

## Research Article

# Cholesky Factorization-Based Adaptive BLAST DFE for Wideband MIMO Channels

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Adaptive equalization of wireless systems operating over time-varying and frequency-selective multiple-input multiple-output (MIMO) channels is considered. A novel equalization structure is proposed, which comprises a cascade of decision feedback equalizer (DFE) stages, each one detecting a single stream. The equalizer filters, as well as the ordering by which the streams are extracted, are updated based on the minimization of a set of least squares (LS) cost functions in a BLAST-like fashion. To ensure numerically robust performance of the proposed algorithm, Cholesky factorization of the equalizer input autocorrelation matrix is applied. Moreover, after showing that the equalization problem possesses an order recursive structure, a computationally efficient scheme is developed. A variation of the method is also described, which is appropriate for slow time-varying conditions. Theoretical analysis of the equalization problem reveals an inherent numerical deficiency, thus justifying our choice of employing a numerically robust algebraic transformation. The performance of the proposed method in terms of convergence, tracking, and bit error rate (BER) is evaluated through extensive computer simulations for time-varying and wideband channels.

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## 1. INTRODUCTION

To exploit the potential spectral efficiency of multiple-input multiple-output (MIMO) wireless communication systems, sophisticated receiver structures should be designed. Most of the MIMO receivers described so far deal with narrowband systems, where the channel is considered flat. Among these receivers, the BLAST (Bell Labs Layered Space-Time) architecture [1] is usually employed in high-rate spatial multiplexing systems. However, to increase transmission rate, the symbol period should be made shorter, thus giving rise to intersymbol interference (ISI). Under these circumstances a MIMO equalizer should be designed in a proper way to compensate for both intersymbol and interstream interference.

Given that interference evolves in space and time, various MIMO DFE architectures have been proposed, corresponding to different detection scenarios [2, 3]. A *first scenario* comprises a parallel architecture, where all transmitted streams are detected simultaneously and hence only decisions on past detected symbols are available at each time instant. Due to this parallel structure, no detection ordering of streams is required. A *second scenario* is to process symbol

streams sequentially in an ordered manner and on a symbol-by-symbol basis [2]. In such a case, past decisions from all streams along with current decisions from already detected streams are used for detecting the current symbol of one of the remaining streams, and so forth. A *third scenario* could be an ordered sequential architecture operating on a packet basis, where at each stage the whole packet of a stream is extracted. Hence, future decisions of already detected streams are also available, when detecting a new stream. In all three scenarios, the available decisions can be either convolved with the corresponding channel impulse response and then subtracted from the received signal, or subtracted from the output of a feedforward filter after they have been convolved with a feedback filter. As shown in [4], these two approaches are mathematically equivalent. (The analysis in [4] is for flat fading channels. It can, however, be rather easily extended for frequency-selective channels.)

In [2] a theoretical framework for designing optimum in the minimum mean square error (MMSE) sense MIMO DFEs for the first and second detection scenarios was presented. In [3] the BLAST concept [1] was extended to frequency-selective channels and a meaningful criterion for

stream ordering was applied. The authors presented three equalizer architectures operating in the time domain: a MIMO DFE for the first scenario, the so-called partially connected (PC) receiver, and a fully connected (FC) receiver, both implementing the third detection scheme. For single carrier systems employing a cyclic prefix (SC-CP), hybrid schemes of these receivers have been suggested in [5–7], where feedforward and feedback filtering were performed in the frequency and time domain, respectively. A MIMO DFE completely implemented in the frequency domain was presented in [8], while an approximation for the ordering process of [3] was described in [9].

All the above equalizer designs assume that the channel is static and primarily that it is known at the receiver, while the detection ordering is predetermined and fixed. However, when the channel impulse response changes within a burst, adaptive channel estimation should be employed, and detection ordering needs to be updated quite frequently, thus leading to an overall prohibitive computational complexity. An adaptive channel estimation-based MIMO DFE for the first detection scenario was presented in [10]. Moreover, adaptive schemes, which perform linear MIMO equalization directly have been recently developed in [11–13]. An adaptive MIMO DFE employing the classical recursive least squares (RLS) algorithm has also been described in [14], dealing with the first detection scenario. Due to their architecture, all these equalizers do not take into consideration any ordering of the input streams, which can be a key factor in improving receiver's performance.

An adaptive BLAST DFE for flat fading MIMO channels has been developed in [15], while a numerically robust and computationally efficient modification of this algorithm was suggested in [16]. In this paper,<sup>1</sup> we develop an adaptive BLAST DFE for frequency-selective MIMO channels following the second detection scenario. The new algorithm originates from a recursive minimization of a set of LS cost functions, and thus exhibits a fast convergence behavior. Moreover, the detection ordering is naturally encapsulated in the LS problem and is efficiently updated at each time instant. By continuously updating both the DFE filters and the ordering of streams, the proposed equalizer can effectively track fast time variations appearing in high mobility applications. In case of slow fading, the ordering can be kept fixed and a variation of the proposed algorithm is presented. Note that, as it will be shown in our analysis, the equalization problem under consideration has, under certain conditions, a potential numerical deficiency. To compensate for this deficiency, in our approach the equalizer filters are designed and updated based on the Cholesky factorization of the equalizer's input autocorrelation matrix. This ensures numerically robust performance of the proposed algorithm, as justified intuitively and verified by simulation results.

To the best of our knowledge, in the limited literature on adaptive MIMO equalization, the proposed method is the only adaptive BLAST DFE for wideband systems. The

application of the idea of [16] in order to develop a computationally efficient DFE for frequency-selective MIMO channels requires a suitable formulation of the equalization problem. Indeed, by properly defining the structure of the DFE filters, we prove that all significant quantities of the algorithm can be efficiently order updated. Moreover, when it comes to wideband channels, the increase in the size of the problem has several implications in convergence, tracking, and numerical behavior of the algorithm that are successfully addressed in this work.

The rest of the paper is organized as follows. In Section 2, the system model is presented and the problem is formulated, while in Section 3 the new algorithm is derived. In Section 4, the proposed method is described in detail, and complexity and robustness issues are discussed. The performance of the algorithm is studied through extensive simulations in Section 5, and the work is concluded in Section 6.

Throughout the paper, we use bold lowercase and capital letters to denote vectors and matrices, respectively. We represent with  $\mathbf{I}_n$  the  $n \times n$  identity matrix, and with  $\mathbf{O}$ ,  $\mathbf{0}$  the all-zero matrix and vector, respectively. Finally, we utilize  $(\cdot)^T$ ,  $(\cdot)^*$ , and  $(\cdot)^H$  for matrix transposition, conjugation, and Hermitian transposition, respectively.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

Let us consider a MIMO communication system operating over a frequency-selective and time-varying wireless channel. The system employs  $M$  transmit and  $N$  receive antennas, with  $M \leq N$ , while spatial multiplexing is assumed for high data rate communication. Assuming perfect carrier recovery and downconversion, the received signals are sampled at the symbol rate and the system can be described via a discrete-time complex baseband model. The transmitted signal at time  $k$  can be described by the vector

$$\underline{\mathbf{s}}(k) = \frac{1}{\sqrt{M}} [s_1(k) \ s_2(k) \ \cdots \ s_M(k)]^T, \quad (1)$$

where  $s_m(k)$ , for  $m = 1, \dots, M$ , are i.i.d. symbols taken from the same finite alphabet. Note that the total average transmit power is fixed and independent of  $M$ .

The sampled impulse response, including the wireless channel and the pulse shaping filters, between transmitter  $m$  and receiver  $n$  at time  $k$ , is denoted by  $h_{nm}(k; l)$ , for  $l = 0, \dots, L$ . The channel length,  $(L + 1)$ , is considered to be common for all subchannels. By assembling the  $l$ th impulse response coefficients from all subchannels into the  $N \times M$  matrices

$$\mathbf{H}(k; l) = \begin{bmatrix} h_{11}(k; l) & \cdots & h_{1M}(k; l) \\ \vdots & \ddots & \vdots \\ h_{N1}(k; l) & \cdots & h_{NM}(k; l) \end{bmatrix}, \quad l = 0, \dots, L. \quad (2)$$

The signal received at the  $N$  receive antennas at time  $k$  can be expressed as

$$\underline{\mathbf{x}}(k) = [x_1(k) \ \cdots \ x_N(k)]^T = \sum_{l=0}^L \mathbf{H}(k; l) \underline{\mathbf{s}}(k-l) + \underline{\mathbf{n}}(k), \quad (3)$$

<sup>1</sup> Part of this work has been presented in [17].

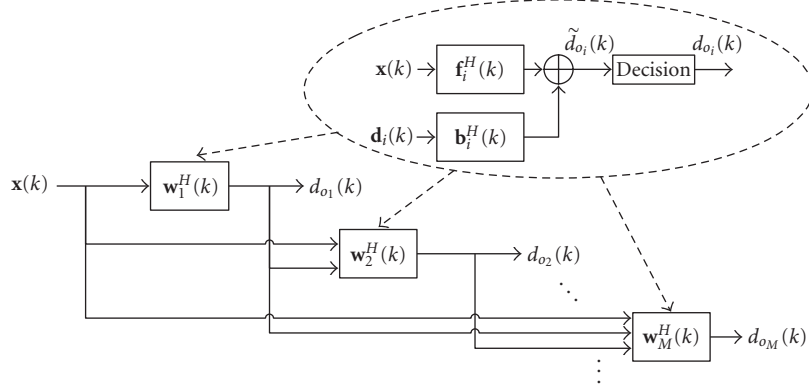


FIGURE 1: Adaptive BLAST MIMO decision feedback equalizer architecture.

where  $\mathbf{n}(k)$  is a  $N \times 1$  vector containing additive white Gaussian noise (AWGN) samples of variance  $\sigma_n^2$ .

The intersymbol and interstream interference involved in the system described by (3) can be mitigated through the equalizer architecture illustrated in Figure 1. The proposed architecture is a structure of  $M$  serially connected stages implementing the second detection scenario described in Section 1. The DFE of the  $i$ th stage equalizes one of the  $M$  symbol streams, according to the assignment  $o_i(k)$ , where  $o_i(k) \in \{1, 2, \dots, M\}$ . The sequence  $\{o_1(k), o_2(k), \dots, o_M(k)\}$  indicates the ordering at which the streams are extracted at time  $k$ , and is adaptively updated in a BLAST manner. Although the ordering of streams depends on time  $k$ , we will skip this notation for the sake of simplicity. Thus, for the rest of the paper,  $o_i$  denotes the stream assigned to the  $i$ th stage at time  $k$ , unless otherwise stated.

As shown in Figure 1, each DFE consists of a multiple-input single-output (MISO) feedforward filter,  $\mathbf{f}_i(k)$ , with  $NK_f$  taps. The input of the feedforward filters is common for all DFEs, and is described by the  $NK_f \times 1$  vector

$$\mathbf{x}(k) = [\mathbf{x}^T(k - K_f + 1) \ \dots \ \mathbf{x}^T(k)]^T. \quad (4)$$

The MISO feedback filter at stage  $i$ ,  $\mathbf{b}_i(k)$ , has a total of  $(MK_b + i - 1)$  taps. Its input consists of  $MK_b$  postcursor decisions from all streams, as well as the current decisions made for the streams already acquired at the previous  $(i - 1)$  stages. If  $\tilde{d}_i(k)$  is the output of the DFE assigned to the  $i$ th stream and  $d_i(k) = f\{\tilde{d}_i(k)\}$  is the corresponding decision device output, that is, the hard decision for the  $i$ th stream, then we define the  $M \times 1$  vector with the default ordering as

$$\mathbf{d}(k) = [d_1(k) \ \dots \ d_M(k)]^T. \quad (5)$$

Hence, the input of the feedback filter at the  $i$ th stage is described by the  $(MK_b + i - 1) \times 1$  vector

$$\mathbf{d}_i(k) = [\mathbf{d}^T(k - K_b) \ \dots \ \mathbf{d}^T(k - 1) \ d_{o_1}(k) \ \dots \ d_{o_{i-1}}(k)]^T, \quad (6)$$

where  $d_{o_i}(k)$  is the decision made at the  $i$ th DFE for the  $o_i$ th stream. As mentioned above, the decision vector  $\mathbf{d}_i(k)$  consists of two parts. The first  $MK_b$  elements correspond to previous decisions placed at the default ordering. The remaining part consists of the  $(i - 1)$  current decisions made at the previous stages, which are stored according to the current ordering.

By using the above definitions, the output of the  $i$ th DFE can be compactly expressed as

$$\tilde{d}_{o_i}(k) = \mathbf{w}_i^H(k) \mathbf{y}_i(k), \quad (7)$$

where

$$\begin{aligned} \mathbf{w}_i(k) &= [\mathbf{f}_i^T(k) \ \mathbf{b}_i^T(k)]^T, \\ \mathbf{y}_i(k) &= [\mathbf{x}^T(k) \ \mathbf{d}_i^T(k)]^T, \end{aligned} \quad i = 1, \dots, M, \quad (8)$$

and, thus, the input of the  $i$ th DFE,  $\mathbf{y}_i(k)$ , is a  $K_i \times 1$  vector with  $K_i = NK_f + MK_b + (i - 1)$ .

To completely describe the proposed equalizer architecture, we need to specify how the detection ordering is determined. To eliminate error propagation effects, we adopt the idea of BLAST [1] as in [3], that is, the streams achieving lower mean squared detection error are extracted at earlier stages. These streams are characterized by higher post detection SNR and since they are taken from the same constellation, they achieve lower bit error rate (BER). By feeding those more reliable decisions into the feedback filters of the next stages, weaker streams can be detected more reliably as well. Obviously, under fast fading conditions not only the equalizer filters, but also the detection ordering should be adapted at each time instant. Next, we follow an LS approach to satisfy both requirements. More specifically, let us assume that the equalizer of the  $i$ th stage should be computed, provided that the DFEs of the previous stages have been determined and symbol decisions have been extracted according to the ordering  $\{o_1, \dots, o_{i-1}\}$ . The remaining streams form the set  $S_i(k) = \{1, \dots, M\} \setminus \{o_1, \dots, o_{i-1}\}$ . To find out which of these streams achieves the lowest squared error and should be detected at the current stage, all the respective equalizers must be updated first. The equalizer  $\mathbf{w}_{i,j}(k)$ , corresponding to the

$i$ th stage and the  $j$ th stream, is the one minimizing the following LS cost function:

$$\mathcal{E}_{i,j}(k) = \sum_{l=1}^k \lambda^{k-l} |d_j(l) - \mathbf{w}_{i,j}^H(k) \mathbf{y}_i(l)|^2, \quad j \in \mathcal{S}_i(k), \quad (9)$$

where  $0 < \lambda \leq 1$  is the usual forgetting factor. After having updated all tentative equalizers,  $\mathbf{w}_{i,j}(k)$  for  $j \in \mathcal{S}_i(k)$ , the one achieving the lowest squared error is finally applied at the current stage. In other words, we set

$$\begin{aligned} o_i &= \arg \min_{j \in \mathcal{S}_i(k)} \mathcal{E}_{i,j}(k), \\ \mathbf{w}_i(k) &= \mathbf{w}_{i,o_i}(k), \\ \mathcal{E}_i(k) &= \mathcal{E}_{i,o_i}(k). \end{aligned} \quad (10)$$

The procedure continues until the last stage is reached. During the next time instant, the  $NM$  subchannels may have been changed significantly, and thus, a new ordering is needed.

Based on the minimization problems defined in (10), we derive an adaptive LS MIMO DFE algorithm, with stream ordering being incorporated in the equalization process. The proposed receiver performs direct equalization of MIMO frequency-selective channels.

### 3. DERIVATION OF THE ALGORITHM

The aim of this work is the efficient solution of the double minimization problem defined in (10), (9). It is well known that minimization of  $\mathcal{E}_{i,j}(k)$  with respect to  $\mathbf{w}_{i,j}(k)$  yields the equalizer vector as the solution of the so-called normal equations [18], that is,

$$\mathbf{w}_{i,j}(k) = \mathbf{\Phi}_i^{-1}(k) \mathbf{z}_{i,j}(k), \quad (11)$$

where  $\mathbf{\Phi}_i(k)$  stands for the  $K_i \times K_i$  exponentially time-averaged input autocorrelation matrix, and  $\mathbf{z}_{i,j}(k)$  for the  $K_i \times 1$  crosscorrelation vector, which are defined as

$$\mathbf{\Phi}_i(k) = \sum_{l=1}^k \lambda^{k-l} \mathbf{y}_i(l) \mathbf{y}_i^H(l), \quad (12)$$

$$\mathbf{z}_{i,j}(k) = \sum_{l=1}^k \lambda^{k-l} \mathbf{y}_i(l) d_j^*(l). \quad (13)$$

As it can be seen from (11) and (13), to compute the tentative equalizers  $\mathbf{w}_{i,j}(k)$  at stage  $i$ , current decisions from all streams must be known. To overcome this causality problem during the decision-directed mode, we assume as in [15], that the decisions at time  $k$  are extracted using the optimum equalizers and detection ordering found at time  $(k-1)$ , that is,

$$\begin{aligned} \tilde{d}_{o_i}(k) &= \mathbf{w}_i^H(k-1) \mathbf{y}_i(k), \\ d_{o_i}(k) &= f\{\tilde{d}_{o_i}(k)\}, \end{aligned} \quad (14)$$

where  $o_i$  here refers to the detection ordering at time  $(k-1)$ .

The system in (11) can be solved recursively by applying directly the conventional RLS algorithm. Two are the main

drawbacks of the approach. First, due to the high number  $(M(M+1)/2)$  of LS problems involved, the computational requirements become prohibitive. Second, as it will be explained in more detail in Section 4, the equalization problem at hand is prone to numerical instabilities, rendering the conventional RLS algorithm rather inappropriate. In order to ensure numerical robustness, a square-root LS algorithm is developed, which stems from the Cholesky factorization of the input autocorrelation matrix [19]. Moreover, to reduce complexity we take advantage of the order recursive structure of the problem, as described in the following analysis.

#### 3.1. Square-root transformations

In the proposed method, all quantities of the original problem are properly modified based on a square-root transformation. More specifically, let  $\mathbf{R}_i(k)$  denote the upper triangular Cholesky factor of  $\mathbf{\Phi}_i(k)$ , that is,  $\mathbf{\Phi}_i(k) = \mathbf{R}_i^H(k) \mathbf{R}_i(k)$ . Then (11) is rewritten as

$$\mathbf{w}_{i,j}(k) = \mathbf{R}_i^{-1}(k) \mathbf{p}_{i,j}(k), \quad (15)$$

where the transformed equalizer coefficients vector  $\mathbf{p}_{i,j}(k)$  is defined as

$$\mathbf{p}_{i,j}(k) = \mathbf{R}_i^{-H}(k) \mathbf{z}_{i,j}(k). \quad (16)$$

By using (11)–(16) in (9), the minimum LS error energy with respect to  $\mathbf{w}_{i,j}(k)$  can be expressed as

$$\mathcal{E}_{i,j}(k) = \left( \sum_{l=1}^k \lambda^{k-l} |d_j(l)|^2 \right) - \|\mathbf{p}_{i,j}(k)\|^2, \quad (17)$$

where  $\|\cdot\|$  denotes the Euclidean norm.

Moreover, by defining the  $M \times M$  matrix

$$\mathbf{Q}(k) = \sum_{l=1}^k \lambda^{k-l} \mathbf{d}(l) \mathbf{d}^H(l) = \lambda \mathbf{Q}(k-1) + \mathbf{d}(k) \mathbf{d}^H(k), \quad (18)$$

it is straightforward to show that

$$\mathcal{E}_{i,j}(k) = q_{j,j}(k) - \|\mathbf{p}_{i,j}(k)\|^2, \quad (19)$$

where  $q_{j,j}(k)$  stands for the  $(j, j)$ th entry of  $\mathbf{Q}(k)$ .

Finally, using the transformations imposed by the Cholesky factorization of the input autocorrelation matrix, (14), which provides the output of the  $i$ th equalizer,  $\tilde{d}_{o_i}(k)$ , is rewritten as

$$\tilde{d}_{o_i}(k) = \mathbf{p}_i^H(k-1) \mathbf{g}_i(k), \quad (20)$$

$$d_{o_i}(k) = f\{\tilde{d}_{o_i}(k)\}. \quad (21)$$

The vector  $\mathbf{g}_i(k)$  appearing in (20) can be considered as the transformed input vector, that is,

$$\mathbf{g}_i(k) = \mathbf{R}_i^{-H}(k-1) \mathbf{y}_i(k), \quad (22)$$

while the optimal transformed equalizer  $\mathbf{p}_i(k-1)$  is related to  $\mathbf{w}_i(k-1)$  via an expression similar to (15).

### 3.2. Order-update recursions

As already mentioned, in order to reduce complexity, we can take advantage of the special structure of the problem at hand. Indeed, by exploiting the order increasing nature of the input vectors between successive stages, that is,

$$\mathbf{y}_i(k) = \begin{bmatrix} \mathbf{y}_{i-1}^T(k) & d_{o_{i-1}}(k) \end{bmatrix}^T, \quad (23)$$

it can be shown that the  $i$ th order Cholesky factor is given by the following expression [20]:

$$\mathbf{R}_i(k) = \begin{bmatrix} \mathbf{R}_{i-1}(k) & \mathbf{p}_{i-1}(k) \\ \mathbf{0}^T & \sqrt{\mathcal{E}_{i-1}(k)} \end{bmatrix}. \quad (24)$$

Furthermore, similarly to [15], it is easily derived from (13) and (18) that

$$\mathbf{z}_{i,j}(k) = \begin{bmatrix} \mathbf{z}_{i-1,j}^T(k) & q_{o_{i-1},j}(k) \end{bmatrix}^T. \quad (25)$$

Thus, by using (16), (24), and (25), we get

$$\begin{aligned} \mathbf{p}_{i,j}(k) &= \begin{bmatrix} \mathbf{R}_{i-1}^{-H}(k) & \mathbf{0} \\ -\frac{\mathbf{p}_{i-1}^H(k)\mathbf{R}_{i-1}^{-H}(k)}{\sqrt{\mathcal{E}_{i-1}(k)}} & \frac{1}{\sqrt{\mathcal{E}_{i-1}(k)}} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{i-1,j}(k) \\ q_{o_{i-1},j}(k) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{p}_{i-1,j}(k) \\ \frac{q_{o_{i-1},j}(k) - \mathbf{p}_{i-1}^H(k)\mathbf{p}_{i-1,j}(k)}{\sqrt{\mathcal{E}_{i-1}(k)}} \end{bmatrix}. \end{aligned} \quad (26)$$

Having computed matrix  $\mathbf{Q}(k)$  from (18), vectors  $\mathbf{p}_{i,j}(k)$  for  $j \in S_i(k)$  are order updated through (26). Then, the LS error energies  $\mathcal{E}_{i,j}(k)$  given by (19) can be efficiently order-updated as well via

$$\mathcal{E}_{i,j}(k) = \mathcal{E}_{i-1,j}(k) - |[\mathbf{p}_{i,j}(k)]_{K_i}|^2, \quad (27)$$

where  $[\mathbf{p}_{i,j}(k)]_{K_i}$  is the last element of  $\mathbf{p}_{i,j}(k)$ . The minimum of these energies is denoted as  $\mathcal{E}_i(k)$ , and the corresponding vector as  $\mathbf{p}_i(k)$ . Note from (27) that computation of  $\mathcal{E}_{i,j}(k)$ , for all  $j \in S_i(k)$ , requires only  $O(1)$  operations.

Furthermore, an efficient order-update formula can be obtained for the transformed input vector  $\mathbf{g}_i(k)$ . By substituting the inverse Cholesky factor  $\mathbf{R}_i^{-H}(k)$  in (22) in the same way as in (26), and using the property of (23), it is easily shown that

$$\mathbf{g}_i(k) = \begin{bmatrix} \mathbf{g}_{i-1}(k) \\ \frac{d_{o_{i-1}}(k) - \tilde{d}_{o_{i-1}}(k)}{\sqrt{\mathcal{E}_{i-1}(k-1)}} \end{bmatrix}. \quad (28)$$

Up to now, order-update expressions have been derived for all algorithmic quantities. To complete the proposed method the initial first-order (i.e., for  $i = 1$ ) terms must be computed at each time instant  $k$ .

### 3.3. Initial time-update recursions

First-order quantities required at the beginning of the algorithm include  $\mathbf{g}_1(k)$ ,  $\mathbf{R}_1^{-1}(k)$  and  $\mathbf{p}_{1,j}(k)$  for  $j = 1, \dots, M$ . Assuming that  $\mathbf{R}_1^{-1}(k-1)$  has already been computed, the transformed input vector for the first stage is given by

$$\mathbf{g}_1(k) = \mathbf{R}_1^{-H}(k-1)\mathbf{y}_1(k). \quad (29)$$

Next, we produce a sequence of  $K_1$  elementary complex Givens rotation matrices, whose product is denoted by  $\mathbf{T}_1(k)$ , according to the following expression:

$$\mathbf{T}_1(k) \begin{bmatrix} -\lambda^{-1/2}\mathbf{g}_1(k) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \alpha_1(k) \end{bmatrix}, \quad (30)$$

where  $\alpha_1(k)$  is the last element of the vector in the right-hand side of (30), and the  $(K_1 + 1) \times (K_1 + 1)$  matrix  $\mathbf{T}_1(k)$  can be expressed as

$$\mathbf{T}_1(k) = \mathbf{T}_1^{(K_1)}(k)\mathbf{T}_1^{(K_1-1)}(k) \cdots \mathbf{T}_1^{(1)}(k), \quad (31)$$

and each elementary matrix is of the form

$$\mathbf{T}_1^{(l)}(k) = \begin{bmatrix} \mathbf{I}_{l-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}^T & c_l(k) & \mathbf{0}^T & -s_l^*(k) \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{K_1-l-1} & \mathbf{0} \\ \mathbf{0}^T & s_l(k) & \mathbf{0}^T & c_l(k) \end{bmatrix}. \quad (32)$$

The  $l$ th elementary matrix  $\mathbf{T}_1^{(l)}(k)$  annihilates the  $l$ th element of  $-\lambda^{-1/2}\mathbf{g}_1(k)$  with respect to the last element of the whole vector, which initially equals 1.<sup>2</sup>

It can be shown (see [20, 21]) that the same rotation matrices can be used for time updating the inverse Cholesky factor as

$$\mathbf{T}_1(k) \begin{bmatrix} \lambda^{-1/2}\mathbf{R}_1^{-H}(k-1) \\ \mathbf{0}^T \end{bmatrix} = \begin{bmatrix} \mathbf{R}_1^{-H}(k) \\ \star \end{bmatrix}, \quad (33)$$

where  $\star$  denotes “do not care” elements. Moreover, and more importantly,  $\mathbf{T}_1(k)$  can be also applied for the time-update of  $\mathbf{p}_{1,j}(k)$ ,  $j = 1, \dots, M$ , that is, [20]

$$\mathbf{T}_1(k) \begin{bmatrix} \lambda^{1/2}\mathbf{p}_{1,j}(k-1) \\ d_j^*(k) \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{1,j}(k) \\ \star \end{bmatrix}. \quad (34)$$

Obviously, it is not necessary to compute matrix  $\mathbf{T}_1(k)$  explicitly. Instead, the pairs of rotation parameters,  $(c_l(k), s_l(k))$  for  $l = 1, \dots, K_1$ , are evaluated from (30) and are then used in rotations (33) and (34).

<sup>2</sup> In a vector rotation  $\begin{bmatrix} c & -s^* \\ s & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ d \end{bmatrix}$ , the rotation parameters are evaluated as  $c = |a|/\sqrt{|a|^2 + |b|^2}$  and  $s = (b^*/\sqrt{|a|^2 + |b|^2})(a/|a|)$ .

*Initialization:* For  $i = 1, \dots, M$ ,  $o_i(0) = i$ ,  $\mathbf{p}_i(0) = \mathbf{0}$ ,  $\mathcal{E}_i(0) = 0$ . For  $j = 1, \dots, M$ ,  $\mathbf{p}_{1,j}(0) = \mathbf{0}$ .  $\mathbf{Q}(0) = \mathbf{O}$ .  $\mathbf{R}^{-1}(0) = \delta^{-1/2}\mathbf{I}$  where  $\delta$  is a small positive constant.

- (1) Compute  $\mathbf{g}_i(k)$  from (29), and  $d_{o_i}(k)$  from (20)-(21).
- (2) Find rotation parameters from (30).
- (3) Time-update the inverse Cholesky factor from (33).
- (4) For  $i = 2, \dots, M$ 
  - (a) Order-update  $\mathbf{g}_i(k)$  from (28).
  - (b) Compute decisions  $d_{o_i}(k)$  from (20)-(21).
- (5) Time-update matrix  $\mathbf{Q}(k)$  by using (18).
- (6) For  $j = 1, \dots, M$ 
  - (a) Time-update  $\mathbf{p}_{1,j}(k)$  by rotation (34).
  - (b) Evaluate  $\mathcal{E}_{1,j}(k)$  from (19).
- (7) Set as  $\mathcal{E}_1(k)$  the minimum, and as  $\mathbf{p}_1(k)$  the corresponding  $\mathbf{p}_{1,j}(k)$ .
- (8) For  $i = 2, \dots, M$ 
  - (a) For  $j \in S_i(k)$ 
    - (i) Order-update  $\mathbf{p}_{i,j}(k)$  from (26).
    - (ii) Evaluate  $\mathcal{E}_{i,j}(k)$  from (27).
  - (b) Set as  $\mathcal{E}_i(k)$  the minimum, and as  $\mathbf{p}_i(k)$  the corresponding  $\mathbf{p}_{i,j}(k)$ .

ALGORITHM 1: The proposed SROC algorithm.

#### 4. MIMO EQUALIZATION ALGORITHMS

The basic steps of the proposed squared root equalization algorithm with Ordered Cancellation (SROC) are summarized in Algorithm 1. During the initial training mode, known symbols are used in place of the hard decisions of the equalizer. Then the equalizer switches to the decision-directed mode, and hard decisions are computed via (21). Moreover, following the generic rule for DFE design, a decision delay should be inserted between equalizer decisions and transmitted symbols. As in [2, 3], we consider a decision delay parameter  $\Delta$  common for all streams, and set it to  $\Delta = K_f - 1$ . Hence, the decision  $d_{o_i}(k)$  corresponds to symbol  $s_{o_i}(k - \Delta)$ .

In case of slow channel variations, the detection ordering may be kept fixed. The SROC algorithm can then be properly modified, leading to the square-root equalizer with Cancellation (SRC), as shown in Algorithm 2. Without loss of generality we assume that detection ordering is the default stream indexing  $\{1, \dots, M\}$ . In Algorithm 2,  $\mathbf{p}_i(k)$  stands for the transformed equalizer coefficients vector of the  $i$ th stage of the DFE, and  $\mathbf{T}_i(k)$ ,  $\mathcal{E}_i(k)$  are the corresponding rotation matrices and LS energies, respectively. The term  $e_i(k)$  is the so-called angle-normalized LS estimation error, which can be used for time updating  $\mathcal{E}_i(k - 1)$  as in Step (2)f of Algorithm 2 [18]. Note that due to the order recursive property of  $\mathbf{g}_i(k)$ , that is, (28), the pairs of rotation parameters  $(c_l(k), s_l(k))$  for  $l = 1, \dots, K_i - 1$  are common for the rotation matrices  $\mathbf{T}_i(k)$  and  $\mathbf{T}_{i-1}(k)$ ,  $i = 2, \dots, M$ . Hence, to evaluate  $\mathbf{T}_i(k)$  at Step (2)d of Algorithm 2 for  $i > 1$ , only the rotation pair  $(c_{K_i}(k), s_{K_i}(k))$  need to be computed according

to

$$\begin{bmatrix} c_{K_i}(k) & -s_{K_i}^*(k) \\ s_{K_i}(k) & c_{K_i}(k) \end{bmatrix} \begin{bmatrix} -\lambda^{-1/2}[\mathbf{g}_i(k)]_{K_i} \\ \alpha_{i-1}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_i(k) \end{bmatrix}. \quad (35)$$

Two important issues closely related to the performance of the algorithms are further discussed below, that is, computational complexity and numerical robustness.

##### 4.1. Computational complexity

The computational complexity of the proposed equalization algorithms in terms of the number of multiplications and additions is shown in Table 1. We observe that, inevitably,  $K_1 M^2$  extra operations are required, in order to achieve ordering update in each iteration. This is, however, traded off with a noticeable improvement of the performance of the algorithm under fast time-varying conditions. Notice that in our derivation, we have taken advantage of the special structure of the problem to reduce the number of required operations.

The methods mostly related to our work are those presented in [3, 14], even though the respective equalization architectures are different. The equalizer of [14] corresponds to the first detection scenario, where no stream ordering is needed. The authors propose a conventional RLS algorithm with a computational complexity slightly lower compared to that of the SRC algorithm. However as will be shown in Section 4.2 and verified by simulations, due to the nature of the equalization problem, the conventional RLS algorithm exhibits severe numerical problems.

$$\begin{aligned}
(1) \mathbf{g}_i(k) &= \mathbf{R}_1^{-H}(k-1)\mathbf{y}_i(k) \\
(2) \text{ For } i &= 1 \text{ to } M \\
\quad (a) \tilde{d}_i(k) &= \mathbf{p}_i^H(k-1)\mathbf{g}_i(k) \\
\quad (b) d_i(k) &= f\{\tilde{d}_i(k)\} \\
\quad (c) \mathbf{T}_i(k) \begin{bmatrix} -\lambda^{-1/2}\mathbf{g}_i(k) \\ 1 \end{bmatrix} &= \begin{bmatrix} \mathbf{0} \\ \alpha_i(k) \end{bmatrix} \\
\quad (d) \mathbf{T}_i(k) \begin{bmatrix} \lambda^{1/2}\mathbf{p}_i(k-1) \\ d_i^*(k) \end{bmatrix} &= \begin{bmatrix} \mathbf{p}_i(k) \\ e_i(k) \end{bmatrix} \\
\quad (e) \mathbf{g}_{i+1}(k) &= \begin{bmatrix} \mathbf{g}_i(k) \\ \frac{d_i(k) - \tilde{d}_i(k)}{\sqrt{\mathcal{E}_i(k-1)}} \end{bmatrix} \\
\quad (f) \mathcal{E}_i(k) &= \lambda\mathcal{E}_i(k-1) + |e_i(k)|^2 \\
(3) \mathbf{T}_1(k) \begin{bmatrix} \lambda^{-1/2}\mathbf{R}_1^{-H}(k-1) \\ \mathbf{0}^T \end{bmatrix} &= \begin{bmatrix} \mathbf{R}_1^{-H}(k) \\ \star \end{bmatrix}
\end{aligned}$$

ALGORITHM 2: The proposed SRC algorithm.

TABLE 1: Computational complexity of the proposed algorithms.

Algorithm	Complex multiplications	Complex additions
SROC-DFE	$\frac{5}{2}K_1^2 + \frac{1}{2}K_1M^2$ $+ \frac{9}{2}K_1M + O(K_1)$	$\frac{3}{2}K_1^2 + \frac{1}{2}K_1M^2$ $+ \frac{5}{2}K_1M + O(K_1)$
SRC-DFE	$\frac{5}{2}K_1^2 + 5K_1M + O(K_1)$	$\frac{3}{2}K_1^2 + 3K_1M + O(K_1)$

In [3], two related equalizers are presented, which perform ordered successive cancellation of past, as well as future decisions from already detected streams. The channel is considered known at the receiver or estimated in the training phase along with the detection ordering, which remains fixed during the decision-directed phase. The computational complexity of these schemes is  $O(MK_1^2)$  without counting in the computations for ordering update, channel estimation, and filtering, that is, it is much higher compared to the complexity of the proposed algorithms.

#### 4.2. Numerical behavior

The numerical behavior of the proposed MIMO equalizers is related to the properties of the autocorrelation matrices  $\Phi_i = \mathbb{E}[\mathbf{y}_i(k)\mathbf{y}_i^H(k)]$  for  $i = 1, \dots, M$ , where  $\mathbb{E}[\cdot]$  is the expectation operator. To study the properties of these matrices, let us assume that the equalizer is designed such that  $\Delta = K_f - 1$ , and  $K_b = L$ , which is a common choice in practice. Moreover, the channel is considered static,  $\mathbf{H}(k; l) = \mathbf{H}(l)$ , and symbol

streams are assumed independent and of unitary variance, that is,  $\mathbb{E}[\mathbf{s}(k)\mathbf{s}^H(k)] = (1/M)\mathbf{I}_M$ .

By ignoring error propagation effects of the DFE, the  $K_i \times K_i$  matrix  $\Phi_i$  can be expressed as

$$\Phi_i = \frac{1}{M} \begin{bmatrix} \bar{\mathbf{H}}\bar{\mathbf{H}}^H + M\sigma_n^2\mathbf{I}_{NK_f} & \bar{\mathbf{H}}_{1,i} \\ \bar{\mathbf{H}}_{1,i}^H & \mathbf{I}_{MK_b+i-1} \end{bmatrix}, \quad (36)$$

where matrix  $\bar{\mathbf{H}}$  can be partitioned in two blocks as follows:

$$\bar{\mathbf{H}} = [\bar{\mathbf{H}}_{1,i} \mid \bar{\mathbf{H}}_{2,i}]. \quad (37)$$

Matrix  $\bar{\mathbf{H}}_{1,i}$  is of dimension  $NK_f \times (MK_b+i-1)$  and is defined as

$$\bar{\mathbf{H}}_{1,i} = \begin{bmatrix} \mathbf{H}(L) & \cdots & \cdots & \mathbf{H}(1) & \mathbf{H}_{\xi_i}(0) \\ \mathbf{0} & \mathbf{H}(L) & \cdots & \cdots & \mathbf{H}(2) & \mathbf{H}_{\xi_i}(1) \\ \vdots & & \ddots & & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}(L) & \cdots & \mathbf{H}(\Delta+1) & \mathbf{H}_{\xi_i}(\Delta) \end{bmatrix} \quad (38)$$

while  $\bar{\mathbf{H}}_{2,i}$  is the  $NK_f \times (MK_f - i + 1)$  matrix

$$\bar{\mathbf{H}}_{2,i} = \begin{bmatrix} \mathbf{H}_{S_i}(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{H}_{S_i}(1) & \mathbf{H}(0) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{S_i}(\Delta) & \mathbf{H}(\Delta-1) & \cdots & \mathbf{H}(0) \end{bmatrix}. \quad (39)$$

The  $N \times (i-1)$  matrix  $\mathbf{H}_{\xi_i}(l)$ ,  $l = 0, \dots, \Delta$  in (38) results from  $\mathbf{H}(l)$  by selecting only those columns corresponding to already detected streams, in the order they have been extracted. Correspondingly, matrix  $\mathbf{H}_{S_i}(l)$  for  $l = 0, \dots, \Delta$ , in (39) consists of the remaining columns of  $\mathbf{H}(l)$ .

In the high SNR region, the effect of noise can be ignored, and for  $\sigma_n^2 = 0$ , (36) is expressed as

$$\Phi_i = \begin{bmatrix} \bar{\mathbf{H}}_{1,i} \\ \mathbf{I} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{H}}_{1,i} \\ \mathbf{I} \end{bmatrix}^H + \begin{bmatrix} \bar{\mathbf{H}}_{2,i} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{H}}_{2,i} \\ \mathbf{0} \end{bmatrix}^H. \quad (40)$$

The rank of the first term in the right-hand side of (40) is  $MK_b + i - 1$ , while the rank of the second term is  $MK_f - i + 1$ . Thus, the rank of matrix  $\Phi_i$  is less than or equal to  $MK_b + MK_f$  [22], rendering  $\Phi_i$  rank deficient. Similar results can be extracted for other practical choices of filter lengths. As a conclusion, in the medium to high SNR region, the autocorrelation matrices involved in the DFE problems exhibit high condition numbers, and hence numerical problems arise. In the proposed square-root implementation of the RLS algorithm, the Cholesky factor  $\mathbf{R}_i$  of  $\Phi_i$  is used instead of  $\Phi_i$ , having a condition number equal to the square root of that of the original autocorrelation matrix. Thus, the proposed algorithm is expected to remain numerically robust for a wide range of operating SNRs and forgetting factors  $\lambda$ . If, instead, the conventional RLS was utilized as the basis of our derivation, numerical problems would be present even for relatively low SNR values.

## 5. PERFORMANCE EVALUATION

The performance of the proposed equalizers was evaluated through extensive computer simulations. More precisely, we considered a  $4 \times 4$  system transmitting uncoded QPSK symbols of duration  $T_s = 0.25 \mu\text{sec}$  over a wireless channel. All transmitter-receiver links were assumed independent, and modeled according to the UMTS Vehicular Channel Model A [23]. This channel model consists of six independent, Rayleigh faded paths, with a power delay profile described in [23]. The physical channel was convolved with a raised cosine pulse having a roll-off factor 0.3, resulting in a channel impulse response with a total channel length  $L = 23$ . The SNR was defined as the expected SNR (over the ensemble of channel realizations) on each receive antenna, while the feed-forward and feedback filters had a temporal span of  $K_f = 20$ , and  $K_b = 10$  taps, respectively.

The new equalizers were compared to the most relevant ones from the existing literature. More specifically, the equalizer proposed in [14], the partially connected DFE (PC-DFE), and the fully connected DFE (FC-DFE) of [3] were tested. To study the convergence and the steady state performance of the equalizers, the Doppler effect was ignored and the channel was kept static for an interval of  $4096T_s$ . The system was operating constantly in the training mode at SNR = 16 dB, while parameter  $\lambda$  was set to 0.995. In Figure 2, the mean square error (MSE) is plotted, that is, the instantaneous squared error at the filter outputs, averaged over all four streams and over 500 independent runs. The RLS adaptation conducted by the algorithms of [3, 14] is susceptible to numerical instability, as verified by the computer simulations shown in Figure 2 for the algorithm of [14]. For medium to high SNRs and small values of  $\lambda$ , the algorithms sooner or later diverge. Thus, in our comparisons, to avoid divergence, we have not used the original algorithms of [3, 14], but their square-root versions, instead.

As expected, each equalizer converges to a different level of steady state MSE due to the different detection scenarios and architectures implemented. Moreover, a training period of  $512T_s$  suffices for all algorithms to converge. Note that the delay introduced due to initial channel and equalizer estimation in the algorithms of [3], is not shown in the figure, and this is the reason why these algorithms seem to converge immediately. Concerning the SRC-DFE, when a random ordering is used, some performance degradation is inevitable, but when the correct stream ordering is applied, it has identical performance to the SROC-DFE.

To study the effects of error propagation, we tested the equalizers under a more practical situation, where a decision directed mode of operation follows a training period of  $512T_s$ . The results shown in Figure 3 indicate that the PC-DFE and FC-DFE are strongly affected, while the two other algorithms remain robust to error propagation.

The tracking performance together with the error propagation effects were studied by simulating a system that operates over a time-varying channel. Assuming operation in the 2.4 GHz band, and a maximum mobile velocity of 100 Km/h, a normalized Doppler frequency  $f_D T_s = 5.5 \cdot 10^{-5}$  was

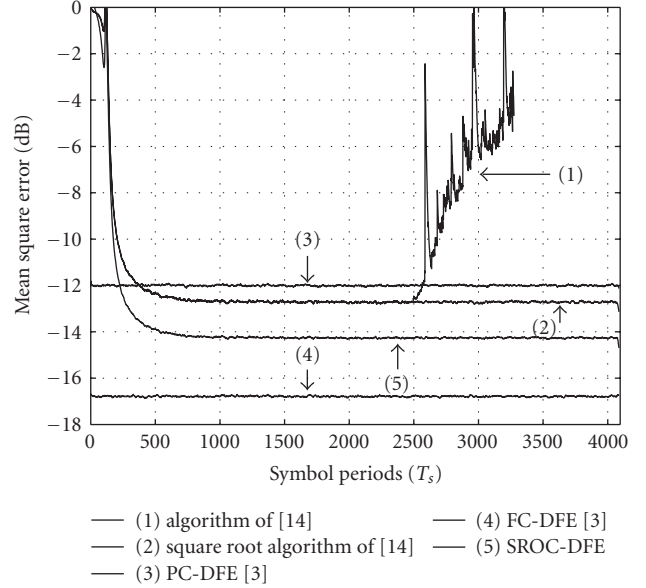


FIGURE 2: Convergence and steady-state performance of equalizers for a  $4 \times 4$  system operating constantly in training mode over a static frequency-selective MIMO channel at SNR = 16 dB.

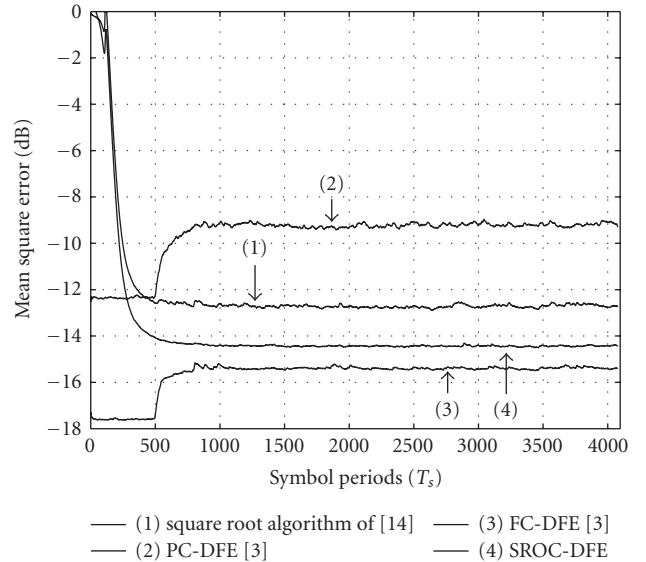


FIGURE 3: Convergence and steady-state performance of equalizers for a  $4 \times 4$  system trained for  $512T_s$ , and operating over a static frequency-selective MIMO channel at SNR = 16 dB.

simulated for all channel paths, by using the Jakes method [23]. The MSE curves obtained for this experiment are illustrated in Figure 4. As shown in this figure, the proposed SROC-DFE successfully tracks channel variations, while the algorithms of [3], seem to be strongly affected by the channel dynamics. Furthermore, a hybrid equalizer was simulated by combining the Algorithms 1-2: during the training period the receiver employs SROC-DFE, while in decision-directed mode switches to the SRC-DFE algorithm with the stream



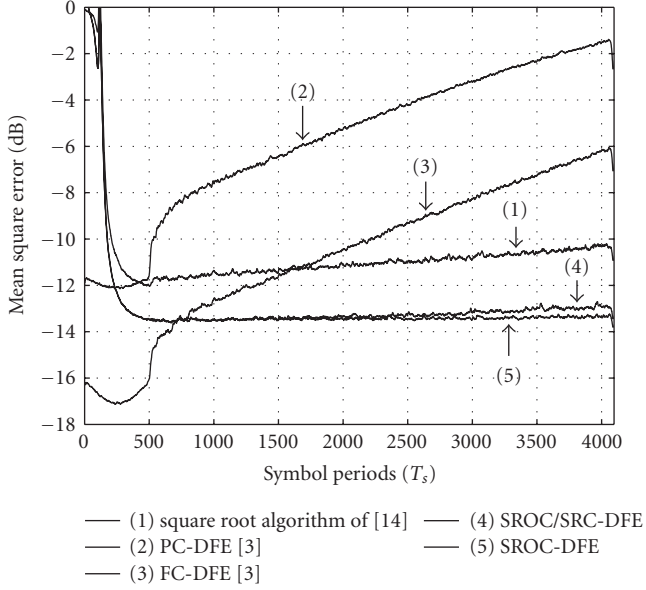


FIGURE 4: Convergence and tracking performance of equalizers for a  $4 \times 4$  system trained for  $512T_s$ , and operating over a time-varying, frequency-selective MIMO channel at SNR = 16 dB. The normalized Doppler frequency is  $5.5 \cdot 10^{-5}$ .

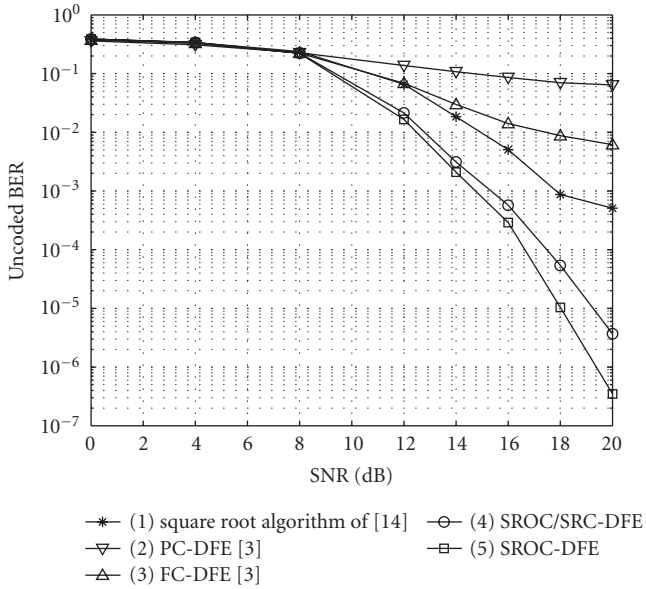


FIGURE 5: Uncoded BER curves of equalizers for a  $4 \times 4$  system trained for  $512T_s$ , and operating over a time-varying frequency-selective MIMO channel with a normalized Doppler frequency of  $5.5 \cdot 10^{-5}$ .

ordering already found. We observe from Figure 4 that by keeping the ordering as determined at the training phase, the MSE increases after a period of time.

The BER performance achieved by the equalizers for the previous experiment is presented in Figure 5. The BER

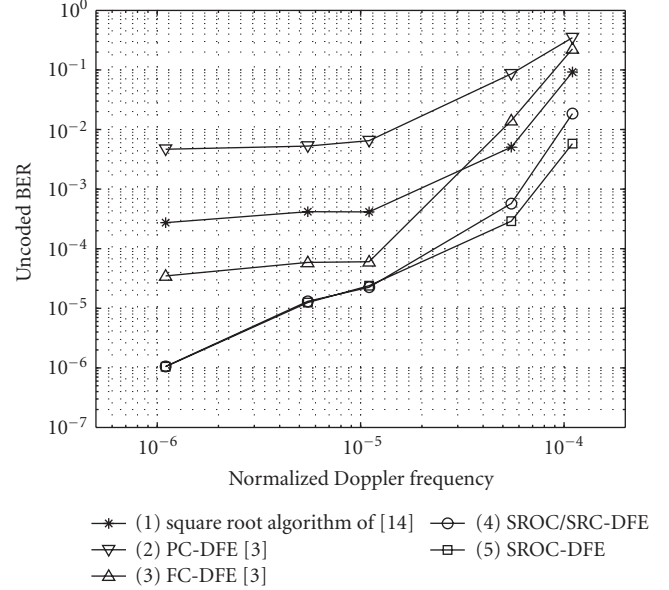


FIGURE 6: Uncoded BER of equalizers for a  $4 \times 4$  system trained for  $512T_s$ , and operating over a time-varying frequency-selective MIMO channel at SNR = 16 dB. The normalized Doppler frequency ranges from  $1 \cdot 10^{-6}$  to  $1 \cdot 10^{-4}$ .

curves indicate that the proposed algorithms can operate efficiently under the severe channel selectivity simulated, while the updating of detection ordering can improve performance at high SNRs at the expense of an increase in computational complexity.

Finally, to evaluate the tracking performance and the necessity of continuously updating both equalizer filters and detection ordering, we performed BER measurements for different channel fading rates. More precisely, the normalized Doppler frequency laid in the range of  $1 \cdot 10^{-6}$  to  $1 \cdot 10^{-4}$  at SNR = 16 dB. Due to the difference in fading rates, parameter  $\lambda$  was tuned to its best value, ranging from 0.9995 down to 0.993. As shown in Figure 6, the proposed algorithms are robust to error propagation effects and can track channel variations for a wide range of channel fading rates. On the other hand, the architecture advantage of the receivers proposed in [3] almost disappears due to channel variations and severe error propagation.

## 6. CONCLUSIONS

A novel adaptive decision feedback equalization method has been developed for wideband MIMO channels. After properly formulating the problem, an LS adaptive algorithm is derived, in which not only the equalizer filters, but also the detection ordering of the input streams are naturally updated at each time instant. Two are the main characteristics of the proposed algorithm. First, the initial RLS solution is transformed according to the Cholesky factorization of the equalizer input autocorrelation matrix. Second, efficient order update expressions are derived for all significant algorithmic quantities. The proposed algorithm is numerically

robust and offers improved convergence and tracking performance at a reasonable computational complexity, compared to other related methods. Extensive simulations have been carried out to confirm our theoretical results.

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