ECE 5984: Power Distribution System Analysis

Lecture 10: Distribution Feeder Analysis

Reference: Textbook, Chapter 10

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Power flow analysis

Known quantities

- Substation LN voltage phasor
- Complex power and voltage ratings, and ZIP percentages for all loads (gens)
- Detailed feeder model

Quantities to be determined

- LN voltage phasors at all nodes
- Line flows and power losses
- Actual power injections

Need for customized PF solvers

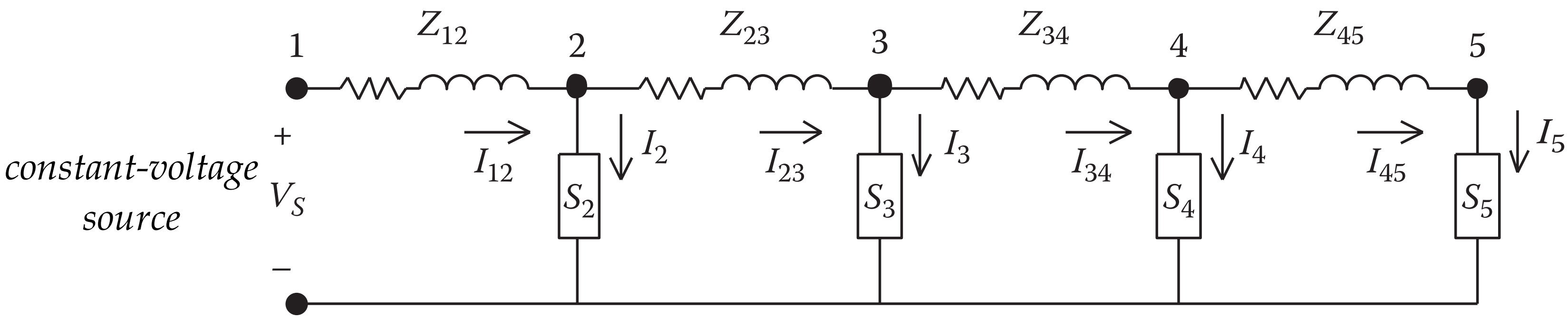
- Unbalanced loads and untransposed lines
- Convergence issues for standard (Newton-Raphson) solvers
- Approximations do not hold
- Exploit radial structure to simplify computations

Power flow solvers

- Giver a feeder and loads, we would like to find the voltages at all buses
- If we know the voltages, we can specify everything else
- Two-port networks relate *linearly* voltages and currents (sending/receiving)
- However, *load currents depend on load voltages* (unless load is an impedance)
- Nonlinear power flow equations are solved by iterative techniques
- *Forward-backward solver*: technique tailored to distribution grids (radial)

E.Bombard, E. Carpaneto, G. Chicco, R. Napoli, 'Convergence of the backward/forward sweep method for the load-flow analysis of radial distribution systems,' *Electrical Power and Energy Systems*, 2000.

Forward-backward solver



- Toy example on line feeder with constant-power loads

- *Backward sweep* updates currents given voltages

$$I_i^{(t+1)} = \left(\frac{S_i}{V_i^{(t)}} \right)^* \quad \text{[load]}$$

$$I_{i-1,i}^{(t+1)} = I_i^{(t+1)} + I_{i,i+1}^{(t+1)} \quad \text{[line]}$$

feeder end to feeder head

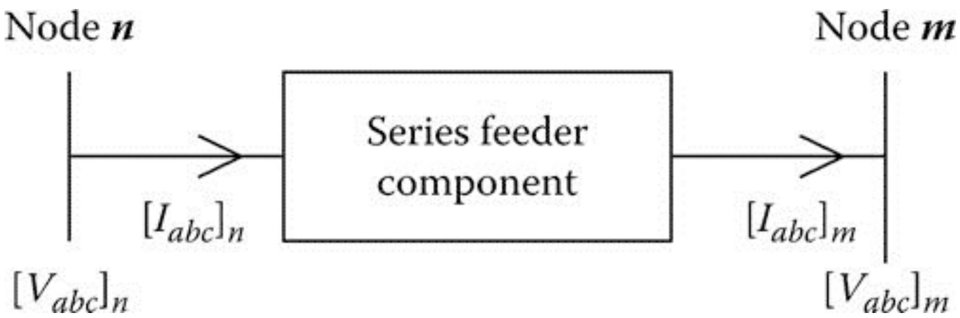
- *Forward sweep* updates voltages given currents

$$V_{i+1}^{(t+1)} = V_i^{(t+1)} + Z_{i,i+1} I_{i,i+1}^{(t+1)}$$

feeder head to feeder end

- Several iterations needed to converge; initialize voltages at nominal

Series components



- Distribution lines, regulators, transformers
- Backward sweep $\mathbf{i}_n = \mathbf{C}\mathbf{v}_m + \mathbf{D}\mathbf{i}_m$
- Forward sweep $\mathbf{v}_m = \mathbf{E}\mathbf{v}_n - \mathbf{F}\mathbf{i}_m$
- Voltages are *LN* ones
- Matrix $\mathbf{C} = \mathbf{0}$ except for
  - long underground lines (due to shunt admittance)
  - grounded Wye-Delta transformers

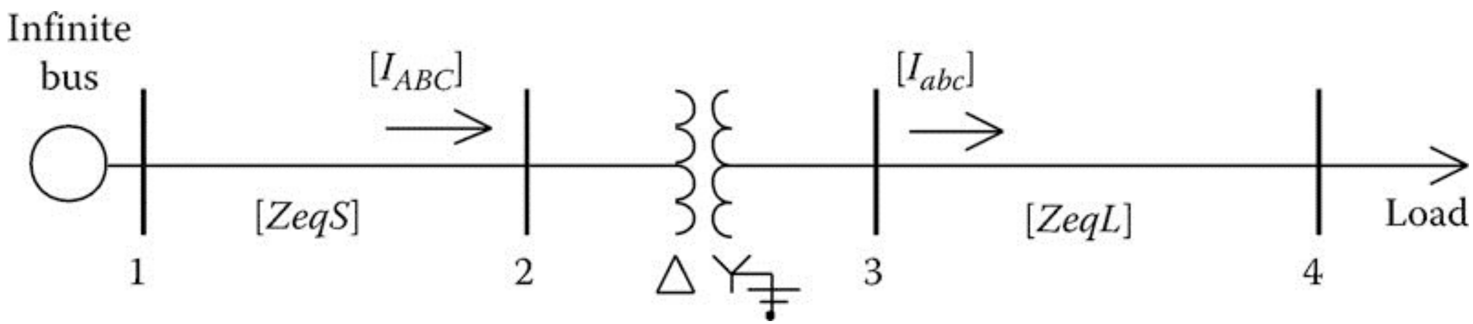
Shunt components

- Spot static loads:
  for ZIP loads, compute currents for each component separately

- Spot induction motors:
  constant-impedance (fixed speed/slip)
  constant-power: compute slip value first

- Capacitor banks:
  constant-impedance loads
  susceptance computed using rated voltage/VAR values

IEEE 4-bus test feeder



$$[ZeqS_{ABC}] = \begin{bmatrix} 0.1414 + j0.5353 & 0.0361 + j0.3225 & 0.0361 + j0.2752 \\ 0.0361 + j0.3225 & 0.1414 + j0.5353 & 0.0361 + j0.2955 \\ 0.0361 + j0.2752 & 0.0361 + j0.2955 & 0.1414 + j0.5353 \end{bmatrix} \Omega$$

single-phase transformer rating

2000 kVA, 12.47-2.4 kV, $Z = 1 + j6\%$ pu

$$[ZeqL_{abc}] = \begin{bmatrix} 0.1907 + j0.5035 & 0.0607 + j0.2302 & 0.0598 + j0.1751 \\ 0.0607 + j0.2302 & 0.1939 + j0.4885 & 0.0614 + j0.1931 \\ 0.0598 + j0.1751 & 0.0614 + j0.1931 & 0.1921 + j0.4970 \end{bmatrix} \Omega$$

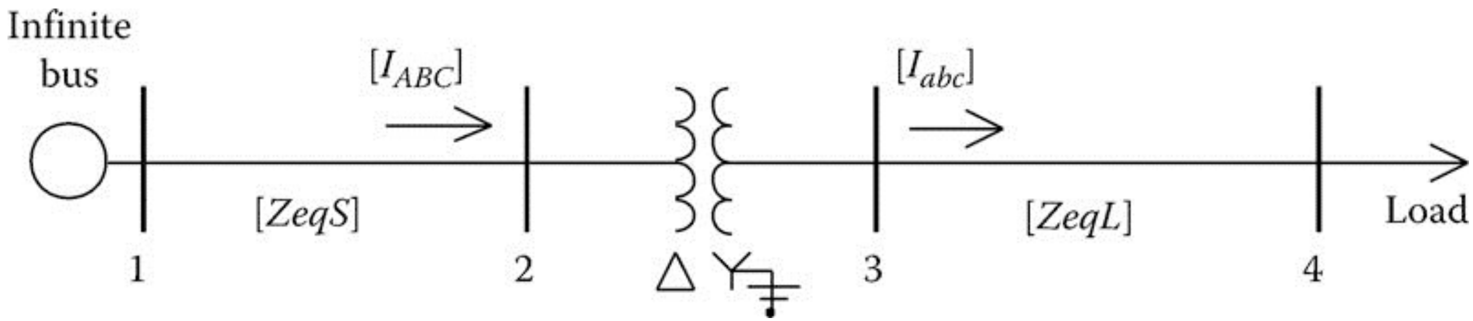
constant power load

$$[Zt_{abc}] = \begin{bmatrix} 0.0288 + j0.1728 & 0 & 0 \\ 0 & 0.0288 + j0.1728 & 0 \\ 0 & 0 & 0.0288 + j0.1728 \end{bmatrix} \Omega$$

$$\mathbf{s}_{\text{load}} = \begin{bmatrix} 750\angle 31.79^\circ \\ 1000\angle 25.84^\circ \\ 1250\angle 18.19^\circ \end{bmatrix} \text{kVA}$$

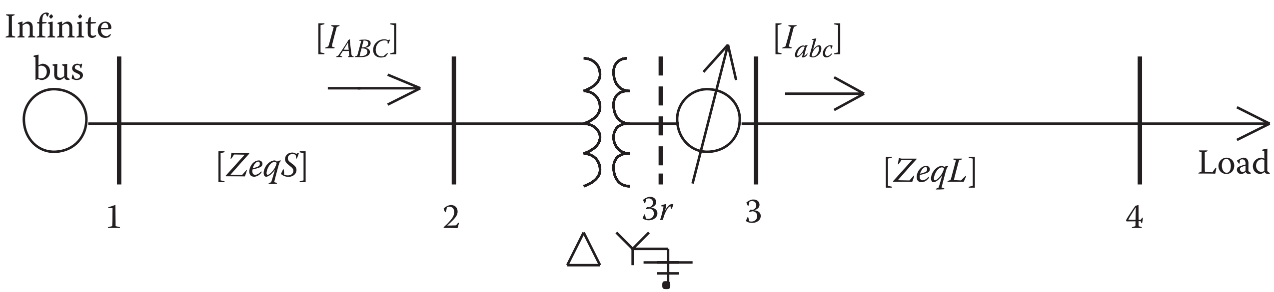
$$n_t = \frac{12.47}{2.4} = 5.19$$

IEEE 4-bus test feeder (cont'd)



- Line models $\mathbf{E} = \mathbf{I},\ \mathbf{F} = \mathbf{Z},\ \mathbf{D} = \mathbf{I}$
- Transformer model (step-down Delta-$\mathrm{Y_G}$) $\mathbf{E} = \frac{1}{n_t}\mathbf{D}_f^\top,\ \mathbf{F} = \mathrm{dg}(\mathbf{z}_t),\ \mathbf{D} = \frac{1}{n_t}\mathbf{D}_f$
- Source voltage $\mathbf{v}_1 = \begin{bmatrix} 7,199.6\angle 0^\circ \\ 7,199.6\angle -120^\circ \\ 7,199.6\angle +120^\circ \end{bmatrix}$
- Modified ladder updates converge within 8 iterations to voltage magnitudes (in 120 V scale) $\mathbf{v}_4^{120} = \begin{bmatrix} 113.9 \\ 110.0 \\ 110.6 \end{bmatrix}$

IEEE 4-bus test feeder (cont'd)



- Bank of 3 SVRs $\mathbf{E} = \mathrm{dg}^{-1}(\mathbf{a}_R),\ \mathbf{F} = \mathbf{0},\ \mathbf{D} = \mathrm{dg}^{-1}(\mathbf{a}_R)$
- LDC settings: $121\text{V},\ 2\text{V bandwidth},\ N_{\text{CT}} = \frac{1000}{5} = 200, N_{\text{PT}} = \frac{2400}{120} = 20$

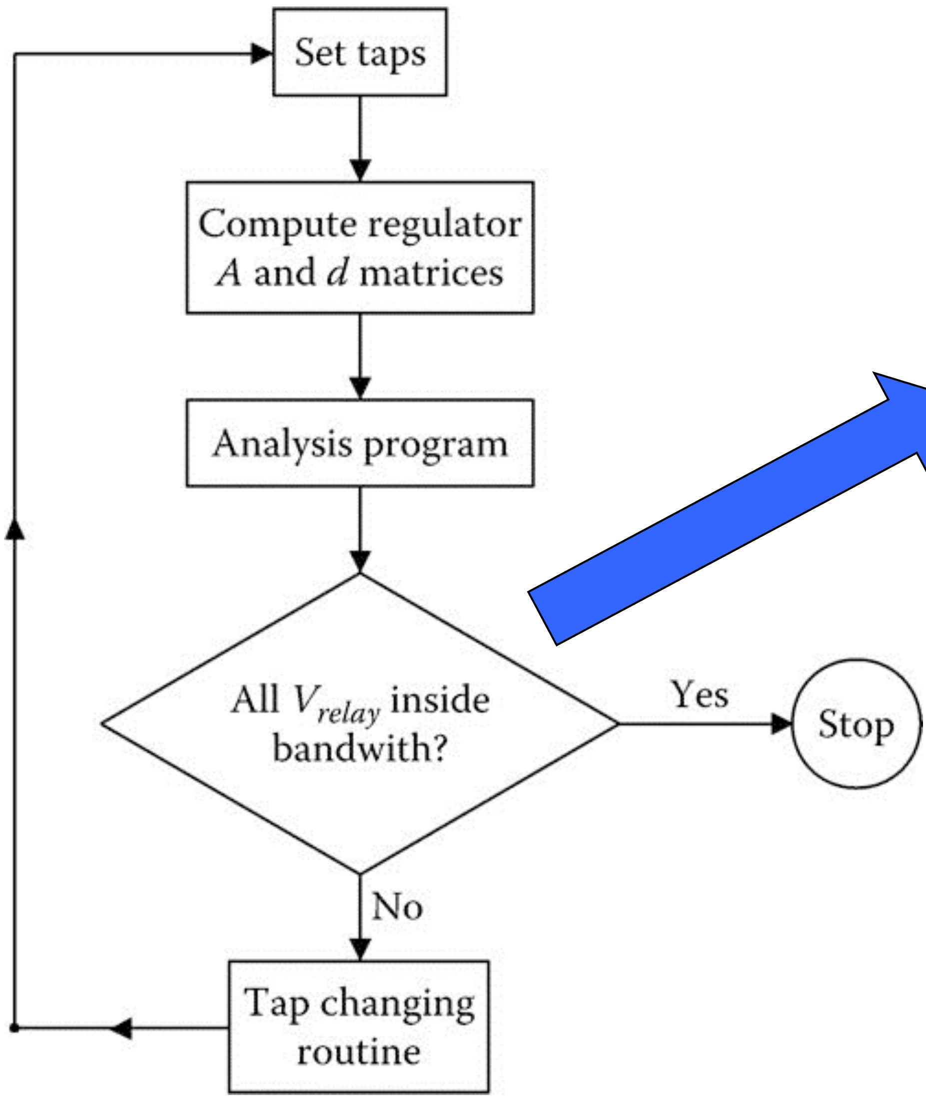
$$Z^\phi = \frac{V_3^\phi - V_4^\phi}{I_4^\phi} \Longrightarrow \mathbf{z} = \begin{bmatrix} 0.14 + j0.18 \\ 0.21 + j0.28 \\ 0.09 + j0.38 \end{bmatrix} \Omega$$

- SVRs share the compensator; heuristically pick

$$Z = \frac{1}{3}\sum_\phi Z^\phi = 0.14 + j0.28\ \Omega \Longrightarrow Z_{\text{LDC}} = 1.46 + j2.84\ \Omega$$

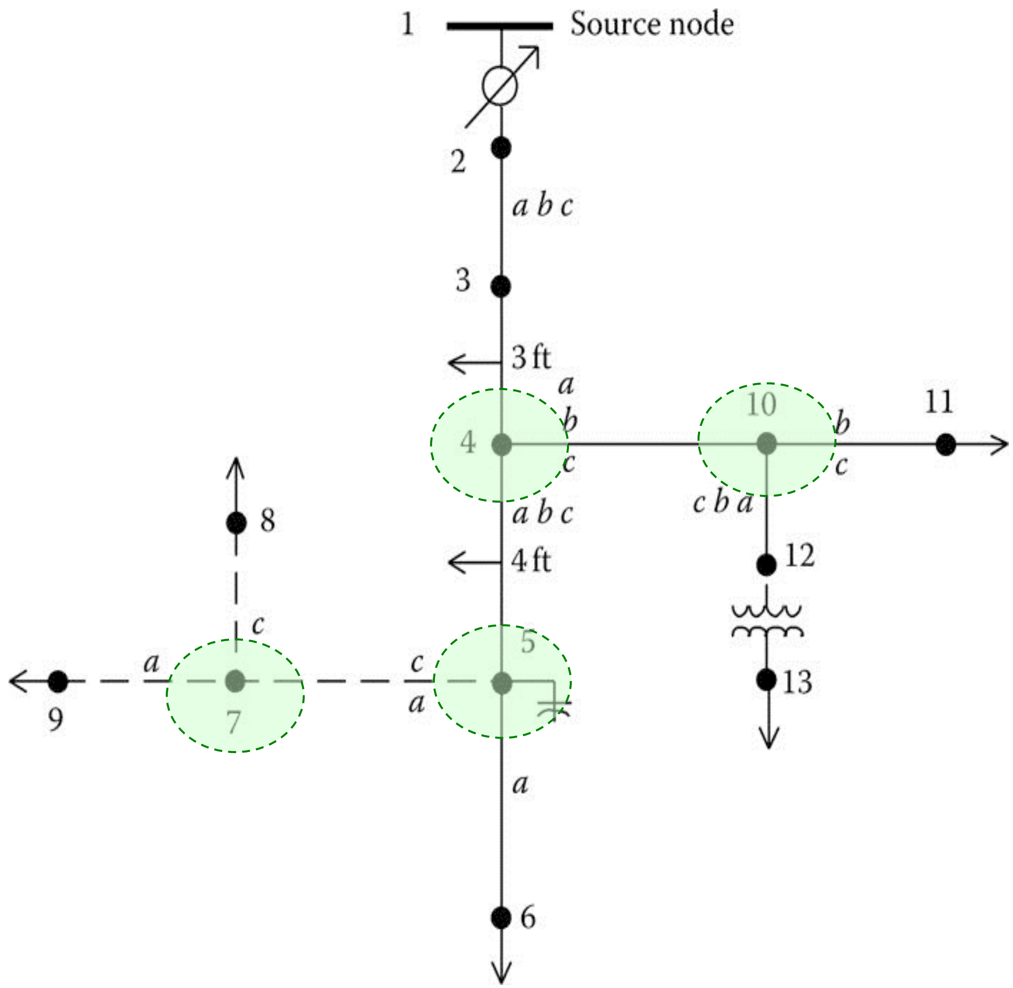
- Taps keep changing; final solution $\mathbf{v}_4^{120} = \begin{bmatrix} 121.0 \\ 119.3 \\ 120.7 \end{bmatrix} \text{V} \quad \text{Tap} = \begin{bmatrix} 9 \\ 11 \\ 12 \end{bmatrix}$

Realistic tap setting



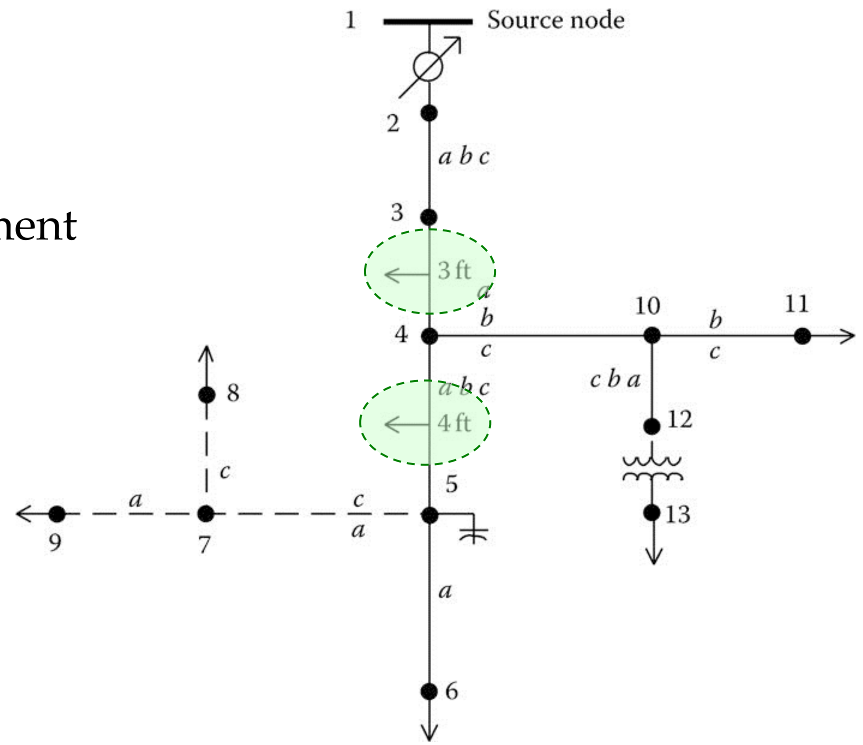
$$Y := \left|\begin{array}{l} \text{for } i \in 1..3 \\ \quad \left| \begin{array}{l} V_{reg_i} \leftarrow \dfrac{VLN_{3_i}}{N_{pt}} \\ I_{reg_i} \leftarrow \dfrac{I_{abc_i}}{CT} \end{array}\right. \\ V_{relay} \leftarrow V_{reg} - Z_{comp} \cdot I_{reg} \\ Tap_1 \leftarrow Tap_1 + 1 \text{ if } \left|V_{relay_1}\right| < 120 \\ Tap_2 \leftarrow Tap_2 + 1 \text{ if } \left|V_{relay_2}\right| < 120 \\ Tap_3 \leftarrow Tap_3 + 1 \text{ if } \left|V_{relay_3}\right| < 120 \\ Out_1 \leftarrow Tap \\ Out \end{array}\right.$$

Junction nodes



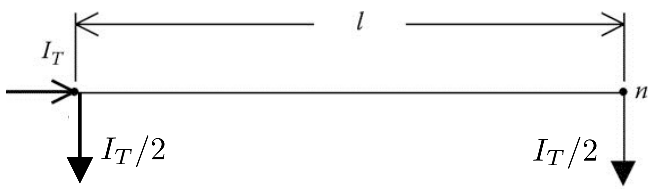
- In backward steps, all currents downstream from junction nodes have to be updated and summed up

Distributed loads

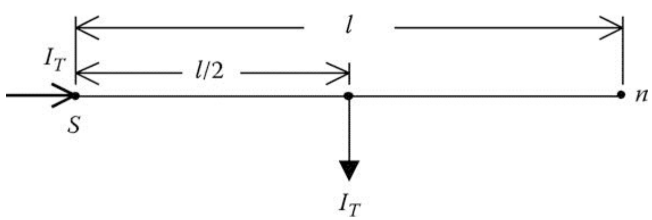


- Loads may be distributed along line segment

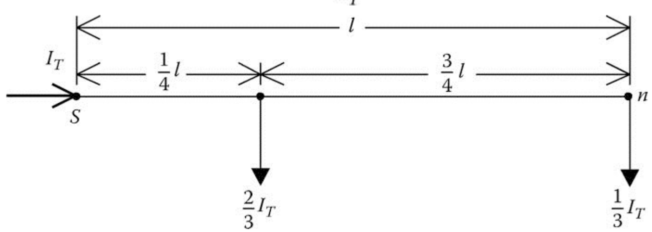
- Three lumping alternatives



1. split load in two ends (heuristic)



2. fix voltage drop



3. fix voltage drop and power losses

Ohmic Losses

- Two alternatives to calculate losses

Sum up the losses per conductor

- Squared current magnitudes times resistances
- Including neutral conductors and dirt
- Neutral and ground currents should be computed; resistances must be known

Real power sent minus real power received

- Due to the way the phase impedance matrix has been calculated, the two are equivalent
- Remark: a phase conductor may exhibit negative losses!