(Lec 8 Feb. 5)

645 Project Ideas

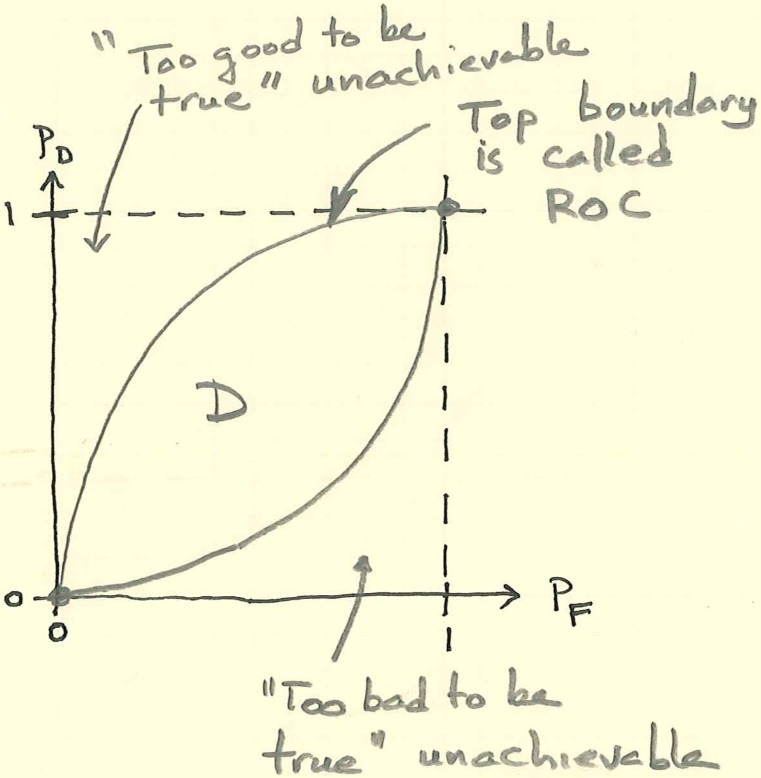
Complexity → Like doing an additional HW assignment.
With outputs

Lecture → slides
Video → youtube.
Problems, Soln.

Ideas
1. Info Theory & Statistics (Cover + Thomas Ch11).
2. Spectrum Estimation → old book S.M. Kay (sinusoidal param est.)
   MUSIC Alg.
   ESPRIT Alg.
3. Poor Book Sequential Det.
   Nonparametric + Robust Est.
4. Wiener Filtering
5. Det + Est in Continuous Time → Van Trees.
6. Kalman Filtering → Multiple Target Tracking
   Particle Filters.
   IMM Alg.

Recall Properties of $\mathcal{D} \triangleq \left\{ \left(P_F(\delta), P_D(\delta)\right) : \delta \text{ is a decision rule} \right\}$

= Domain of feasible tests for a binary hypothesis testing problem.



(P1) $(0,0) \in \mathcal{D}$

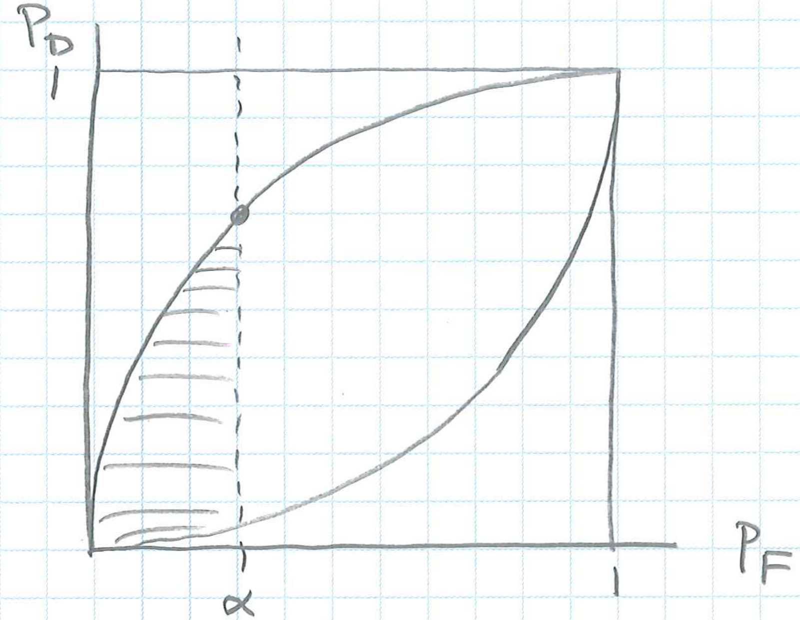
(P2) $(1,1) \in \mathcal{D}$

(P3) If $(\alpha, \beta) \in \mathcal{D} \Rightarrow (1-\alpha, 1-\beta) \in \mathcal{D}$ — This is why the "too bad to be true" points are not achievable

(P4) $\mathcal{D}$ is convex

All of the binary decision rules that we care about... e.g. Bayes rules, minimax rules, NP rules must "live" inside of $\mathcal{D}$ or on its boundary.

NP Solves:



What Does Bayes Do? Both Bayes & NP are LRTs

$$L(y) = \frac{f_1(y)}{f_0(y)} \qquad y \in \Gamma = \text{assume both hyp. supported on } \Gamma.$$

If we substitute Y in for its values y $\Longrightarrow$ get new rv

Now L has cdfs/pdfs that dep. on H_0, H_1. $\quad L(Y) = L$

$$F_{L,0}(\ell) = \int_0^{\ell} f_{L,0}(\lambda)\, d\lambda \quad \dots \quad F_{L,1}(\ell) \;\&\; f_{L,1}(r)$$

$$P_D(\tau) = 1 - F_{L,1}(\tau) = \int_\tau^\infty f_{L,1}(\lambda)\, d\lambda$$

$$P_F(\tau) = 1 - F_{L,0}(\tau) = \int_\tau^\infty f_{L,0}(\lambda)\, d\lambda$$

As τ varies from 0 to $+\infty$ the Point $\left(P_F(\tau), P_D(\tau)\right)$ will trace the ROC from $(1,1)$ to $(0,0)$.

Now take derivative wrt τ

$$\frac{dP_F}{d\tau}(\tau) = \frac{d}{d\tau}\int_\tau^\infty f_{L,0}(\lambda)\, d\lambda = -f_{L,0}(\tau)$$

$$\frac{dP_D}{d\tau}(\tau) = -f_{L,1}(\tau).$$

Claim $$\underbrace{\frac{dP_D}{dP_F}}_{\text{slope of ROC}} = \frac{f_{L,1}(\tau)}{f_{L,0}(\tau)} = \tau$$

$$\frac{dP_D}{d\tau} = -f_{L,1}(\tau) \qquad \frac{dP_F}{d\tau} = -f_{L,0}(\tau)$$

Take ratio, cancel the $d\tau$

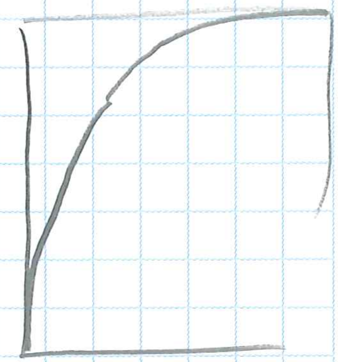
$$\frac{dP_D/\cancel{d\tau}}{dP_F/\cancel{d\tau}} = \frac{-f_{L,1}(\tau)}{-f_{L,0}(\tau)}$$

$$P_D(\tau) = \int_{\Gamma(\tau)} f_1(y) \frac{f_0(y)}{f_0(y)} dy \qquad \Gamma(\tau) = \{ y \in \Gamma : L(y) \geq \tau \}.$$

$$= \int_{\Gamma(\tau)} L(y) f_0(y)\, dy = \int_{\tau}^{\infty} \ell f_{L,0}(\ell)\, d\ell$$

$$\frac{dP_D}{d\tau} = -\tau f_{L,0}(\tau) = -f_{L,1}(\tau)$$

Consequences



$$\frac{dP_D}{dP_F} = 0 \quad @ \quad P_F = 1 \quad (\text{ie as } \tau \to 0)$$

$$\frac{dP_D}{dP_F} = +\infty \quad @ \quad P_F = 0 \quad (\text{ie as } \tau \to \infty)$$

$$E_1\{L\} = 1$$

$$E_1\{L^n\} = E_0\{L^{n+1}\}$$

$$E_1\{L\} - E_0\{L\} = \mathrm{Var}_0\{L\}.$$