Watt/VAR curve in a mixed-integer linear formulation

- For simplicity, consider only the right half-plane

$$q(p) = \max\left\{-\bar{q}, \min\left\{0, \frac{-\bar{q}}{p_4 - p_3}(p - p_3)\right\}\right\}$$



- Design parameters could be the breakpoints, or equivalently the slopes and intercepts

- Using slopes and intercepts [2]

$$-\delta_2\bar{q} - \delta_3 M \leq \beta p + \gamma \leq \delta_1 M - \delta_3\bar{q}$$
$$q = \delta_2(\beta p + \gamma) - \delta_3\bar{q}$$
$$\delta_1 + \delta_2 + \delta_3 = 1$$
$$(\delta_1, \delta_2, \delta_3) \in \{0, 1\}^3$$

Binary variables pick among the three regions.
Can one use fewer binary variables?

- Try using (p_1, p_2) as design variables

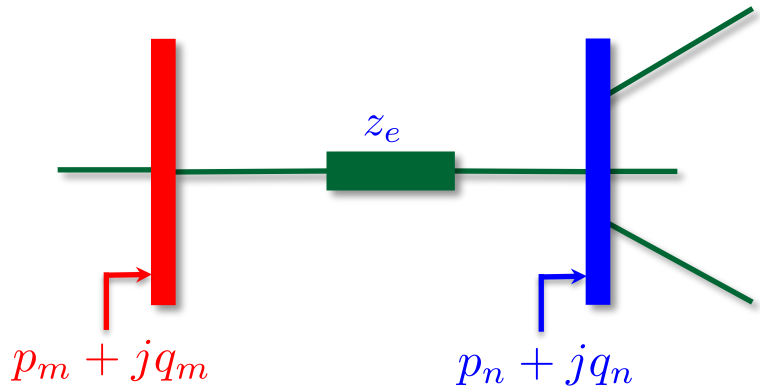
Topology reconfiguration

- Why reconfigure without an outage? Topology determines (**R**,**X**)

$$\|\mathbf{Rp} + \mathbf{Xq}\|_2^2 \quad \text{and} \quad \mathbf{p}^\top \mathbf{Rp} + \mathbf{q}^\top \mathbf{Xq}$$

voltage regulation *power loss reduction*

- Modify power flow model to include line status



$$y_e = \begin{cases} 1, & \text{line } e \text{ closed} \\ 0, & \text{otherwise} \end{cases}$$

$$y_e(\underline{P}_e + j\underline{Q}_e) \leq S_e \leq y_e\left(\bar{P}_e + j\bar{Q}_e\right)$$

- Relating injections to flows $\quad s_n = \sum_{e:(n,m)\in\mathcal{E}} S_e - \sum_{e:(m,n)\in\mathcal{E}} S_e$

- Approximate voltage drop $\quad y_e(u_m - u_n - 2\text{Re}\{z_e S_e^*\}) = 0$

variable products handled exactly
via McCormick linearization

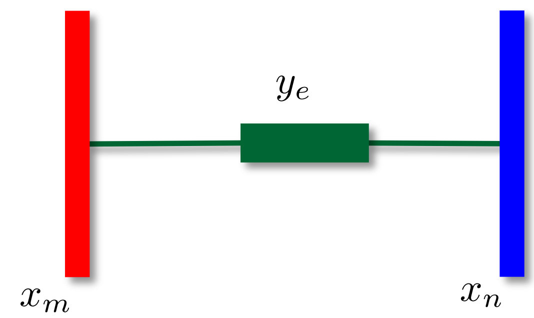
Distribution system restoration

- Some buses and lines become unavailable. So? need extra bus constraints

voltage and injection are zero
for de-energized buses

$$x_n \underline{u} \leq u_n \leq x_n \overline{u}$$
$$x_n(\underline{p}_n + j\underline{q}_n) \leq s_n \leq x_n(\overline{p}_n + j\overline{q}_n)$$

$$x_n = \begin{cases} 1, & \text{bus } n \text{ energized} \\ 0, & \text{otherwise} \end{cases}$$



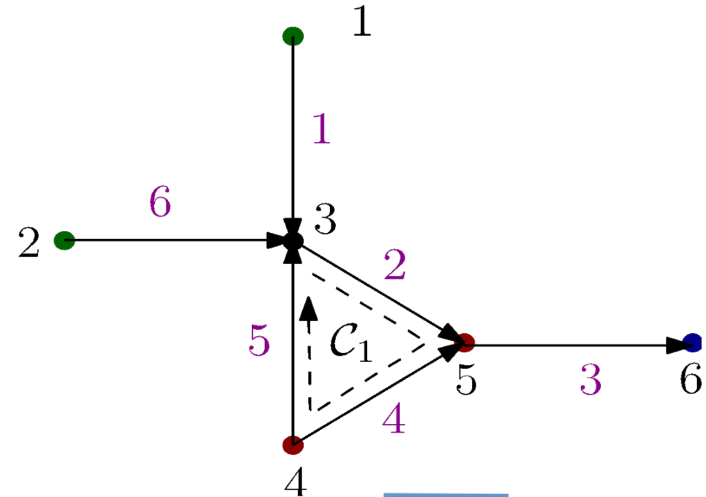
- Coupling between buses and line statuses

$$|x_m - x_n| \leq 1 - y_e, \quad \forall e : (m,n) \in E$$

- Other models to be added: voltage regulators, capacitors, (non)black-start generators

- Both reconfiguration and restoration problems seek *radial* topologies

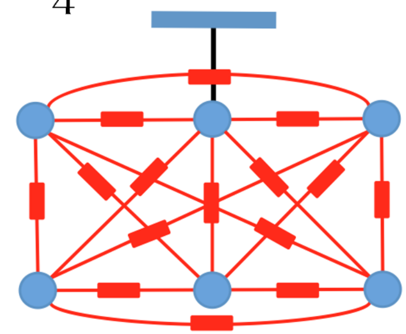
Enforcing radiality



- Cycle elimination

- Define cycle indicator vectors [3]

$$\mathbf{n}_{C_1} = [0\ 1\ 0\ 0\ 1\ 1\ 0]^\top$$
$$\mathbf{y}^\top \mathbf{n}_C \leq \mathbf{1}^\top \mathbf{n}_C - 1, \forall C$$



- Problem? Number of constraints may grow exponentially

- Radiality with one component = Connectivity + fix # of edges

commodity flow model

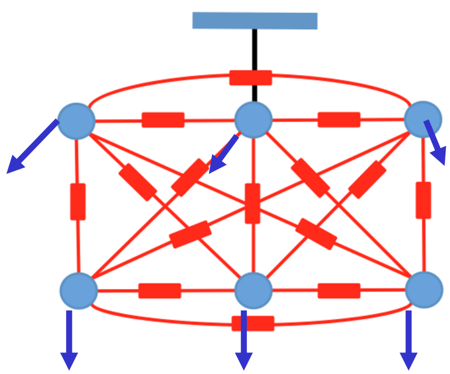
$$\mathbf{y}^\top \mathbf{1} = \mathbf{x}^\top \mathbf{1} - 1$$

Virtual commodity flow model

- Consider a *virtual* commodity flowing on the network graph
- Set a demand of one unit on all *non-substation* buses
- Enforce flow balance (KCL) on all nodes, allowing flows only on active lines [4]

$$\sum_{i \sim j} f_{ij} = 1, \ \forall \ j$$

$$-Ny_e \leq f_e \leq Ny_e, \ \forall \ e$$



- Consequences
  - all demands supplied by substation
  - all nodes have a path to substation, ensuring connectivity
- Say for restoration, one seeks a single connected island

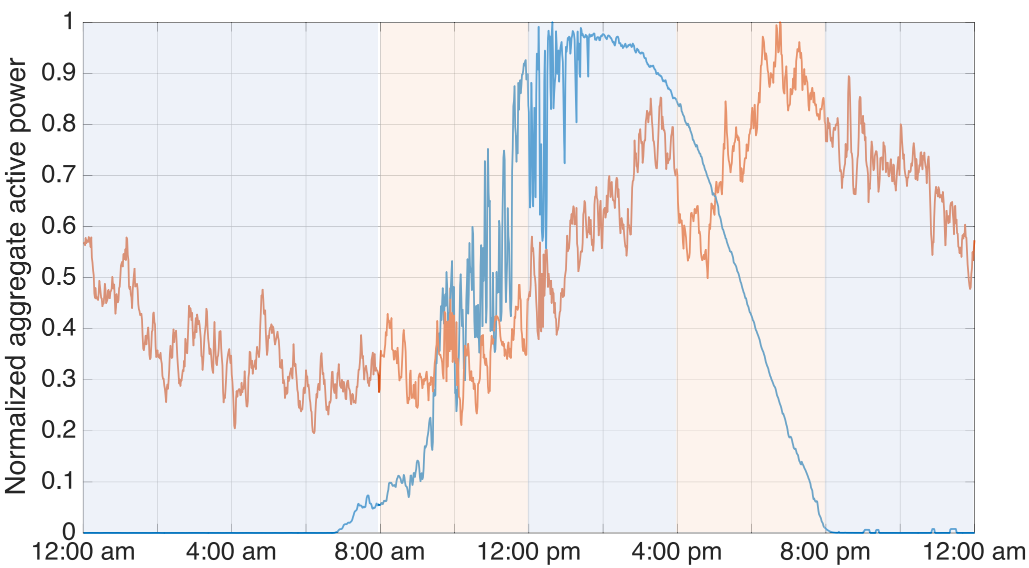
$$\sum_{i \sim j} f_{ij} = x_j, \ \forall \ j$$

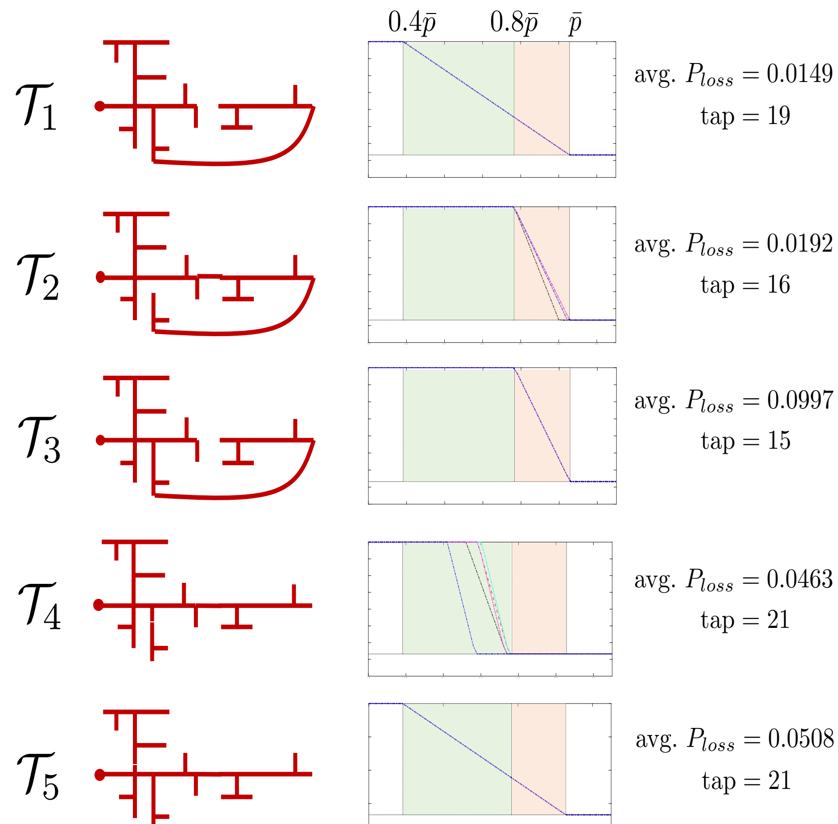
How to allow for multiple islands? [4]

Topology reconfiguration example

- Formulation variants: deterministic/stochastic, multi-timescale
- Optimize over slow (topology, inverter curves), and fast (voltage, power) variables [2]

$$\begin{aligned} \min \quad & \sum_{t\in\mathcal{T}} \sum_{e\in\mathcal{E}} r_e(P_{e,t}^2 + Q_{e,t}^2) \\ \text{over} \quad & \boldsymbol{\omega}_1, \{\boldsymbol{\omega}_2^t\}_{t=1}^{T} \\ \text{s.to} \quad & \text{power flow and topological constraints} \end{aligned}$$





System restoration formulation

- *Single-step*: Given post-outage statuses x^0 and y^0, maximize total load served [3]

$$\begin{aligned} \min \quad & \sum_{i \in \mathcal{N}_L} p_i + \lambda \mathbf{1}^\top |\mathbf{y} - \mathbf{y}^0| \\ \text{s.to} \quad & \text{power flow constraints, topological} \\ & \text{and coordination constraints}, \mathbf{x} \geq \mathbf{x}^0 \end{aligned}$$

- Regularization to promote topologies with fewer switching operations

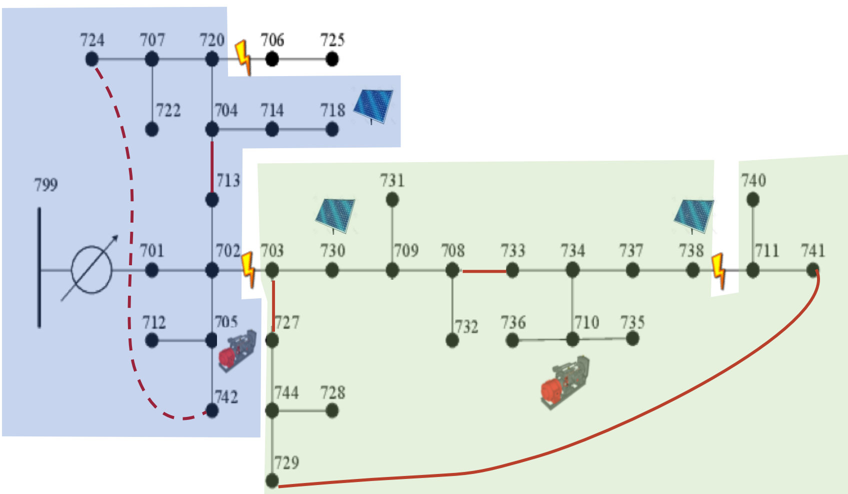
- *T-step*: Several additional constraints

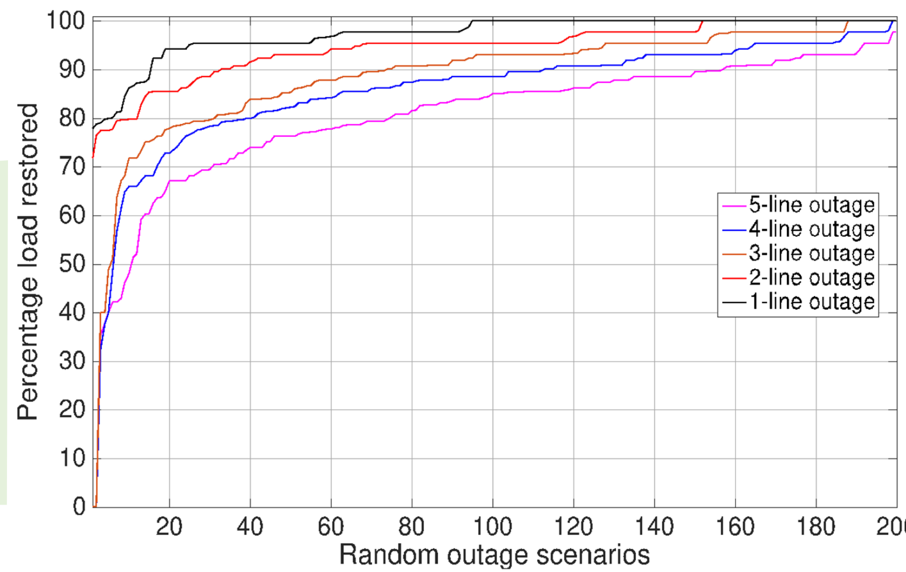
$$\min \quad \sum_{t=1}^{T} \sum_{i} \mu_i^t p_i + \lambda \mathbf{1}^\top |\mathbf{y}^t - \mathbf{y}^{t-1}|$$

  - radiality at the end
  - ramping
  - cold-load pick-up
  - irreversible actions

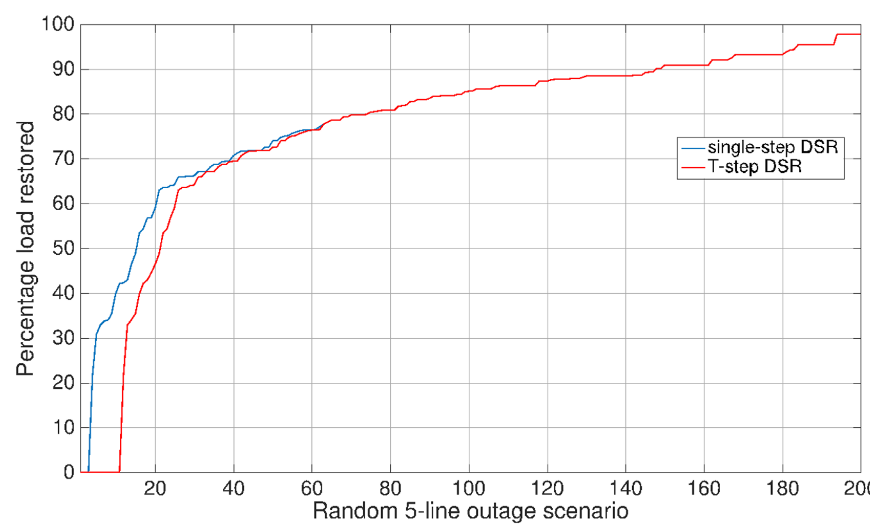
System restoration results

- *Single-step restoration*



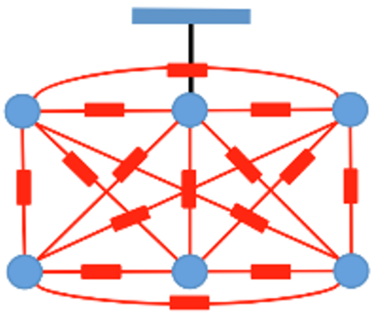


- *T-step restoration*

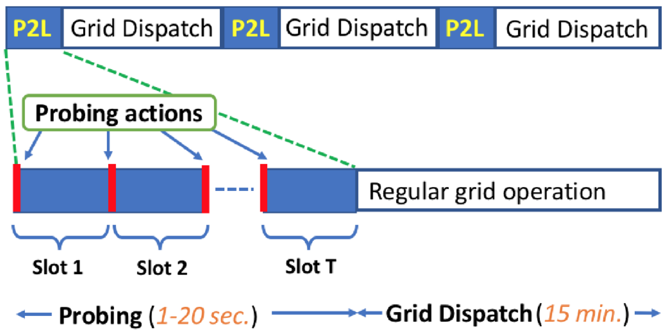


- Lesser load restored; *why?*

Topology identification



- Find out which lines are active
- Find the impedance of those lines $z = r + jx$
- How can we do this? recall *LDF* model $\mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + v_0\mathbf{1}$
  - Voltage drops are approximately *linearly* related to power injections
  - Given (**v**,**p**,**q**), can we find (**R**,**X**)?
- *Grid probing* with inverters [5]
  - perturb injections at probing buses [how?]
  - record voltage responses (magnitudes, phasors)
  - repeat over T probing actions spaced 1-2′ apart



- Probing can be done using other grid apparatus (capacitors, VR)

Identifiability analysis

- Rewrite LDF with inverted matrices and noisy differential quantities

$$\tilde{\mathbf{v}}_t = \mathbf{G}^{-1}\tilde{\mathbf{p}}_t + \mathbf{B}^{-1}\tilde{\mathbf{q}}_t + \mathbf{n}_t$$
$$\tilde{\mathbf{p}}_t = \mathbf{p}_t - \mathbf{p}_{t-1}$$

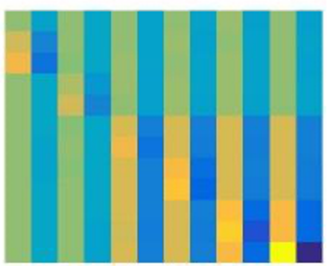
$\mathbf{n}_t$:
1. *modeling error*
2. *metering noise*
3. *unmodeled (load) variations*

- Probe the grid over T periods (only active power), collect quantities in matrices [6]

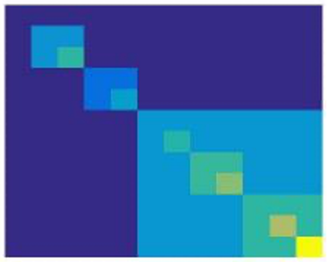
$$\tilde{\mathbf{v}}_t = \mathbf{G}^{-1}\mathbf{p}_t + \mathbf{n}_t \quad \Rightarrow \quad \mathbf{V} = \mathbf{G}^{-1}\mathbf{P} + \mathbf{N}$$

- Inverted data model $\mathbf{P} = \mathbf{G}\mathbf{V} + \mathbf{E}$

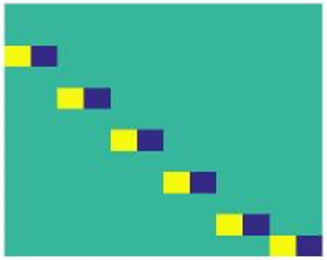
known ($\mathbf{P}$) known ($\mathbf{V}$)



$\mathbf{V}$



$\mathbf{G}^{-1}$



$\mathbf{P}$

If all leaf buses are probed and voltages are collected everywhere, the system is identifiable

References

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[2] M. K. Singh, V. Kekatos, S. Taheri, K. P. Schneider, and C.-C. Liu, "Enforcing radiality constraints for DER-aided power distribution grid reconfiguration," arXiv preprint 1910.03020, 2019.

[3] M. K. Singh, V. Kekatos, and C. Liu, "Optimal distribution system restoration with microgrids and distributed generators," in *Proc. IEEE PES General Meeting*, Atlanta, GA, Aug. 2019, pp. 1–5.

[4] S. Lei, C. Chen, Y. Song, and Y. Hou, "Radiality constraints for resilient reconfiguration of distribution systems: formulation and application to microgrid formation," *IEEE Trans. Smart Grid*, vol. 11, no. 5, pp. 3944– 3956, Sep. 2020.

[5] G. Cavraro and V. Kekatos, "Graph Algorithms for Topology Identification using Power Grid Probing," *IEEE Control Systems Letters*, vol. 2, no. 4, pp. 689-694, Oct. 2018.

[6] S. Taheri, V. Kekatos, and G. Cavraro 'An MILP Approach for Distribution Grid Topology Identification using Inverter Probing,' *in Proc. IEEE PowerTech*, Milan, Italy, Jun. 2019.