Natural Signals for Navigation: Position and Orientation from the Local Magnetic Field, Sun Vector and the Gravity Vector

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Position and Orientation from Natural Signals

- History: Natural fields, landmarks, and a clock have always been used
  - Animals—insects and fish to newts, birds and mammals
  - Medieval navigators—the compass, the sextant and the astrolabe
- Limits of the old methods
  - They are slow—fine for sailing ships, insects and birds
  - They cannot be automated as they stand
- Advantages of today's technology
  - Extremely good clocks and timing circuits
  - Much better sensors—still improving with Moore's law

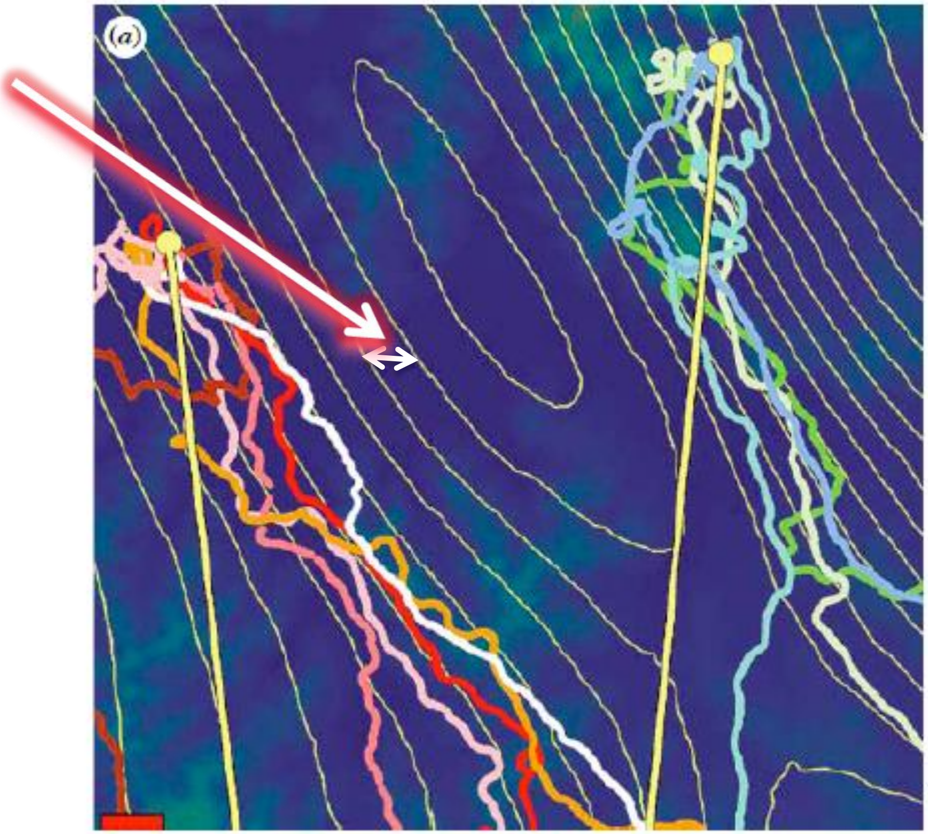
- TO ATTAIN **GPS-LIKE NAVIGATION** FOR *UAVS* AND *WEAPONS*, WE NEED TO TACKLE BIASES AND NOISE, TO OBTAIN ACCURACY, CONTINUITY, AND REPEATABILITY.

Outline

- *Geomagnetic field*
- Sun vector/vectors to moon and stars
- Gravity vector—accelerometers

Motivation: Pigeon Navigation

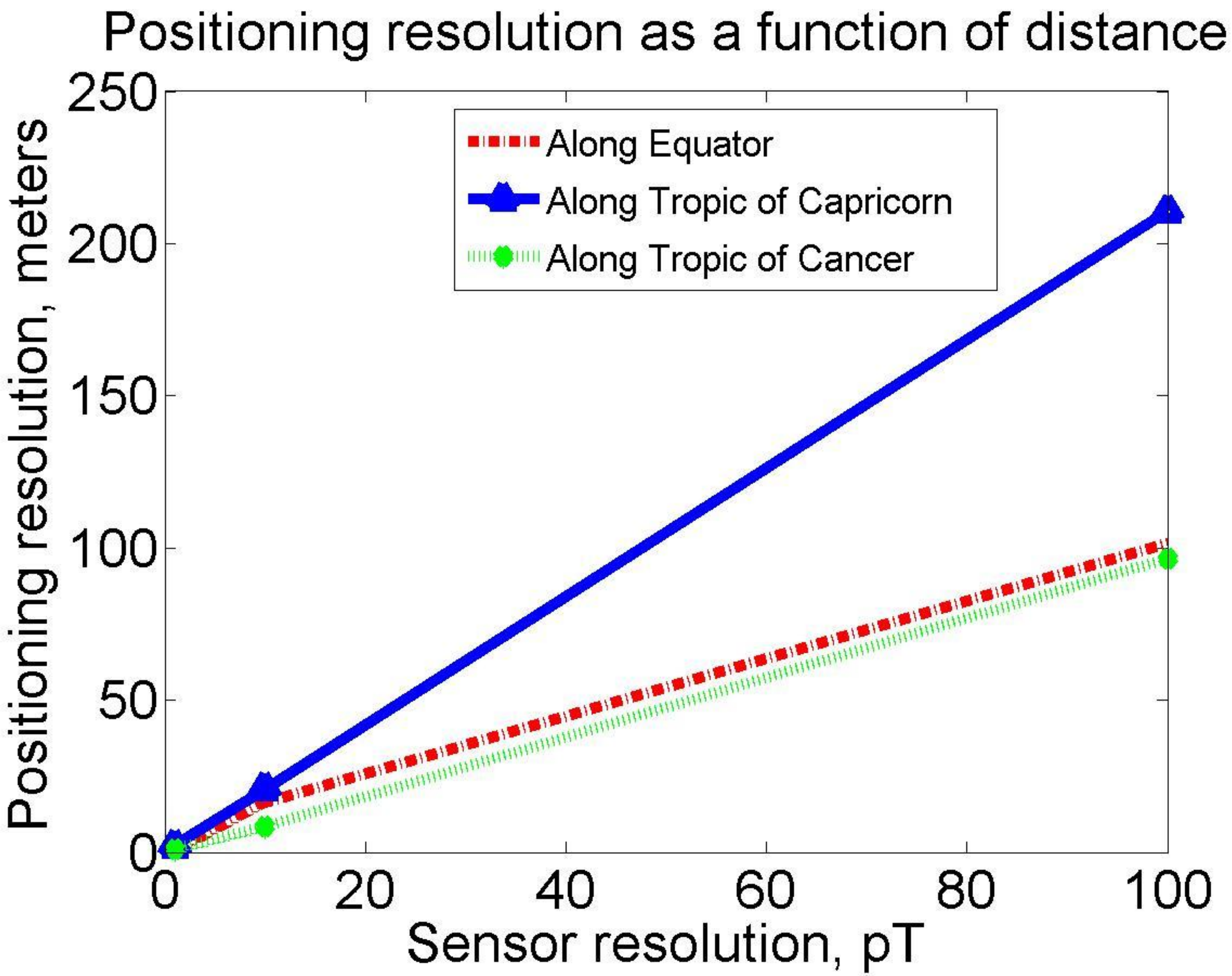
10 nT isopleths



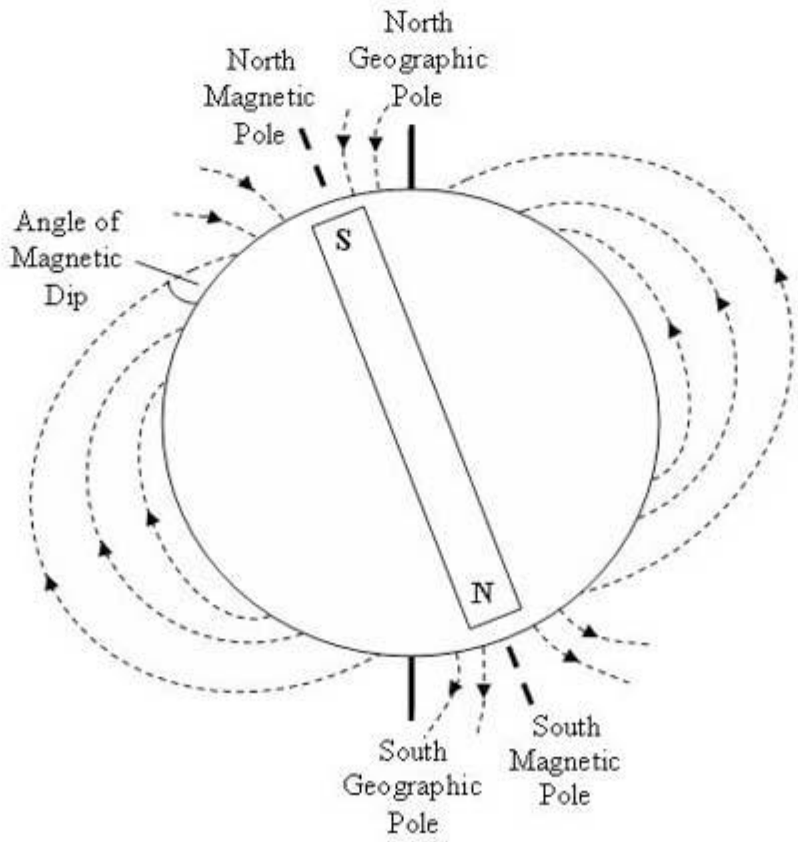
[1]

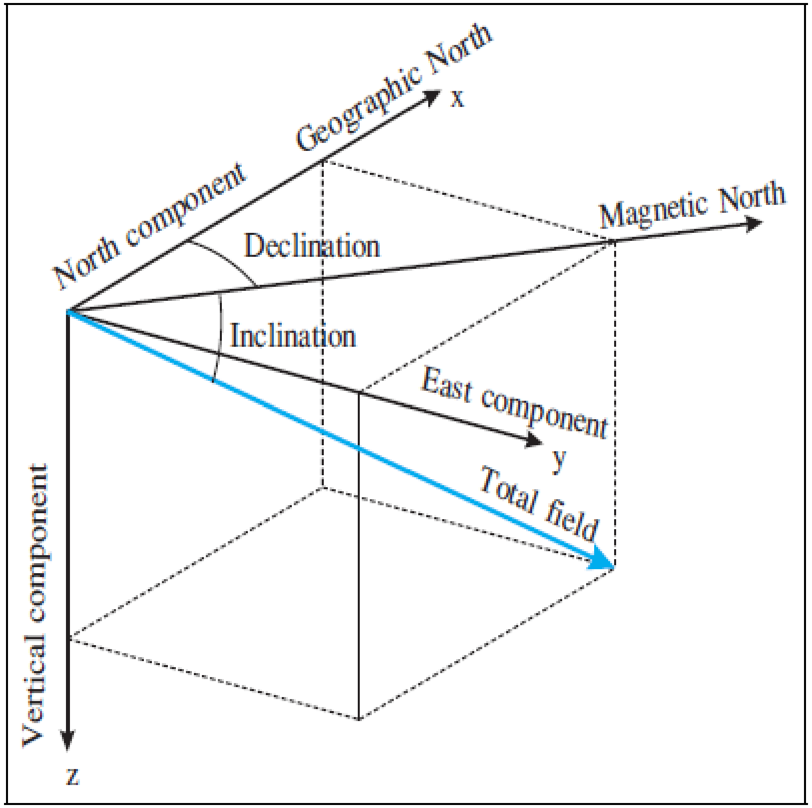
[1] T. Dennis, et al., *Evidence that pigeons orient to geomagnetic intensity during homing*, Proc. R. Soc., 2007

Geomagnetic Field

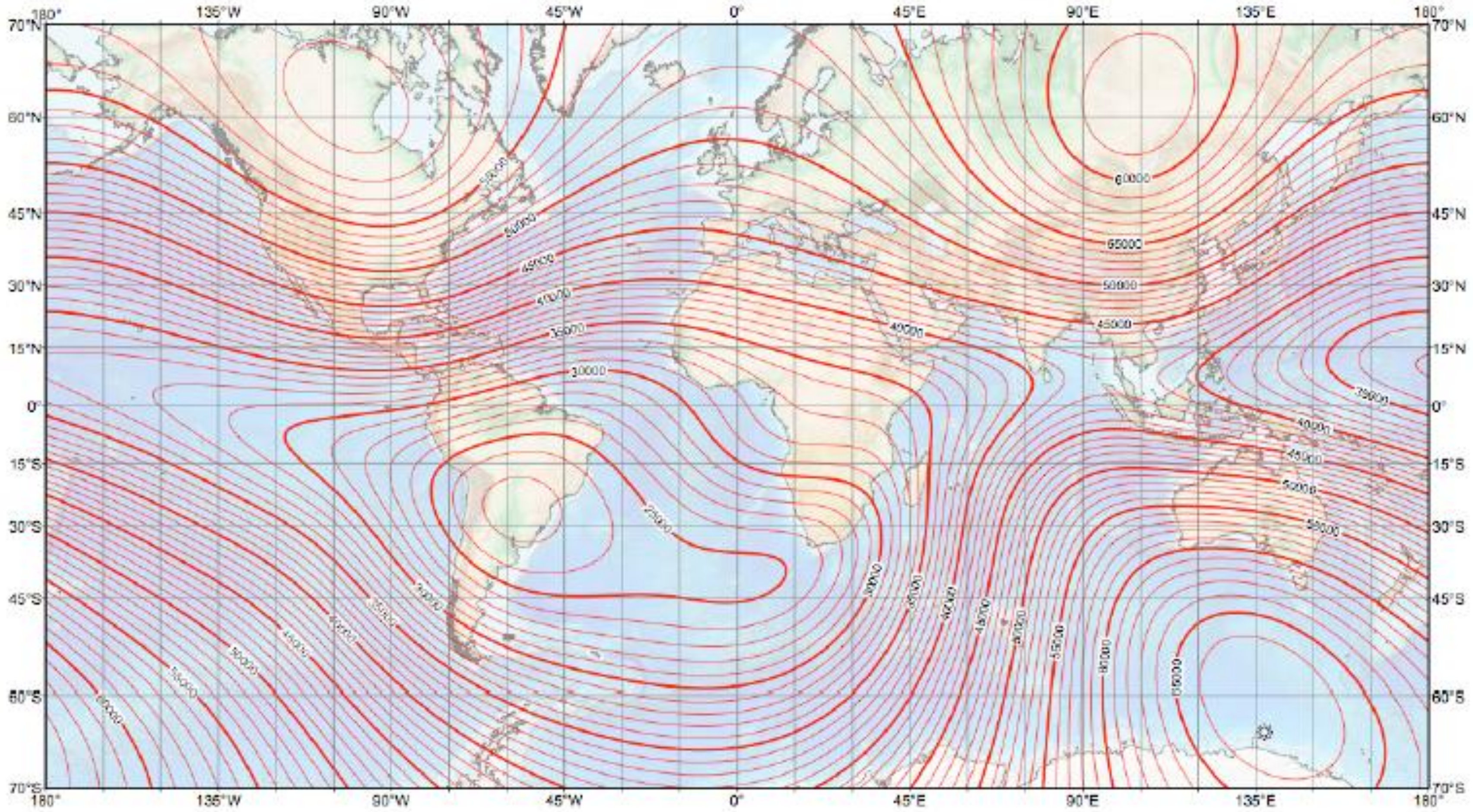


The Geomagnetic Field

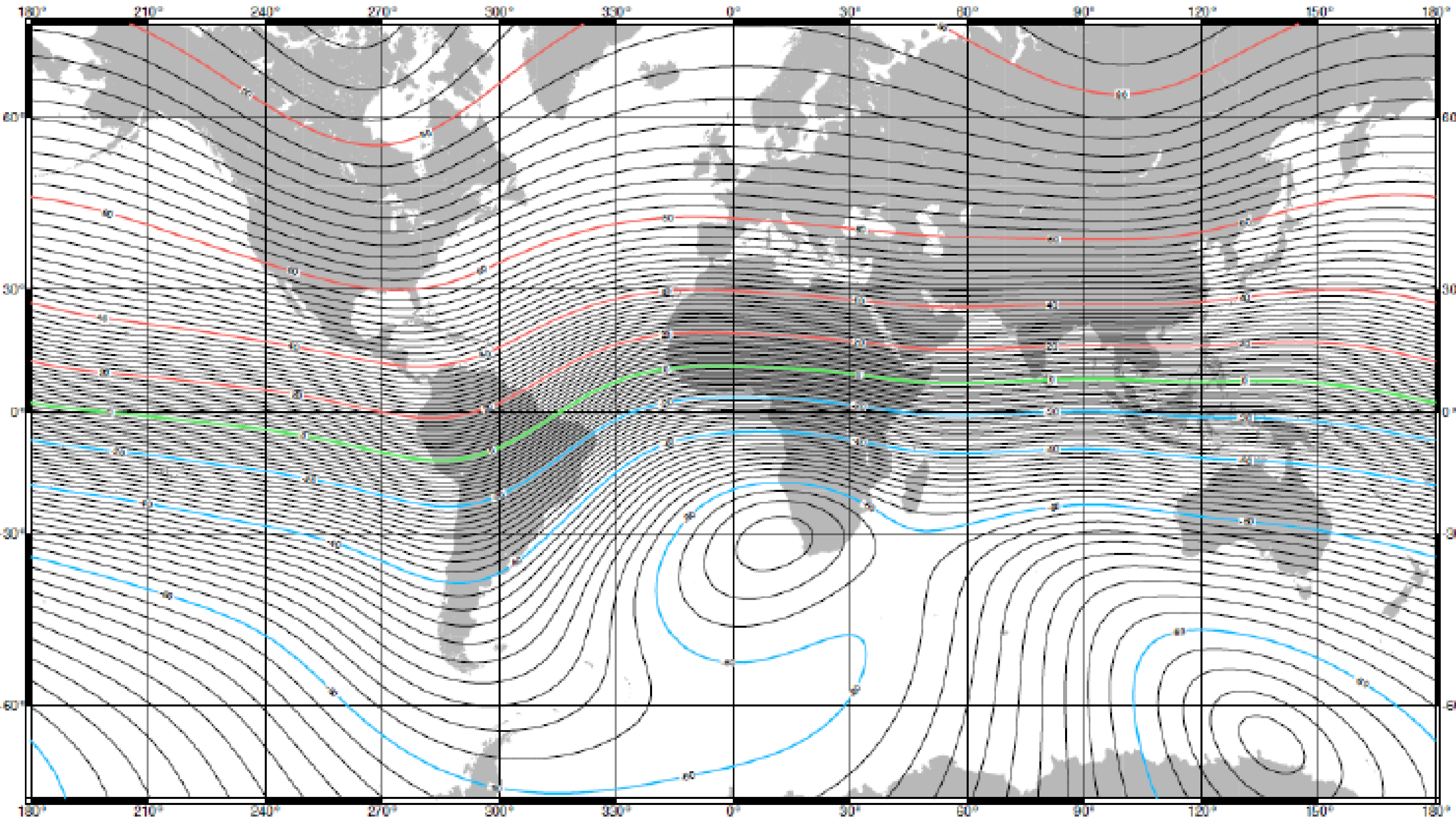




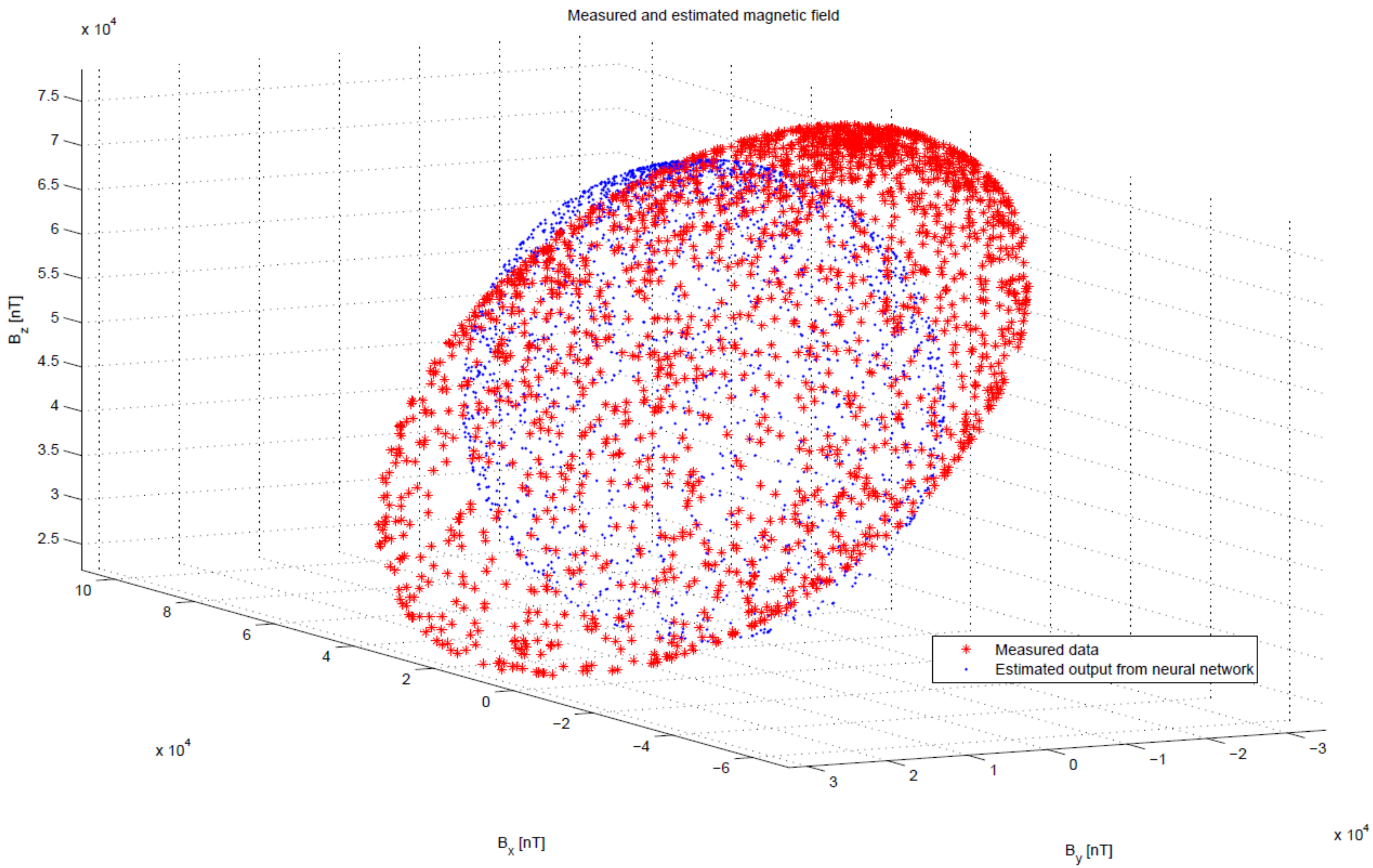
Geomagnetic Field: Intensity



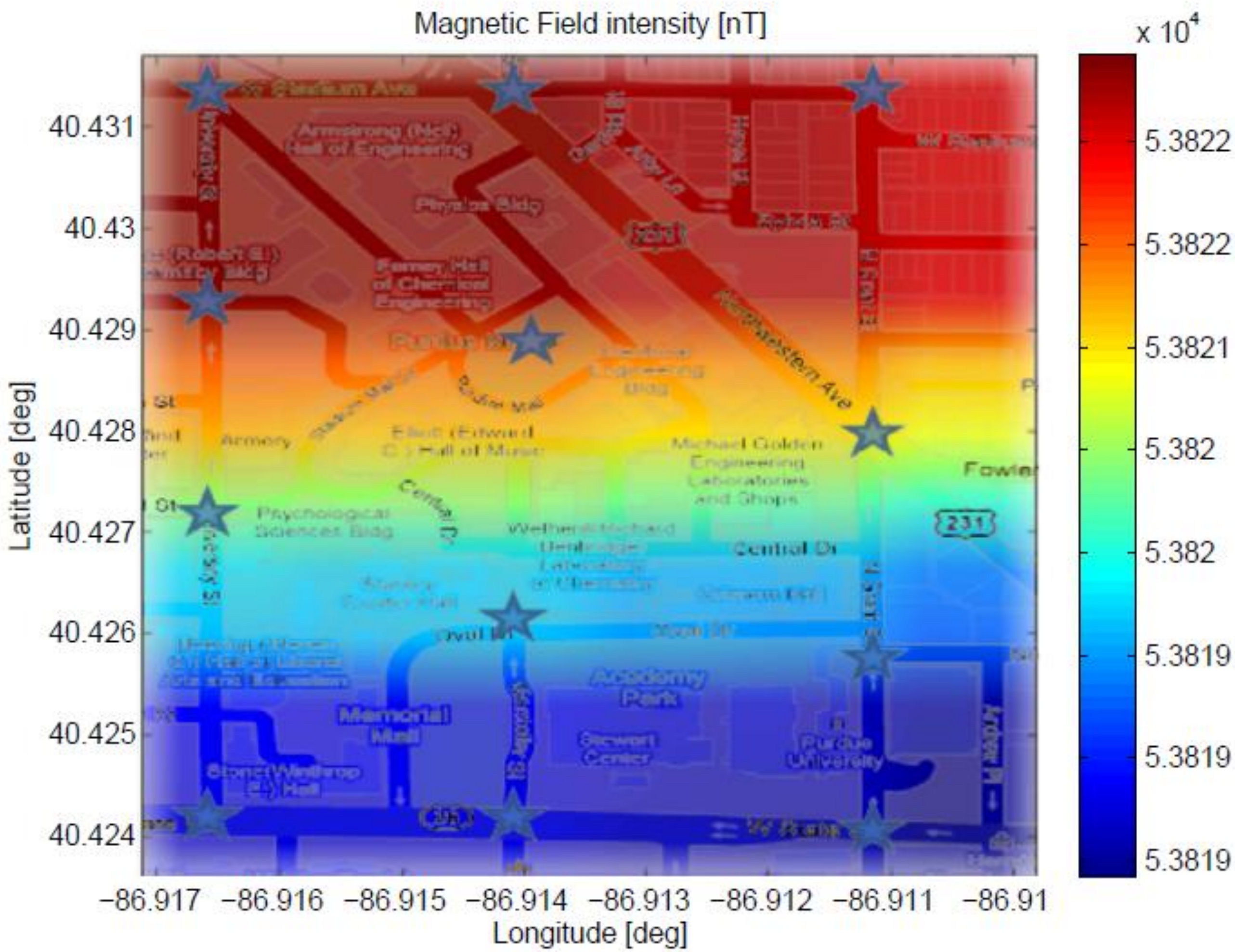
Geomagnetic Field: Inclination



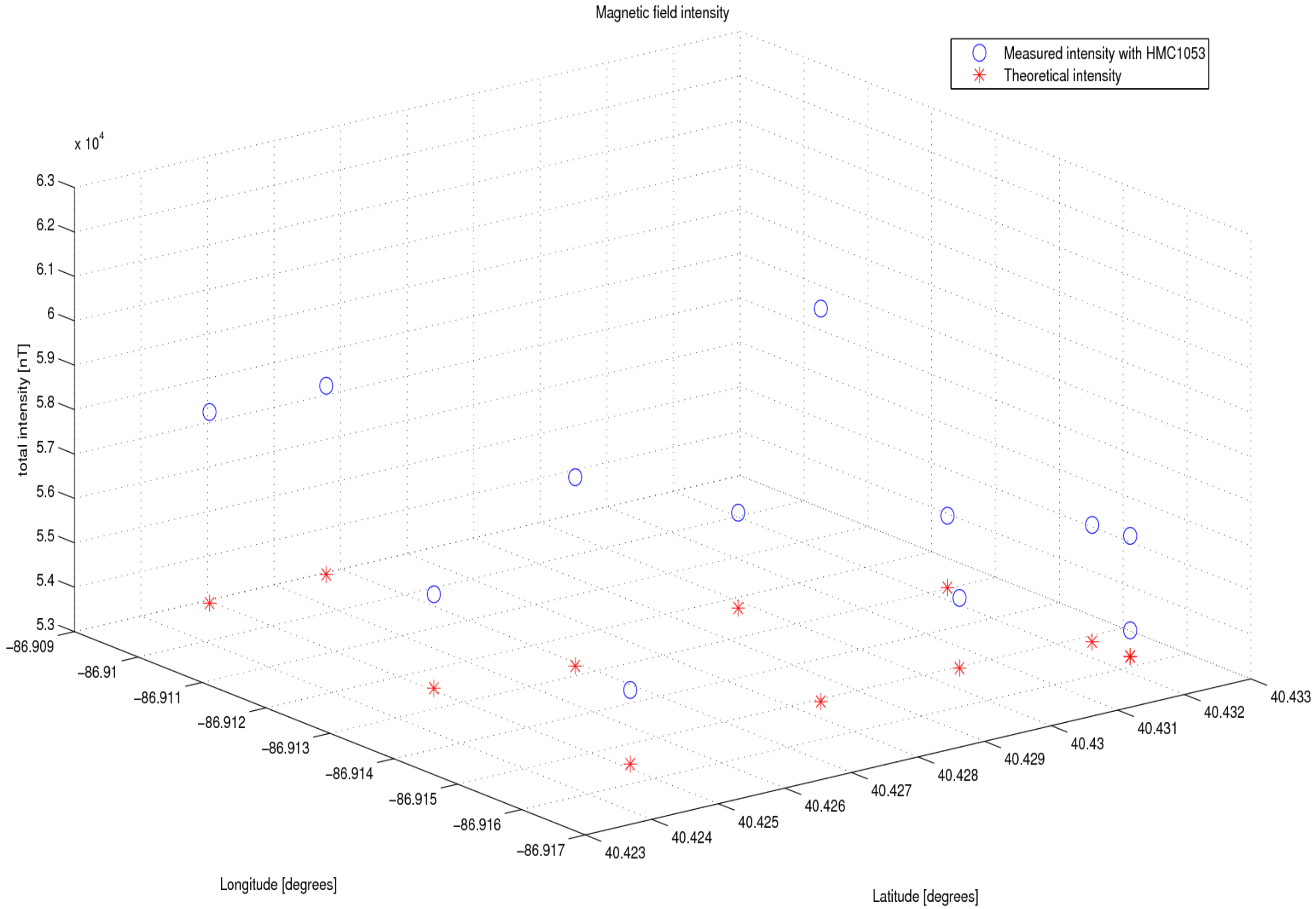
Example for Calibration (HMC1053)



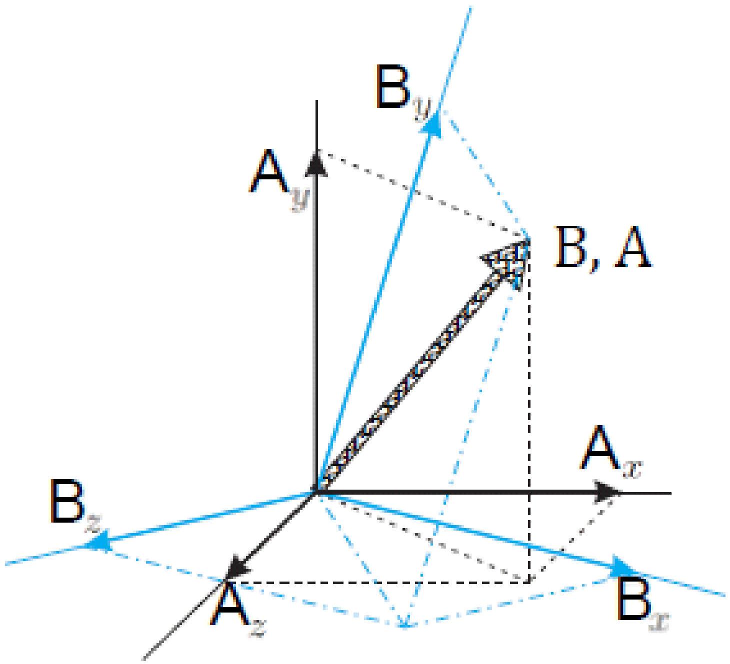
Location Determination



Location - Measurements



Rotation Estimation



$$\bar{H}^B = M_A^B \bar{H}^A$$

$$\begin{bmatrix} 0 \\ \bar{H}^B \end{bmatrix} = q_A^B \begin{bmatrix} 0 \\ \bar{H}^A \end{bmatrix} \left(q_A^B\right)^*$$

$$M = 2 \begin{bmatrix} q_1^2 + q_0^2 - \frac{1}{2} & q_1 q_2 - q_3 q_0 & q_1 q_3 + q_2 q_0 \\ q_1 q_2 + q_0 q_3 & q_2^2 + q_0^2 - \frac{1}{2} & q_2 q_3 - q_1 q_0 \\ q_3 q_1 - q_2 q_0 & q_3 q_2 + q_1 q_0 & q_3^2 + q_0^2 - \frac{1}{2} \end{bmatrix}$$

Rotation Estimation

- Cayley transformation

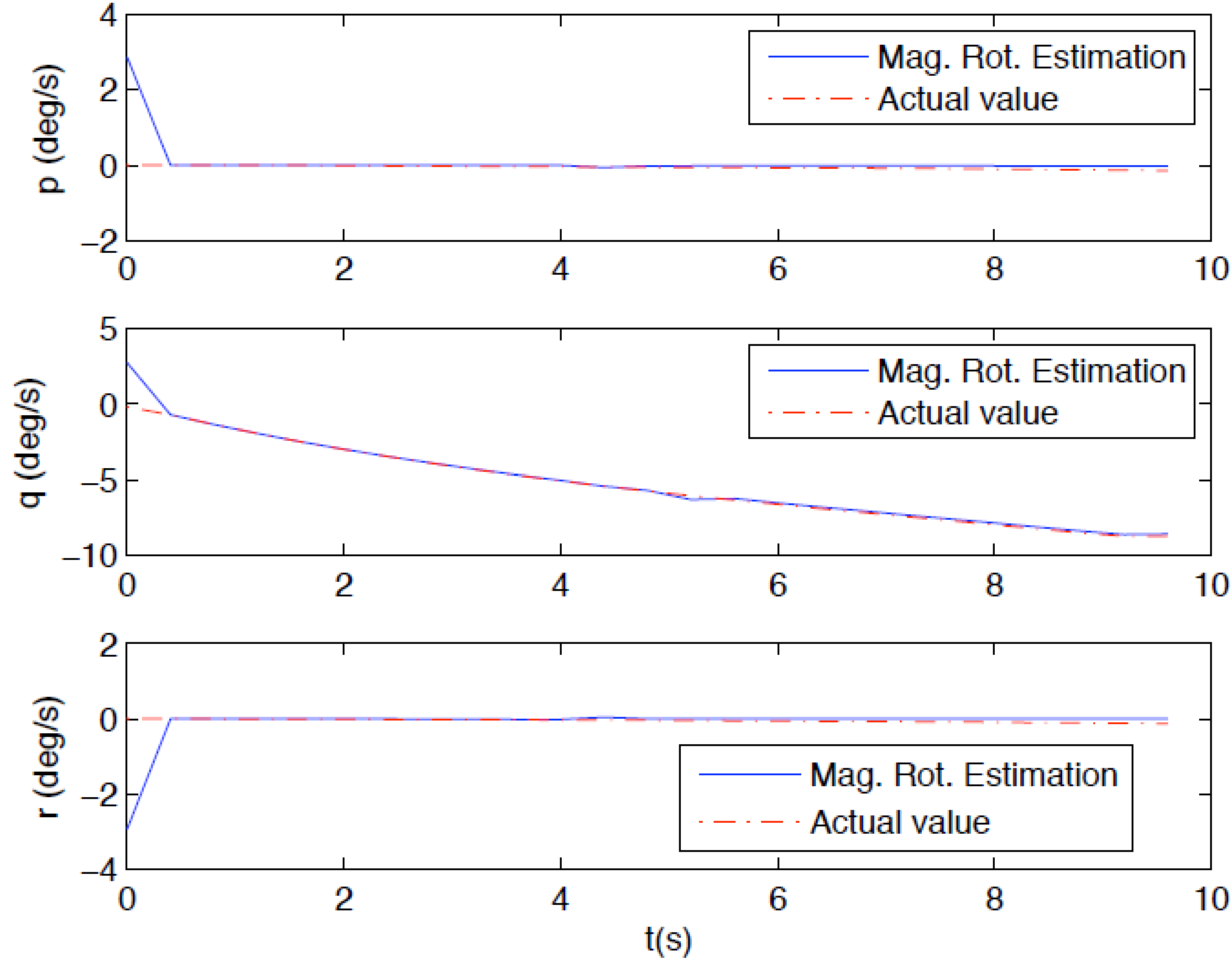
$$M_A^B \overline{H}^A = (I - A)^{-1} (I + A) \overline{H}^A = \overline{H}^B$$

- Set of equations -- Computational time reduction

$$(I + A) \overline{H}^A = (I - A) \overline{H}^B$$

$$A = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

Rotation Estimation: F16 Model



Outline

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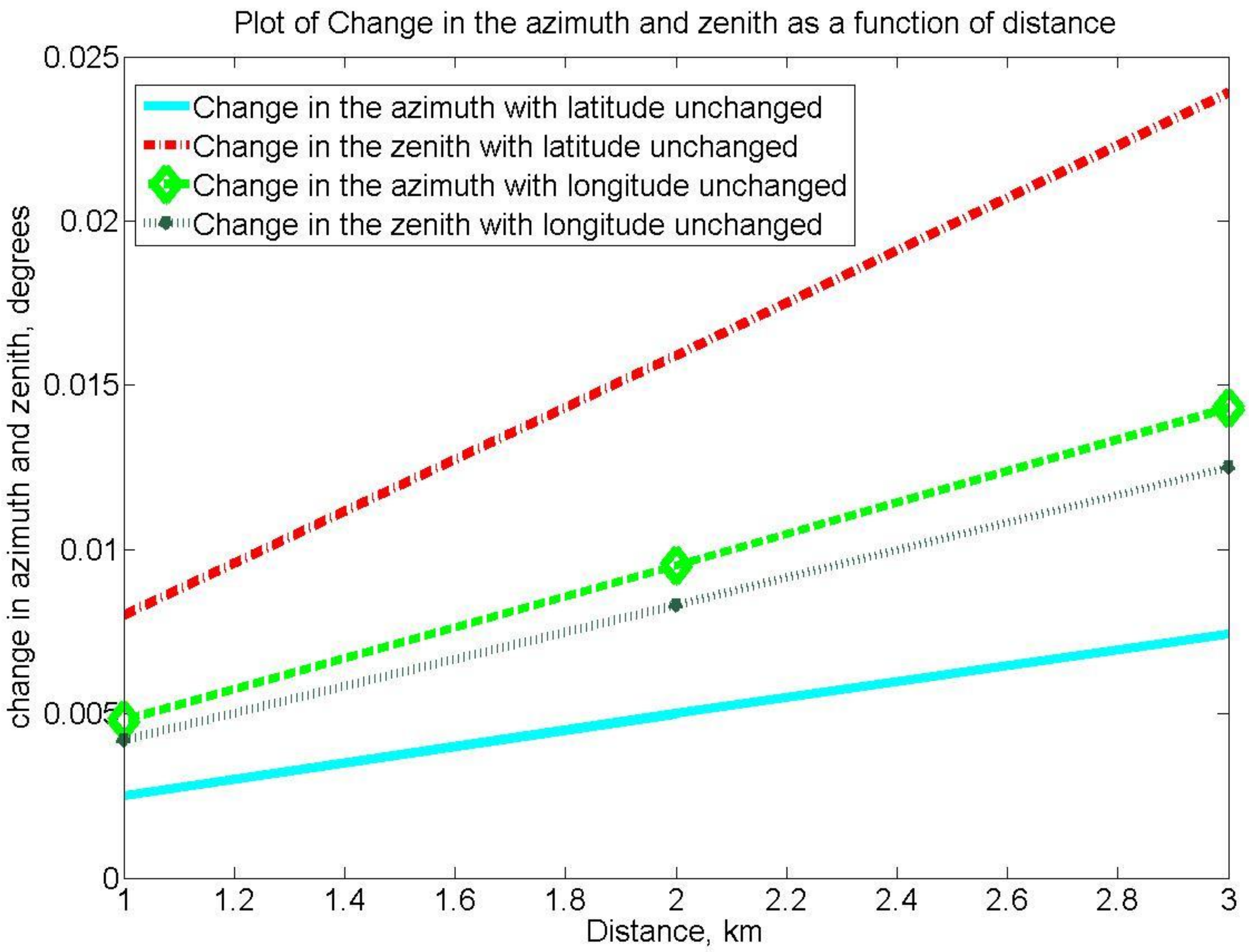
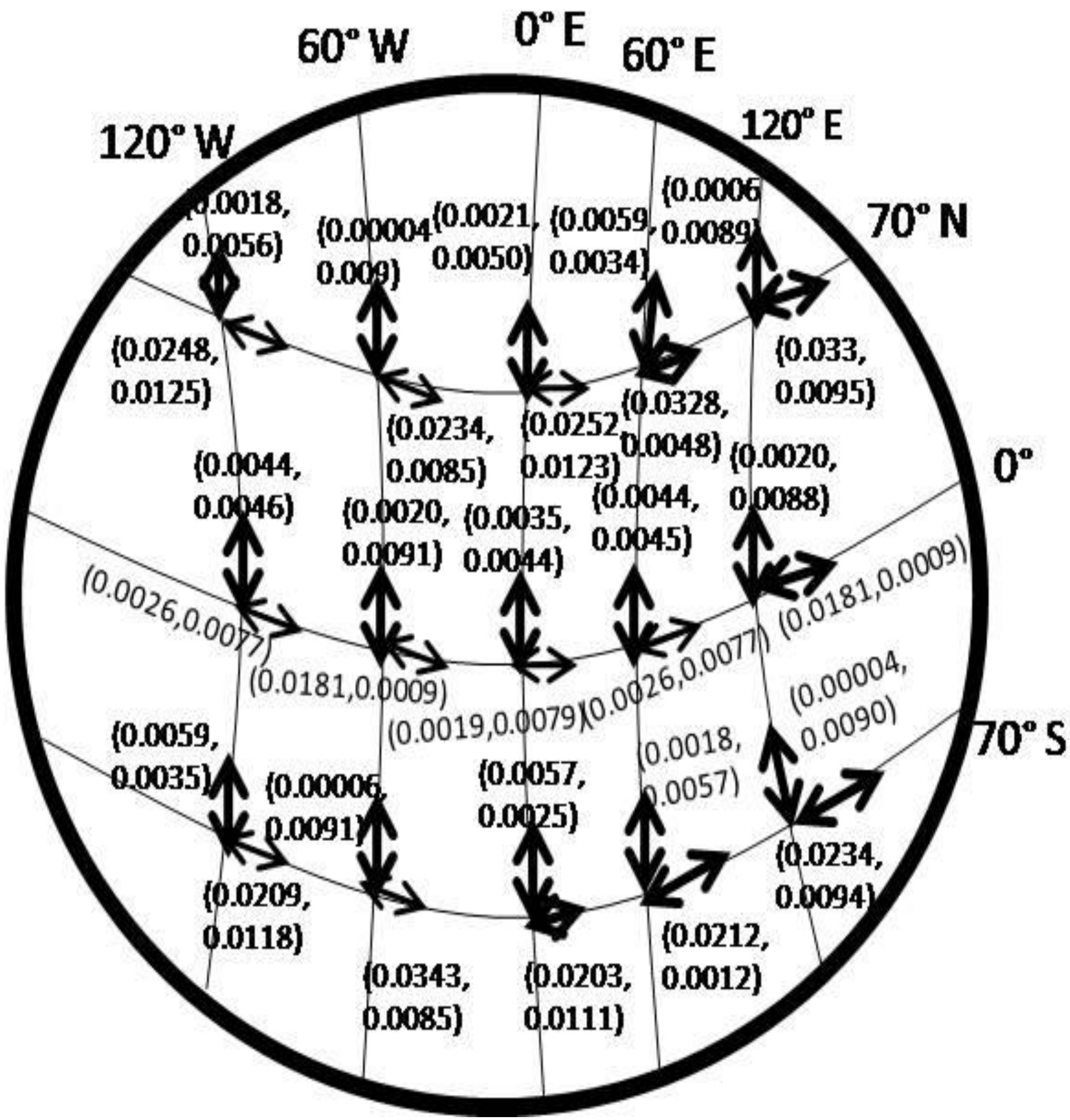
- *Sun vector/vectors to moon and stars*

- Gravity vector—accelerometers

Miniaturizing the Spherical Sundial



Using the Moon

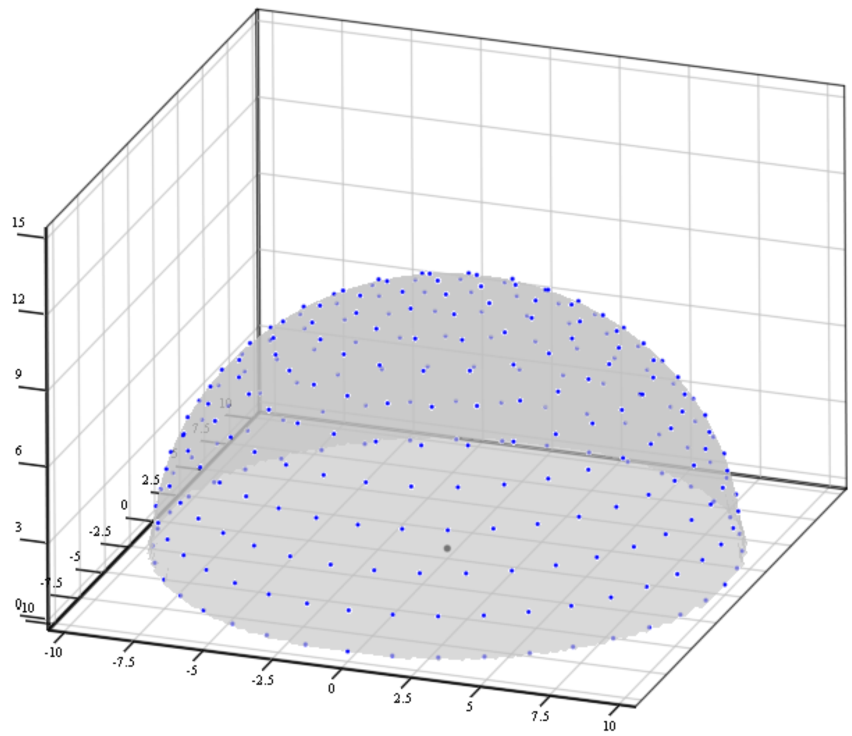


Prior Art

- Sun sensors have been used for several decades in space-related projects (e.g. Martian robotic vehicles).
  - Unsurprisingly, the technology used in these sensors is designed for operation in space, not on Earth.
  - Certain properties of space-based solar sensors (e.g. low update rates, small field of view, etc.) are not acceptable for our purposes.
- Sun sensors are also used on heliostats for sun-tracking.
  - But these sensors are large, have too many moving parts, and consume more power than is acceptable for small autonomous vehicles.

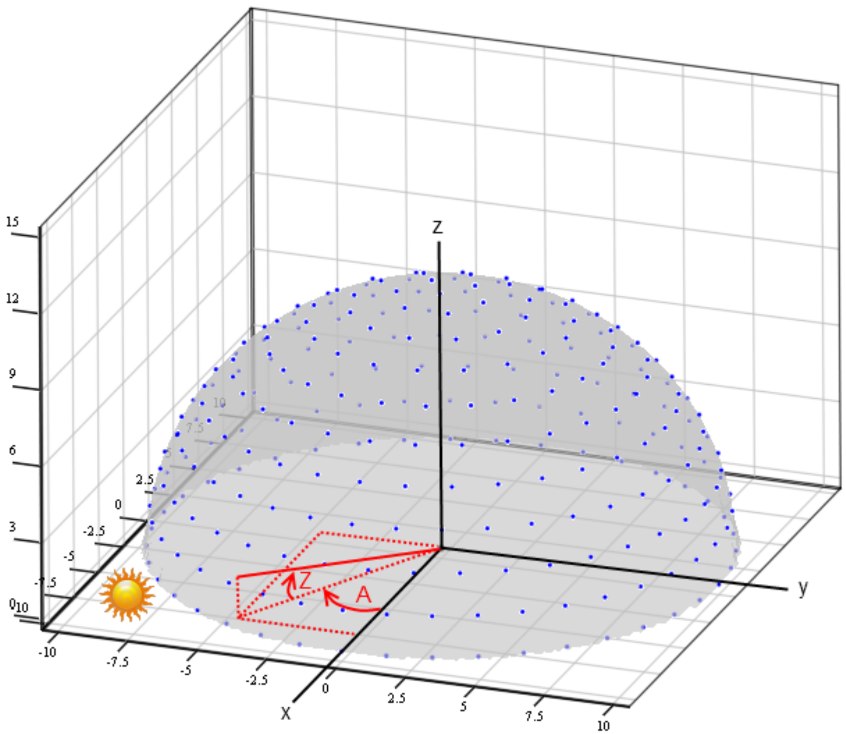
Basic Idea

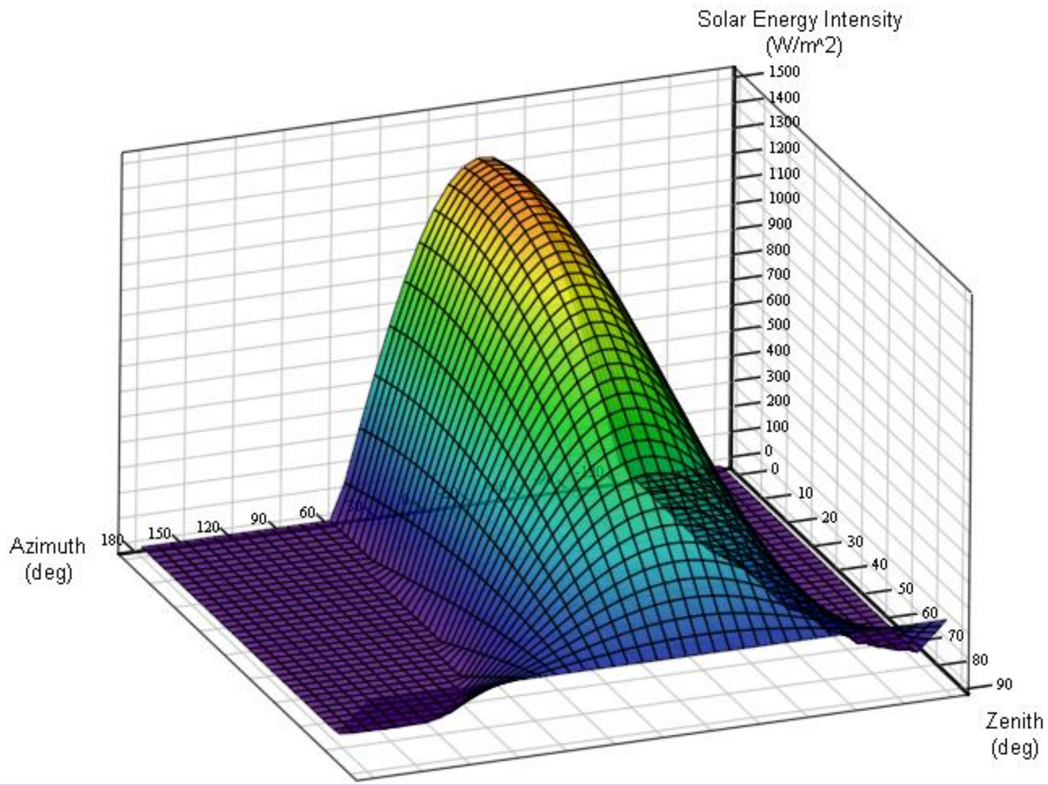
- Photosensitive pixels (blue dots) are distributed around a hemisphere.
- The solar energy incident on each pixel is a function of the sun’s angle of incidence for that pixel.
- By analyzing the energy distribution of all the pixels, the sun vector can be extracted.



An Example

- Sun is at A = -30°, Z = 10°
- Solar intensity (W/m^2) is a sinusoidal function of a pixel's location on the hemisphere.



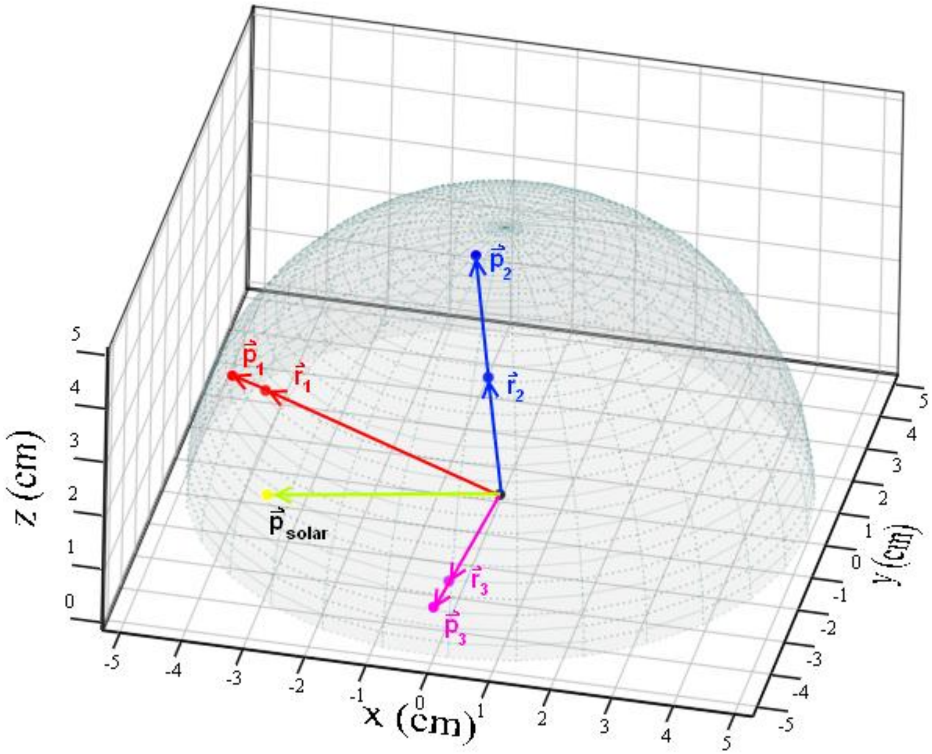


Three Pixel Theorem

- *Theorem*: Any three pixels uniquely determine the sun vector with zero error, provided that the following conditions are met:
  1) The three pixels are each illuminated by the sun's (direct) radiation.
  2) No noise or interference sources are present.
  3) The orientation vector of each pixel is equal to the normal vector of the hemisphere's surface at each pixel's location.
  4) The incident solar radiation at the time is known.
  5) The plane containing the three pixels does not intersect the origin of the sensor.
- This theorem only applies to a hemispherical sensor, but similar results may hold for other geometries.

Three Pixel Theorem: Definitions

- Consider three illuminated pixels with positions specified by vectors $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{p_3}$.
- Let the sun's position be specified by an unknown vector, $\mathbf{p_{solar}}$.
- Let the (known) solar irradiation be I_{solar}.
- Let the intensity recorded by each pixel be I_1, I_2, and I_3.

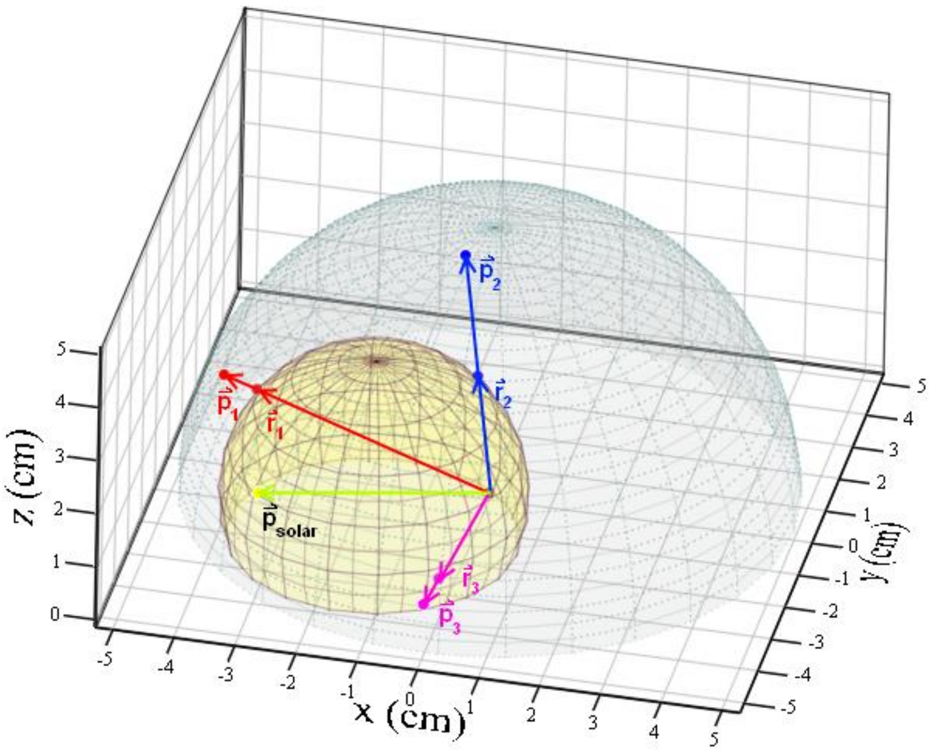


Three Pixel Theorem: Inner Vectors

- Define *inner vectors,* ***r₁*, *r₂*,** *and* ***r₃*** *as:*

$$\vec{r}_i = R_{sensor}\frac{I_i}{I_{solar}}\vec{e}_i$$

- Where $\mathbf{e_i}$ is the unit vector in the direction of the ith pixel.
- The incident solar radiation, I_{solar}, must be known (Condition 4).
- Then it can be shown that these vectors always lie on a sphere of radius $R_{sensor}/2$ that also intersects the origin of the sensor and $\mathbf{p_{solar}}$.
  - This sphere is called the *inner sphere.*



Three Pixel Theorem: Matrix Form

- Let x, y, and z denote the components of each vector, the subscripts 1-3 denote the inner vectors $\mathbf{r_1}$-$\mathbf{r_3}$, and the subscript 0 denote the vector to the center of the inner sphere.
- Then define the following matrix and vectors:

$$A = \begin{pmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ x_2 - x_3 & y_2 - y_3 & z_2 - z_3 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \quad \vec{x} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 \\ x_2^2 + y_2^2 + z_2^2 - x_3^2 - y_3^2 - z_3^2 \\ x_3^2 + y_3^2 + z_3^2 - x_1^2 - y_1^2 - z_1^2 \\ x_1^2 + y_1^2 + z_1^2 \\ x_2^2 + y_2^2 + z_2^2 \\ x_3^2 + y_3^2 + z_3^2 \end{pmatrix}$$

- Then it can be shown that the sun vector, $\mathbf{p}_{\mathbf{solar}}$, is given by:

$$\vec{p}_{solar} = 2(\vec{x}) = (A^T A)^{-1}(A^T)\vec{b}$$

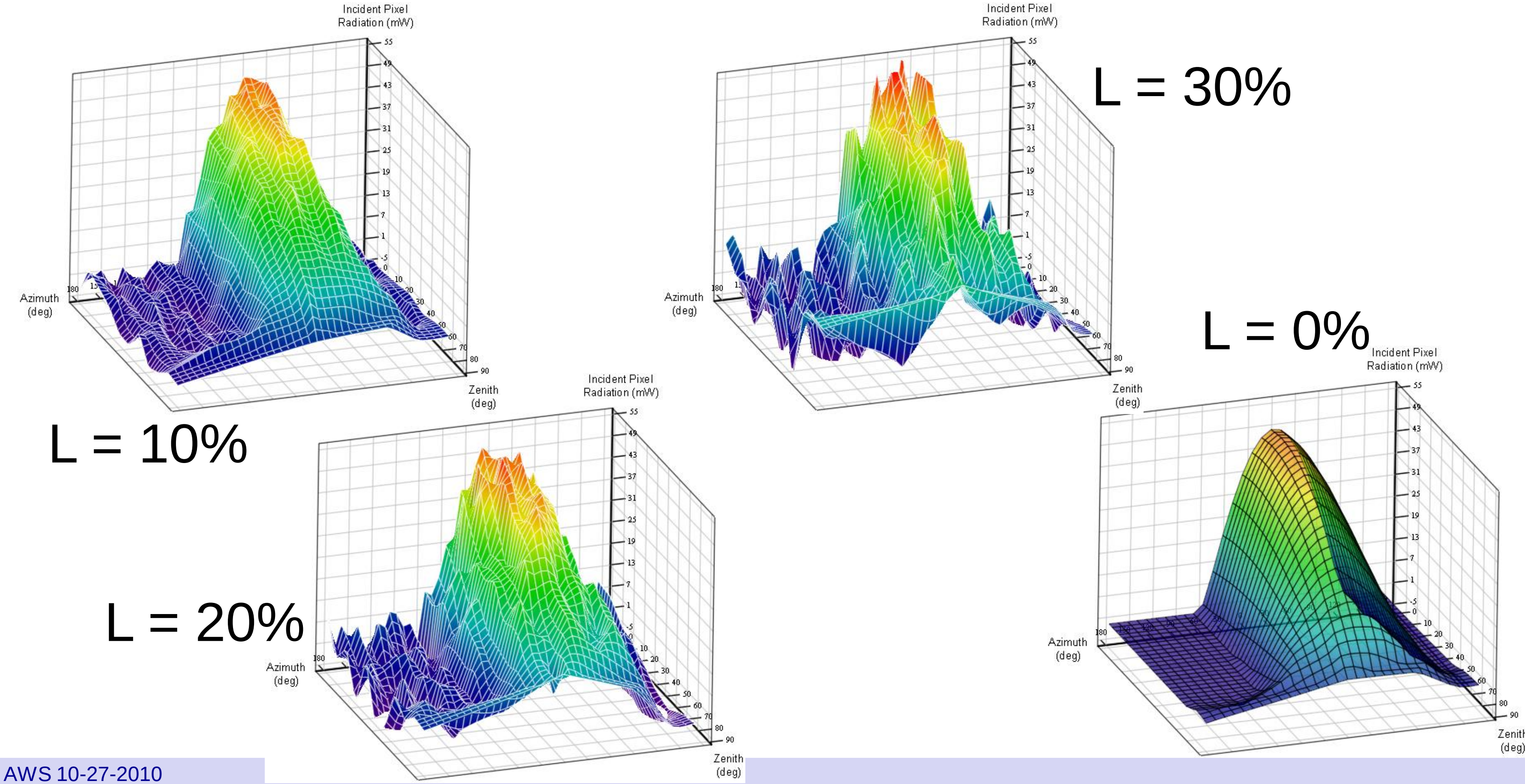
- The matrix ($A^T A$) is invertible only when the plane containing the three pixels does not intersect the origin (Condition 5).

Challenge: Interference

- Reflected sunlight from the body of the vehicle/aircraft
- Light reflected from natural features (e.g. lakes, rivers, clouds)
- Shadowing (partially or wholly obscured sensor)
- Standard noise sources

Noise Effects

- True solar azimuth, zenith at A = -30°, Z = 10°
- N = 275 pixels
- Noise level, L, is specified as a percent of the peak solar intensity value.



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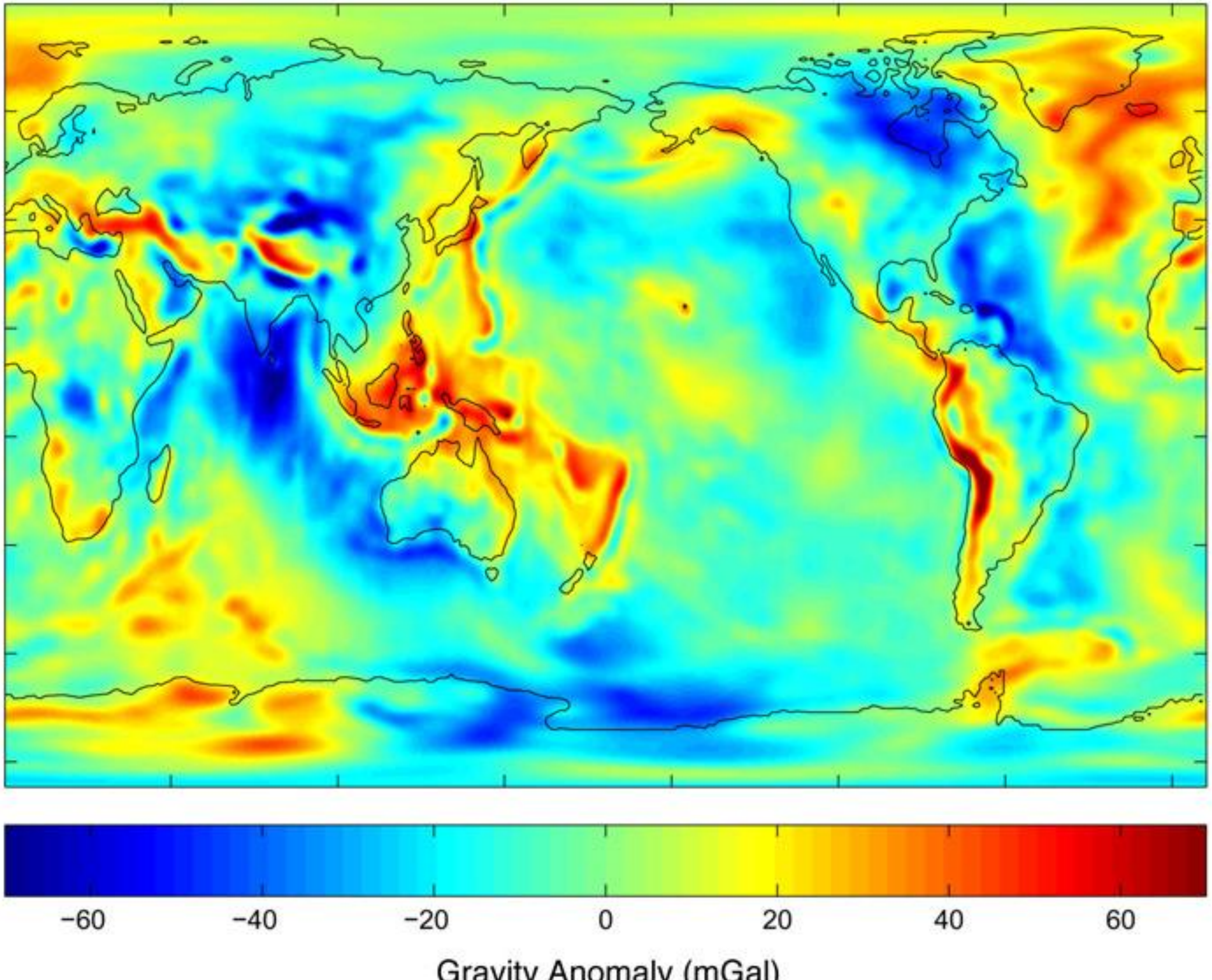
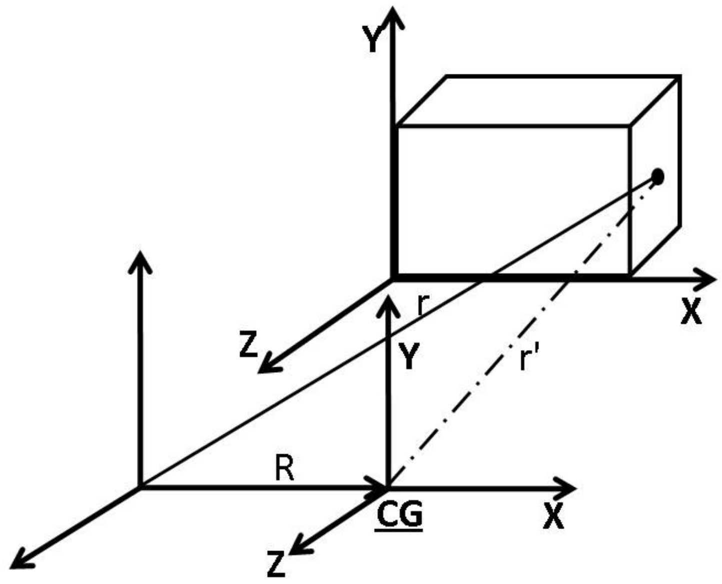
- *Gravity vector—accelerometers*

Gravity Vector



g: good for orientation but not for positioning *with present day technology*

Accelerometer IMUs



- The measurement equations for this set up can be obtained as follows:

$$\vec{r} = \vec{R} + \vec{r}'$$

- Vectorially, we can write the equation as:

$$\frac{d^2 r}{dt^2} = \frac{d^2 R}{dt^2} + \omega \times \omega \times r' + 2\omega \times \frac{dr'}{dt} + \alpha \times r'$$

- Accelerations and angular velocities are uniform for a rigid body.
- The measurement equations for the setup are as follows:

$$a_x - r_x(\omega_y^2 + \omega_z^2) + r_y(\omega_x\omega_y - \alpha_z) + r_z(\omega_x\omega_z + \alpha_y) = a_{xm}$$

Error Propagation

- Sources of error in our setup:
  - Relative positions and orientations of accelerometers
  - Shifts in Center of Gravity
  - Accelerometer drifts and errors

- We seek to establish various properties of these errors, namely if these errors can be calibrated out, how these propagate and if they are dominant.

Error Propagation – Worst Case

- If a_m represents the measured acceleration, and a_{xm}, a_{ym} and a_{zm} represent the measured values along the x, y and z directions respectively, then the error calculated is as follows:

$$\delta a_m = \left|\frac{\partial a_{xm}}{\partial r}\delta r\right| + \left|\frac{\partial a_{ym}}{\partial r}\delta r\right| + \left|\frac{\partial a_{zm}}{\partial r}\delta r\right|$$

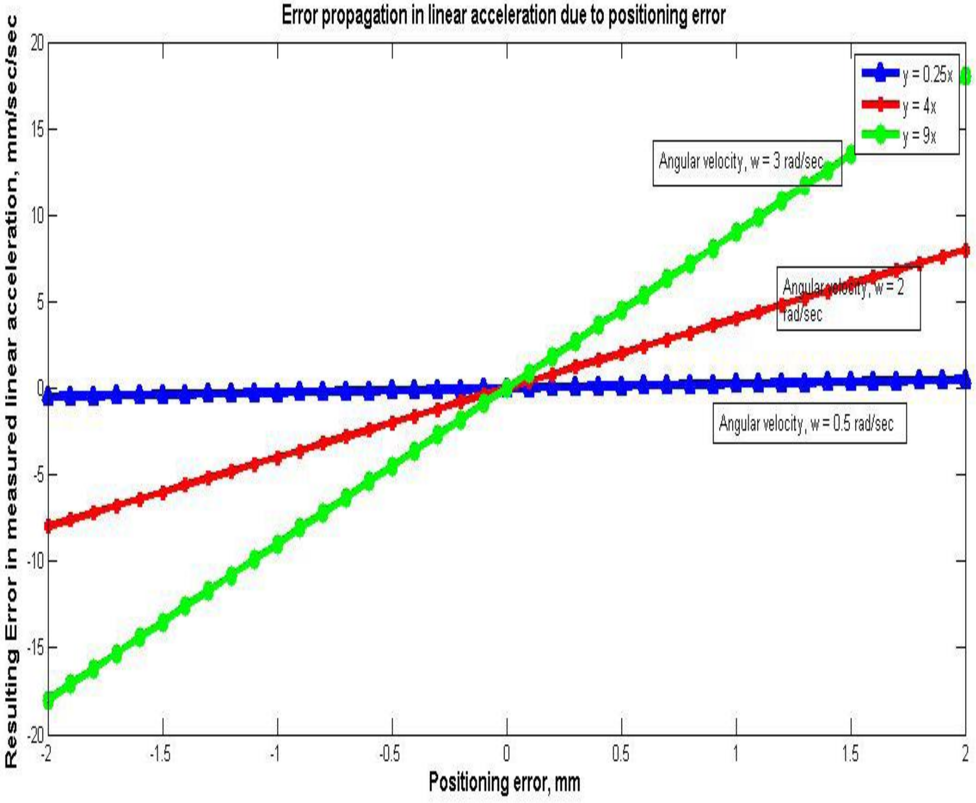
$$\delta a_m = [(|\omega_y^2+\omega_z^2| + |\omega_x\omega_y-\alpha_z| + |\omega_x\omega_z+\alpha_y|) + (|\omega_x^2+\omega_z^2| + |\omega_y\omega_z-\alpha_x| + |\omega_x\omega_y+\alpha_z|) +$$
$$(|\omega_x^2+\omega_y^2| + |\omega_x\omega_z-\alpha_y| + |\omega_y\omega_z+\alpha_x|)]\delta r$$

- The angular accelerations are assumed to be negligible as compared to the product $\omega_x\omega_y$, $\omega_y\omega_z$ and $\omega_z\omega_y$

Error Blowup

- Simplified expression:

$$\delta a_m = [2(\omega_x^2 + \omega_y^2 + \omega_z^2) + 2\,|\,\omega_x\omega_y + \omega_y\omega_z + \omega_x\omega_z\,|]\,\delta r$$



Open Problems and Approaches

- Dynamic calibration of magnetic sensors—possibly using at least two 3-axis magnetometers
- Sun sensor construction using cell phone camera imagers.
- Determining accelerometer configurations amenable to self-calibration.
- Data agglomeration for improved positioning

ALL FEASIBLE NOW (WITH $$$ AND EFFORT)

Questions

Backup

The Geomagnetic Field: External Component

- Carl Gauss proved that 95% of the Earth's magnetic field is internal and 5% is external
- The external magnetic field
  - Mainly from solar activity
  - Variations from 100 up to 1000 nT
  - Several models exist Paraboloid model. Mead-Fairfield model and Tsyganenko model
  - For the Paraboloid model:
    - Estimation of the external field by using estimation location, time, Disturbance storm time index and Auroral Electro jet index, solar wind velocity and density

Magnetometer Calibration

- Magnetometers are used:
  - To remove gyro drift error
  - To provide more reliable heading information
  - Help in GPS signal loss

- Calibration of Magnetometers:
  - When the reference heading is known, swinging procedure – Bowditch
  - Reference heading is unknown; Caruso showed that the magnetometer measures a circle (noise – free)
  - Alonso and Shuster’s TWO-STEP algorithm

Ideas

- What are the positions that will yield all the 6 or 9 quantities instantaneously?
- How does the error propagate in different geometric configurations?
- What is the configuration for minimal error propagation?

Geomagnetic Field: Declination