

# An Adaptive Decision Feedback Equalizer for Time-Varying Frequency Selective MIMO Channels

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**Abstract**—A new adaptive equalizer for wireless systems operating over time-varying and frequency selective multiple-input multiple-output (MIMO) channels is proposed. The equalizer consists of a number of decision feedback equalizer (DFE) stages, each one detecting a single stream. The equalizer filters, as well as the ordering by which the streams are extracted, are updated according to a LS cost function in a VBLAST-like fashion. By taking advantage of the underlying order recursive problem structure, a computationally efficient and numerically robust algorithm is developed based on the Cholesky factorization of the input data autocorrelation matrix. The convergence and tracking capabilities of the equalizer are studied through extensive computer simulations, and its BER performance is evaluated for hostile, time and frequency selective channels.

## I. INTRODUCTION

Equalization of wireless MIMO communication channels is a challenging task mainly due to the fact that the respective MIMO equalizers should cope with intersymbol, as well as interstream interference. A theoretical framework for designing optimum in the Minimum Mean Square Error (MMSE) sense finite-length MIMO DFEs was given in [1], where three detection scenarios were considered. The first scenario assumes that at each time instant, only previous decisions from all streams are available. In the second and third scenarios, current decisions from already detected streams are also exploited by the feedback filters. In [2], the VBLAST concept [3] was extended for frequency selective channels and two equalizer architectures with ordered successive cancellation were proposed. Frequency domain MIMO DFEs have also been considered in [4], [5]. All the above equalizers assume that the channel is static and known at the receiver, while the detection ordering is predetermined and fixed. In a time-varying environment, however, adaptive channel estimation should be employed, and detection ordering need to be updated quite frequently, thus leading to an overall prohibitive computational complexity. To the best of our knowledge, the only adaptive MIMO DFE is the one of [6], where a recursive LS (RLS) algorithm is employed for the first scenario of [1].

In this paper, we extend our previous work of [7] and develop an adaptive DFE for frequency selective MIMO

channels. The proposed method performs direct equalization, without making use of any channel estimation. By properly formulating the problem and taking advantage of its special structure (as was originally done in [8] for flat fading channels), we end up with a computationally efficient equalization scheme for the second scenario of [1]. The new algorithm originates from a set of LS cost functions and thus exhibits a fast convergence behavior. Moreover, it is expected to be numerically robust, since the equalizer filters are designed and updated based on the Cholesky factorization of the equalizers' input autocorrelation matrix. Finally, in the proposed method both the equalizer filters and detection ordering are updated at each time instant, rendering it appropriate for fast fading conditions.

## II. CHANNEL MODEL

Let us consider a MIMO communication system operating over a frequency selective and time-varying wireless channel. The system employs  $M$  transmit and  $N$  receive antennas, with  $M \leq N$ , while spatial multiplexing is assumed for high data rate communication. The received signals are sampled at the symbol rate and the system can be described via a discrete-time complex baseband model. The transmitted signal at time  $k$  can be expressed as

$$\underline{s}(k) = \frac{1}{\sqrt{M}} [ s_1(k) \ s_2(k) \ \dots \ s_M(k) ]^T \quad (1)$$

where  $s_m(k)$ , for  $m = 1, \dots, M$ , are i.i.d. symbols taken from a finite alphabet. Note that the total average transmit power is kept fixed and independent of  $M$ . Operators  $(\cdot)^T$  and  $(\cdot)^H$  denote transposition and Hermitian transposition, respectively.

The sampled impulse response, including pulse shaping filters, between transmitter  $m$  and receiver  $n$  at time  $k$ , is denoted by  $h_{nm}(k; l)$ , for  $l=0, \dots, L$ . The channel length  $(L+1)$  is considered to be common for all subchannels. By assembling the  $l$ -th impulse response coefficients from all subchannels into the  $N \times M$  matrices

$$\mathbf{H}(k; l) = \begin{bmatrix} h_{11}(k; l) & \dots & h_{1M}(k; l) \\ \vdots & \ddots & \vdots \\ h_{N1}(k; l) & \dots & h_{NM}(k; l) \end{bmatrix}, \quad l = 0, \dots, L \quad (2)$$

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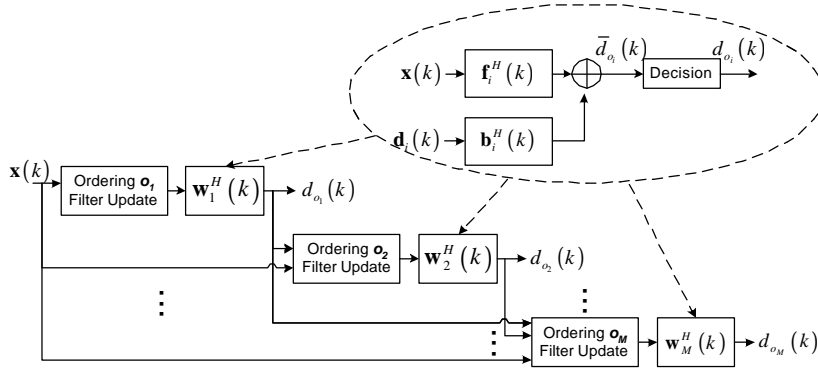


Fig. 1. Adaptive VBLAST MIMO Decision Feedback Equalizer Architecture.

the signal received at the  $N$  receive antennas at time  $k$  is expressed as follows:

$$\underline{\mathbf{x}}(k) = [x_1(k) \dots x_N(k)]^T = \sum_{l=0}^L \mathbf{H}(k; l) \underline{\mathbf{s}}(k-l) + \underline{\mathbf{n}}(k) \quad (3)$$

where  $\underline{\mathbf{n}}(k)$  is a  $N \times 1$  vector containing additive white Gaussian noise (AWGN) samples of variance  $\sigma^2$ .

### III. EQUALIZER ARCHITECTURE

The intersymbol and interstream interference involved in the system described by (3) can be mitigated through the equalizer architecture illustrated in Fig. 1. The proposed architecture is a structure of  $M$  serially connected DFEs. The DFE of the  $i$ -th stage equalizes one of the  $M$  symbol streams, according to the assignment  $o_i(k)$ , where  $o_i(k) \in \{1, 2, \dots, M\}$ . The sequence  $\{o_1(k), o_2(k), \dots, o_M(k)\}$  indicates the ordering at which the streams are extracted at time  $k$ , and is adaptively updated in a VBLAST manner. Although the ordering of streams depends on time  $k$ , we will skip this notation for the sake of simplicity. Thus, for the rest of the paper,  $o_i$  denotes the stream assigned to the  $i$ -th stage at time  $k$ , unless otherwise stated.

Each DFE consists of a feedforward filter,  $\mathbf{f}_i(k)$ , with a temporal span of  $K_f$  taps. The input of the feedforward filters is common for all DFEs, and is described by the  $NK_f \times 1$  vector

$$\mathbf{x}(k) = [\underline{\mathbf{x}}^T(k - K_f + 1) \quad \dots \quad \underline{\mathbf{x}}^T(k)]^T. \quad (4)$$

The feedback filter at stage  $i$ ,  $\mathbf{b}_i(k)$ , has a temporal span of  $K_b$ , or  $K_b + 1$  taps. At each stage,  $K_b$  postcursor decisions from all streams are available. Furthermore, the DFE of the  $i$ -th stage can exploit the current decisions made for the streams already acquired at the previous  $(i-1)$  stages. If  $\tilde{d}_{o_i}(k)$  is the output of the  $i$ -th DFE and  $d_{o_i}(k) = f\{\tilde{d}_{o_i}\}$  is the decision device output, then vector

$$\underline{\mathbf{d}}_o(k) = [d_{o_1}(k) \quad \dots \quad d_{o_M}(k)]^T \quad (5)$$

contains the decisions made for all streams at time  $k$ , permuted according to the current ordering. Hence, the input of the  $i$ -th feedback filter is described by the  $(MK_b + i - 1) \times 1$  vector

$$\underline{\mathbf{d}}_i(k) = [\underline{\mathbf{d}}_o^T(k - K_b) \quad \dots \quad \underline{\mathbf{d}}_o^T(k - 1) \quad d_{o_1}(k) \quad \dots \quad d_{o_{i-1}}(k)]^T.$$

By using the above definitions, the output of the  $i$ -th DFE can be compactly expressed as

$$\tilde{d}_{o_i}(k) = \mathbf{w}_i^H(k) \mathbf{y}_i(k) \quad (6)$$

where

$$\begin{aligned} \mathbf{w}_i(k) &= [\mathbf{f}_i^T(k) \quad \mathbf{b}_i^T(k)]^T \\ \mathbf{y}_i(k) &= [\mathbf{x}^T(k) \quad \underline{\mathbf{d}}_i^T(k)]^T, \quad i = 1, \dots, M, \end{aligned} \quad (7)$$

and, thus, the input of the  $i$ -th DFE,  $\mathbf{y}_i(k)$ , is of dimension  $K_i = NK_f + MK_b + (i - 1)$ .

To completely describe the proposed equalizer architecture, we need to specify how the detection ordering is determined. Following the idea of VBLAST, the streams achieving lower mean squared detection error must be obtained at earlier stages. By feeding those more reliable decisions into the feedback filters of the next stages, weaker streams can be detected more reliably as well. Apparently, under fast fading conditions not only the equalizer filters, but also the detection ordering should be adapted at each time instant.

Next, we follow a LS approach to satisfy both requirements. More specifically, let us assume that the equalizer of the  $i$ -th stage should be updated, provided that the DFEs of the previous stages have been determined and symbol decisions have been extracted according to the ordering  $\{o_1, \dots, o_{i-1}\}$ . The remaining streams form the set  $S_i(k) = \{1, \dots, M\} \setminus \{o_1, \dots, o_{i-1}\}$ . To find out which of these streams will be detected at the current stage, all the respective equalizers must be updated first. Each equalizer  $\mathbf{w}_{i,j}(k)$ , corresponding to the  $j$ -th stream, is updated as the minimizing argument of the following LS cost function:

$$\mathcal{E}_{i,j}(k) = \sum_{l=1}^k \lambda^{k-l} |d_j(l) - \mathbf{w}_{i,j}^H(k) \mathbf{y}_i(l)|^2, \quad j \in S_i(k) \quad (8)$$

where  $\lambda$  is the usual forgetting factor. After having computed all tentative equalizers,  $\mathbf{w}_{i,j}(k)$  for  $j \in S_i(k)$ , the one achieving the lowest squared error is finally applied at the current stage. In other words, we set

$$\begin{aligned} o_i &= \arg \min_{j \in S_i(k)} \mathcal{E}_{i,j}(k), \\ \mathbf{w}_i(k) &= \mathbf{w}_{i,o_i}(k), \quad \mathcal{E}_i(k) = \mathcal{E}_{i,o_i}(k) \end{aligned} \quad (9)$$

The procedure continues until the last stage is reached. During the next time instant, the  $NM$  subchannels may have been changed significantly, and thus, a new ordering is needed.

It is not difficult to show that the above minimization is equivalent to a VBLAST-type ordered cancellation scheme as it was also the case with the two equalizers of [2]. In [2], however, to perform cancellation, channel estimation is required. In the following section we develop a new computationally efficient adaptive algorithm for direct equalization of MIMO frequency selective channels.

#### IV. DERIVATION OF THE ALGORITHM

It is well known that minimization of  $\mathcal{E}_{i,j}(k)$  in (8) with respect to  $\mathbf{w}_{i,j}(k)$  leads to the following solution:

$$\mathbf{w}_{i,j}(k) = \Phi_i^{-1}(k) \mathbf{z}_{i,j}(k) \quad (10)$$

where  $\Phi_i(k)$  stands for the  $K_i \times K_i$  exponentially time-averaged input data autocorrelation matrix, and  $\mathbf{z}_{i,j}(k)$  for the  $K_i \times 1$  crosscorrelation vector, which are defined as

$$\Phi_i(k) = \sum_{l=1}^k \lambda^{k-l} \mathbf{y}_i(l) \mathbf{y}_i^H(l), \quad (11)$$

$$\mathbf{z}_{i,j}(k) = \sum_{l=1}^k \lambda^{k-l} \mathbf{y}_i(l) d_j^*(l). \quad (12)$$

As it can be seen from (10) and (12), to update the tentative equalizers  $\mathbf{w}_{i,j}(k)$  at stage  $i$ , current decisions from all streams must be known. To overcome this causality problem during the decision-directed mode, we assume as in [8], that the decisions at time  $k$  are extracted using the optimum equalizers and detection ordering found at time  $(k-1)$ , i.e.

$$\bar{d}_{o_i}(k) = \mathbf{w}_i^H(k-1) \mathbf{y}_i(k), \quad d_{o_i}(k) = f\{\bar{d}_{o_i}(k)\}$$

where  $o_i$  refers to the detection ordering at time  $(k-1)$ .

##### A. Square-Root Transformations

Estimation of the equalizers in (10) can be carried out by applying the conventional RLS approach. However, in order to ensure numerical robustness, a square-root algorithm is developed, which stems from the Cholesky factorization of the input autocorrelation matrix. Moreover, to reduce complexity we take advantage of the order recursive structure of the problem as described in the following analysis.

Let  $\mathbf{R}_i(k)$  denote the upper triangular Cholesky factor of  $\Phi_i(k)$ , i.e.  $\Phi_i(k) = \mathbf{R}_i^H(k) \mathbf{R}_i(k)$ . Then (10) is transformed to

$$\mathbf{w}_{i,j}(k) = \mathbf{R}_i^{-1}(k) \mathbf{p}_{i,j}(k) \quad (13)$$

where  $\mathbf{p}_{i,j}(k)$  is defined as

$$\mathbf{p}_{i,j}(k) = \mathbf{R}_i^{-1}(k) \mathbf{z}_{i,j}(k). \quad (14)$$

By using (10)-(14) in (8), the minimum LS error energy with respect to  $\mathbf{w}_{i,j}(k)$  can be expressed as

$$\mathcal{E}_{i,j}(k) = \left( \sum_{l=1}^k \lambda^{k-l} |d_j(l)|^2 \right) - |\mathbf{p}_{i,j}(k)|^2. \quad (15)$$

Moreover, by defining the  $M \times M$  matrix

$$\mathbf{Q}(k) = \sum_{l=1}^k \lambda^{k-l} \underline{\mathbf{d}}(l) \underline{\mathbf{d}}^H(l) = \lambda \mathbf{Q}(k-1) + \underline{\mathbf{d}}(k) \underline{\mathbf{d}}^H(k) \quad (16)$$

where  $\underline{\mathbf{d}}(k) = [d_1(k) \ \dots \ d_M(k)]^T$ , it is straightforward to show that

$$\mathcal{E}_{i,j}(k) = q_{j,j}(k) - |\mathbf{p}_{i,j}(k)|^2 \quad (17)$$

where  $q_{j,j}(k)$  stands for the  $(j,j)$ -th entry of  $\mathbf{Q}(k)$ .

Finally, using the transformations enforced by the Cholesky factorization of the input autocorrelation matrix, the output of the  $i$ -th equalizer,  $\bar{d}_{o_i}(k)$ , is described by

$$\bar{d}_{o_i}(k) = \mathbf{p}_i^H(k-1) \mathbf{g}_i(k), \quad d_{o_i}(k) = f\{\bar{d}_{o_i}(k)\} \quad (18)$$

where

$$\mathbf{g}_i(k) = \mathbf{R}_i^{-H}(k-1) \mathbf{y}_i(k) \quad (19)$$

is the transformed input vector, and  $\mathbf{p}_i(k-1)$  corresponds to  $\mathbf{w}_i(k-1)$  via an expression similar to (13).

##### B. Order-Update Recursions

To reduce complexity, we may exploit the order increasing nature of the input vectors between successive stages, i.e.,

$$\mathbf{y}_i(k) = [ \mathbf{y}_{i-1}^T(k) \quad d_{o_{i-1}}(k) ]^T. \quad (20)$$

It can be shown [8], [9], that the same property holds true for the correlation quantities of the  $i$ -th stage, since

$$\mathbf{R}_i(k) = \begin{bmatrix} \mathbf{R}_{i-1}(k) & \mathbf{p}_{i-1}(k) \\ \mathbf{0}^T & \sqrt{\mathcal{E}_{i-1}(k)} \end{bmatrix} \quad (21)$$

and

$$\mathbf{z}_{i,j}(k) = [ \mathbf{z}_{i-1,j}^T(k) \quad q_{o_{i-1},j}(k) ]^T. \quad (22)$$

Moreover, by using (14), (21), and (22), we get:

$$\begin{aligned} \mathbf{p}_{i,j}(k) &= \begin{bmatrix} \mathbf{R}_{i-1}^{-H}(k) & \mathbf{0} \\ -\frac{\mathbf{p}_{i-1}^H(k) \mathbf{R}_{i-1}^{-H}(k)}{\sqrt{\mathcal{E}_{i-1}(k)}} & \frac{1}{\sqrt{\mathcal{E}_{i-1}(k)}} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{i-1,j}(k) \\ q_{o_{i-1},j}(k) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{p}_{i-1,j}(k) \\ \frac{q_{o_{i-1},j}(k) - \mathbf{p}_{i-1}^H(k) \mathbf{p}_{i-1,j}(k)}{\sqrt{\mathcal{E}_{i-1}(k)}} \end{bmatrix}. \end{aligned} \quad (23)$$

Having computed matrix  $\mathbf{Q}(k)$  from (16), vectors  $\mathbf{p}_{i,j}(k)$  for  $j \in S_i(k)$  are order updated through (23). Then, the LS error energies  $\mathcal{E}_{i,j}(k)$  given by (17) can be efficiently order-updated as well through

$$\mathcal{E}_{i,j}(k) = \mathcal{E}_{i-1,j}(k) - \left| [\mathbf{p}_{i,j}(k)]_{K_i} \right|^2 \quad (24)$$

where  $[\mathbf{p}_{i,j}(k)]_{K_i}$  is the last element of  $\mathbf{p}_{i,j}(k)$ . The minimum of these energies is denoted as  $\mathcal{E}_i(k)$ , and the corresponding vector as  $\mathbf{p}_i(k)$ .

Furthermore, an efficient order update operation can be applied to the transformed input vector  $\mathbf{g}_i(k)$ . By substituting

the inverse Cholesky factor  $\mathbf{R}_i^{-H}(k)$  in (19) as performed in (23), and using the property of (20), it is easily shown that

$$\mathbf{g}_i(k) = \begin{bmatrix} \mathbf{g}_{i-1}(k) \\ \frac{d_{o_{i-1}}(k) - \bar{d}_{o_{i-1}}(k)}{\sqrt{\mathcal{E}_{i-1}(k-1)}} \end{bmatrix}. \quad (25)$$

Thus, if  $\mathbf{g}_1(k)$  is available,  $\mathbf{g}_i(k)$  can be order updated.

### C. Initial Time-Update Recursions

To complete the proposed algorithm, the involved first order quantities, i.e. for  $i=1$ , must be computed at each time instant  $k$ . More precisely, vectors  $\mathbf{p}_{1,j}(k)$  for  $j = 1, \dots, M$  can be time updated using the recursions described below. If  $\mathbf{R}_1^{-1}(k-1)$  has already been computed, then the transformed input vector for the first stage is given by

$$\mathbf{g}_1(k) = \mathbf{R}_1^{-H}(k-1)\mathbf{y}_1(k). \quad (26)$$

Next, we produce a sequence of  $K_1$  elementary complex Givens rotation matrices, whose product is denoted by  $\mathbf{T}(k)$ , according to the following expression:

$$\mathbf{T}(k) \begin{bmatrix} -\frac{\mathbf{g}_1(k)}{\sqrt{\lambda}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \star \end{bmatrix} \quad (27)$$

where  $\star$  denotes a ‘don’t care’ element. The  $l$ -th elementary matrix,  $l=1, 2, \dots, K_1$ , annihilates the  $l$ -th element of  $-\frac{\mathbf{g}_1(k)}{\sqrt{\lambda}}$  with respect to the last element of the whole vector, which initially equals 1. It can be shown [9], that the same rotation matrices can be used for time updating the inverse Cholesky factor as

$$\mathbf{T}(k) \begin{bmatrix} \lambda^{-1/2}\mathbf{R}_1^{-H}(k-1) \\ \mathbf{0}^T \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^{-H}(k) \\ \star \end{bmatrix}. \quad (28)$$

Moreover, and more importantly,  $\mathbf{T}(k)$  can be applied for the time-update of  $\mathbf{p}_{1,j}(k)$  for  $j = 1, \dots, M$  as well, i.e., [9]

$$\mathbf{T}(k) \begin{bmatrix} \lambda^{1/2}\mathbf{p}_{1,j}(k-1) \\ \hat{d}_j^*(k) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{1,j}(k) \\ \star \end{bmatrix}. \quad (29)$$

Note that it is not necessary to compute matrix  $\mathbf{T}(k)$  explicitly. Instead, the rotation parameters are calculated from (27) and are then used in rotations (28) and (29).

### D. The Algorithm

The basic steps of the proposed equalization algorithm are summarized in Table I. During the initial training mode, step 4 of the equalizer is skipped, since the respective decisions are replaced by known training symbols. After convergence, the equalizer switches to decision-directed mode, and the decisions are computed as described in step 4. Following the generic rule for DFE design, a decision delay should be inserted between equalizer decisions and transmitted symbols. As in [1], [2], we consider a decision delay parameter  $\Delta$  common for all streams, and set it to  $\Delta=K_f-1$ . Hence, the decision  $d_{o_i}(k)$  corresponds to symbol  $s_{o_i}(k-\Delta)$ .

The computational complexity of the new algorithm is  $O(K_1^2+M^2K_1)$  complex multiply-add operations per symbol period, where  $K_1=NK_f+MK_b$ . Note that when the channel

TABLE I  
SUMMARY OF THE PROPOSED ALGORITHM

<p><i>Initialization:</i> For <math>i = 1, \dots, M</math>, <math>o_i(0)=i</math>, <math>\mathbf{p}_i(0) = \mathbf{0}</math>, <math>\mathcal{E}_i(0)=0</math>. For <math>j = 1, \dots, M</math>, <math>\mathbf{p}_{1,j}(0) = \mathbf{0}</math>. <math>\mathbf{Q}(0)=\mathbf{0}</math>. <math>\mathbf{R}^{-1}(0)=\delta^{-1}\mathbf{I}</math> where <math>\delta</math> is a small positive constant.</p> <ol style="list-style-type: none"> <li>1) Compute <math>\mathbf{g}_1(k)</math> from (26).</li> <li>2) Find rotation parameters from (27).</li> <li>3) Time update the inverse Cholesky factor from (28).</li> <li>4) Order update <math>\mathbf{g}_i(k)</math> from (25), and compute decisions <math>d_{o_i}(k)</math> from (18).</li> <li>5) Time update matrix <math>\mathbf{Q}(k)</math> by using (16).</li> <li>6) Time update <math>\mathbf{p}_{1,j}(k)</math> for <math>j = 1, \dots, M</math> by rotation (29).</li> <li>7) Evaluate <math>\mathcal{E}_{1,j}(k)</math> for <math>j = 1, \dots, M</math> from (17).</li> <li>8) Set as <math>\mathcal{E}_1(k)</math> the minimum, and as <math>\mathbf{p}_1(k)</math> the corresponding <math>\mathbf{p}_{1,j}(k)</math>.</li> <li>9) For <math>i = 2, \dots, M</math> <ol style="list-style-type: none"> <li>a) Order update <math>\mathbf{p}_{i,j}(k)</math> and <math>\mathcal{E}_{i,j}(k) \forall j \in S_i(k)</math>, from (23) and (24).</li> <li>b) Set as <math>\mathcal{E}_i(k)</math> the minimum, and as <math>\mathbf{p}_i(k)</math> the corresponding <math>\mathbf{p}_{i,j}(k)</math>.</li> </ol> </li> </ol>
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changes in a rather slow rate, detection ordering can be updated less frequently, and thus its complexity can be further reduced to  $O(K_1^2+MK_1)$ .

Although the proposed equalizer looks similar to those in [6], [2], they differ in architecture. The equalizer of [6] corresponds to the first scenario of [1], in which ordered cancellation is not performed. Its computational complexity is  $O(K_1^2+MK_1)$ , however its performance is seriously affected as shown in Section V. The two equalizers presented in [2] perform ordered successive cancellation of past, as well as future decisions from already detected streams, but the channel is considered known at the receiver and the detection ordering is computed once. Their computational complexity is  $O(MK_1^2)$  without accounting for ordering update, channel estimation, and filtering.

## V. PERFORMANCE EVALUATION

The performance of the proposed equalizer was evaluated through extensive computer simulations. More specifically, we considered a system transmitting uncoded QPSK symbols of duration  $T_s=0.25\mu sec$  over a wireless channel modeled according to the UMTS Vehicular Channel Model A [10]. This channel model consists of six independent, Rayleigh faded paths, with a power delay profile described in [10], and a RMS delay spread of  $1.48T_s$ . The physical channel was convolved with a raised cosine pulse of roll-off factor 0.3. The SNR was defined as the expected SNR (over the ensemble of channel realizations) on each receive antenna. The feedforward and feedback filters had a temporal span of  $K_f=20$ , and  $K_b=10$  taps respectively, and  $\lambda=0.995$  was used.

Initially, to study the convergence of the equalizer, the Doppler effect was ignored and the channel was kept static for an interval of  $4096T_s$ . An  $M=N=3$  antenna configuration operating at SNR=16dB was simulated, while the system was in training mode. Three different equalizer algorithms were tested: (1) the proposed algorithm, (2) the proposed algorithm with a randomly selected ordering that was kept

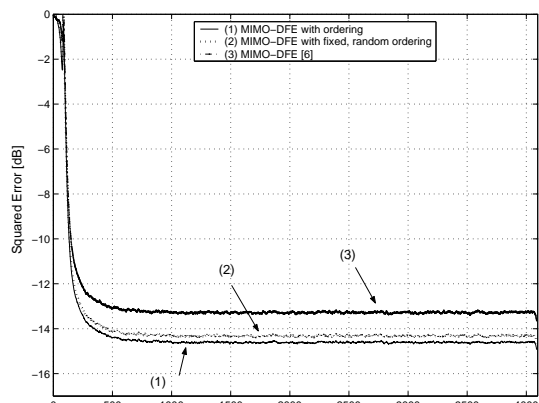


Fig. 2. Convergence of different equalizer architectures for a system operating in training mode over a  $3 \times 3$  static MIMO channel at  $\text{SNR}=16\text{dB}$ .

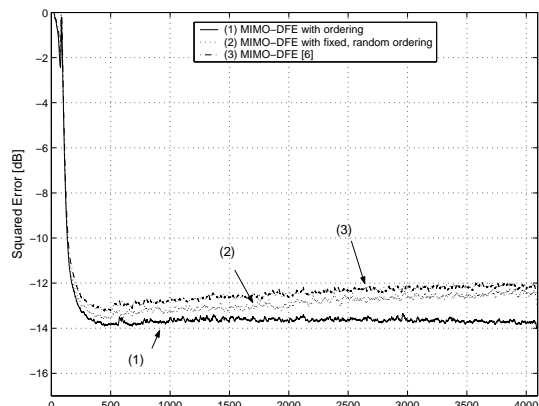


Fig. 3. Tracking performance of different equalizer architectures for a system operating over a  $3 \times 3$  time-varying MIMO channel at  $\text{SNR}=16\text{dB}$ .

fixed, and (3) the equalizer of [6]. Square-root RLS adaptation was used by all three equalizers, similar to the approach described in this paper. The equalizers of [2] were not included since an adaptive implementation was neither available, nor straightforward. In Fig. 2, the Mean Square Error (MSE) is plotted, i.e., the instantaneous squared error at the filter outputs averaged over all streams and over 500 independent runs. As expected, all three schemes converge very fast, however, the proposed method achieves the lowest MSE.

The tracking performance and the error propagation effects in decision-directed mode were studied by simulating a system that operates over a  $3 \times 3$  time-varying channel. Assuming operation in the 2.4GHz band, and a maximum mobile velocity of 100Km/h, a normalized Doppler frequency  $f_D T_s = 5.5 \cdot 10^{-5}$  was simulated by using the Jakes method. Moreover, the initial 512 symbol periods were used for training, and the rest of the equalizer parameters were as described above. In Fig. 3, the error curves for the three equalizers are plotted as the average of 500 runs. As shown in the figure, the proposed algorithm tracks very effectively channel variations and its superiority is further enhanced compared to the other two methods. This fact indicates the significance of tracking optimal detection ordering for time-varying channels. Note that in comparison to the MSE of Fig. 2, a degradation of less than 1dB is caused due to error propagation effects for the new algorithm.

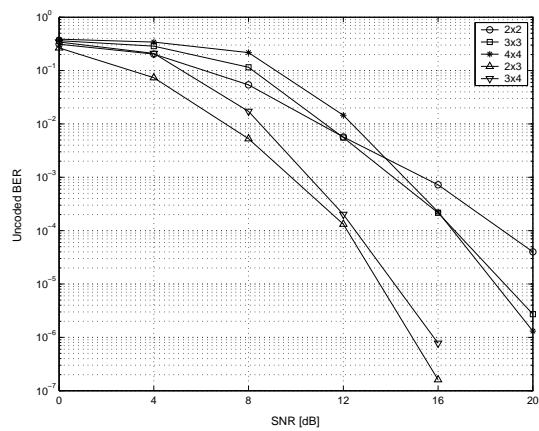


Fig. 4. Uncoded BER curves for a time and frequency selective channel.

Finally, the equalizer of Table I was evaluated in terms of uncoded BER for five MIMO configurations. The conclusion drawn by the BER curves of Fig. 4 is that the proposed equalizer can operate under the hostile environment simulated. Moreover, it is indicated that error propagation effects can degrade system's performance at low SNR.

## VI. CONCLUSIONS

A new adaptive equalizer for time and frequency selective MIMO channels has been derived. Extending the ideas of [8], [7], to the frequency selective channel case, we arrived at a square-root RLS adaptive MIMO DFE. By efficiently updating equalizer filters and detection ordering, the proposed algorithm offers improved convergence and tracking performance at reasonable computational complexity. Simulation results indicate the applicability of the equalizer in practical systems.

## REFERENCES

- [1] N. Al-Dhahir and A. H. Sayed, "The finite length multi-input multi-output MMSE-DFE," *IEEE Trans. Signal Processing*, vol. 48, no. 10, pp. 2921–2936, Oct. 2000.
- [2] A. Lozano and C. Papadias, "Layered space-time receivers for frequency selective wireless channels," *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 65–73, Jan. 2002.
- [3] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communications employing multi-element arrays," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1841–1852, Nov. 1999.
- [4] X. Zhu and R. Murch, "Layered space-frequency equalization in a single-carrier MIMO system for frequency-selective channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 3, pp. 701–708, May 2004.
- [5] R. Kalbasi, P. Dinis, D. Falconer, and A. Banihashemi, "Hybrid time-frequency layered space-time receivers for severe time-dispersive channels," in *Proc. IEEE Workshop on Signal Processing Advances on Wireless Communications*, Lisbon, July 2004.
- [6] A. Maleki-Tehrani, B. Hassibi, and J. M. Cioffi, "Adaptive equalization of multiple-input multiple-output (MIMO) channels," in *Proc. IEEE Int. Conf. Communications*, New Orleans, LA, June 2000, pp. 1670–1674.
- [7] A. A. Rontogiannis, V. Kekatos, and K. Berberidis, "A square-root adaptive V-BLAST algorithm for fast time-varying MIMO channels," *IEEE Signal Processing Lett.*, vol. 13, no. 5, pp. 265–268, May 2006.
- [8] J. Choi, H. Yu, and Y. H. Lee, "Adaptive MIMO decision feedback equalization for receivers with time-varying channels," *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4295–4303, Nov. 2005.
- [9] A. A. Rontogiannis and S. Theodoridis, "New fast QR decomposition least squares adaptive algorithms," *IEEE Trans. Signal Processing*, vol. 46, no. 8, pp. 2113–2121, Aug. 1998.
- [10] "Selection procedures for the choice of radio transmission technologies of the UMTS," ETSI, Technical Report 101.112, Apr. 1998.