Microgrid Dispatch and Price of Reliability Using Stochastic Approximation

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Abstract—When properly operated, microgrids can facilitate the integration of stochastic renewable energy without compromising service reliability. However, in the context of multi-stage dispatching, finding the *optimal day-ahead* energy procurement that accounts for the variability of real-time operation is a *computationally challenging* task. This paper develops a computationally efficient *two-stage* economic dispatch scheme for a microgrid that exchanges energy with an external power system. The scheme is designed to minimize the generation and energy exchange costs, while setting limits on the microgrid-wide expected load not served. The day-ahead variables, which are the solution to the first stage, are found using a stochastic approximation saddle-point algorithm. The proposed algorithm is asymptotically convergent and can be efficiently implemented upon drawing samples from the distribution of the real-time state variables (wind energy, demand, and energy prices). Numerical tests using the IEEE 14-bus power system benchmark verify that the proposed scheme outperforms all other tested alternatives, even for very high wind power penetration.

Index Terms—Smart microgrids, stochastic approximation, saddle-point problem, dual subgradient.

I. INTRODUCTION

Grid efficiency and supply reliability are two desirable yet contradicting features of contemporary power systems. The increasing penetration of renewables renders energy markets even more sensitive to this tradeoff. Microgrids are envisioned to provide resiliency against time-variability and seamless integration of distributed energy resources [5], [8]. Nonetheless, their optimal operation is challenging due to the uncertainty of stochastic generation and demand, especially when microgrids engage in energy transactions with an external power system.

Since uncertainty decreases as the decision time approaches the operation time, power systems are typically dispatched in multiple stages [9]. At a day-ahead stage, the microgrid operator aims at minimizing its generation cost, including energy transactions with external systems. At the same time, reliability is traded for redispatching and curtailment costs, which are expected to occur during subsequent decision stages. The goal of this paper is to design computationally efficient day-ahead dispatch schemes guaranteeing that the *expected* load not served (ELNS) does not exceed a pre-specified level.

Redispatching during real-time operation is a common recourse action, whereas wind curtailment and load shedding are emergency actions to maintain grid stability in situations of unexpected energy surplus or deficit, respectively [1], [10]. Common metrics to quantify load shedding effects are the ELNS, the loss of load probability (LOLP), and the value of lost load (VOLL) [2], [16]. With worst-case formulations (see [18] and references therein), reliability usually comes at the cost of excessive conservativeness. Replacing worst-case designs with contingency probabilities constraints has been shown to yield non-negligible savings at no reliability sacrifices [3]. Hence, stochastic rather than worst-case approaches may be more desirable.

To that end, the expected costs of preventive actions and post-disturbance corrective actions are minimized in [2]; yet network constraints have been ignored. The nature of successive grid redispatching makes microgrid operation a multi-stage sequential decision problem, which thus calls for stochastic programming techniques [6]. A suitable stochastic programming framework is risk-limiting dispatch [15], [13], [17]. In particular, [17] considers a two-stage risk-limiting economic dispatch problem under the presumptions of mild congestion or low-variance stochastic generation.

Dispatching a microgrid under a multi-stage stochastic problem is considered here under practical conditions. We first postulate a two-stage stochastic economic dispatch that incorporates network and ELNS constraints, as well as curtailment and load shedding penalties (Section II). The problem is subsequently solved by adopting a stochastic approximation technique to optimize the convex-concave function involved (Section III). The derived stochastic primal-dual algorithm relies on data samples drawn from the related joint probability density function (pdf) of renewable generation, load, and prices. Efficient iterates involving convex quadratic programs are guaranteed to converge to the optimal day-ahead decisions and provide the price of reliability as a Lagrange multiplier. Numerical tests using the IEEE 14-bus grid corroborate the validity of our findings (Section IV), by comparing the developed scheme with worst-case and static spinning reserve approaches for different wind power penetration levels. Regarding *notation*, lower- (upper-) case boldface letters denote column vectors (matrices). Symbol $^\top$ denotes transposition, $\mathbb{E}$ expectation, while $\mathbf{0}$ and $\mathbf{1}$ the all-zeros and all-ones vectors.

II. PROBLEM STATEMENT

Consider a grid comprising conventional (e.g., thermal) and renewable generation, as well as loads to be served under specific reliability guarantees. The microgrid consists of $N+1$ buses indexed by n, and it is connected to an external power system through an interconnection bus indexed by $n = 0$. The interconnection bus serves also as the reference bus.

For every operation interval, energy trading takes place at two stages. At the first or *day-ahead stage*, the microgrid operator commits to exchange energy p_0 with the external system at the day-ahead (DA) price β. Since p_0 is the injection at bus 0, it is positive when the microgrid is importing energy from the external system; and negative, otherwise. In real-time operation, actual loads and renewable generation differ from their day-ahead predictions. Hence, the microgrid operator redispatches internal generation and adjusts the energy exchange with the external grid to $p_0 + \delta_0$. The additional energy exchange δ_0 incurs a real-time (RT) transaction cost. Note that restraining p_0 and δ_0 to be 0 permits to analyze an islanded microgrid case.

To model power injections, define the set of all but the interconnection buses $\mathcal{N} := \{1, \ldots, N\}$. Let $\mathbf{g}$ be the vector of conventional generation for all $n \in \mathcal{N}$ during the day-ahead stage; and $\boldsymbol{\delta}_g$ its correction in real-time. The load demand and the generated renewable energy over the nodal set $\mathcal{N}$ comprise vectors $\mathbf{d}$ and $\mathbf{w}$, which from a day-ahead perspective are modeled as stochastic. When the microgrid experiences conditions of extreme energy deficit or surplus, the operator may decide to shed load by reducing $\mathbf{d}$ to $\mathbf{d} - \boldsymbol{\delta}_d$, or to curtail renewable energy from $\mathbf{w}$ to $\mathbf{w} - \boldsymbol{\delta}_w$. For brevity, the RT deviations $(\boldsymbol{\delta}_g, \boldsymbol{\delta}_d, \boldsymbol{\delta}_w, \delta_0)$ are collectively denoted by $\boldsymbol{\delta}$. The vector of nodal power injections is then

$$\mathbf{p} = (\mathbf{g} + \boldsymbol{\delta}_g) + (\mathbf{w} - \boldsymbol{\delta}_w) - (\mathbf{d} - \boldsymbol{\delta}_d). \tag{1}$$

Financially, the internal generation cost for the microgrid is denoted by the convex functions $C(\mathbf{g})$ and $R(\boldsymbol{\delta}_g)$ for the day-ahead and the real-time stages, respectively. Shed load is penalized with the VOLL incurring quadratic cost

$$P(\boldsymbol{\delta}_d) := \boldsymbol{\delta}_d^\top \mathbf{V}_d \boldsymbol{\delta}_d \tag{2}$$

where $\mathbf{V}_d$ is a diagonal matrix with the per-bus positive shedding penalties on its main diagonal. In the other extreme, curtailed renewable energy entails a relatively small, linear penalty $\mathbf{v}_w^\top \boldsymbol{\delta}_w$. Regarding energy transactions with the external system, the DA price β is assumed fixed and known, whereas the RT cost is stochastic. In particular, the real-time price of buying (selling) energy from the external grid is γ^B (γ^S). To avoid arbitrage, it is assumed that $\gamma^S < \gamma^B$ and $\mathbb{E}[\gamma^S] < \beta < \mathbb{E}[\gamma^B]$; see e.g., [15], [17]. Using the notation $[x]_+ := \max\{0, x\}$, the cost of carrying out real-time transactions with the external grid becomes

$$T(\delta_0) := \gamma^B [\delta_0]_+ - \gamma^S [-\delta_0]_+, \tag{3}$$

which is stochastic due to randomness of (γ^B, γ^S). Granted $\gamma^S < \gamma^B$, the transaction cost can be also expressed as $T(\delta_0) = \max\{\gamma^B \delta_0, \gamma^S \delta_0\}$, which is certainly convex [18].

Taking into account the internal generation cost, the transaction cost with the external grid, and the related RT penalties, the microgrid operator must decide $(p_0, \mathbf{g})$ at the day-ahead stage. To that end, the operator aims at solving the problem:

$$\min_{p_0, \mathbf{g}, \{\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi})\}} \quad \mathbb{E}_{\boldsymbol{\xi}} \left[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w \right]$$

$$+ C(\mathbf{g}) + \beta p_0 \tag{4a}$$

$$\text{s.to} \quad \mathbf{p} = \mathbf{g} + \boldsymbol{\delta}_g + \mathbf{w} - \boldsymbol{\delta}_w - \mathbf{d} + \boldsymbol{\delta}_d \tag{4b}$$

$$\mathbf{1}^\top \mathbf{p} + p_0 + \delta_0 = 0 \tag{4c}$$

$$|\mathbf{H}\mathbf{p}| \leq \mathbf{f}^{\max} \tag{4d}$$

$$0 \leq \boldsymbol{\delta}_w \leq \mathbf{w} \tag{4e}$$

$$0 \leq \boldsymbol{\delta}_d \leq \mathbf{d} \tag{4f}$$

$$0 \leq \boldsymbol{\delta}_g \leq \boldsymbol{\delta}_g^{\max} \tag{4g}$$

$$0 \leq \mathbf{g} \leq \mathbf{g}^{\max} \tag{4h}$$

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta. \tag{4i}$$

The expectation operator in (4) is over the joint pdf of the involved random variables (namely renewable generation, demand, and real-time costs) that are collectively denoted by $\boldsymbol{\xi}$. This pdf may not be necessarily known. The equalities in (4b)-(4c) result from power balance constraints. Constraint (4d) guarantees that power flows on transmission lines do not exceed line capacities. Specifically, based on the linearized DC power flow model, power flows can be expressed as $\mathbf{H}\mathbf{p}$ with $\mathbf{H}$ being the matrix of power transfer distribution factors [9]. To prevent line outages from overheating, constraint (4d) bounds absolute flows by the known vector of line capacities $\mathbf{f}^{\max}$. The box constraints in (4e)–(4h) capture operational limits for conventional generation, load shedding, and wind power curtailment, accordingly. The ELNS constraint in (4i) guarantees that the average value of load shedding is smaller than a prescribed level η.

As a two-stage stochastic problem, (4) involves the day-ahead variables $(p_0, \mathbf{g})$ and infinitely many real-time variables $(\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi}))$. Constraints (4c)–(4g) apply for each pair $(\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi}))$, whereas the ELNS constraint in (4i) couples all second-stage variables. Second-stage variables will be oftentimes denoted simply by $(\boldsymbol{\delta}, \mathbf{p})$. Although $\mathbf{p}$ can be eliminated by substituting (4b) into (4c), it contributes to present the problem solution in a compact form.

The formulation in (4) controls system reliability in two ways: i) via the penalty function $P(\boldsymbol{\delta}_d)$, and ii) by requiring network-wide shedding to be smaller than a prescribed level η. Albeit seemingly redundant, the two mechanisms complement each other. If the expected revenue from selling power to the external system is high, the microgrid operator could shed a significant amount of load. The ELNS constraint prevents the operator from misusing load shedding simply to make profit from market opportunities, whereas the VOLL-weighted penalty term implements different priorities across buses.

III. PROBLEM SOLUTION

The optimal solution to (4) is found in two phases. In the first phase, the first-stage variables $(p_0, \mathbf{g})$ are assumed known

and the goal is to obtain the optimal real-time variables $\{\boldsymbol{\delta}, \mathbf{p}\}$ (for all $\boldsymbol{\xi}$) as a function of $(p_0, \mathbf{g})$. This is accomplished in Section III-A. In the second phase, the optimal real-time policies found in the first phase are used to obtain the optimal values of p_0 and $\mathbf{g}$ (Section III-B).

A. Second-Stage Problem: Real-Time Operation

In this stage, $(p_0, \mathbf{g})$ are fixed and the operator solves

$$f(p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \ \mathbb{E}_{\boldsymbol{\xi}}[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w] \quad (5\text{a})$$

$$\text{s.to } (4\text{b}) - (4\text{g}) \ \forall \ \boldsymbol{\xi} \quad (5\text{b})$$

$$\mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d] \leq \eta. \quad (5\text{c})$$

To resolve the coupling across RT variables introduced by (5c), we will resort to a dual decomposition approach. To be concrete, if ν is the Lagrange multiplier associated with (5c), the *partial* Lagrangian function for (5) is

$$\mathscr{L}(\{\boldsymbol{\delta}, \mathbf{p}\}, \nu; p_0, \mathbf{g}) := \mathbb{E}_{\boldsymbol{\xi}}\big[R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) \\ + \mathbf{v}_w^\top \boldsymbol{\delta}_w + \nu(\mathbf{1}^\top \boldsymbol{\delta}_d - \eta)\big] \quad (6)$$

the corresponding dual function is

$$\mathscr{D}(\nu; p_0, \mathbf{g}) := \min_{\{\boldsymbol{\delta}, \mathbf{p}\}} \ \mathscr{L}(\{\boldsymbol{\delta}, \mathbf{p}\}, \nu; p_0, \mathbf{g}) \quad (7)$$

$$\text{s.to } (4\text{b}) - (4\text{g}) \ \forall \boldsymbol{\xi}$$

and the associated dual problem is

$$\nu^* := \arg\max_{\nu \geq 0} \ \mathscr{D}(\nu; p_0, \mathbf{g}). \quad (8)$$

Since the problem in (5) is convex and the constraint (5c) is linear, strong duality guarantees that $f(p_0, \mathbf{g}) = \mathscr{D}(\nu^*; p_0, \mathbf{g})$. By the KKT optimality conditions and given the strict convexity of $\mathscr{L}(\{\boldsymbol{\delta}, \mathbf{p}\}, \nu; p_0, \mathbf{g})$ with respect to (w.r.t.) $\boldsymbol{\delta}_d$ [cf. (2)], the optimal second-stage variables $(\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi}))$ can be found as the minimizers of $\mathscr{L}(\{\boldsymbol{\delta}, \mathbf{p}\}, \nu^*; p_0, \mathbf{g})$ in (6). After dualizing (5c), the minimization of the Lagrangian can be performed separately for each pair of primal variables $(\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi}))$ as

$$\min_{\boldsymbol{\delta}(\boldsymbol{\xi}), \mathbf{p}(\boldsymbol{\xi})} \ R(\boldsymbol{\delta}_g) + T(\delta_0) + P(\boldsymbol{\delta}_d) + \mathbf{v}_w^\top \boldsymbol{\delta}_w + \nu \mathbf{1}^\top \boldsymbol{\delta}_d \quad (9)$$

$$\text{s.to } (4\text{b}) - (4\text{g}).$$

In other words, the minimization of the Lagrangian in (7) amounts to solving infinitely many instances of (9), one per $\boldsymbol{\xi}$. When $R(\boldsymbol{\delta}_g)$ is chosen as a linear, quadratic, or piecewise-linear function, then (9) is a convex problem, and it can be solved as a linearly-constrained quadratic program.

B. First-Stage Problem: Day-Ahead Operation

Having found the optimal second-stage variables for given $(\nu, p_0, \mathbf{g})$ we next develop an algorithm to obtain $(\nu^*, p_0^*, \mathbf{g}^*)$. Let us define $h(\nu, p_0, \mathbf{g})$ as

$$h(\nu, p_0, \mathbf{g}) := C(\mathbf{g}) + \beta p_0 + \mathscr{D}(\nu; p_0, \mathbf{g}). \quad (10)$$

Recalling the original problem (4) and leveraging the zero duality gap between (5) and (8), it follows that

$$\min_{p_0, \mathbf{g} \in \mathcal{G}} C(\mathbf{g}) + \beta p_0 + f(p_0, \mathbf{g}) = \min_{p_0, \mathbf{g} \in \mathcal{G}} \max_{\nu \geq 0} h(\nu, p_0, \mathbf{g}) \quad (11)$$

where $\mathcal{G} := \{\mathbf{g} : \mathbf{0} \leq \mathbf{g} \leq \mathbf{g}^{\max}\}$.

Observe that function $h(\nu, p_0, \mathbf{g})$ is convex-concave. To see that, note that $\mathscr{D}$ is concave w.r.t. ν for all $(p_0, \mathbf{g})$ as a dual function. Moreover, $\mathscr{D}$ is a convex function of $(p_0, \mathbf{g})$ for every $\nu \geq 0$. To see that, fix ν in the definition of $\mathscr{D}$ in (7), and observe that $\mathscr{D}$ is a perturbation function of $(p_0, \mathbf{g})$ [4], [7]. Hence, $\mathscr{D}(\nu; p_0, \mathbf{g})$ is a convex-concave function. Because $C(\mathbf{g}) + \beta p_0$ is convex in terms of $(p_0, \mathbf{g})$, function $h(\nu, p_0, \mathbf{g})$ is convex-concave too [cf. (10)]. Therefore and given the stochasticity of $\boldsymbol{\xi}$, the problem at the right hand side of (11) is a stochastic convex-concave saddle point problem [4], [11].

It is clear that $(p_0^*, \mathbf{g}^*, \nu^*)$ is a saddle point of $h(\nu, p_0, \mathbf{g})$. We next deploy a primal-dual subgradient scheme for finding it [12]. The partial super-/sub- gradients of h are

$$\partial_\nu h = \mathbb{E}_{\boldsymbol{\xi}}[\mathbf{1}^\top \boldsymbol{\delta}_d^*(\boldsymbol{\xi}; \nu, p_0, \mathbf{g})] - \eta \quad (12\text{a})$$

$$\partial_{p_0} h = \beta - \mathbb{E}_{\boldsymbol{\xi}}[\lambda^*(\boldsymbol{\xi}; \nu, p_0, \mathbf{g})] \quad (12\text{b})$$

$$\partial_{\mathbf{g}} h = \partial_{\mathbf{g}} C(\mathbf{g}) + \mathbb{E}_{\boldsymbol{\xi}}[\boldsymbol{\theta}^*(\boldsymbol{\xi}; \nu, p_0, \mathbf{g})] \quad (12\text{c})$$

where $\partial_{\mathbf{g}} C(\mathbf{g})$ is the subgradient of C evaluated at $\mathbf{g}$; $\lambda^*(\boldsymbol{\xi}; \nu, p_0, \mathbf{g})$ is an optimal Lagrange multiplier associated with the power balance constraint (4c) in (9); and $\boldsymbol{\theta}^*(\boldsymbol{\xi}; \nu, p_0, \mathbf{g})$ is an optimal Lagrange multiplier vector corresponding to constraint (4b) in (9). Note that the multipliers have been written as a function of $\boldsymbol{\xi}$ because their optimum value depends on the specific realization $\boldsymbol{\xi}$. The partial sub-gradient in (12a) holds by definition of the dual function, while the ones in (12b)–(12c) follow upon viewing $\mathscr{D}$ as a perturbation function [4], [7].

Using a standard primal-dual subgradient method for solving (11) would require calculating the expectations in (12). This amounts to solving problem (9) for all possible $\boldsymbol{\xi}$ and numerically integrating over the pdf of $\boldsymbol{\xi}$; the whole process being repeated at each iteration k. Such a task would be intractable even if the joint pdf were known. If only a set of uniformly drawn samples $\{\boldsymbol{\xi}_k\}$ is available, a more practical yet still computationally intensive approach is approximating the expectations by sample averages. Differently, to reduce complexity, we will rely on the saddle-point mirror stochastic approximation methodology proposed in [11]. For the problem at hand, the methodology involves the ensuing stochastic primal-dual subgradient iterations indexed by k:

$$\nu^{k+1} := \big[\nu^k + \mu_k\big(\mathbf{1}^\top \boldsymbol{\delta}_d^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k) - \eta\big)\big]_+ \quad (13\text{a})$$

$$p_0^{k+1} := p_0^k - \varepsilon_k\big(\beta - \lambda^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\big) \quad (13\text{b})$$

$$\mathbf{g}^{k+1} := \big[\mathbf{g}^k - \varepsilon_k\big(\partial_{\mathbf{g}} C(\mathbf{g}^k) + \boldsymbol{\theta}^*(\boldsymbol{\xi}_k; \nu^k, p_0^k, \mathbf{g}^k)\big)\big]_{\mathcal{G}} \quad (13\text{c})$$

where ε_k and μ_k are step sizes, and $[\cdot]_{\mathcal{G}}$ is the projection operator onto the set $\mathcal{G}$. The actual output of the algorithm at iteration k is the (possibly weighted) average of the m_k most recent iterates for each of the variables $(\nu, p_0, \mathbf{g})$.

The step size sequences $\{\mu_k, \varepsilon_k\}$ can be selected using two alternatives. The first alternative involves constant step sizes and averaging of all iterates. The second alternative involves step sizes diminishing across iterations as $O(1/\sqrt{k})$ and the output is a weighted average of the m_k last iterates. The first

alternative is useful to provide an approximate solution when the number of iterations K is fixed (due to e.g., computational limitations). The second one is more suitable when the number of iterations is unlimited or unknown a priori, and its output asymptotically converges to their optimal values as $k \to \infty$ [11, Sec. 3.1].

C. Economic Interpretation

It is well-known that the multipliers $\lambda^*(\boldsymbol{\xi}; \nu^*, p_0^*, \mathbf{g}^*)$ associated with the nodal balance constraints in (4c) can be viewed as locational marginal prices (LMPs) [9]. More interestingly, for the problem at hand the optimality condition $\mathbb{E}[\partial_{p_0} h(\nu^*, p_0^*, \mathbf{g}^*)] = 0$ yields $\mathbb{E}_{\boldsymbol{\xi}}[\lambda^*(\boldsymbol{\xi}; \nu^*, p_0^*, \mathbf{g}^*)] = \beta$. This expression reveals that for optimal two-stage operation, the *expected* value of the internal real-time LMP at the interconnection bus should be equal to the DA price of energy.

Regarding the multiplier ν associated with the ELNS constraint, (9) reveals that ν acts as a reliability-controlling parameter. It can also be seen as the *price of reliability* because it represents the *marginal* operation *cost* associated with a small variation of η. This allows to analyze the sensitivity of the optimal net cost relative to changes in the reliability level η, enabling anticipating the effects of regulatory changes.

IV. NUMERICAL TESTS

The proposed scheme has been numerically tested on the IEEE 14-bus grid [14] complemented with synthetic stochastic generation data to yield illustrative results. The external power system replaces the generator at bus 1; the rest of the conventional generators remain unchanged from the original benchmark, whereas the synthetic renewable generation has been co-located with demand. For the stochastic demand, samples were drawn from independent Gaussian distributions having the nominal demands $\mathbf{d}^B$ as means and standard deviations $0.3\mathbf{d}^B$, while negative values were truncated to zero. The stochastic renewable generation is sampled from a uniform distribution in the range $[\mathbf{0}, 2p_w\mathbf{d}^B]$, where the wind penetration factor p_w indicates the ratio between the average demand and average wind generation. Regarding the prices, β is set to 3000 and γ^B and γ^S are uniformly distributed in $[4500, 8500]$ and $[750, 1500]$, respectively. The wind spilling penalty is $v_w = 1000 \cdot \mathbf{1}$ and the load shedding penalty matrix is $\mathbf{V_d} = 5000 \cdot \mathbf{I}$. All prices and penalties are in terms of \$ per unit (p.u.). The ELNS limit η has been fixed to 0.01.

The proposed method (PM) implements the iterations in (13) with constant stepsize and running averaging iterates as described in [11, Sec. 3.1]. Since the number of iterations is finite, the output of the algorithm approximates the global solution. Once $(p_0^*, \mathbf{g}^*, \nu^*)$ are estimated, the RT dispatch is designed for each realization of ξ as the solution to (9).

The PM is compared with three alternative methods (AM). (AM1) a simple deterministic dispatch scheme that designs $(p_0, \mathbf{g})$ for the expected wind generation and demand, neglecting the variability of the wind, demand, and RT prices (this scheme would be optimal if such variables were deterministic). (AM2) the deterministic dispatch (AM1) along with a static
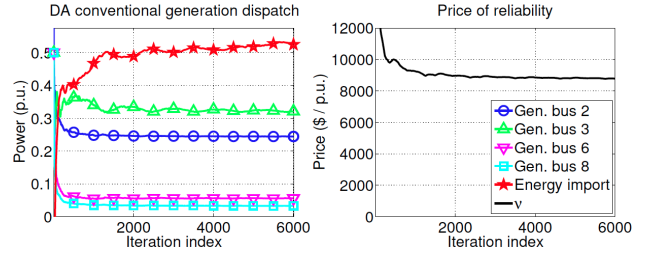

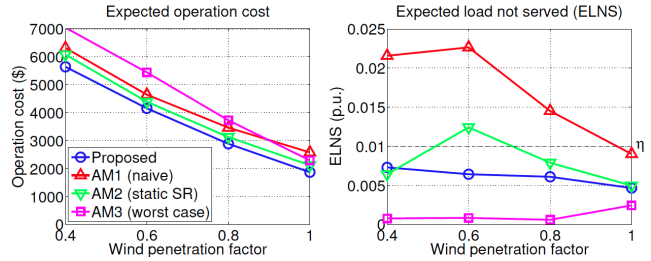Fig. 1. Convergence of the first-stage primal, and dual variables.

Fig. 2. Expected cost and ELNS as a function of p_w for the tested methods.

spinning-reserve allocation that adds $0.1 \cdot \mathbf{g}^{max}$ to the conventional generation at every generation bus. (AM3) a dispatch scheme that designs $(p_0, \mathbf{g})$ for an approximate worst-case scenario where $\mathbf{w} = \mathbf{0}$, $\mathbf{d} = 1.3 \cdot \mathbf{d}^B$, and the buy (sell) RT price is the highest (lowest). The RT dispatch for the three AMs is designed by first minimizing $\mathbf{1}^\top \boldsymbol{\delta}_d$, and then the RT operation cost.

Figure 1 shows the convergence of the running averages of $p_0, \mathbf{g}$, and ν for a wind penetration factor of 0.8. The expected operation cost and ELNS are estimated via a Monte Carlo simulation with a set of randomly generated samples of ξ (different from those used to run the optimization). Figure 2 shows the network cost and ELNS for the PS as well as the three AM. Confirming the findings in [3], the stochastic method clearly outperforms the alternative methods, which either violate the ELNS constraint or turn out to be over-conservative and less efficient.

V. CONCLUSIONS

A two-stage stochastic, distribution-free, optimal dispatch and procurement scheme has been proposed. The first-stage variables corresponded to the power procured in the day-ahead operation, and the dispatch of the conventional generation units. Second-stage variables were the powers that are actually generated by the dispatchable generators, and real-time adjustments such as the shed load, the spilled wind, and the power traded in the real-time market. Relying on a provably convergent stochastic approximation saddle-point algorithm, the developed scheme efficiently finds the optimal primal and dual variables for the first stage. Once the first-stage variables are found, the problem decomposes across realizations of ξ, and the real-time (second stage) solution can be easily found by solving a single convex problem.

REFERENCES

[1] K. H. Abdul-Rahman, H. Alarian, M. Rothleder, P. Ristanovic, B. Vesovic, and B. Lu, "Enhanced system reliability using flexible ramp constraint in CAISO market," in *Proc. IEEE Power & Energy Society General Meeting*, San Diego, CA, Jul. 2012.

[2] F. Bouffard and F. D. Galiana, "Stochastic security for operations planning with significant wind power generation," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 306–316, 2008.

[3] F. Bouffard, F. D. Galiana, and A. J. Conejo, "Market-clearing with stochastic security–Part II: Case studies," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1827–1835, 2005.

[4] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY: Cambridge University Press, 2004.

[5] X. Chen, H. Dinh, and B. Wang, "Cascading failures in smart grid-benefits of distributed generation," in *Proc. IEEE Intl. Conf. on Smart Grid Commun.*, Gaithersburg, MD, Oct. 2010, pp. 73–78.

[6] A. J. Conejo, M. Carrión, and J. M. Morales, *Decision making under uncertainty in electricity markets*. Springer, 2010, vol. 1.

[7] A. J. Conejo, E. Castillo, R. Minguez, and R. Garcia-Bertrand, *Decomposition techniques in mathematical programming*. Springer, 2006.

[8] G. B. Giannakis, V. Kekatos, N. Gatsis, S.-J. Kim, H. Zhu, and B. Wollenberg, "Monitoring and optimization for power grids: A signal processing perspective," *IEEE Signal Processing Mag.*, vol. 30, no. 5, pp. 107–128, 2013.

[9] A. Gómez-Expósito, A. J. Conejo, and C. Canizares, Eds., *Electric Energy Systems, Analysis and Operation*. Boca Raton, FL: CRC Press, 2009.

[10] J. M. Morales, A. J. Conejo, and J. Pérez-Ruiz, "Economic valuation of reserves in power systems with high penetration of wind power," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 900–910, 2009.

[11] A. Nemirovski, A. Juditsky, G. Lan, and A. Shapiro, "Robust stochastic approximation approach to stochastic programming," *SIAM Journal on Optimization*, vol. 19, no. 4, pp. 1574–1609, 2009.

[12] Y. Nesterov, "Primal-dual subgradient methods for convex problems," *Mathematical Programming*, vol. 120, no. 1, pp. 221–259, 2009.

[13] R. Rajagopal, E. Bitar, P. Varaiya, and F. Wu, "Risk-limiting dispatch for integrating renewable power," *Intl. Journal of Electrical Power & Energy Systems*, vol. 44, no. 1, pp. 615–628, 2013.

[14] Power systems test case archive. Univ. of Washington. [Online]. Available: http://www.ee.washington.edu/research/pstca/

[15] P. P. Varaiya, F. F. Wu, and J. W. Bialek, "Smart operation of smart grid: Risk-limiting dispatch," *Proc. IEEE*, vol. 99, no. 1, pp. 40–57, 2011.

[16] C. Wu, G. Hug, and S. Kar, "Risk-limiting economic dispatch for electricity markets with flexible ramping products," 2015, (unpublished). [Online]. Available: arXiv:1409.5173v2

[17] B. Zhang, R. Rajagopal, and D. Tse, "Network risk limiting dispatch: Optimal control and price of uncertainty," *IEEE Trans. Automat. Contr.*, vol. 59, no. 9, pp. 2442–2456, Sep. 2014.

[18] Y. Zhang, N. Gatsis, and G. B. Giannakis, "Robust energy management for microgrids with high-penetration renewables," *IEEE Trans. Sustain. Energy*, vol. 4, no. 4, pp. 944–953, Oct. 2013.